Name: _______________________________

Final Exam,
Physics 117-Spring 2003, Wed. 5/21/2003
Instructor: Dr. S. Liberati

GENERAL INSTRUCTIONS

• There are a total of twelve problems in this exam.
• All problems carry equal weights.
• Do all the problems by writing on the exam book (continue to work on the back of each page if you run out of room).
• Write your name (in capital letters) on every page of the exam.
• Purely numerical answers will not be accepted. Explain with symbols or words your line of reasoning. Corrected formulae count more than corrected numbers.

What you can do

• You may look at your text book and lecture notes in taking the exam
• Use a calculator

What you can’t do

• Speak with nearby colleagues
• Use any wireless device during the exam

Hints to do well

• Read carefully the problem before to compute. Before to start you must have clear in your mind what you need to arrive to the answer.
• Do problems with symbols first (introduce them if you have to). Only put in numbers at the end.
• Check your answers for dimensional correctness.
• If you are not absolutely sure about a problem, please write down what you understand so that partial credit can be given.

Honor Pledge: Please sign at the end of the statement below confirming that you will abide by the University of Maryland Honor Pledge
"I pledge on my honor that I have not given or received any unauthorized assistance on this assignment/examination."

Signature: ________________________________
**Exercise 1**

David wants to play a trick on his sister Helen. He is standing on the balcony of their house 5 m high from the ground and she is in the garden working on her snowman horizontally 15 meters from David (see figure). David wants to hit his sister with a snowball without being seen. To do this, his plan is to not aim directly at her but to just launch the ball horizontally and use gravity.

A) How fast does David have to throw the snowball in order to hit Helen, if he throws it horizontally? (i.e. no vertical speed)

Approximate \( g \) as \( g = 10 \text{ m/s}^2 \) and neglect air resistance.

As usual we can treat the horizontal and vertical motions as independent.

Neglecting the friction of air the snowball has constant velocity horizontally and accelerated motion vertically (with acceleration downward equal to \( g \)).

The ball will hit the ground (or Helen) after a time

\[
 t = \sqrt{\frac{2d}{g}} = \sqrt{\frac{10 \text{ m}}{10 \text{ m/s}^2}} = 1 \text{ s}
\]

David must throw the ball in such a way that in this lapse of time it covers exactly the 15 meters between him and Helen.

\[
 v_{\text{horiz}} = \frac{h}{t} = 15 \frac{m}{s}
\]

B) Imagine that David and Helen do not live on Earth but on a planet whose mass is the same as the Earth’s mass but whose radius is two times the Earth one. In this case how fast does David have to launch horizontally the ball in order to hit Helen?
We know that the gravitational acceleration on the surface of Earth $g$ is defined as

$$ g = G \frac{M_{earth}}{R_{earth}^2} $$

If the imaginary planet has the same mass but double the radius then the gravitational acceleration on its surface will be

$$ g_{planet} = G \frac{M_{planet}}{R_{planet}^2} = G \frac{M_{earth}}{4 \cdot R_{earth}^2} = \frac{1}{4} g_{earth} $$

In order to answer the question we just need to recompute the quantities with this new $g$.

$$ t = \sqrt{\frac{2d}{g}} = 2 \cdot \sqrt{\frac{10m}{10m/s^2}} = 2s $$

David must throw the ball in such a way that in this lapse of time it covers exactly the 15 meters between him and Helen.

$$ v_{horiz} = \frac{h}{t} = 7.5 \frac{m}{s} $$
**Exercise 2**

A racing car has a mass of 1200 Kg. At the start the car accelerates on a straight lane from 0 to 180 Km/h in 4 seconds. (Neglect air friction. 1 hour=3600 s)

1) What is the force exerted by the car engine?

First of all \( 180 \text{Km/h} = 180 \frac{1000}{3600} \text{m/s} = 50 \text{ m/s} \)

From Newton's second law

\[
F = ma \quad a = \frac{\Delta v}{\Delta t} = \frac{50}{4} \frac{m}{s} = 12.5 \text{ m/s}^2
\]

\[
F = 1200 \text{ Kg} \cdot 12.5 \text{ m/s}^2 = 15000 \text{ N}
\]

2) If the car covers 100 meters in the above 4 seconds and we assume the force to be constant in this lapse of time what is the work done by the engine?

The work done by the engine can be derived as

\[
W = F \cdot \Delta l \quad \text{or} \quad W = KE
\]

\[
W = F \cdot \Delta l = 15000 \text{ N} \cdot 100 \text{ m} = 15 \times 10^5 \text{ N} \cdot \text{m}
\]

\[
W = KE = \frac{1}{2} m v^2 = 600 \text{ Kg} \cdot 2500 \text{ m}^2/\text{s}^2 = 15 \times 10^5 \text{ J}
\]
Exercise 3

A 1000 Kg satellite is sent to explore the moon of a distant planet. The satellite is positioned in a circular orbit with a radius of 3000 Km from the center of the moon. The satellite completes a revolution around the moon in 8600 seconds. Calculate:

- The centripetal acceleration of the satellite

\[
\begin{align*}
a &= \frac{v^2}{r} \\
\text{We need } v &= \frac{2\pi R}{T} \\
&= \frac{2 \cdot 3.14 \cdot 3000 \times 10^3}{8600} \\
&= 2.2 \times 10^3 \text{ m/s} \\
a &= \frac{(2.2 \times 10^3 \text{ m/s})^2}{3 \times 10^6} \\
&= 1.6 \text{ m/s}^2
\end{align*}
\]

- The gravitational force between the satellite and the moon (assume no other force act on the satellite)

The centripetal force is provided by gravity so

\[
F_{grav} = ma_{centrip} = 1.6 \times 10^3 \text{ N}
\]

- The mass of this moon (\(G=6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{Kg}^2\))

\[
F_{grav} = ma_{centrip} = G \frac{Mm}{R^2} \\
M = \frac{R^2 a_{centrip}}{G} = \frac{9 \times 10^{12} \text{ m}^2 \cdot 1.6 \text{ m/s}^2}{6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{Kg}^2} \\
= 2.16 \times 10^{23} \text{ Kg}
\]
Exercise 4

A cannon fixed on the ground is placed in front of a cart with an empty cylinder of the right size to catch a cannon ball. The cannon shoots a 100 Kg ball into the cylinder on the cart which before the impact was at rest.

1) Knowing that the ball before the impact had a speed of 60 m/s and that after the impact the system of the cart plus ball has a speed of 10 m/s, calculate the mass of the cart.

No external forces so the momentum is conserved

\[ p_i = p_f \]

\[ m_{\text{ball}} v_{\text{ball, init}} = (m_{\text{ball}} + M_{\text{cart}})v_{\text{final}} \]

\[ m v_{\text{init}} = M v_{\text{final}} \]

\[ M = m \frac{v_{\text{init}} - v_{\text{final}}}{v_{\text{final}}} = 100 \text{ Kg} \left( \frac{60 - 10}{10} \right) \text{m/s} = 500 \text{ Kg} \]

2) What is the change in kinetic energy before and after the collision?

\[ KE = KE_f - KE_i \]

\[ KE = \frac{1}{2} (m_{\text{ball}} + M_{\text{cart}})v_{\text{final}}^2 - \frac{1}{2} m_{\text{ball}}v_{\text{ball, init}}^2 \]

\[ KE = 300 \text{ m}^2/\text{s}^2 \times 50 \text{ Kg} \times 3600 \text{ m}^2/\text{s}^2 = 150000 \text{ J} \]

3) Imagine that the kinetic energy lost is all converted into heat given to the ball+cart system. If both the cart and the ball are made of aluminum [Aluminum specific heat \( c_{\text{al}} = 900 \text{ J/(Kg·K)} \)] what is the change \( \Delta T \) in their temperature?

\[ Q = KE = 150000 \text{ J} \]

\[ c(m + M)\Delta T = Q \]

\[ \Delta T = \frac{Q}{c(m + M)} = \frac{150000 \text{ J}}{900 \text{ J/(Kg·K)} \times 600 \text{ Kg}} = 0.27 \text{ K} \]
In a special race a bobsled is pushed on a straight and leveled track by four athletes. The bobsled has a mass of 480 Kg and each athlete has a mass of 80 Kg. The cart starts from rest and its speed at the end of the leveled track (before the athletes jump in it) is 5 m/s.

1) If the leveled track is 20 meters long what is the work done by each athlete?

The way the question was formulated was misleading. This question was counted just 1 point in the case of error, 3 in the case of correct answer.

\[ W = \Delta KE = KE_f - KE_i = \frac{1}{2} m_{bob} v_{\text{final}}^2 - \frac{1}{2} m_{bob} v_{\text{init}}^2 \]

\[ KE = 240 \text{ Kg} \times 25 \text{ m}^2/\text{s}^2 = 6000 \text{ J} \]

\[ W_{\text{ath}} = \frac{\Delta KE}{4} = 1500 \text{ J} \]

At the end of the straight part (say at point A) the athletes jump on the bobsled. The bobsled (now with the four athletes on board) is at an altitude of 50 meters (and has still a speed of 5 m/s). The rest of the track is similar to that of a roller coaster (see figure). Calculate:

2) The speed of the racers at B (h_B=20 m)
We need to use conservation of mechanical energy
\[ ME = KE + GPE = \text{const} \quad \square \quad KE + GPE_{A} = KE + GPE_{B} \]
\[ M_{\text{total}} = m_{\text{bob}} + 4 \cdot m_{\text{ath}} = (480 + 320) \text{ Kg} = 800 \text{ Kg} \]
\[ \frac{1}{2} M v_{A}^2 + Mgh_{A} = \frac{1}{2} M v_{B}^2 + Mgh_{B} \quad \square \quad v_{B}^2 = v_{A}^2 + 2g(h_{A} - h_{B}) \]
\[ v_{B} = \sqrt{v_{A}^2 + 2g(h_{A} - h_{B})} = \sqrt{25 + 600} = 25 \text{ m/s} \]

3) The height of point C (\( v_{C} = 15 \text{ m/s} \))

We need to use conservation of mechanical energy
\[ ME = KE + GPE = \text{const} \quad \square \quad KE + GPE_{A} = KE + GPE_{C} \]
\[ \frac{1}{2} M v_{A}^2 + Mgh_{A} = \frac{1}{2} M v_{C}^2 + Mgh_{C} \quad \square \quad h_{C} = h_{A} + \frac{1}{2g} (v_{A}^2 - v_{C}^2) \]
\[ h_{C} = h_{A} + \frac{1}{2g} (v_{A}^2 - v_{C}^2) = 50 \text{ m} + \frac{1}{20} (25 - 225) \text{ m} = 40 \text{ m} \]

4) Knowing that after C the track is leveled again determine the work that the brakes have to do in order to stop completely the bob.

\[ W = \square KE \quad \square \quad \frac{1}{2} M v_{C}^2 = \frac{1}{2} 800 \text{ Kg} \cdot 225 (\text{m/s})^2 = 90,000 \text{ J} \]
Exercise 6

The float in a toilet tank is a sphere of volume 520 cm$^3$ and mass 10 gr.

1) What is the buoyancy force (in Newtons) on the float when it is completely submerged? (Remember that the density of fresh water is $D_{\text{water}} = 1$ gr/cm$^3$)

\[
F_{\text{buoy}} = m_{\text{disp,water}} g
\]

\[
m_{\text{disp,water}} = D_{\text{water}} V_{\text{float}} = 520 \text{ gr}
\]

\[
F_{\text{buoy}} = m_{\text{disp,water}} g = 520 \times 10^{-3} \text{ Kg} \times 10 \text{ m/s}^2 = 5.2 \text{ N}
\]

2) If the float must have an upward buoyancy force of 3.0 N to shut off the ballcock valve, what percentage of the float volume must be submerged?

\[
F_{\text{buoy}} = m_{\text{disp,water}} g = 3 \text{ N}
\]

\[
m_{\text{disp,water}} = D_{\text{water}} V_{\text{immersed}} = F_{\text{buoy}} / g = 0.3 \text{ Kg}
\]

\[
V_{\text{immersed}} = \frac{0.3 \text{ Kg}}{D_{\text{water}}} = \frac{300 \text{ gr}}{1 \text{ gr/cm}^3} = 300 \text{ cm}^3
\]

\[
\frac{300}{520} \times 0.58 \times V_{\text{immersed}} \times 58\% = V_{\text{float}}
\]

3) If the component of the net force orthogonal to the arm is 2.6 N and the arm length $R = 20$ cm what is the torque that shuts off the ballcock valve?

\[
\tau = F_{\text{buoy, normal}} \times R = 2.6 \text{ N} \times 20 \times 10^{-3} \text{ m} = 0.52 \text{ N} \cdot \text{m}
\]
Exercise 7

You want to use a Joule apparatus (see figure) to heat up 2 Kg of water. To do this you drop several times a weight of 83.72 Kg from an altitude of 5 meters. Assume that all the mechanical work is converted in heat and that no heat loss occurs anywhere.

1) What is the change in temperature of the water for each weight drop.

\[ \Delta T = \frac{Q}{cm} \]
\[ Q = W = mgh = 83.72 \text{ Kg} \times 10 \text{ m/s}^2 \times 5 \text{ m} = 4186 \text{ J} \]
\[ \Delta T_{water} = \frac{Q}{cm_{water}} = \frac{4186 \text{ J}}{4186 \text{ J/(Kg} \cdot K) \div 2 \text{ Kg}} = 0.5 \text{ K} \]

2) If the initial temperature of the water is 20 °C how many times does one need to drop the weight to reach the boiling temperature?

For each drop the temperature increases by

\[ \Delta T_{water} = \frac{Q}{cm_{water}} = \frac{4186 \text{ J}}{4186 \text{ J/(Kg} \cdot K) \div 2 \text{ Kg}} = 0.5 \text{ K} \]

A total \[ \Delta T_{water} = 373 \text{ K} \div 293 \text{ K} = 80 \text{ K} \] requires 160 droppings.

3) Once the boiling temperature of water is reached at least how many times does one need to drop the weight in order to convert all the water into water vapor? (latent heat for water vaporization \( L = 2257 \text{ KJ/kg} \))
For each drop the work converted in heat is
\[ W = Q = 4186 \, J \]
In order to completely vaporize the water one need
\[ Q_{\text{vap}} = Lm_{\text{wat}} = 4514 \, KJ \]
So one needs to drop the weight for at least \( N \) times where
\[ N = \frac{Q_{\text{vap}}}{W_{\text{each,drop}}} \geq 1079 \]
Exercise 8

A certain heat engine operates between the temperatures of 700K and 290K.

1) If 6000 Joules of heat enter the engine every cycle what is the amount of heat exhausted at each cycle in the case that the efficiency of the engine equals the ideal one?

\[
\eta_{\text{ideal}} = 1 - \frac{T_{\text{cold}}}{T_{\text{hot}}} \quad \eta_{\text{real}} = 1 - \frac{Q_{\text{out}}}{Q_{\text{in}}} \quad \frac{T_{\text{cold}}}{T_{\text{hot}}} = \frac{Q_{\text{out}}}{Q_{\text{in}}} = \frac{290}{700} \quad 6000 \ J \div 2460 \ J
\]

2) In the above limit (real efficiency\[\eta\]ideal efficiency) what is the mechanical work extracted at each cycle?

\[W = Q_{\text{in}} - Q_{\text{out}} = 6000J \div 2460J = 3540 \ J\]

3) If this work is then introduced in a heat pump with coefficient of performance 4 what is the heat released in the hot region (house, see figure) at each cycle?

\[W = Q_{\text{in}} - Q_{\text{out}} = 6000J \div 2486J = 3540 \ J\]

\[\text{COP} = \frac{Q_{\text{pump, in}}}{W} \quad Q_{\text{pump, in}} = 4 \times 3540 \ J = 14160 \ J\]

\[Q_{\text{pump, out}} = W + Q_{\text{pump, in}} = 17770 \ J\]
Exercise 9

A child weighs 400 N standing on Earth.

1) What is the apparent weight of the child if he stands on a scale in an elevator accelerating upward at 0.25 g?
   (hint: this is classical relativity, use your physical intuition!)

   The child experience a fictitious force pushing him downward
   (opposite direction w.r.t. elevator acceleration) so the apparent
   weight of the child will be
   \[
   \text{Weight}_{\text{apparent}} = \text{Weight}_{\text{real}} + F_{\text{inertial}} = m_{\text{child}}g + m_{\text{child}}a
   \]
   \[
   m_{\text{child}} = \frac{\text{Weight}_{\text{real}}}{g} = 40 \text{ Kg} \quad \text{so}
   \]
   \[
   \text{Weight}_{\text{apparent}} = 400 \text{ N} + 100 \text{ N} = 500 \text{ N}
   \]

2) What is the apparent weight of the child if he stands on a scale in an elevator accelerating downward at 0.25 g?

   The child experience a fictitious force pushing him upward
   (opposite direction w.r.t. elevator acceleration) so the apparent
   weight of the child will be
   \[
   \text{Weight}_{\text{apparent}} = \text{Weight}_{\text{real}} - F_{\text{inertial}} = m_{\text{child}}g + m_{\text{child}}a
   \]
   \[
   \text{Weight}_{\text{apparent}} = 400 \text{ N} - 100 \text{ N} = 300 \text{ N}
   \]

3) What is the apparent weight of the child if he stands on a scale in the elevator which moves with constant speed=20 m/s?
   (Assume g=10 m/s²)

   The child is in an inertial system so there is no fictitious force
   so the weight of the child will still be its normal one.
Exercise 10

An astronaut measures the length of her giant cylindrical space station using a laser beam and a very accurate clock. The astronaut measures the time that the laser light takes to travel from one side of the base to the other one and back (two way trip).

1) If the light of the laser takes $10^{-4}$ seconds for the two way trip from side to side of the cylinder, how long is the base? (Assume $c=3\cdot10^5$ km/s)

Double the length of the base is given by the speed of light multiplied by the time taken by the laser:

$$2L = c \cdot t = 3 \cdot 10^5 \text{Km/s} \cdot 10^{-4} \text{s} \quad L = 15 \text{Km}$$

The base station moves at 80% of the speed of light ($v=0.8 \cdot c$) with respect to a nearby planet. (see picture)

2) For an observer on the planet how long does it take for the laser to cross the base?

The observer on the planet see the clock on the base (that for him is moving) slower than his one:

$$\frac{t_{\text{moving}}}{t_{\text{rest}}} = \frac{1}{\sqrt{1 - \frac{0.8 \cdot c}{c}}}$$

so $t_{\text{moving}} = 1.67 \cdot 10^{-4}$ s

3) How long will the space colony appear to be?

The observer on the planet see the length of the base contracted by the same relativistic factor:

$$\frac{l_{\text{moving}}}{l_{\text{rest}}} = \frac{15 \text{Km}}{1.67} = 9 \text{Km}$$
Exercise 11

Two positive charges $Q_a = +1$ C and $Q_b$ are separated by a distance $R = 9$ cm.

1) What is the value of $Q_b$ if the electric field is zero at a point between the charges, 3 cm from charge $Q_a$? (See picture)

\[
E = k \frac{Q_a}{r_1^2} - \frac{Q_b}{r_2^2} = 0 \quad \frac{Q_a}{r_1^2} = \frac{Q_b}{r_2^2} \quad \frac{Q_a}{r_1^2} (R r_1)^2 = Q_b
\]

\[Q_b = 1C \frac{6^2}{9} = 4C\]

2) With this configuration of charges (i.e. same positions and $Q_b$ as obtained above), what is the electric field value midway between the two charges? ($k_e = 9 \cdot 10^9$ Nm$^2$/C$^2$)

\[
E = k \frac{Q_a}{r_1^2} - \frac{Q_b}{r_2^2} = 9 \cdot 10^9 \text{ Nm}^2/\text{C}^2 \frac{3 \text{ C}}{(0.045 \text{ m})^2} = \frac{27}{2025} \cdot 10^{15} \text{ N/C} = 13 \cdot 10^{12} \text{ N/C}
\]

3) What is the net force acting on a -2 mC (N.B. mC=milliCoulomb=10$^{-3}$ C) charge in this point equidistant from $Q_a$ and $Q_b$?

\[
F = E \cdot q = [13 \cdot 10^{12} \text{ N/C}] \cdot [2 \cdot 10^{-3} \text{ C}] = [26 \cdot 10^9 \text{ N}]
\]
**Exercise 12**

An electron is in a stable orbit around the nucleus of a hydrogen atom. The radius of the electron orbit is 47.7 $\times 10^{-11}$ m. (Planck constant $h=6.63 \times 10^{-34}$ J$\cdot$s)

1) Knowing that the radius of the innermost orbit is $r_1=5.3 \times 10^{-11}$ m determine the angular momentum of the electron.

$$L_n = n \frac{h}{2\pi}$$

we need to determine $n$

$$r_n = n^2 r_1$$

if $r_n = 47.7 \times 10^{11}$ m then $n = \sqrt{\frac{r_n}{r_1}} = \sqrt{9} = 3$

$$L_3 = \frac{3h}{2\pi} 3.17 \times 10^{34} \text{ J} \cdot \text{s}$$

2) Look at the figure below. From the previous question you should be able to identify on which energy level is the electron. If this electron now jumps to the ground level ($n=1$) what will be the frequency of the photon emitted? What is the corresponding wavelength? (N.B. 1 eV $= 1.6 \times 10^{-19}$ J. The speed of light is $c=3 \times 10^8$ m/s)

\[
\begin{array}{c|c}
\text{n} & \text{E (eV)} \\
\hline
\infty & 0 \\
4 & -0.85 \\
3 & -1.5 \\
2 & -3.4 \\
1 & -13.6 \\
\end{array}
\]

\[
E(n=3, n=1) = 1.5 \times (13.6) = 12.1 \text{ eV}
\]

$1 \text{ eV} = 1.6 \times 10^{19} \text{ J}$

$E = 19.36 \times 10^{19} \text{ J}$

$f = E / h = 2.92 \times 10^{15} \text{ Hz}$

$l = c / f = (3 \times 10^8 \text{ m/s}) / (2.92 \times 10^{15} \text{ s}) = 1.03 \times 10^7 \text{ m}$