3) In this problem we are given the numerical data $m = 62 \text{ kg}$, $v_0 = 5.5 \text{ m/s}$, $\Delta t = 1.65 \text{ s}$ and $v_0 = 1.1 \text{ m/s}$. It is simple to calculate the change in momentum

$$\Delta p = m[v_f - v_0] = 62 \text{ kg}[-1.1 - (-5.5)] \frac{m}{s} = 272.8 \text{ N s}$$

and the Impulse-Momentum Theorem states

$$\Delta p = F \Delta t \rightarrow F = \frac{\Delta p}{\Delta t} = \frac{272.8 \text{ N s}}{1.65 \text{ s}} = 165.3 \text{ N}$$

Since the force is a positive number, it acts upward.

4) The numerical data for this problem are $m = 1 \text{ kg}$ and $|\vec{v}| = 30 \frac{m}{s}$. But since the arrows are fired in different directions, we need to treat the different momenta with a spreadsheet.

<table>
<thead>
<tr>
<th>Momentum</th>
<th>$x$-component</th>
<th>$y$-component</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m \vec{v}_1$</td>
<td>$30 \text{ kg} \frac{m}{s}$</td>
<td>0</td>
</tr>
<tr>
<td>$m \vec{v}_2$</td>
<td>0</td>
<td>$-30 \text{ kg} \frac{m}{s}$</td>
</tr>
<tr>
<td>$m \vec{v}_1 + m \vec{v}_2$</td>
<td>$30 \text{ kg} \frac{m}{s}$</td>
<td>$-30 \text{ kg} \frac{m}{s}$</td>
</tr>
</tbody>
</table>

This makes an angle of $45^o$ south of east as seen from

$$\tan\theta = \frac{-30 \text{ kg} \frac{m}{s}}{30 \text{ kg} \frac{m}{s}} = -1 \rightarrow \theta = -45^o$$

5) We are given the numerical data $m = 0.35 \text{ kg}$, $v_0 = 4.0 \frac{m}{s}$, $v_f = -21 \frac{m}{s}$. It is simple to calculate the change in momentum

$$\Delta p = (0.35 \text{ kg})[-21 - 4] \frac{m}{s} = -8.75 \text{ N s}$$

But the impulse $I = \Delta p = -8.75 \text{ N s}$. 
13) We can use one of the kinematic equations to deduce the speed of the ball just before impact.

\[
(v_f)^2 = (v_0)^2 + 2g(y_f - y_0)
\]

\[
\rightarrow (v_f)^2 = 0 + 2(-9.8 \frac{m}{s^2})(0.1 - 1.2 m)
\]

\[
v_f = \sqrt{23.5 \frac{m}{s}} = 4.8 \frac{m}{s}
\]

Now we can use the same equation to find out how fast the ball was moving after the rebound

\[
(V_f)^2 = (V_0)^2 + 2g(Y_f - Y_0)
\]

because we know that \(V_f = 0\) when \(Y_f = 0.7 \text{ m}\) so that

\[
(0)^2 = (V_0)^2 + 2(-9.8 \frac{m}{s^2})(0.7 m - 0)
\]

\[
\rightarrow (V_0)^2 = 2(-9.8 \frac{m}{s^2})(0.7 m) = 13.72 \frac{m^2}{s^2}
\]

\[
V_0 = \sqrt{13.72} \frac{m}{s} = 3.7 \frac{m}{s}
\]

The impulse \(I\) is just equal to \(\Delta p\) so

\[
I = m[v_{after} - v_{before}] = (0.5 kg)[3.7 - (-4.8)] \frac{m}{s} = (0.5 kg)(8.5 \frac{m}{s}) = 4.2 Ns
\]

Since the impulse is a positive number, it acts upward.

14) This problem is little tricky. The Impulse-Momentum Theorem can also be written as

\[
\overline{F} = \Delta p = m \frac{\Delta v}{\Delta t} = m \frac{\Delta v}{\Delta t} \Delta v
\]

When a grain of sand is dropped from a height of 2 m, it has a \(y\)-component of velocity just before landing of

\[
v_y = -\sqrt{2gH} = -\sqrt{2(9.8 \frac{m}{s^2})(2 \text{ m})} = -6.26 \frac{m}{s}
\]

After landing on the truck, its \(y\)-component of velocity = 0. So that

\[
\Delta v_y = [0 - (-6.26 \frac{m}{s})] = 6.26 \frac{m}{s}
\]

The problem states that the sand hits the truck as a rate of \(55 \frac{kg}{s}\), which tells us

\[
\frac{m}{\Delta t} = 55 \frac{kg}{s}
\]

So that finally we conclude

\[
\overline{F} = (55 \frac{kg}{s})(6.26 \frac{m}{s}) = 344.3 N
\]
19) This is really a momentum conservation problem. Before the bullet is fired the total momentum is zero. This means that afterward we must have

\[ 0 = m_b v_b + M_{W+G} V_{W+G} \]

\[ \rightarrow V_{W+G} = - \frac{m_b}{M_{W+G}} v_b \]

where the numerical data for part (a.) are \( m_b = 0.01 \text{ kg}, \ v_b = 720 \text{ m/s} \) and \( M_{W+G} = 51 \text{ kg} \)

\[ V_{W+G} = - \frac{m_b}{M_{W+G}} v_b = - \frac{0.01 \text{ kg}}{51 \text{ kg}} (720 \text{ m/s}) \]

\[ = - \frac{7.2}{51} \text{ m/s} = - 0.14 \text{ m/s} \]

For (b.) we simply change \( m_b \rightarrow 5.0 \times 10^{-4} \text{ kg} \) so that

\[ V_{W+G} = - \frac{m_b}{M_{W+G}} v_b = - \frac{5 \times 10^{-4} \text{ kg}}{51 \text{ kg}} (720 \text{ m/s}) \]

\[ = - 7.1 \times 10^{-3} \text{ m/s} \]

20) This problem is solved using conservation of momentum and conservation of energy. Since before they pushed off they were at rest, the total momentum is 0. So we have

\[ 0 = m_{ed} V_{ed} + M_A V_A \rightarrow V_{ed} = - \left( \frac{M_A}{m_{ed}} \right) V_A \]

From many previous problems and energy conservation we know that the two velocities are related to the two height via

\[ V_{ed} = \sqrt{2 g H_{ed}}, \ V_A = \sqrt{2 g H_A} \]

\[ \rightarrow (V_{ed})^2 = 2 g H_{ed}, \ (V_A)^2 = 2 g H_A \]

Now we see

\[ H_{ed} = \frac{1}{2 g} (V_{ed})^2 = \frac{1}{2 g} \left( \frac{M_A}{m_{ed}} \right)^2 (V_A)^2 \]

\[ = \frac{1}{2 g} \left( \frac{M_A}{m_{ed}} \right)^2 (2 g H_A) = \left( \frac{M_A}{m_{ed}} \right)^2 H_A \]

\[ H_{ed} = \left( \frac{120}{78} \right)^2 (0.65 \text{ m}) = 1.5 \text{ m} \]

32) This is problem that we can simply solve using a spreadsheet for the components of the momentum. Before the collision we see
and afterward we see

\[
\begin{array}{|c|c|c|}
\hline
\text{Momentum} & \text{x-component} & \text{y-component} \\
\hline
m_A \vec{v}_A & (0.025 \text{ kg}) (5.5) \frac{\text{m}}{\text{s}} & 0 \\
\hline
m_B \vec{v}_B' & (0.05 \text{ kg}) (v_B') \cos(37^\circ) & -(0.05 \text{ kg}) (v_B') \sin(37^\circ) \\
\hline
\end{array}
\]

We now set the total x-component of the momentum before the collision equal to the total x-component of the momentum after the collision.

\[
(0.025 \text{ kg}) (5.5) \frac{\text{m}}{\text{s}} = (0.025 \text{ kg}) (v_A') \cos(65^\circ) + (0.05 \text{ kg}) (v_B') \cos(37^\circ)
\]

\[
(5.5) \frac{\text{m}}{\text{s}} = (v_A') \cos(65^\circ) + 2 (v_B') \cos(37^\circ)
\]

We also set the total y-component of the momentum before the collision equal to the total x-component of the momentum after the collision.

\[
0 = (0.025 \text{ kg}) (v_A') \sin(65^\circ) - (0.05 \text{ kg}) (v_B') \sin(37^\circ)
\]

\[
0 = (v_A') \sin(65^\circ) - 2 (v_B') \sin(37^\circ)
\]

At this stage, we have two unknowns \(v_A'\) and \(v_B'\) and the two equations

\[
(5.5) \frac{\text{m}}{\text{s}} = (v_A') \cos(65^\circ) + 2 (v_B') \cos(37^\circ)
\]

\[
0 = (v_A') \sin(65^\circ) - 2 (v_B') \sin(37^\circ)
\]

we multiply the first by \(\sin(37^\circ)\), the second \(\cos(37^\circ)\) and add them together to find

\[
(5.5) \sin(37^\circ) \frac{\text{m}}{\text{s}} = (v_A') \left[ \cos(65^\circ) \sin(37^\circ) + \sin(65^\circ) \cos(37^\circ) \right]
\]

\[
(5.5) \sin(37^\circ) \frac{\text{m}}{\text{s}} = (v_A') \sin(102^\circ)
\]

\[
v_A' = \frac{(5.5) \sin(37^\circ) \frac{\text{m}}{\text{s}}}{\sin(102^\circ)} = \frac{(5.5) (0.6) \frac{\text{m}}{\text{s}}}{0.99} = 3.4 \frac{\text{m}}{\text{s}}
\]

From the second of the two equations that we are trying to solve, we find

\[
v_B' = \frac{(5.5) \sin(65^\circ) \frac{\text{m}}{\text{s}}}{2 \sin(102^\circ)} = \frac{(5.5) (0.9) \frac{\text{m}}{\text{s}}}{2 (0.99)} = 2.6 \frac{\text{m}}{\text{s}}
\]

34) To solve this problem we once again use a spreadsheet for the different components of the momentum before the collision
Momentum

\begin{array}{|c|c|c|}
\hline
\text{Momentum} & x\text{-component} & y\text{-component} \\
\hline
m_1 \vec{v}_1 & 0 & -m_A V_1 \\
m_2 \vec{v}_2 & m_B V_2 \cos(30^\circ) & m_B V_2 \sin(30^\circ) \\
m_3 \vec{v}_3 & -m_B V_3 \cos(30^\circ) & m_B V_3 \sin(30^\circ) \\
\hline
\end{array}

The numerical data for this problem are

\[ m_A = 2.5 \times 10^{-3} \text{ kg} \quad m_B = 4.5 \times 10^{-3} \text{ kg} \quad V_1 = 575 \frac{m}{s} \]

Since after the collision all three bullets are at rest, there is no net momentum. Therefore, adding the entries from the \( x \)-column leads to

\[ 0 = m_B V_2 \cos(30^\circ) - m_B V_3 \cos(30^\circ) \quad \rightarrow \quad V_2 = V_3 \]

There also no momentum after adding all the entries from the \( y \)-column

\[ 0 = -m_A V_1 + m_B V_2 \cos(30^\circ) + m_B V_3 \cos(30^\circ) \]

\[ 0 = -m_A V_1 + 2m_B V_2 \cos(30^\circ) \]

\[ 0 = - (2.5 \times 10^{-3} \text{ kg}) (575 \frac{m}{s}) + 2(4.5 \times 10^{-3} \text{ kg}) V_2 (0.86) \]

\[ \rightarrow \quad V_2 = \frac{(2.5 \times 10^{-3} \text{ kg}) (575 \frac{m}{s})}{2(4.5 \times 10^{-3} \text{ kg}) (0.86)} = 319 \frac{m}{s} \]

40) Momentum conservation gives

\[ m (-4 \frac{m}{s}) + m (7 \frac{m}{s}) = m (v'_A) + m (v'_B) \]

and energy conservation gives

\[ \frac{1}{2} m (-4 \frac{m}{s})^2 + \frac{1}{2} m (7 \frac{m}{s})^2 = \frac{1}{2} m (v'_A)^2 + \frac{1}{2} m (v'_B)^2 \]

The factors of \( m \) can be divided out of each equation and we are left with

\[ -4 \frac{m}{s^2} + 7 \frac{m}{s^2} = (v'_A)^2 + (v'_B)^2 \]

\[ 16 \frac{m^2}{s^2} + 49 \frac{m^2}{s^2} = (v'_A)^2 + (v'_B)^2 \]

In other words

\[ 3 \frac{m}{s} = (v'_A) + (v'_B) \]

\[ 65 \frac{m^2}{s^2} = (v'_A)^2 + (v'_B)^2 \]

If we square the first equation we find

\[ 9 \frac{m^2}{s^2} = \left[ (v'_A)^2 + (v'_B)^2 \right] + 2 v'_A v'_B \]

\[ 9 \frac{m^2}{s^2} = \left[ 65 \frac{m^2}{s^2} \right] + 2 v'_A v'_B \]

\[ -56 \frac{m^2}{s^2} = 2 v'_A v'_B \]

\[ -28 \frac{m^2}{s^2} = v'_A v'_B \]
Now we multiply the same first equation by $v_A'$

\[
(3 \frac{m}{s}) v_A' = (v_A')^2 + v_B' v_A'
\]

\[
(3 \frac{m}{s}) v_A' = (v_A')^2 - 28 \frac{m^2}{s^2}
\]

\[0 = (v_A')^2 - (3 \frac{m}{s}) v_A' - 28 \frac{m^2}{s^2}\]

Now we multiply the same first equation by $v_B'$

\[
(3 \frac{m}{s}) v_B' = v_B' v_A' + (v_B')^2
\]

\[
(3 \frac{m}{s}) v_B' = (v_B')^2 - 28 \frac{m^2}{s^2}
\]

\[0 = (v_B')^2 - (3 \frac{m}{s}) v_B' - 28 \frac{m^2}{s^2}\]

Thus $v_A'$ and $v_B'$ satisfy the same quadratic equation. We use the quadratic formula to find the solutions of this equation.

\[
v_A' = \frac{-(-3 \frac{m}{s}) \pm \sqrt{(-3 \frac{m}{s})^2 - 4 \left(3 \frac{m}{s} \right)^2}}{2}
\]

\[= \frac{3 \pm \sqrt{(9 + 112)}}{2} \frac{m}{s} = \frac{3 \pm 11}{2} \frac{m}{s}
\]

\[= 7 \frac{m}{s} \text{ or } -4 \frac{m}{s}\]

The first ball before the collision had a velocity of $-4 \frac{m}{s}$ and the second ball before the collision had a velocity of $7 \frac{m}{s}$ so after the collision we find

\[v_A' = 7 \frac{m}{s} \text{ and } v_B' = -4 \frac{m}{s}\]

Both balls are travelling opposite to their original directions.