Exam I, Part A: Multiple Choice

This Part A of Exam I consists of 8 problems worth 10 points each and comprises 67% of the full exam. For each problem fill in the circle next to the letter of your chosen answer on the NCS answer sheet using a #2 pencil, making sure that the number of the line corresponds to the number of the problem you are answering. There is only one correct answer for each problem, and no subtractions will be made for wrong answers. Use only lines 1 through 8 on the NCS sheet.

1. The spring in a spring gun has a spring constant, \( k = 300 \text{ N/m} \), and is compressed 0.10 m when fully cocked. What is the "launch speed" it imparts to a 15-g marble upon release, most nearly?

   a. 10 m/s  
   b. 14 m/s  
   c. 20 m/s  
   d. 44 m/s  
   e. 200 m/s

\[
\text{Conservation of ME for conservative force of spring:} \\
\text{Initial energy:} \\
\frac{1}{2}kx^2 = \frac{1}{2}m_{\text{marble}}v_f^2 \\
\text{Final energy:} \\
kx = \frac{1}{2}m_{\text{marble}}v_f^2 \\
v_f = \sqrt{\frac{2kx}{m_{\text{marble}}}} = \sqrt{\frac{2 \cdot 300 \text{ N/m}(0.10 \text{ m})}{0.015 \text{ kg}}} = 14.1 \text{ m/s} = v_f
\]

\( b \) is correct.

2. A simple pendulum, 1.00 m in length and of mass 0.5 kg, is released from rest when the support string is at an angle of 35.0° from the vertical. What is the speed of the suspended mass at the bottom of the swing? Ignore air resistance. \( (g = 9.80 \text{ m/s}^2) \)

\[
\text{Cons of ME. For conservative force of gravity, } PE = mgh \\
\text{Initially at rest, } h_0 = h \text{ (initial height)} \\
\text{Finally at min, } h = h_0 \\
h - h_0 = L - L \cos 35^\circ = (1.00 \text{ m}) - 0.8192 = 0.1808 \text{ m} \\
mgh = \frac{1}{2}mv_f^2 \\
v_f = \sqrt{2g(h-h_0)} = \sqrt{1.88 \text{ m/s}^2 \cdot 0.1808} = 1.88 \text{ m/s} = v_f
\]

\( c \) is correct.
3. A railroad freight car, mass 15,000 kg, is allowed to coast along a level track at a speed of 2.0 m/s. It collides and couples with a 50,000-kg loaded second car, initially at rest and with brakes released. What percentage of the initial kinetic energy of the 15,000-kg car remains as kinetic energy in the two coupled cars after collision, most nearly?

- a. 7%
- b. 14%
- c. 23%
- d. 46%
- e. 100%

\[
\frac{KE_f}{KE_i} = \frac{1}{2} \left( \frac{15,000 \cdot 2.0}{2} \right)^2 = 0.230 = 23\% \quad \text{(c) is correct)}
\]

4. A miniature, spring-loaded, radio-controlled gun is mounted on an air puck. The gun's bullet has a mass of 5.00 g, and the gun and puck have a combined mass of 120 g. With the system initially at rest, the radio-controlled trigger releases the bullet, causing the puck and empty gun to move with a speed of 0.500 m/s. Of the total kinetic energy of the gun-puck-bullet system, what percentage is in the bullet?

- a. 4.0%
- b. 50%
- c. 70%
- d. 96%
- e. 100%

\[
\frac{(KE_f)_B}{(KE_f)_{tot}} = \frac{\frac{1}{2} m_B v_B^2}{\frac{1}{2} m_B v_B^2 + \frac{1}{2} M_6 v_6^2} = \frac{1}{1 + \frac{M_6 v_6^2}{m_B v_B^2}} = \frac{1}{1 + (0.04)^2} = 0.96\%
\]

\[
\text{(d) is correct)}
\]
5. Geosynchronous satellites orbit the Earth at a distance of 42,000 km from the Earth's center. Their angular velocity at this height is the same as the rotation of the Earth, so they appear stationary at certain locations in the sky. What is the force acting on a 1500-kg satellite at this height, most nearly?

a. 15,000 N
b. 1500 N
c. 300 N
d. 150 N
e. 30 N

\[ F = \frac{GMm}{r^2} = \frac{GME}{RE^2} = mg \frac{RE}{v^2} \]

Since \( (\frac{GME}{RE})^2 = \frac{a}{g} \), then \( 2\pi RE = 40,000 \text{ km} \Rightarrow RE = 6.37 \times 10^3 \text{ km} \).

Then \( F = mg \cdot (\frac{6.37}{42000})^2 = 338.1 \text{ N} \)  

\( \square \) is correct

6. An object of mass 0.50 kg is transported to the surface of Planet X where the object's weight is measured to be 20 N. The radius of the planet is \( 4.0 \times 10^6 \) m. What free fall acceleration will the 0.50-kg object experience when transported to a distance of \( 2.0 \times 10^6 \) m above the surface of this planet, approximately?

a. 90 m/s\(^2\)
b. 40 m/s\(^2\)
c. 24 m/s\(^2\)
d. 18 m/s\(^2\)
e. 10 m/s\(^2\)

Let \( g_x \) be the gravity on X-planet. \( M = 0.50 \text{ kg} \)

\[ W = 20 \text{ N} = Mg_x \Rightarrow g_x = \frac{W}{M} = 40 \text{ m/s}^2 \text{ at surface of X} \]

Surface of X is distance \( R_v = 4.0 \times 10^6 \text{ m} \) from center.

Point at \( 2.0 \times 10^6 \text{ m} \) above X is at distance \( r = h + R_v = 6.0 \times 10^6 \text{ m} \) from center of X. Since \( a = \frac{F_x}{m} = \frac{GMX}{r^2} \), decreases on \( \frac{1}{r^2} \) as \( r \) increases.

\[ a = \left( \frac{4.0 \times 10^6}{6.0 \times 10^6} \right)^2 \cdot g_x = (0.444)(40) = 17.78 \text{ m/s}^2 \text{ is correct at } r = 6.0 \times 10^6 \text{ m} \]

\( \square \) is correct

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7. A bucket of water with total mass 23 kg is attached to a rope, which in turn is wound around a 0.050-m radius cylinder at the top of a well. A crank with a turning radius of 0.25 m is attached to the end of the cylinder and the moment of inertia of cylinder and crank is 0.12 kg·m². If the bucket is raised to the top of the well and released, what is the acceleration of the bucket as it falls toward the bottom of the well? (Assume rope’s mass is negligible, that cylinder turns on frictionless bearings and that \( g = 9.8 \text{ m/s}^2 \).)

- \( a. \) 3.2 m/s²
- \( b. \) 4.8 m/s²
- \( c. \) 6.2 m/s²
- \( d. \) 7.4 m/s²
- \( e. \) 9.8 m/s²

**Solution:**

**Ang Acceleration of cylinder/crank:** 
\[ \alpha = \frac{T - R}{I} \]

**Vertical Accele of bucket:** 
\[ F_{net} = Ma = Mg - T \]

**Finding \( a \):**

1. \( TR = I \alpha/R \Rightarrow T = I \alpha R^2 \).
2. \( Ma = -I \alpha/R^2 + Mg \Rightarrow a(M + I/R^2) = Mg \)
3. So \( a = Mg/M + I/R^2 = \frac{23 \times 9.8}{23 + (12)(0.05)} = 3.17 \text{ m/s}^2 \)

**8.** An astronaut is on a 100-m lifeline outside a spaceship, circling the ship with an angular velocity of 0.100 rad/s. How far inward can she be pulled before the centripetal acceleration reaches 5g = 49 m/s²?

- \( a. \) 10.1 m
- \( b. \) 50.0 m
- \( c. \) 72.7 m
- \( d. \) 89.9 m
- \( e. \) It can never reach 5g.

**Solution:**

\[ \Omega f = \frac{m \omega_f^2}{r_f} = \frac{m(0.100)^2}{r_f} \]

To keep \( 5g > \frac{F_c}{m} = \frac{m \omega_i^2}{r_i} = \frac{m(0.100)^2}{r_i} \)

\[ \frac{r_f}{r_i}^3 > \frac{\omega_f^2}{\omega_i^2} 5g = \frac{100}{5g} = \frac{100}{49} \]

\[ \omega_f^3 > 2.04 \times 10^4 \Rightarrow \omega_f > 27.3 \text{ m/s} \]

\[ \text{She can be pulled } 100 - 27.3 = 72.7 \text{ m inward} \]

\[ \Rightarrow \text{IS correct} \]
A 2.6-g bullet leaves the barrel of a gun at a speed of 350 m/s.

(a) Find its kinetic energy. (5 points)

(b) Find the average force exerted by the expanding gases on the bullet as the bullet moves the length of the 47-cm-long barrel. (5 points)

\[ KE = \frac{1}{2} m v^2 = \frac{1}{2} (2.6 \times 10^{-3}) (350)^2 = 1.59 \times 10^2 \text{ J} = KE \]

\[ F_{\Delta x} = W_{\text{NET}} = \Delta (KE) \text{ by WORK-ENERGY THEOREM} \]

\[ = (KE_f) - (KE)_0 \] force bullet started at rest

\[ F_{\text{AVG}} = \frac{W_{\text{NET}}}{\Delta x} = \frac{(KE_f)}{\Delta x} = \frac{1.59 \times 10^2 J}{0.47 \text{ m}} = 3.39 \times 10^2 \text{ N} = F_{\text{AVG}} \]
A high diver of mass 70 kg jumps off a board 10 m above the water. Exactly 1 sec after entering the water his downward motion has stopped.

a) What net average upward force acted upon him while in the water? (5 points)

b) What average upward force did the water exert upon him? (5 points)
11. A 700-kg satellite is in a circular orbit about Earth at a height above Earth equal to Earth’s mean radius.

(a) Find the satellite’s centripetal acceleration in units of $g = 9.8 \text{m/s}^2$. (5 points)

(b) Kepler’s third law states that $T^2$, the square of satellite’s period, is proportional to $R^3$, the cube of the satellite’s orbital radius. If this satellite’s height above the earth had been three Earth’s radii instead of one Earth radius, by what factor would the period have changed? (5 points)

\[
\frac{F_g}{m} = \frac{m \cdot a_c}{m} = \frac{G \cdot M_E \cdot M}{m \cdot r^2} \quad \text{and} \quad r = R_E + h = 2R_E
\]

Then (a) $a_c = \frac{G \cdot M_E}{4(Re)^2} = \frac{g}{4}$, since $\frac{G \cdot M_E}{(Re)^2} = g = 9.8 \text{m/s}^2$

(b) $T^2 = (\text{const}) R^3$. If $h$ increases from $R_E + 3R_E$, then $R$ increases from $R_E + R_E = 2R_E$ to $R_E + 3R_E = 4R_E$: i.e. $R$ doubles.

Therefore $T_1^2 \propto R_1^3 \Rightarrow T_2^2 \propto (2R_1)^3 = 8R_1^3 = 8T_1^2$

Then $T_2 = \sqrt{8}T_1$ increases by $\sqrt{8} = 2.83 \times$ (b)
12. Two football players each weighing 70 kg are at opposite edges of a playground merry-go-round rotating at an initial angular velocity of 5 radians per second. The players, initially 2 m from the center, both walk 1 m inward towards the center at which point they are just 1 m from the center. For simplicity, assume that merry go round is massless.

a) What is the final angular velocity of the merry-go-round? (5 points)

\[ I_i \omega_i = I_f \omega_f \]

Thus \[ \omega_f = \frac{I_i \omega_i}{I_f} \]

and \[ r_i = 2 \text{ m}; r_f = 1 \text{ m} \]

\[ I_f = 2Mr_f^2 \]

\[ I_i = 2Mr_i^2 \]

\[ \omega_f = \frac{2Mr_i^2 \cdot 5}{(2 \times 70 \cdot 1)} \]

b) Compute the physical work that the football players performed on the merry-go-round, including the sign, as they walked towards the center? (Hint: Use the Work-Energy theorem.) (5 points)

\[ \Delta (KE) = (KE)_f - (KE)_i = \frac{1}{2}I_f \omega_f^2 - \frac{1}{2}I_i \omega_i^2 \]

\[ = \frac{1}{2} (2 \times 70 \cdot 1 \times 20)^2 - \frac{1}{2} (2 \times 70 \cdot 4)^2 \]

\[ = 70 (440 - 4.25) = 70 (395.75) \]

\[ = 2.71 \times 10^3 \text{ J} = \text{Work Done} \]

****** End of Exam II******