Phys 623, Introduction to Quantum Mechanics II, Spring 1999, Dr. Yakovenko

Final Examination

Saturday, May 22, 1999, 8–10 a.m., room 1410

1. [**10 points**] The shell model of nuclear structure assumes that protons and neutrons move in an effective spherically-symmetrical potential well produced by nuclear forces. The energy levels are labeled by the orbital angular momentum l and the radial quantum number N. The lowest energy levels are 1s and 1p (see p. 317 of Schwabl). These levels are completely filled by 2+6=8 protons and 2+6=8 neutrons in the oxygen nucleus ${}^{16}_{8}\mathrm{O}_8$ (see the nomenclature on p. 318 of Schwabl). Obviously, because the energy shells are completely filled, the oxygen nucleus has zero magnetic moment.

Calculate the magnetic moment (in nuclear magnetons μ_N) of the ${}^{15}_{7}\mathrm{N}_8$ nitrogen nucleus. In this nucleus, one proton in the ${}^2P_{1/2}$ state (see the notation on p. 217 of Schwabl) is missing from the 1p shell compared to oxygen. The magnetic moment of a free proton is $\mu_p = 2.8\mu_N$.

2. An electron of mass m is in the ground state of a spherically-symmetrical harmonic potential of frequency Ω. The electron is subject to a weak spatially-uniform time-dependent electric field polarized along the z axis and being gradually turned on over a time τ:

$$\boldsymbol{\mathcal{E}}(t) = \begin{cases} 2\mathbf{e}_z\mathcal{E}_0\, e^{(t-t_0)/\tau}\cos(\omega t), & t < t_0 \\ 2\mathbf{e}_z\mathcal{E}_0 \cos(\omega t), & t > t_0 \end{cases} \tag{1}$$

(a) [**3 points**] Calculate the matrix element of the perturbation between the ground state and an excited electron state.

(b) [**4 points**] In the first order of perturbation theory, calculate the probability $P(t)$ of electron transition to an excited state at $t < t_0$.

In the answer for $P(t)$, there may be some terms that oscillate in time with the frequency ω. Average those terms over time, assuming that $2\pi/\omega \ll \Delta t \ll \tau$, where Δt is the time of observation. Sketch the answer for the averaged probability $\bar{P}$ at $t = t_0$ as a function of ω, assuming that $1/\tau \ll \omega, \Omega$.

(c) [**4 points**] Using the result of Problem 2b, calculate the transition rate $\Gamma = d\bar{P}(t)/dt$ at $t = t_0$. Show how the Fermi golden rule in recovered in the limit $\tau \to \infty$.

3. *Qualifier, January 1999, II-3*

A particle of mass m scatters from a spherically symmetric potential

$$V(r) = \frac{A}{r^2}, \tag{2}$$

where A is a constant.

(a) [**6 points**] Find the scattering amplitude $f(\theta)$ in the Born approximation.

(b) **[6 points]** Find the exact partial wave phase shifts δ_l.

Hint: $V(r)$ looks a lot like the centrifugal potential.

(c) **[6 points]** Show that the Born approximation is valid at any energy provided the condition $mA/\hbar^2 \ll 1$ holds.

Suggestion: You may want to replace $V(r)$ by the finite range potential $V(r)e^{-r/a}$ and take the limit $a \to \infty$.

(d) **[6 points]** Show that your results for parts 3a and 3b give the same value for $f(\theta)$ in the limit $mA/\hbar^2 \ll 1$.

Possibly useful information:

The asymptotic forms of the spherical Bessel and Neumann functions $j_\nu(x)$ and $n_\nu(x)$ for any fixed complex ν and $|x| \to \infty$ are

$$j_\nu(x) \sim \frac{1}{x}\sin(x - \nu\pi/2), \qquad n_l(x) \sim -\frac{1}{x}\cos(x - \nu\pi/2).$$

Possibly useful integral and sum:

$$\int_0^\infty \frac{\sin x}{x}\,dx = \frac{\pi}{2}, \qquad \sum_{n=0}^{\infty} P_n(\cos\theta) = \frac{1}{2\sin(\theta/2)}.$$