Section I. Multiple Choice Questions

Each question in this section is worth eight (8) points. You should NOT take more than two minutes per question. If you do, it is advisable to continue on to the next question!

(1.) (a.) Since the kinematic equation for the ball is \( v_f = -80 \frac{m}{s} + (-16 \frac{m}{s^2})t \), we see that as \( t \) increases the speed (given by \(|v_f|\)) increases.

(2.) (d.) A scalar and a vector cannot be added.

(3.) (b.) For an object thrown, the horizontal velocity never changes from the initial horizontal velocity. The initial horizontal velocity is just \( 16 \cos(60^\circ) \frac{m}{s} = 8 \frac{m}{s} \).

(4.) (c.) (2.46) (9.8 \frac{m}{s^2}) This follows from a result given in a lecture. The acceleration of gravity \( g \) on Earth’s surface is equal to \( g = 9.8 \frac{m}{s^2} = G \frac{M_E}{(R_E)^2} \) but for Jupiter

\[
g_J = G \left( \frac{M_J}{(R_J)^2} \right) = G \left( \frac{314.5 M_E}{(11.31 R_E)^2} \right)
\]

\[= \left( \frac{314.5}{(11.31)^2} \right) G \left( \frac{M_E}{(R_E)^2} \right) = 2.46 g\]

(5.) (d.) All of the situations described in (a.), (b.) and (c.) can occur by making a choice of coordinates.

Section III. Analytical Questions

Problem (1.)

A bug is located at \((1m, 2m)\) at \(t = 0s\). The same bug is observed to be located at \((2m, -3m)\) at \(t = 3s\). Finally it is observed to be located at \((-3m, -1m)\) at \(t = 3.5s\).
(a.) The average velocity between \( t = 0 \) s and \( t = 3 \) s is given by
\[
\bar{v}_x = \left( \frac{2 - 1}{3 - 0} \right) \frac{m}{s} = 0.66 \frac{m}{s}, \quad \bar{v}_y = \left( \frac{-3 - 2}{3 - 0} \right) \frac{m}{s} = -1.66 \frac{m}{s}
\]

(b.) The average velocity between \( t = 3 \) s and \( t = 3.5 \) s is given by
\[
\bar{v}_x = \left( \frac{-3 - 2}{3.5 - 3} \right) \frac{m}{s} = 10 \frac{m}{s}, \quad \bar{v}_y = \left( \frac{-1 + 3}{3.5 - 3} \right) \frac{m}{s} = 4 \frac{m}{s}
\]

(c.) The average average acceleration between \( t = 3 \) s and \( t = 3.5 \) s may be defined by
\[
\bar{a}_x = \left( \frac{10 - 0.66}{3.5 - 3} \right) \frac{m}{s^2} = 18.68 \frac{m}{s^2}, \quad \bar{a}_y = \left( \frac{4 + 1.66}{3.5 - 3} \right) \frac{m}{s^2} = 11.32 \frac{m}{s^2}
\]

Problem (2.)

The kinematic position equations for cars #1 and cars #2 are given by
\[
x_{f,1} = \frac{1}{2} (0.6) (9.8 \frac{m}{s^2}) t^2, \quad x_{f,2} = 45 m + \frac{1}{2} (0.3) (9.8 \frac{m}{s^2}) t^2
\]

If there is a tie then
\[
\frac{1}{2} (0.6) (9.8 \frac{m}{s^2}) t^2 = 45 m + \frac{1}{2} (0.3) (9.8 \frac{m}{s^2}) t^2 \rightarrow \\
\frac{1}{2} (0.3) (9.8 \frac{m}{s^2}) t^2 = 45 m \rightarrow \\
\frac{3 (98)}{200} \frac{m}{s^2} t^2 = 45 m \rightarrow t = \left( \sqrt{\frac{3000}{98}} \right) s = 5.5 s
\]

The kinematic velocity equations for cars #1 and cars #2 are given by
\[
v_{f,1} = (0.6) (9.8 \frac{m}{s^2}) (5.5 s), \quad v_{f,2} = (0.3) (9.8 \frac{m}{s^2}) (5.5 s)
\]
\[
v_{f,1} = 32.5 \frac{m}{s}, \quad v_{f,2} = 16.2 \frac{m}{s}
\]

Problem (3.)

The kinematic equation for the y-velocity of the ball has \( v_{f,y} = 0 \) at the highest point of the ball's flight. Therefore
\[
0 = 196 \frac{m}{s} - (9.8 \frac{m}{s^2}) t \rightarrow t = \left( \frac{196}{9.8} \right) \frac{m}{s} = 20 s
\]
is the time it takes to reach that point. This implies that the ball is in the air for 40 s. This is impossible.

Problem (4.)

This problem is solved by setting up a table as was done in many of the homework problems

<table>
<thead>
<tr>
<th>Vector</th>
<th>$x$-component</th>
<th>$y$-component</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\vec{d}_1$</td>
<td>0 m</td>
<td>$-3 m$</td>
</tr>
<tr>
<td>$\vec{d}_2$</td>
<td>$-4 m$</td>
<td>0 m</td>
</tr>
<tr>
<td>$\vec{d}_3$</td>
<td>0 m</td>
<td>10 m</td>
</tr>
<tr>
<td>$\vec{d}_4$</td>
<td>$-2 m$</td>
<td>0 m</td>
</tr>
<tr>
<td>$\vec{d}_5$</td>
<td>0 m</td>
<td>8 m</td>
</tr>
<tr>
<td>$\vec{d}_6$</td>
<td>2 m</td>
<td>0 m</td>
</tr>
<tr>
<td>$\vec{d}_1 + ... + \vec{d}_6$</td>
<td>$-4 m$</td>
<td>15 m</td>
</tr>
</tbody>
</table>

So magnitude of the total displacement vector is

$$
| \vec{d}_1 + ... + \vec{d}_6 | = \sqrt{16 + 225} ~ m = \sqrt{241} ~ m = 15.52 ~ m
$$

and the angle above the $y$-axis in the third quadrant is just

$$
tan(\theta) = \frac{15}{4} \quad \rightarrow \quad \theta = tan^{-1}\left(\frac{15}{4}\right) = 75.1^\circ
$$

Problem (5.)

Let the tension in the string on the left be denoted by $T_1$ and name the angle $\theta_1$ between the vertical and the first string. Let the tension in the string on the right be denoted by $T_2$ and name the angle $\theta_2$ between the vertical and the second string. Finally let the tension in the vertical string be named $T_3$.  

3
(a.) For the 5 kg mass after drawing a force diagram, Newton’s second law gives the first two equations

\[ m_1 a_{1,x} = 0 \quad , \quad m_1 a_{1,y} = T_3 - m_1 g \]

For the 50 kg mass after drawing a force diagram, Newton’s second law gives the third and fourth equation

\[ m_2 a_{2,x} = -T_2 + \mu N_f \quad , \quad m_2 a_{1,y} = N_f - m_2 g \]

From the place where the strings are tied

\[ T_1 \cos(\theta_1) + T_2 \cos(\theta_2) = T_3 \]
\[ -T_1 \sin(\theta_1) + T_2 \sin(\theta_2) = 0 \]

(b.) Since we have an equilibrium, it follows that all accelerations are equal to zero. So from the second equation above we see \( T_3 = m_1 g \) and from the fourth equation \( N_f = m_2 g \). Using this information, the third equation implies \( T_2 = \mu m_2 g \) and using the conditions where the strings are tied gives

\[ T_1 \cos(\theta_1) + \mu m_2 g \cos(\theta_2) = m_1 g \]
\[ -T_1 \sin(\theta_1) + \mu m_2 g \sin(\theta_2) = 0 \]

We multiply the first of these by \( \sin(\theta_1) \), multiply the second by \( \cos(\theta_1) \) and then add the two equations together

\[
\begin{align*}
T_1 \cos(\theta_1) \sin(\theta_1) + \mu m_2 g \cos(\theta_2) \sin(\theta_1) &= m_1 g \sin(\theta_1) \\
-T_1 \sin(\theta_1) \cos(\theta_1) + \mu m_2 g \sin(\theta_2) \cos(\theta_1) &= 0
\end{align*}
\]

\[
\mu m_2 g \cos(\theta_2) \sin(\theta_1) + \mu m_2 g \sin(\theta_2) \cos(\theta_1) = m_1 g \sin(\theta_1) \]

\[
\mu m_2 g \left[ \cos(\theta_2) \sin(\theta_1) + \sin(\theta_2) \cos(\theta_1) \right] = m_1 g \sin(\theta_1)
\]

\[
\mu = \frac{m_1 \sin(\theta_1)}{m_2 \left[ \cos(\theta_2) \sin(\theta_1) + \sin(\theta_2) \cos(\theta_1) \right]}, \quad \mu = \frac{\sin(\theta_1)}{10 \left[ \sin(\theta_1 + \theta_2) \right]}
\]

\[
\begin{align*}
T_2 &= \frac{(49 N) \sin(\theta_1)}{\sin(\theta_1 + \theta_2)} \quad , \quad T_1 &= \frac{(49 N) \sin(\theta_2)}{\sin(\theta_1 + \theta_2)}
\end{align*}
\]

(c.) We already know \( T_3 = (5 \text{ kg}) \left( 9.8 \frac{m}{s^2} \right) = 49 \text{ N} \). For \( T_2 \) and \( T_1 \) we find

\[
\begin{align*}
T_2 &= \frac{(49 N) \sin(\theta_1)}{\sin(\theta_1 + \theta_2)} \quad , \quad T_1 &= \frac{(49 N) \sin(\theta_2)}{\sin(\theta_1 + \theta_2)}
\end{align*}
\]
Problem (6.)

The radius of the Moon must be \( R = (3.48)/2 \times 10^6 \) m = 1.74 \( \times \) 10^6 m. The circumference of the Moon is then given by

\[
C = 2\pi R
\]

and if it takes time \( T \) to completely go around then the speed is

\[
v = \frac{2\pi R}{T}
\]

and also the acceleration must be

\[
a = \frac{v^2}{R} = \frac{4\pi^2 R}{T^2}
\]

This acceleration is to be set equal to the acceleration of gravity at the Moon surface

\[
\frac{1}{6}g = \frac{4\pi^2 R}{T^2}
\]

\[
T^2 = \frac{24\pi^2 R}{g}
\]

\[
T = \pi \sqrt{\frac{24R}{g}}
\]

\[
= (3.14) \sqrt{\frac{24(1.74 \times 10^6)}{9.8}} {s} = 6485 \text{ s} = 1.8 \text{ hrs}.
\]