Resonant Transducer

Figure 1 illustrates the principle of a resonant transducer [Paik, 1972]. The antenna with mass $M$ receives a tiny “hammer blow” from the GW. If the resonance frequency of the small mass $m$ is tuned to that of the antenna, the antenna begins to drive the resonator, transferring its entire energy to the small mass. The displacement of the transducer then becomes $(M/m)^{1/2}$ times larger than the initial displacement of the antenna. The energy flows back and forth between the two masses with a beat period of $(2\pi/\omega_a) (M/m)^{1/2}$.

Since the energy coupling constant of the transducer, $\beta_S$, is inversely proportional to the mass that it is coupled to, the resonant transducer improves $\beta_S$ by the ratio of $M/m$. However, since it takes half the beat period for the signal to appear fully at the output of the transducer, the detection bandwidth is restricted to

$$\Delta \omega_S \approx \omega_a (m/M)^{1/2}. \tag{1}$$

To obtain the largest $\Delta \omega_S$, one must choose an optimum value of the transducer mass, $m_{opt}$, which satisfies the two conditions simultaneously:

$$\left(\frac{\Delta \omega_S}{\omega_S}\right)_{max} \approx \beta_S (m_{opt}) \approx \left(\frac{m_{opt}}{M}\right)^{1/2}. \tag{2}$$

For the superconducting inductive transducer [Paik, 1976] of Figure 2,

$$\beta_S = \frac{2\eta B^2 S}{1 + \gamma + \mu_0 m \omega_a^2 d}, \tag{3}$$

where $\gamma \equiv L_3 (L_1^{-1} + L_2^{-1})$, $\eta$ is the fraction of the electrical energy coupled to the SQUID, $S$ is the area of each (pancake) sensing coil, $d$ is the gap
between each sensing coil and the transducer mass, $B$ is the dc magnetic field stored in the gap, and $\mu_0$ is the permeability of vacuum. For the single-mode Maryland transducer presently mounted on ALLEGRO, $m = 0.62$ kg, $S = 3.5 \times 10^{-3}$ m$^2$, $d = 2.5 \times 10^{-5}$ m, $B = 0.12$ T ($H_{c1}$ of niobium at 4.5 K), $\omega_0/2\pi = 915$ Hz, $\eta \approx 0.4$, and $\gamma \approx 1$, which yields $\beta_S \approx 0.03$. With the antenna mass $M = 1150$ kg, we find $(m/M)^{1/2} = 0.023$. Thus the transducer mass is close to optimum and allows $\Delta \omega_S/\omega_S \approx 0.03$.

The resonant transducer concept can be extended further by using a cascade of $n$ resonators with geometrically decreasing masses [Richard, 1979]. Since the beat frequency is determined by the ratio of neighboring masses, Eq. (2) is modified to

$$\left(\frac{\Delta \omega_S}{\omega_S}\right)_{\text{max}} \approx \beta_S (m_{\text{opt}}) \approx \left(m_{\text{opt}}/M\right)^{1/(2n-2)}.$$  \hspace{1cm} (4)

In principle, $\Delta \omega_S/\omega_S$ arbitrarily close to unity can be obtained by increasing $n$. The resulting increase in $\Delta \omega_S$, however, is slow beyond $n = 3$, while the hardware becomes very complex with increasing $n$. The practical limit for $n$ appears to be 3 for most cases of interest.

The parameters chosen for the two-mode Maryland transducer under construction are $m = 0.050$ kg, $S = 1.8 \times 10^{-3}$ m$^2$, and $d = 5.0 \times 10^{-5}$ m, with the others unchanged. This leads to $\beta_S \approx 0.10$ and $(m/M)^{1/4} = 0.081$. Again, the transducer mass is close to optimum and allows $\Delta \omega_S/\omega_S \approx 0.10$ (close to a 100-Hz bandwidth).

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