Section I. Multiple Choice Questions

Each question in this section is worth eight (8) points. You should NOT take more than two minutes per question. If you do, it is advisable to continue on to the next question!

(1.) (a.) Since \( a = v^2/R \) and \( v = \frac{2\pi R}{T} \) implies \( a = \frac{4\pi^2 R}{T^2} \), we find \( a = 4\pi^2 × \frac{4m}{(4s)^2} = \pi^2 \frac{m}{s^2} = 9.87 \frac{m}{s^2} \).

(2.) (a.) As with the problem on the practice exam, the velocities and heights are related by \( (V_1)^2 = 2g H_1 \) and \( (V_2)^2 = 2g H_2 \) so that \( \frac{H_1}{H_2} = \frac{(V_1)^2}{(V_2)^2} = \frac{[2 \frac{m}{s}]^2}{[6 \frac{m}{s}]^2} = 1/9 \).

(3.) (b.) Since both start from rest \( W_1 = \frac{1}{2} m_1 (V_1)^2 \) and \( W_2 = \frac{1}{2} m_2 (V_2)^2 \) and since \( W_1 = W_2 \) it follows that \( 3kg (V_1)^2 = 27kg (V_2)^2 \) or \( V_1 = 3V_2 \).

(4.) (b.) Here we have the data; \( \omega_0 = 2\pi \frac{rad}{s} \), \( \omega_f = 4\pi \frac{rad}{s} \), \( \theta_0 = 0 \) and \( \theta_f = 6\pi \) rad and one of the basic angular kinematic equations is \( (\omega_f)^2 = (\omega_0)^2 + 2\alpha (\Delta \theta) \). Putting in the numbers yields \( \alpha = \pi \frac{rad}{s^2} \), also since \( \omega = \omega_0 + \alpha t = 2\pi \frac{rad}{s} + \left( \pi \frac{rad}{s^2} \right) t \) we see that the value of \( t = 2 \) s works.

(5.) (d.) We can find the work done by using \( W = \Delta KE = \frac{1}{2} m [0 - (v_0)^2] \) \( = -\frac{1}{2} (2000 \) kg \( ) (6 \frac{m}{s})^2 = -36000 \) J. The time over which the collision occurs \( \Delta t = 0.12 \) s. So the power delivered is \( |W|/\Delta t = 300,000 \) J.

Section II. Analytical Questions

Problem (1.)

From Newton’s second law it follows that \( m a = F \) and the problem gives the
force and acceleration. Therefore, it must be so that

\[ m (9 \frac{m}{s^2} t^2) = 36 \left( \frac{N}{s^3} t^3 \right) \rightarrow m = 4 t \frac{kg}{s} \]

and we see that the mass depends on the time \( t \). The problem also states that the bucket can only hold 16 kg when full. So we have

\[ 16 \, kg = 4 t \frac{kg}{s} \rightarrow t = 4 \, s \]

Problem (2.)

This is a momentum conservation problem. The momenta for ball #1 and ball #2 before the collision are given by

<table>
<thead>
<tr>
<th>Momentum</th>
<th>( x )-comp.</th>
<th>( y )-comp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \vec{p}_1 )</td>
<td>( 10 , kg \left( \frac{m}{s} \right) )</td>
<td>0</td>
</tr>
<tr>
<td>( \vec{p}_2 )</td>
<td>0</td>
<td>( 5 , kg \left( \frac{m}{s} \right) )</td>
</tr>
<tr>
<td>( \vec{P}_{tot}^{Before} )</td>
<td>( 70 \left( \frac{kg , m}{s} \right) )</td>
<td>( 70 \left( \frac{kg , m}{s} \right) )</td>
</tr>
</tbody>
</table>

After the collision the two balls are stuck together so that \( M_t = 15 \, kg \) but we don’t know the \( x \)-component of velocity \( V'_{x} \) nor the \( y \)-component of velocity \( V'_{y} \)

<table>
<thead>
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<th>( y )-comp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \vec{P}_{tot}^{After} )</td>
<td>( 15 , kg , V'_{x} )</td>
<td>( 15 , kg , V'_{y} )</td>
</tr>
</tbody>
</table>

Momentum conservation implies that

\[ 15 \, kg \, V'_{x} = 70 \left( \frac{kg \, m}{s} \right) , \quad 15 \, kg \, V'_{y} = 70 \left( \frac{kg \, m}{s} \right) , \]

\[ V'_{x} = \frac{14}{3} \left( \frac{m}{s} \right) , \quad V'_{y} = \frac{14}{3} \left( \frac{m}{s} \right) , \]

and to find the angle we note

\[ tan \theta = \frac{V'_{y}}{V'_{x}} = \frac{(14/3)}{(14/3)} = 1 \rightarrow \theta = 45^\circ. \]

Problem (3.)
In general for a planet going around the Sun with speed $V$ and at radius $R$, we have
\[ \frac{GM_S}{R^2} = \frac{V^2}{R} \rightarrow GM_S = V^2 R \]
but also we have $V = R \omega$, so that
\[ GM_S = (\omega)^2 R^3 \]
If we call Earth planet # 2 and Mercury planet # 1 then
\[ GM_S = (\omega_1)^2 (R_1)^3 \quad \& \quad GM_S = (\omega_2)^2 (R_2)^3 \]
or equivalently
\[ (\omega_1)^2 (R_1)^3 = (\omega_2)^2 (R_2)^3 \]
so finally
\[
(\omega_1)^2 = \frac{(R_2)^3}{(R_1)^3} (\omega_2)^2 = \frac{(1.5 \times 10^{11})^3}{(5.8 \times 10^{10})^3} \left(2 \times 10^{-7} \text{ rad/s}\right)^2 \\
= \frac{(15 \times 10^{10})^3}{(5.8 \times 10^{10})^3} \left(2 \times 10^{-7} \text{ rad/s}\right)^2 = \frac{(15)^3}{(5.8)^3} \left(2 \times 10^{-7} \text{ rad/s}\right)^2 \\
= (2.59)^3 \left(2 \times 10^{-7} \text{ rad/s}\right)^2 \rightarrow \\
\omega_1 = \left[ \sqrt{(2.59)^3} \right] \times 2 \times 10^{-7} \text{ rad/s} = 4.16 \times 2 \times 10^{-7} \text{ rad/s} \\
= 8.32 \times 10^{-7} \text{ rad/s}
\]

**Problem (4.)**

We first can find the linear velocity
\[ V = \frac{120 \text{ cm}}{10 \text{ s}} = 12 \text{ cm/s} = \]
and the angular speed and linear speed are related by $V = R \omega$ so that $\omega = V/R$ or
\[ \omega = \frac{12}{4} \frac{\text{ rad}}{\text{s}} = = 3 \frac{\text{ rad}}{\text{s}} \]

**Problem (5.)**
Even though this problem takes place on an incline, its solution proceeds exactly as on a flat surface. The problem gives that the kinetic energy is changed by $2\ J$ so we have the equation

$$2\ J = \Delta KE = W$$

where the last part follows from the Work-Energy Theorem. But the work is also given by $W = F\ s\ \cos\theta = -F\ s$ since the angle between the friction and the motion is $180^\circ$. So it also the case that

$$2\ J = -F \times (10\ m) \rightarrow F = -\frac{1}{5}\ kg\ \frac{m}{s^2}$$

$$\rightarrow \ Ma = -\frac{1}{5}\ kg\ \frac{m}{s^2} \rightarrow (6\ kg)\ a = -\frac{1}{5}\ kg\ \frac{m}{s^2}$$

$$\rightarrow \ a = -\frac{1}{30}\ \frac{m}{s^2}$$

(a.) One of the basic kinematic equations reads $(v_f)^2 = (v_0)^2 + 2\ a\ \Delta x$ and when it comes to rest $v_f = 0$ so

$$0 = (15\ \frac{m}{s})^2 + 2\left(-\frac{1}{30}\ \frac{m}{s^2}\right)\ \Delta x \rightarrow$$

$$\Delta x = (15)^3\ m = 3,375\ m$$

(b.) Another one of the basic kinematic equations reads $v_f = v_0 + a\ t$, so that we have

$$0 = 15\ \frac{m}{s} + (-\frac{1}{30}\ \frac{m}{s^2})\ t \rightarrow$$

$$t = 2(15)^2\ s = 450\ s$$

Problem (6.)

This problem simply uses conservation of energy. If we assume that the mass begins from rest, then we must have before it starts to swing downward

$$E_{Before} = KE_{Before} + PE_{Before} = 0 + mg\ H$$

and at the bottom of its swing

$$E_{After} = KE_{After} + PE_{After} = \frac{1}{2}\ m\ V^2 + 0$$

so that the conversation law implies $V^2 = 2\ g\ H$. The centripetal acceleration is given by $a_C = V^2/R$ so we find

$$a_C = \frac{2\ g\ H}{R} \rightarrow H = \frac{a_C}{2\ g}\ R$$

$$H = \frac{20\ \frac{m}{s^2}}{2(9.8\ \frac{m}{s^2})}[2.5\ m] = 1.02[2.5\ m] = 2.55\ m$$