Theme Music:
Coldplay
High Speed

Cartoon:
Jim Unger
Herman

“How could I have been doing 70 miles an hour when I’ve only been driving for ten minutes?”
Outline

■ Recap
  – position
  – velocity
■ Acceleration
■ With vectors!
■ ILD 2: What if something just doesn’t make sense? Acceleration at the peak.
What have we learned?
Representations and consistency

- Visualizing where an object is moving $\rightarrow$ a position graph at different times
- Visualizing how fast an object is moving $\rightarrow$ a velocity graph at different times
- Position graph $\rightarrow$ velocity graph
- Velocity graph $\rightarrow$ position graph

\[
\text{slopes } v = \frac{\Delta x}{\Delta t}
\]
\[
\text{areas } \Delta x = v \Delta t
\]
Average Acceleration

- We need to keep track not only of the fact that something is moving but how that motion is changing.
- Define the **average acceleration** by

\[
\langle \vec{a} \rangle = \frac{\text{change in velocity}}{\text{time it took to make the change}}
\]

\[
\langle \vec{a} \rangle = \frac{\Delta \vec{v}}{\Delta t}
\]
Instantaneous acceleration

- Sometimes (often) an object will move so that sometimes it speeds up or slows down at different rates.
- We want to be able to describe this change in motion also.
- If we consider small enough time intervals, the change in velocity will look uniform — for a little while at least.
Instantaneous acceleration

- If we consider a small enough time interval so that the object is (approximately) in uniformly accelerated motion during that time interval, we can define the “acceleration at the instant at the center of the time interval” by

\[ \ddot{a}(t) = \frac{d\vec{v}}{dt} \]

\[ \ddot{a}(t) = \frac{\vec{v}(t + \Delta t/2) - \vec{v}(t - \Delta t/2)}{\Delta t} \]
Velocity to acceleration

\[ a(t) = \frac{dv}{dt} = \frac{v(t + \frac{\Delta t}{2}) - v(t - \frac{\Delta t}{2})}{\Delta t} \]
Acceleration to velocity

\[ dv = a(t) \, dt \]

change in velocity over a small time interval

\[ v = \sum dv = \int a(t) \, dt \]

sum ("\( \Delta \)"") in the changes in velocity over many small time intervals.
Working with Vectors

\[ \vec{r}_2 = \vec{r}_1 + \Delta \vec{r} \]

\[ \Delta \vec{r} = \vec{r}_2 - \vec{r}_1 \]
Kinematics with Vectors

\[ \Delta r_1, \Delta r_2, \Delta r_3 \]

\[ \frac{\Delta r_1}{\Delta t} = v_1, \quad \Delta r_2/\Delta t = v_2, \quad \Delta r_3/\Delta t = v_3 \]

\[ \Delta v_1, \Delta v_2 \]

\[ \frac{\Delta v_1}{\Delta t} = a_1, \quad \frac{\Delta v_2}{\Delta t} = a_2 \]

\[ \text{Position} \]

\[ \text{Velocity} \]

\[ \text{Acceleration} \]
What have we learned?

- **Position**  \[ \hat{r} = x \hat{i} \]  (where \( x \) is a signed length)

- **Velocity**  \[ \langle \vec{v} \rangle = \frac{\Delta \vec{r}}{\Delta t} \]  \[ \vec{v} = \frac{d\vec{r}}{dt} \]

- **Acceleration**  \[ \langle \vec{a} \rangle = \frac{\Delta \vec{v}}{\Delta t} \]  \[ \vec{a} = \frac{d\vec{v}}{dt} \]

- **Seeing from the motion**

- **Seeing consistency (graphs & equations)**
ILD 2

What if something just doesn’t make sense?

Acceleration at the peak