8.7 R. Wilson & J M Pasachoff, Physics

TIDAL FORCES AND THE ROCHE LIMIT

When the gravitational field is uniform, all parts of a freely falling object experience exactly the same gravitational acceleration (Fig. 8–39*a*). But when the field is nonuniform, the acceleration of gravity varies in magnitude and/or direction from place to place (Fig. 8–39*b*). The result is a **differential force**—depending not on the strength of gravity but on how gravity varies from place to place—that tends to stretch or compress the object.

differential force

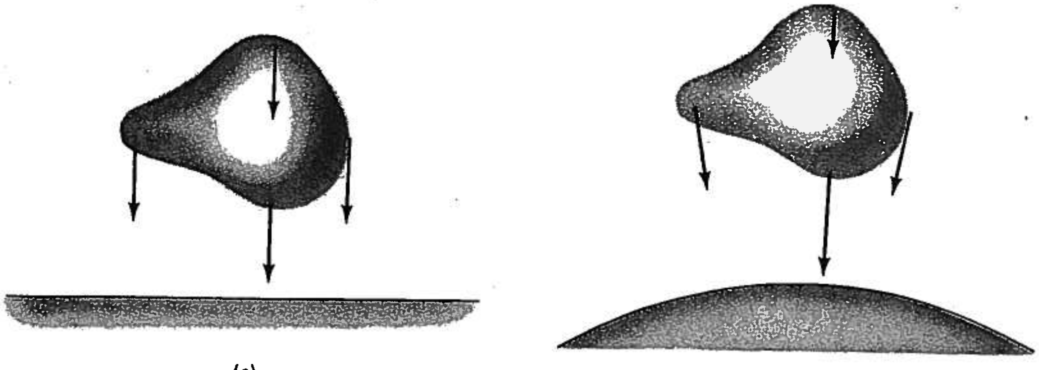

(*a*) (*b*)

Fig. 8–39
(*a*) In a uniform gravitational field, all parts of an object experience the same gravitational acceleration. (*b*) When the field is not uniform, gravity acts differently on the different parts of the object. The result is a differential force that tends to stretch an object along the field and compress it at right angles to the field.

Ocean tides are an important manifestation of this differential force (Fig. 8–40). Because of the association with ocean tides, the differential forces of gravity are usually called **tidal forces.** In reality, the effects of continents and sea floor topography greatly alter the simplified tidal picture of Fig. 8–40. And the differential force of the sun, although weaker than that of the moon, still has a significant effect. As a result, tides are highest when the sun, moon, and earth are in a line and weakest when they make a right angle.

tidal forces

The force of the sun's gravity on earth is far greater than that of the moon (see Problem 37). Why, then, should the moon's gravity be the dominant influence on the tides? The answer lies in the very rapid drop-off of tidal forces with distance. Consider an object consisting of two small masses m separated by a distance $2a$, and located some distance r from a gravitating mass M (Fig. 8–41). The tidal force arises from the *difference* in gravitational field across the object. With the object oriented as in Fig. 8–41, the gravitational force points in the same direction at both ends of the object; its magnitude at either end is given by Equation 8–1, using $r \pm a$ for the distance. So the tidal force on the object—the difference between

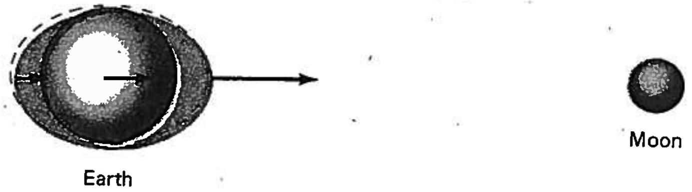


Fig. 8–40
Ocean tides result primarily from the differential force of the moon's gravity. The gravitational force is strongest on the moonward ocean, weaker on the solid earth, and weakest on the distant ocean. Thus the water on the moonward side is pulled away from the solid earth, while earth itself is pulled away from the distant water. Two tidal bulges result, so that a given location experiences two high tides each day as earth rotates. (Not shown is the differential force associated with the sun's gravity, which is weaker but also important.)

the gravitational forces on either end—is

$$F_T = \frac{GMm}{(r-a)^2} - \frac{GMm}{(r+a)^2} = GMm\,\frac{(r+a)^2 - (r-a)^2}{(r-a)^2(r+a)^2}.$$

Expanding both terms in the numerator, and noting that $(r-a)(r+a) = r^2 - a^2$, we have

$$F_T = GMm\frac{(r^2+2ra+a^2) - (r^2-2ra+a^2)}{(r^2-a^2)^2} = \frac{4GMmra}{(r^2-a^2)^2}.$$

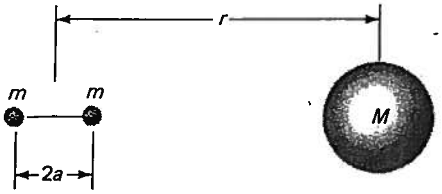


Fig. 8–41
The tidal force on an object arises from the difference in gravitational field strength across the object.

If the object's size $2a$ is small compared with the distance r from the gravitating object—as is the case with earth's diameter in relation to the earth-moon distance—then we can neglect a^2 compared with r^2 in the denominator, so that the tidal force becomes very nearly

$$F_T = \frac{4GMma}{r^3}. \qquad \text{(tidal force)} \qquad (8\text{–}16)$$

This equation shows that the tidal force drops off as the inverse cube of the distance—much more rapidly than the gravitational force itself. For this reason the tidal force of the moon is greater than that of the more massive but more distant sun.

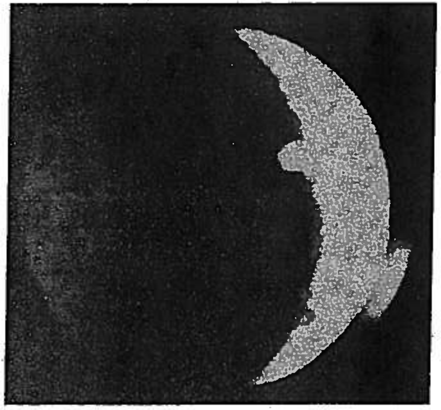


Fig. 8–42
Two volcanic eruptions on Jupiter's moon Io are visible in this image from Voyager I. The volcanic plume at right extends for 250 km. The bright spot on the terminator—the line between night and day—is another erupting volcano. Io's volcanism is thought to arise from heat generated internally by tidal stresses.

Tidal forces exert stresses on any extended object in a nonuniform gravitational field, and the astronomical consequences of tidal forces are therefore numerous. The extensive volcanic activity of Jupiter's moon Io (Fig. 8–42) is thought to arise from heat generated by tidal stresses. Tidal forces can even become strong enough to break an object apart. This happened to the Comet Biela in 1846, when it was split into two pieces by the strong tidal forces at perihelion.

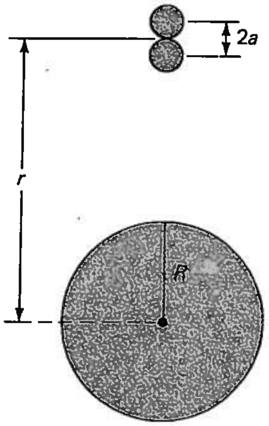


Fig. 8–43
Two small spheres attracted by their own gravity, but pulled apart by the tidal force of the planet they orbit. Which dominates depends on how close they are to the planet.

Since astronomical objects are generally held together by their own gravity, we can ask when tidal forces might become strong enough to overcome the internal gravitational force. Figure 8–43 shows an object consisting of two small spheres of mass m and radius a in free fall a distance r from a planet of mass M. Since the small masses are spherical, they attract each other gravitationally as though they were point masses located at their centers, a distance $2a$ apart. So the magnitude of the attractive force between them is

$$F_g = \frac{Gm^2}{(2a)^2} = \frac{Gm^2}{4a^2}.$$

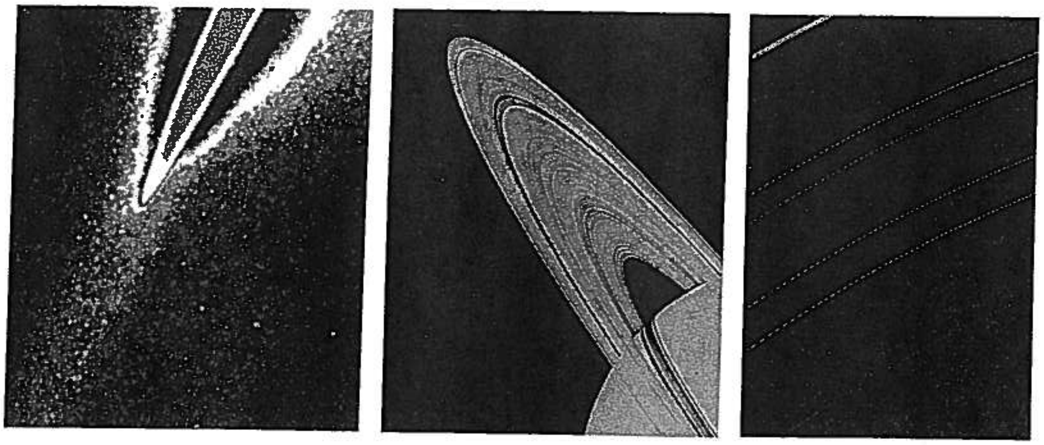
In the configuration of Fig. 8–43, the tidal force associated with differences in the planet's gravity tends to pull the spheres apart; since the spheres are small compared with their distance to the planet's center, and their own centers are a distance $2a$ apart, the magnitude F_T of the tidal force is given by Equation 8–16:

$$F_T = \frac{4GMma}{r^3}.$$

The object will be pulled apart if $F_T > F_g$, so that

$$\frac{4GMma}{r^3} > \frac{Gm^2}{4a^2}$$

ig. 8–44
he rings of a planet lie within the lanet's Roche limit, where tidal forces vercome the gravitational forces that ould hold a moon together. Here, left right, are the rings of Jupiter, Saturn, nd Uranus.



or when

$$r^3 < \frac{16Ma^3}{m}.$$

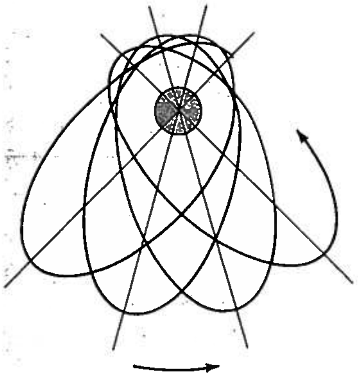
If we let ρ_p be the density of the planet, R its radius, and ρ_s the density of its satellite—the composite object we are considering—then $M=\frac{4}{3}\pi R^3\rho_p$ and $m=\frac{4}{3}\pi a^3\rho_s$. Then our condition for tidal break-up becomes

$$r < \left(\frac{16\rho_p}{\rho_s}\right)^{1/3} R. \qquad (8–17)$$

Roche limit

The critical radius r for tidal break-up is called the **Roche limit,** after the French scientist Edouard Roche, who suggested in 1848 that a moon would break up within this distance of a planet's center. Equation 8–17 shows that the Roche limit occurs at about 2.5 times the planet's radius when planet and satellite densities are comparable. A less simplified analysis would show the Roche limit depends also on the composition of the planet and satellite and whether the satellite material is liquid or solid.

Within the Roche limits of the planets Jupiter, Saturn, Uranus, and Neptune we find rings rather than moons (Fig. 8–44), showing that the tidal force prevents material within that limit from collapsing gravitationally to form moons. Beyond their Roche limits, all these planets have moons. We regularly place artificial satellites in orbit within earth's Roche limit, but these are held together by bolts, welds, and the strengths of materials rather than by gravitational forces, so that our force analysis does not apply.



8–45
shift in Mercury's perihelion results orbit that does not quite close on . Both the shift and ellipticity of rbit are greatly exaggerated; the al orbit is nearly circular, and the elion shift amounts to only 43 nds of arc per century.

8.8

GRAVITY AND THE GENERAL THEORY OF RELATIVITY

Although Newton's theory of gravity provided a brilliantly successful explanation of planetary motion, and today guides space probes like Voyager on billion-mile journeys to the outer planets, that theory is not perfectly correct. In very strong gravitational fields deviations from Newtonian predictions become evident. For decades the only evidence for this was a very slight shift in the position of the perihelion of Mercury's orbit (Fig. 8–45),