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Information to energy conversion in an electronic Maxwell's demon and thermodynamics of measurements.

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Publications: PRL 104, 220601 (2010)
EPL 96, 6704 (2011)
PRB 84, 245448 (2011)

Outline

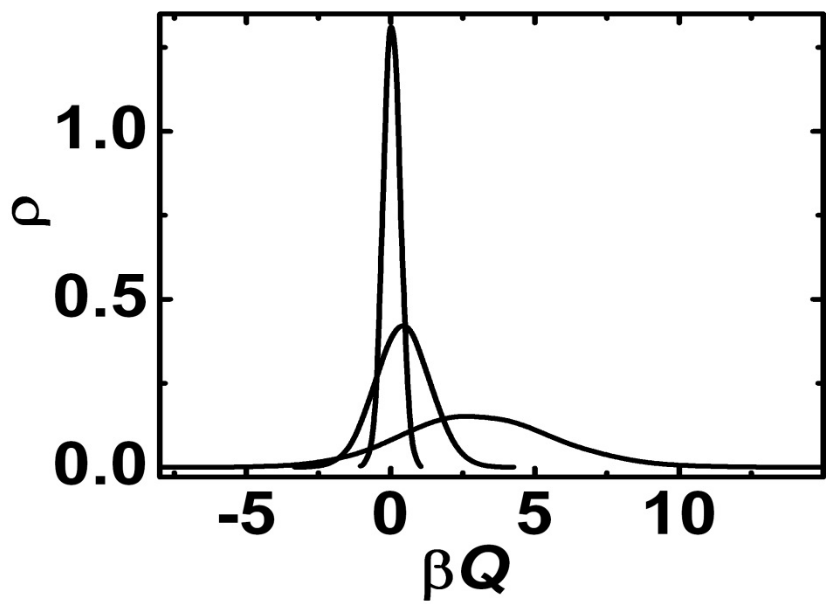
1. Statistical distribution of the heat generated in adiabatic transitions: classical thermal fluctuation-dissipation theorem (FDT).
2. Driven single-electron tunneling (SET) transitions as prototype of reversible information processing.
3. Electronic Maxwell's demons based on the SET pump and nSQUID array.
4. Detector properties in Maxwell's demon operation and thermodynamics of quantum measurements.

Distribution of heat generated in adiabatic transitions

- reversible logic operations are adiabatic transitions;
- potential reversible circuits are based on small, mesoscopic or nano, structures

→ large role of fluctuations

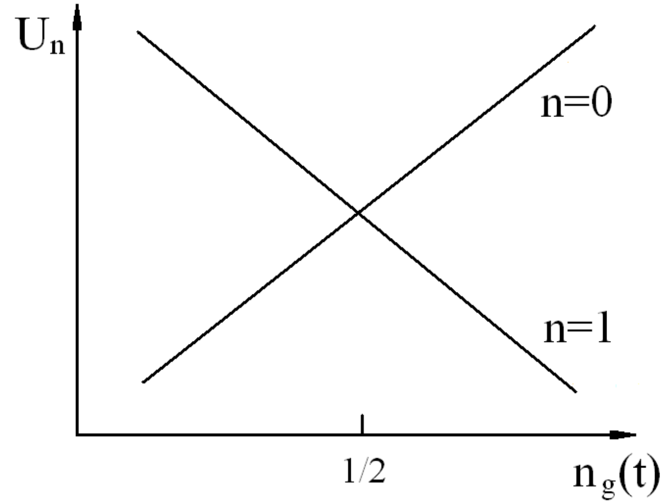
Example: driven SET transitions

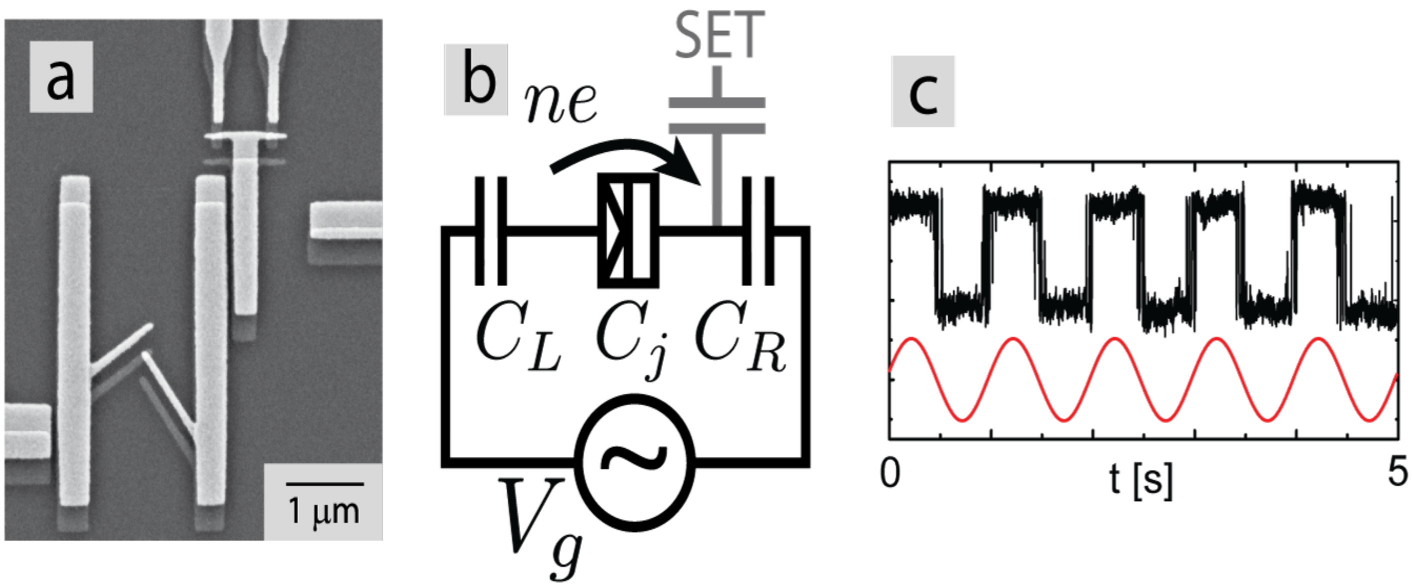


Driven single-electron tunneling (SET) transitions

$$U(n) = U_0(n) - 2E_C n_g(t) n,$$

$$U_0(n) = E_C n^2.$$

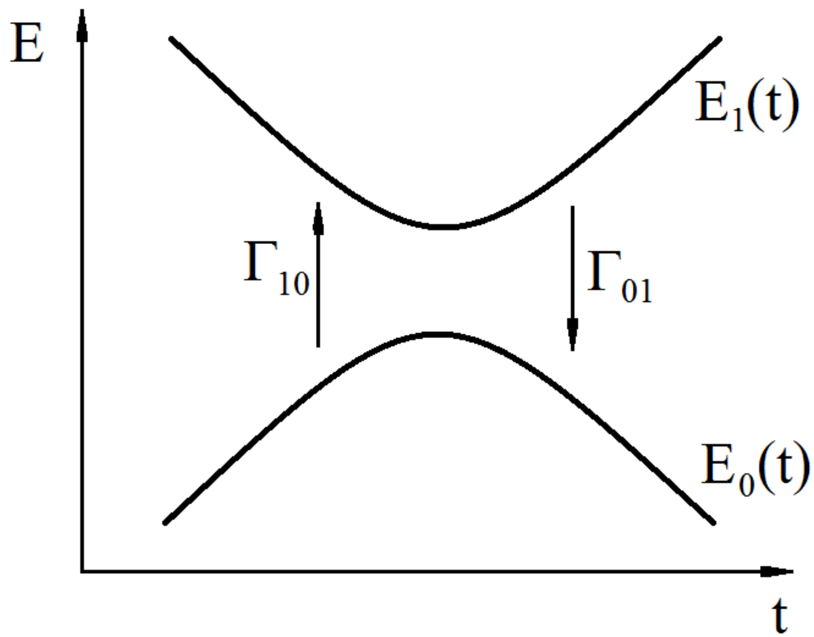




O.-P. Saira *et al.*, PRB (2010); and t.b.p.

Noise of the generated heat in driven adiabatic evolution

$$\sigma_Q^2 = 2T\langle Q\rangle$$



Slow evolution of a system of levels $E_n(t)$ weakly interacting with an equilibrium reservoir, between the two stationary configurations

$$\dot{p}_m = \sum_n \left[p_n \Gamma_{mn} - p_m \Gamma_{nm}\right], \qquad \dot{p} = \Gamma\, p.$$

$$p^{(0)} : \Gamma\, p^{(0)} = 0$$ - local equilibrium

Heat generated in the reservoir in this evolution

$$Q = \sum_{jumps} \left(E_n(t) - E_m(t)\right), \quad \langle Q\rangle^{(tot)} = -T\Delta S + \langle Q\rangle.$$

Average generated irreversible heat

$$\langle Q \rangle = (-1/T)\int dt \sum_{n,m} \dot{E}_n (\Gamma^{-1})_{nm} \dot{E}_m p_m^{(0)} .$$

Γ^{-1} – "group" inverse: Γ^{-1}: $\Gamma\,\Gamma^{-1}\Gamma = \Gamma$; $\Gamma^{-1}\Gamma\,\Gamma^{-1} = \Gamma^{-1}$; $\Gamma^{-1}\Gamma = \Gamma\,\Gamma^{-1}$.

Noise in generated heat

$$\sigma_Q^2 = \langle \tilde{Q}^2 \rangle = -2\int dt \sum_{n,m} \dot{E}_n (\Gamma^{-1})_{nm} \dot{E}_m p_m^{(0)} = 2T\langle Q \rangle .$$

Two-state system

$$\sigma_Q^2 = (1/2)\int dt \frac{\dot{\varepsilon}^2}{\Gamma_{01}+\Gamma_{10}} \cosh^{-2}(\varepsilon/2T), \qquad \varepsilon = E_1(t) - E_2(t) .$$

Conclusion: irreversibly generated heat vanishes not only on average but for each individual transition protocol.

Jarzynski equality and statistics of the generated heat

$$\left\langle \exp\{-(W_{th} - \Delta F)/T\} \right\rangle = 1.$$

In the case of "deterministic" transitions, ΔF=ΔU, and

$$\left\langle \exp\left\{-Q/T\right\} \right\rangle = 1\,.$$

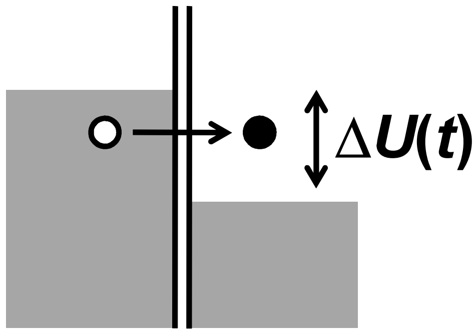
In the limit of adiabatic switching, this relation and thermal FDT

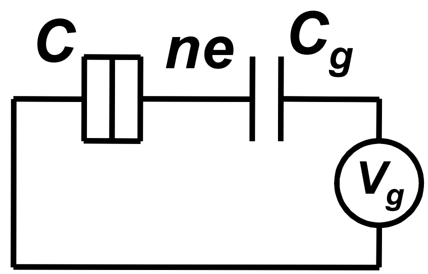
$$\sigma_Q^2 \equiv \left\langle \tilde{Q}^2 \right\rangle = 2T\langle Q \rangle.$$

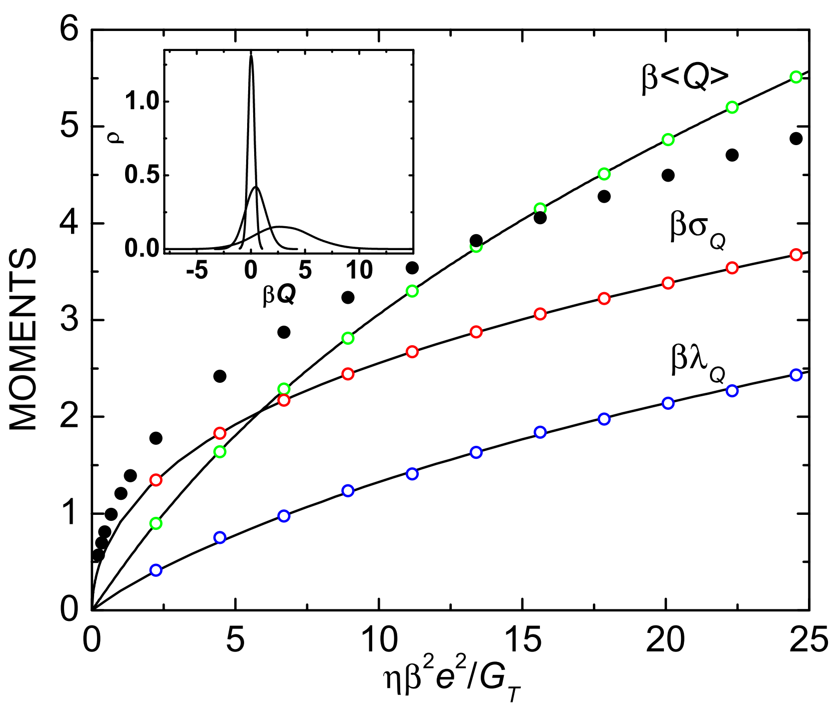
imply Gaussian probability density of the heat distribution

$$\rho(Q) = (1/4\pi T\langle Q \rangle)^{1/2} e^{-(Q-\langle Q \rangle)^2/4T\langle Q \rangle}.$$

Statistics of heat in driven SET transitions





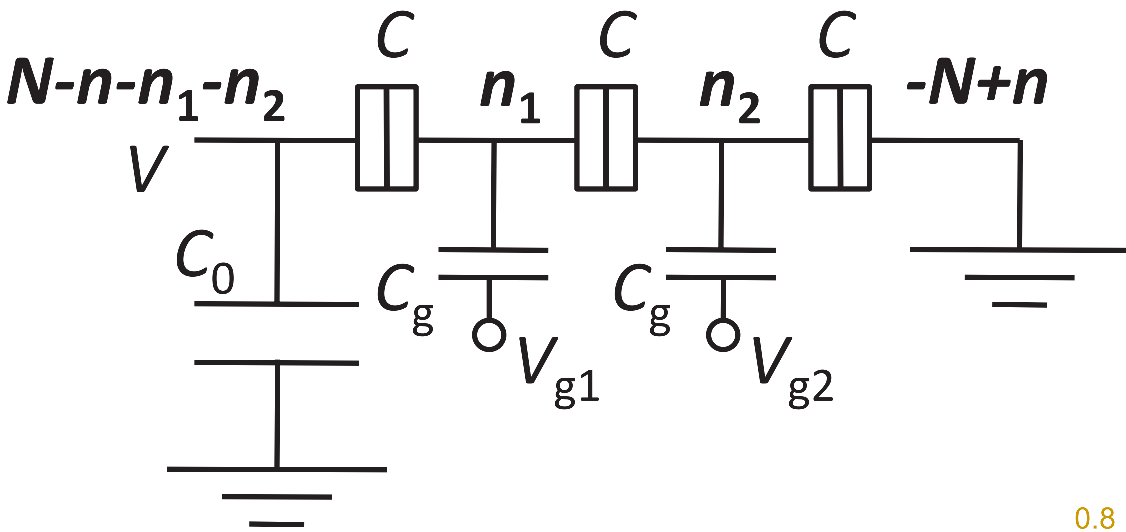


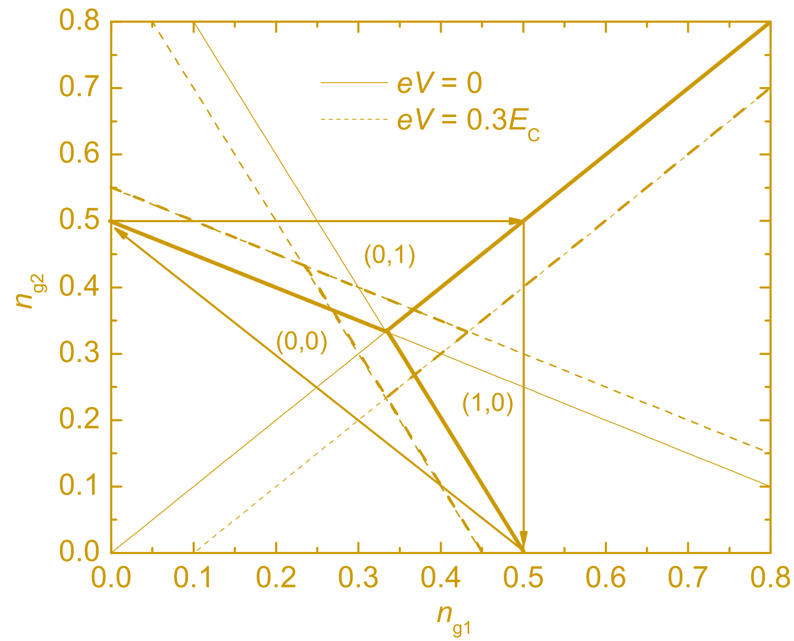
Thermodynamics of the transitions

$$Q = -\sum_j \Delta U(t_j) = E_C \int dt(2n_g(t)-1)I(t)$$

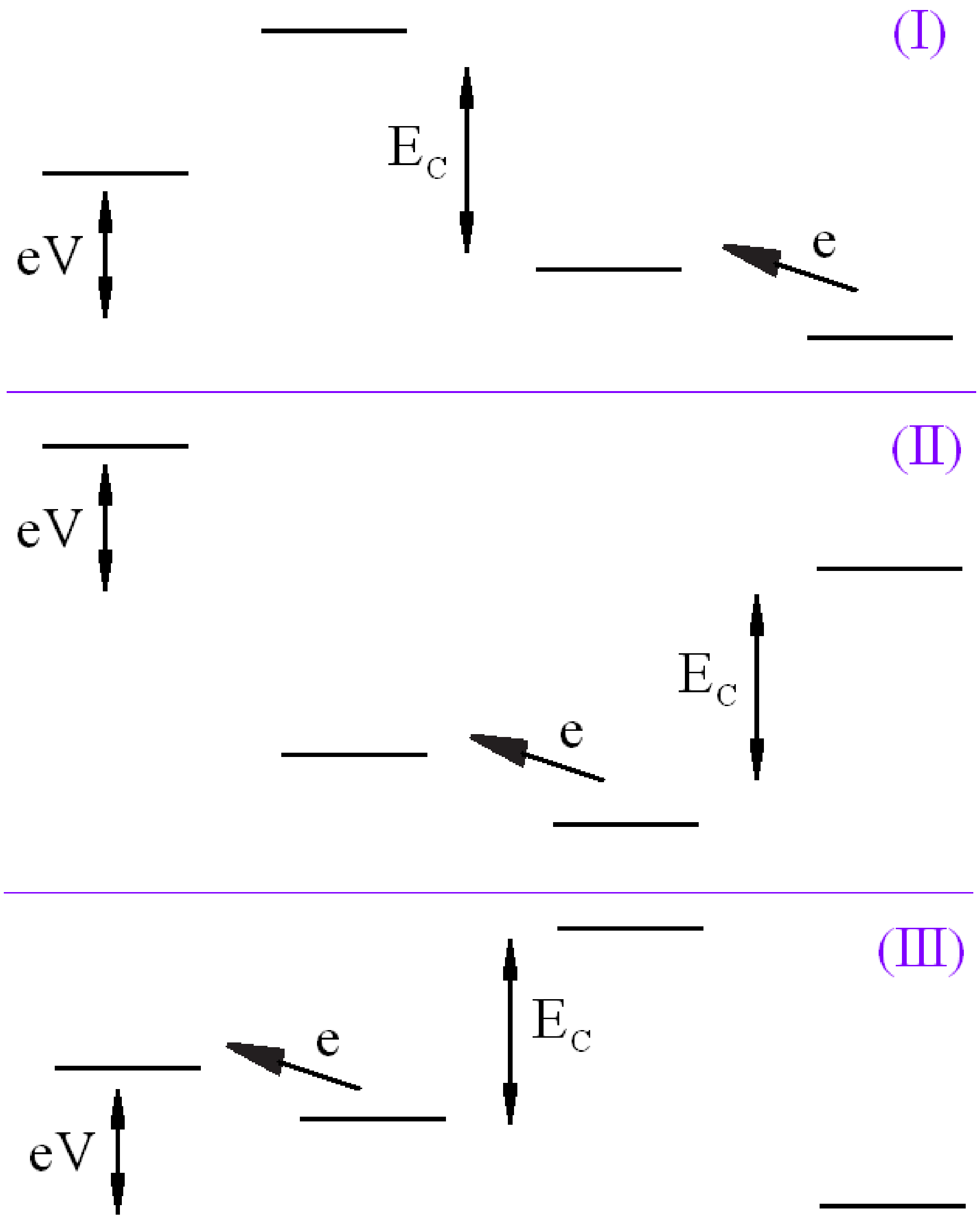
$$\Delta U = -Q + W_{th},\ W_{th} = -2E_C \int n dn_g(t)$$

Electronic Maxwell's demon based on SET pump
"Information-to-energy conversion"





Alternative approach: G. Schaller *et al.*, PRB (2011).

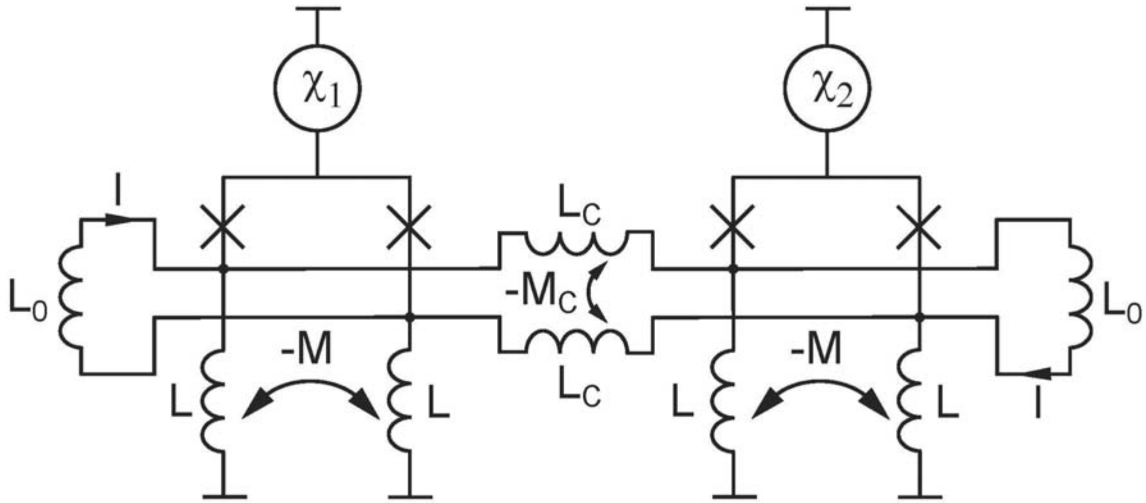


Generated power:

$$P = eV / 3\Gamma(eV/3).$$

Demon inverts the Landauer principle: bit of information gained in a measurement can be used to convert roughly $k_B T$ of thermal energy into free energy.

Maxwell's demon based on nSQUID array



For M/L≈1 dynamics of individual nSQUIDs reduces to that of the differential phase φ describing the circulating current:

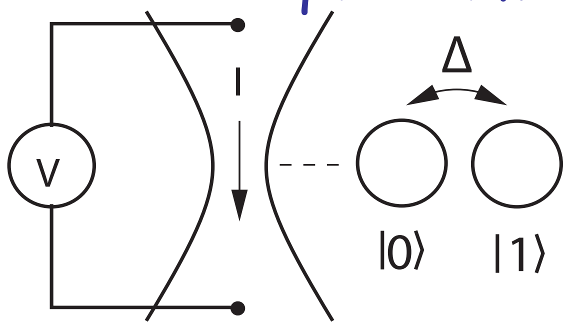
$$H = \frac{Q^2}{C} + \frac{\Phi_0^{\ 2}}{4\pi^2}\frac{(\varphi-\varphi_e)^2}{2L} - 2E_J \cos\chi \cos\varphi ,$$

$$[\varphi, Q] = 2ei.$$

Detector properties for demon operation

Qualitative similarity to quantum measurements: trade-off between the information acquisition and back-action.

Standard quantum detector set-up:



Heisenberg uncertainty relation for detectors:

$$S_I S_V \geq (\hbar\lambda / 4\pi)^2 .$$

Quantitative requirements for Maxwell's demon:

- Error-free and rapid detection:

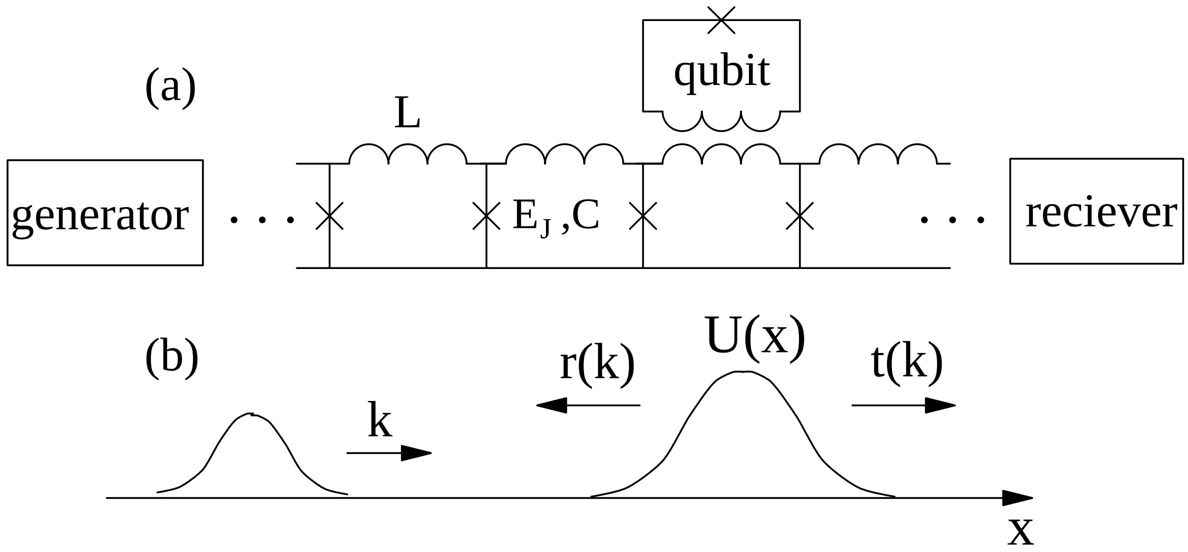
$$S_I / \lambda^2 << e^2 \tau / 8\pi , \quad \Gamma << \tau^{-1}.$$

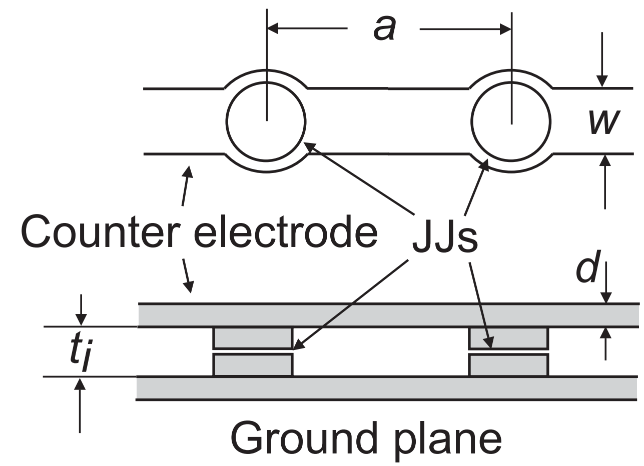
- No back-action excitations

$$\Gamma = \frac{G_T}{e^2} \int d\varepsilon_i \, d\varepsilon_f \, f(\varepsilon_i)[1 - f(\varepsilon_f)] \, \frac{\gamma / \pi}{(\varepsilon_i - \varepsilon_f - \Delta U)^2 + \gamma^2}, \quad \gamma = \pi e^2 S_V / \hbar ,$$

$$S_V << \hbar T / [e^2 \ln(\omega_C / T)].$$

JTL-based magnetic flux detector





The detector employs ballistic propagation of individual fluxons in a JTL. The measured system controls the fluxon scattering potential:

$$H = H_0 + \sigma_z \delta U(x) + \frac{mv^2}{2} + U(x), \qquad m \approx \frac{2}{\lambda_J}\left(\frac{\hbar}{e}\right)^2 C, \quad \lambda_J = \left(\frac{\hbar}{2eI_C L}\right)^{1/2}.$$

D.V. A., K. Rabenstein, V.K. Semenov,
PRB 73, 094504 (2006).

Conclusions

- Gaussian distribution of the generated heat in the reversible transformations with the width related to average by classical thermal FDT.
- SET structures can be developed into a prototype of thermodynamically reversible devices and a promising tool for studying basic thermodynamics, e.g., non-equilibrium fluctuation relations, demonstration of the Maxwell's demon, … ; but at low frequencies.
- nSQUID arrays would add an advantage of developed support electronics allowing the high-frequency operation both for the development of practical reversible circuits and for fundamental studies of the dynamics of information/entropy in electronic devices, e.g., in thermodynamics of quantum measurements.