

Everyday Phenomenon

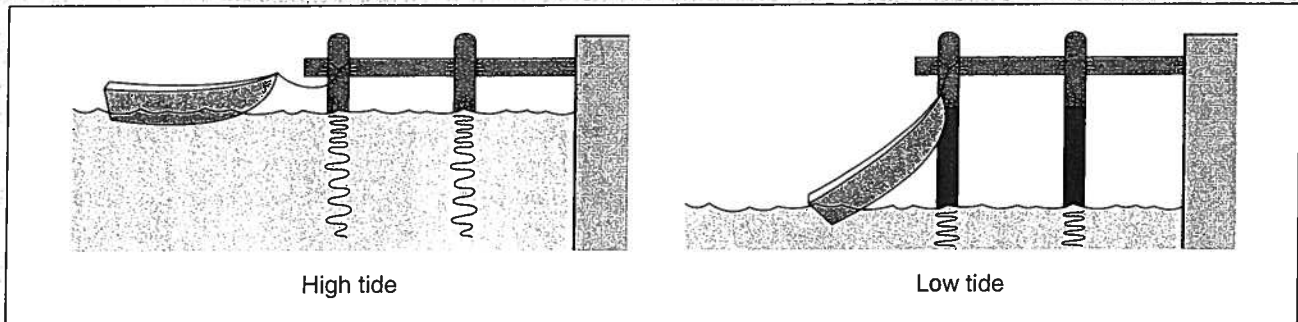
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Explaining the Tides

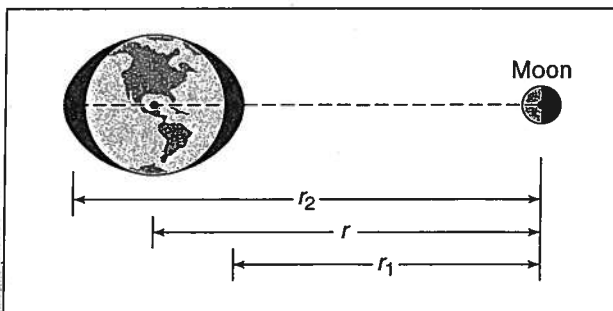
The Situation. Anyone who has lived near the ocean is familiar with the regular variation of the tides. Roughly twice a day the tides go in and go out again. The actual cycle of two high tides and two low tides is closer to 25 hours. Sometimes high tide is higher and low tide is lower than at other times—these times correspond to the full moon or the new moon.

The times when high tides and low tides happen shift from day to day because of their 25-hour cycle, but the pattern repeats monthly. How do we explain this behavior?

The Analysis. The monthly cycle and the correlations of the highest tides with the phase of the moon suggest a lunar influence. Both the moon and the sun exert gravitational forces on the earth. The sun exerts the stronger force because of its much larger mass, but the moon is much closer, and variations in its distance from the earth may be significant. The gravitational force depends on $1/r^2$, so its strength will vary as the distance r varies, as indicated in the drawing on page 95.



High tide and low tide produce different water levels at the dock.



Because it depends on distance, the gravitational force per unit mass exerted by the moon on different parts of the earth (and water in the oceans) gets weaker as we move from the side nearer the moon to the far side. (The bulges here are greatly exaggerated.)

Since water is a fluid (except when frozen), the water that makes up the oceans moves over the more rigid crust of the earth. The primary force acting on the water is the gravitational attraction of the earth that holds the water to the earth's surface. The gravitational force exerted by the moon on the water is also significant, however, and its strength per unit mass is greatest on the side of the earth closest to the moon and weakest on the opposite side of the earth because of the difference in distance.

This difference in strength of the moon's pull produces a bulge in the water surface on both sides of the earth. The bulge on the side nearest the moon results from the water being pulled toward the moon by a stronger force per unit

mass than the force per unit mass exerted on the rest of the earth. This produces a high tide. The water will flow to the top of the dock.

On the opposite side of the earth, the earth is being pulled by the moon with a stronger force per unit mass than the water. Since the earth is pulled away (slightly) from the water, this also produces a high tide. The forces exerted by the moon are small compared to the force that the water and the earth exert on each other but are still large enough to produce the tides.

When the sun and the moon both line up with the earth during the new moon or full moon, the sun also contributes to this difference in forces and produces bulges on either side of the earth, adding to those produced by the moon. The highest tides occur during a full moon or a new moon because of this combination of the moon and sun.

Why is the cycle 25 hours rather than 24 hours? The high-tide bulges occur on either side of the earth along the line joining the moon and the earth. The earth rotates underneath these bulges with a period of 24 hours, but in this time, the moon also moves, since it orbits the earth with a period of 27.3 days. In one day, therefore, the moon has moved through roughly $1/27$ of its orbital cycle, causing the time when the moon again lines up with a given point on the earth to be a little longer than 1 day. This additional time is approximately $1/27$ of 24 hours, or a little less than an hour.

This model was conceived by Newton and accounts neatly for the major features of the tides. The variation of the gravitational force with distance is the key to the explanation.