STEPHEN J. WALLACE
Research Professor of Physics

Education:

B.Sc.	Case Institute of Technology	1961
M.Sc.	University of Washington, Seattle	1969
Ph.D.	University of Washington, Seattle	1971

Experience in Higher Education:

1972-74	Harvard University	Research Associate
1974-78	University of Maryland	Assistant Professor
1978-83	University of Maryland	Associate Professor
1983-09	University of Maryland	Professor
2009-	University of Maryland	Research Professor
1989	Hebrew Univ. of Jerusalem	Sheinbrum Visit. Professor
1989	University of Utrecht	Visiting Professor
1994-99	University of Maryland	Chair, Department of Physics
1999-00	Jefferson Natl. Accelerator Facility	Visiting Professor

Honors and Awards

Appointed Donders Chair, Inst. of Theoretical Physics, Univ. of Utrecht,
The Netherlands, 1989
Fellow, American Physical Society, 1990

Synergistic Activities:

Member, Executive Committee, American Physical Society, Nuclear Physics Div., 1993-95
Southeastern Universities Research Association:
- Chair, Jefferson Laboratory Committee, 2000-02
- Vice-Chair, Board of Trustees, 2002-03
- Chair, Board of Trustees, 2004-05
- Past-Chair, Board of Trustees, 2006-07

Committee on Broadcast Journalism Award, American Physical Society, 2001-03
DNP Fellowship Committee, 2002
Chair, Committee on Constitution and Bylaws, American Physical Society, 2002
Divisional Associate Editor, Physical Review Letters, 2002-04
NSAC Subcommittee on Performance Measures for Nuclear Physics, 2003
NSF Panel for Nuclear Physics, 2004
Jefferson Science Associates:
- Chair, Programs Committee, 2007-09
- Member, Board of Directors, 2007-09
- Member, Search Committee for Lab Director, 2007

Recent Research

Progress in the past year includes one paper published and another paper submitted for publication, a talk at the Lattice 2010 Conference that is published online, and two invited talks at workshops held at Jefferson Laboratory. Progress also has been made in developing a Hamiltonian analysis of lattice spectra.

A. *Nucleon, Δ and Ω Excited States in $N_f = 2+1$ Lattice QCD*

The analysis of excited state spectra of the N, Δ and Ω baryons was performed at Maryland by Professor Wallace using lattices and matrices of correlation functions calculated by collaborators in the Hadron Spectrum Collaboration. The results are now published in Physical Review. [1] An anisotropic $16^3 \times 128$ lattice was used with spatial spacing $a_s = 0.12$ fm and time spacing $a_t = 0.035$ fm. Two light quarks and one strange quark were used. Baryon operators were constructed following the methodology of Ref. [2] with spatial structure incorporated through displacements of the quark fields relative to one another. Using sets of 10 to 12 operators in each irreducible representation (irrep) of the octahedral group to make matrices of correlation functions, and solving for the eigenvalues using the generalized eigenvalue problem, excited states were determined. Figure 1 shows the results for the excited states of the nucleon at $m_\pi = 392$, 438 and 521 MeV. Note that the patterns visible in the lattice spectra resemble the patterns of experimental resonance masses. However, the large number of lattice states with overlapping energies obscures the patterns for subductions of high-spin states to the lattice irreps. An exception is a pair of states near 2000 MeV, one from H_u and one from G_{2u}, that provides a match to the subduction pattern of a $\frac{5}{2}^-$ state.

B. *Nucleon, Delta and Omega Excited State Spectra at Three Pion Mass Values*

Professor Wallace presented the results of the lattice QCD work described in Subsec. 0 A at the Lattice 2010 Conference held at Villasimius, Sardinia. [4] The results of the work also were highlighted in a plenary talk by Christian Hoelbling on light-hadron spectroscopy. [5]

An interesting result was the determination of the Ω-baryon excited state spectra shown in Fig. 2. The Ω was included because its ground-state mass is used to set the scale. However, very little is known about the spins of excited states from experiment.

The lattice results for Ω are expected to be close to realistic because it has only strange quarks in its valence structure, and in our simulations the strange-quark mass is tuned to its physical value. Although the pion mass is well above the physical value, the influence of the light sea quarks is not expected to be large in the excited states of Ω. Note that the negative-parity Ω spectra generally have a gap near 2500 MeV with no lattice states. This is the energy region of the experimentally-known excited states, which suggests that they are likely to be positive-parity states. However, multiparticle states with $S =$ -3, such as ΞK, should show up in the excited state spectra of Ω and their role needs to be understood before excited states can be identified clearly. That will require an analysis with multiparticle operators included.

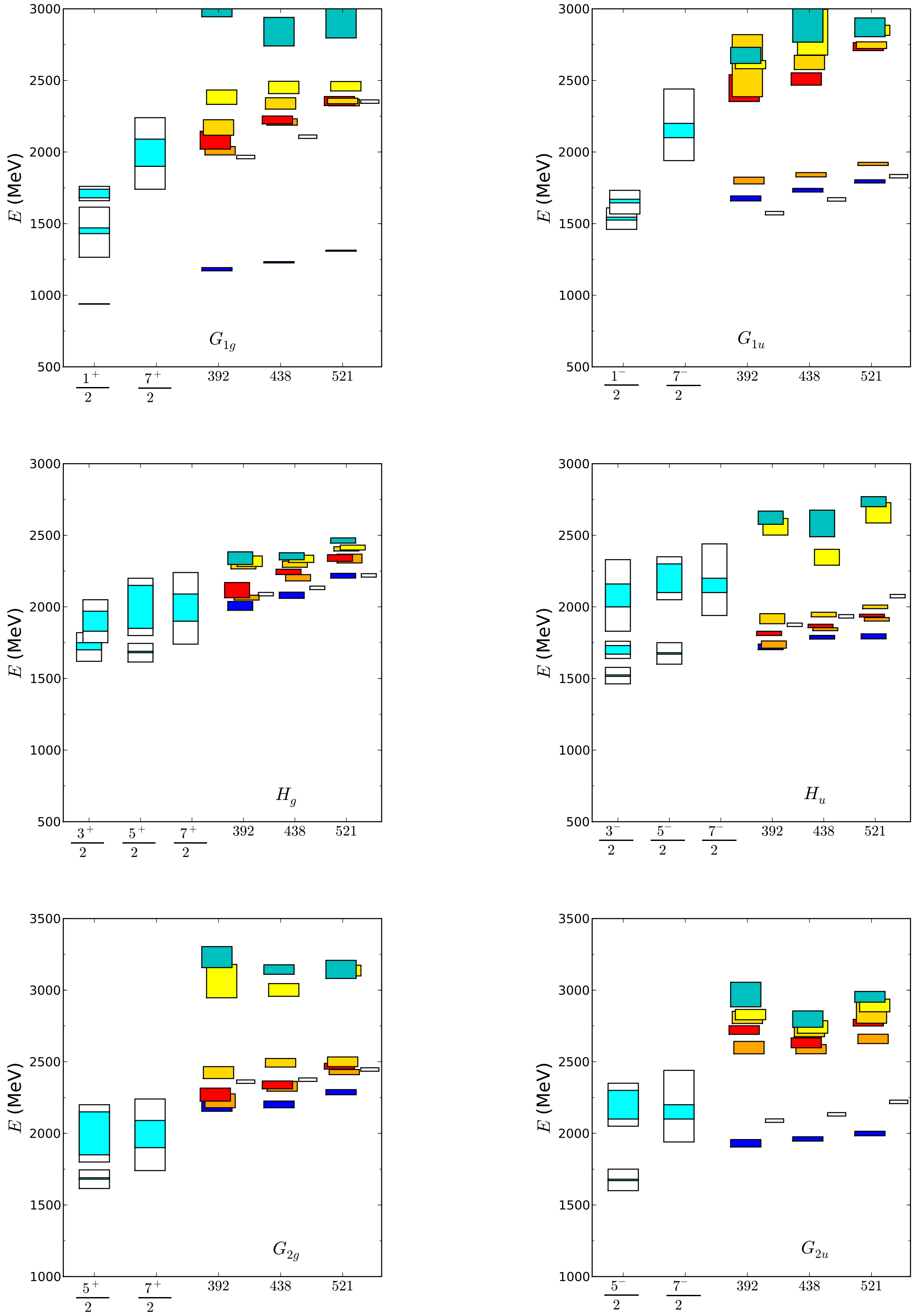

FIG. 1: Nucleon effective energies are shown for the six irreducible representations of the octahedral group: G_{1g}, H_g, G_{2g}, G_{1u}, H_u and G_{2u}, where subscript g is for positive parity and subscript u is for negative parity. Experimental spectra [3] are shown in the columns labeled by $J^P = \frac{1}{2}^{\pm}$ to $\frac{7}{2}^{\pm}$. Lattice spectra are shown by colored boxes with height equal to 2σ. Open boxes show thresholds for multiparticle states.

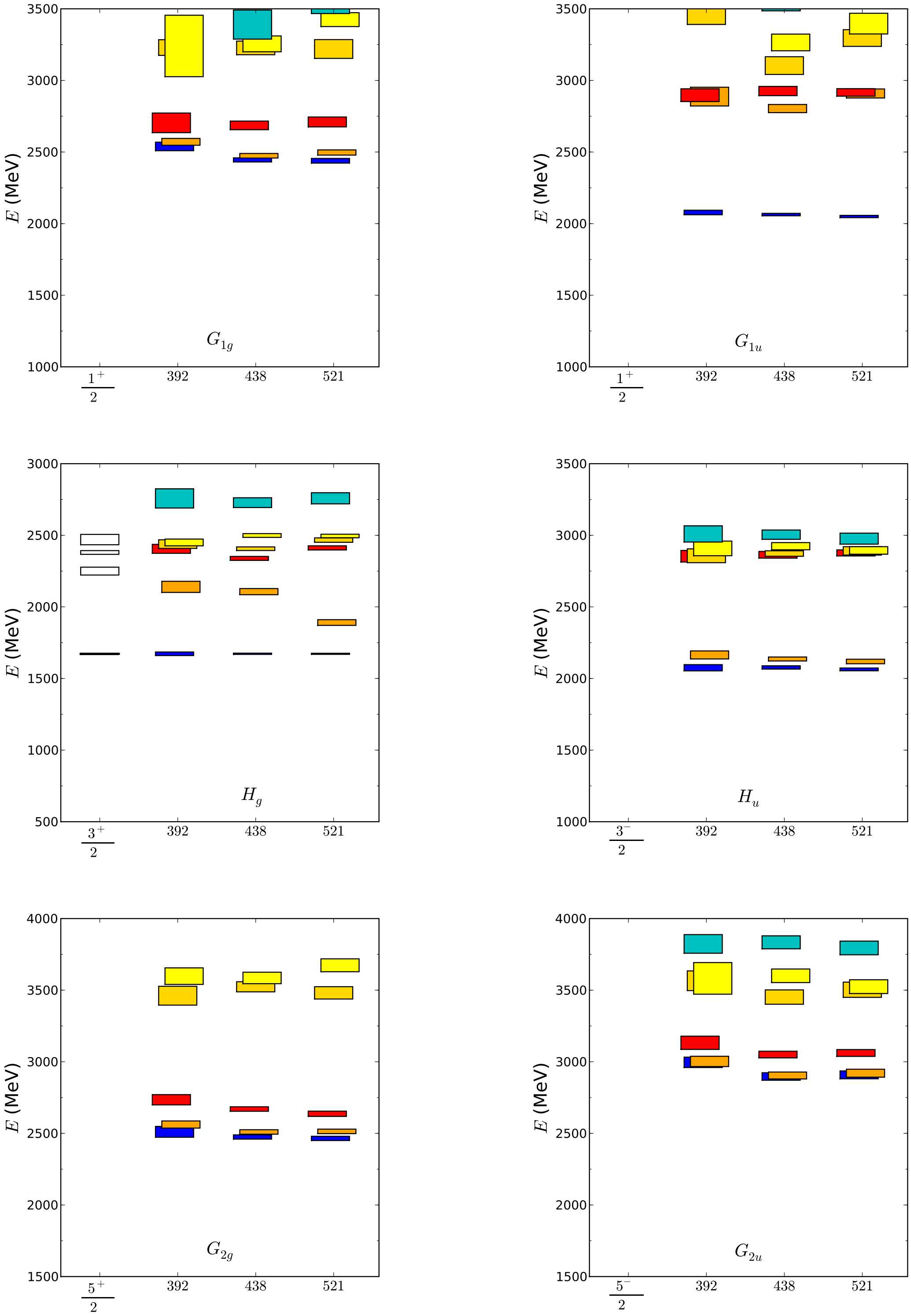

FIG. 2: Ω-baryon effective energies are shown for the six irreducible representations of the octahedral group: G_{1g}, H_g, G_{2g}, G_{1u}, H_u and G_{2u}, where subscript g is for positive parity and subscript u is for negative parity. Experimental spectra [3] are shown in the column labeled by $J^P = \frac{3}{2}^+$, however the spins of the excited states are unknown. Lattice spectra are shown by colored boxes with height equal to 2σ.

C. Spin Identification for Excited State Baryon Spectroscopy from Lattice QCD

Robust spin identification in baryon spectroscopy has been realized in a recent paper that has been submitted for publication. [6] A remarkable finding of this work is that approximate rotational invariance is realized in lattice calculations using an action and baryon operators that are smooth on the scale of 1 fm. The work follows similar work for mesons that provided identification of integer spin states. [7]

New baryon operators were developed that have known half-integer spins in the continuum limit. Orbital angular momentum was included by use of covariant derivatives of the quark fields that transform as the $L = 1$ spherical harmonics Y_1^{+1}, Y_1^0 and Y_1^{-1} as follows,

$$\begin{aligned} \hat{D}^{(+)} &= \frac{i}{\sqrt{2}}\left(\hat{D}_x + i\hat{D}_y\right), \\ \hat{D}^{(0)} &= -i\hat{D}_z, \\ \hat{D}^{(-)} &= -\frac{i}{\sqrt{2}}\left(\hat{D}_x - i\hat{D}_y\right), \end{aligned} \tag{1}$$

where subscripts x, y and z refer to the spatial directions of the lattice. Two such covariant derivatives can be coupled to $L = 2$ using standard Clebsch-Gordan coefficients, allowing spins up to $\frac{7}{2}$ to be realized in the continuum.

The continuum operators were then subduced to the lattice irreps using subduction matrices. Subduction matrices were developed for half-integer spins up to $\frac{9}{2}$ using the complete set of Clebsch-Gordan coefficients for the octahedral group that we developed in Ref. [8]. One can understand the subduction matrices as the transformations of quantum-mechanical states from the $SU(2)$ continuum basis states, $|J, M\rangle$, to octahedral basis states, $|\Lambda, r; [J]\rangle$, where Λ and r denote the octahedral irrep and row, respectively, and $[J]$ denotes the spin in the continuum limit. The transformation is

$$|\Lambda, r; [J]\rangle = \sum_M |J, M\rangle S^{J,M}_{\Lambda,r} \tag{2}$$

where the subduction matrix is defined by the overlap of states corresponding to the same J,

$$S^{J,M}_{\Lambda,r} = \langle J, M|\Lambda, r; [J]\rangle. \tag{3}$$

Operators with good J in the continuum are subduced to the lattice irreps using the same matrices,

$$\mathcal{O}_i^{\Lambda,r,[J]} = \sum_M \mathcal{O}_i^{J,M} S^{J,M}_{\Lambda,r}. \tag{4}$$

In the continuum, correlation matrices based on such operators are block-diagonal with respect to J because of rotational invariance, as follows,

$$\langle 0|\mathcal{O}_i^{\Lambda,r,[J]}(t)\mathcal{O}_j^{\Lambda',r',[J']}(0)|0\rangle \propto \delta_{J,J'}. \tag{5}$$

Two tests of approximate rotational invariance have been performed. The first is a test of block-diagonality of the matrix of correlations functions at time slice 5 as shown in Fig. 3. The plot is for the nucleon H_u irrep using 48 operators, 28 of them subduced from $J = \frac{3}{2}$, 16 subduced from $J = \frac{5}{2}$ and 4 subduced from $J = \frac{7}{2}$. Qualitatively, one sees that the matrix is block diagonal to a good approximation.

The second test is whether the couplings between different J values are sufficiently small that they have little effect on spectra. This test uses spins identified by use of the spectral weights of the

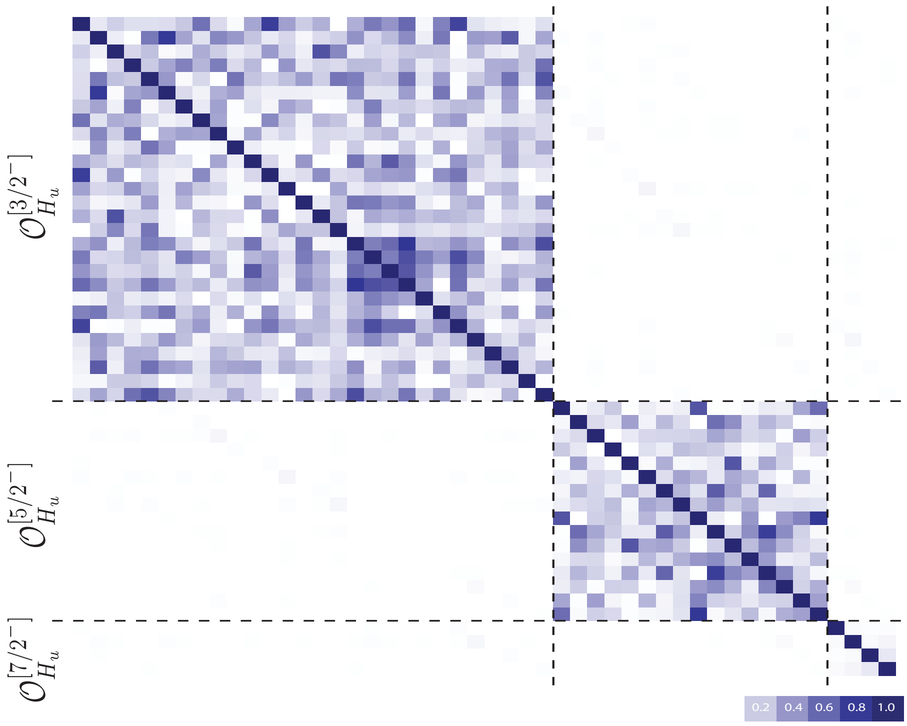

FIG. 3: Block-diagonality test of matrix of correlation functions.

new operators in each state as in Ref. [7]. The left column of Fig. 4 that is labeled "all" shows the energies of the lowest 12 states obtained using all 48 H_u operators. The spins of the resulting states have been identified as $J = \frac{3}{2}$ (red boxes), $J = \frac{5}{2}$ (green boxes) or $J = \frac{7}{2}$ (blue boxes). The other columns show the energies based on the subset of operators subduced only from a single J value. One sees that the states of each J have essentially the same energies whether determined from all 48 H_u operators or from a subset that omits the $J \neq J'$ couplings. This is striking because even a small coupling is expected to cause highly-excited states in the H_u irrep to decay to the ground state, which is the lowest $J = \frac{3}{2}$ state. That is not observed.

The approximate rotational invariance realized in our lattice calculations using the new operators provides the underlying reason why spins can be identified in a robust manner. The first spin-identified spectrum for the nucleon and Δ excited states is shown in Fig. 5 based on $m_\pi = 392$ MeV. A noteworthy feature of the lattice spectra is the absence up to rather high excitation energy of any clear evidence for the parity-doubling that was suggested in Ref. [9].

D. Excited Hadronic States and the Deconfinement Transition Workshop, Jefferson Laboratory, Feb 23-25, 2011

This meeting brought together physicists working on modeling relativistic heavy-ion collisions using hadronic resonance gas models and physicists working on hadron spectroscopy. Professor Wallace presented the recent lattice QCD results of the Hadron Spectrum Collaboration on baryon spectroscopy.

With the spins identified, it has become possible to identify bands of excited baryon states that correspond to $SU(6) \times O(3)$ multiplets. The reason is because low-lying states generally are dominated by a few operators with well-defined $SU(6) \times O(3)$ quantum numbers. Figure 6 shows two bands inside shaded boxes. The lowest band of 5 negative-parity nucleon excited states on the right side of the plot includes two $N\frac{1}{2}^-$ states, two $N\frac{3}{2}^-$ states and one $N\frac{5}{2}^-$ state. These states are

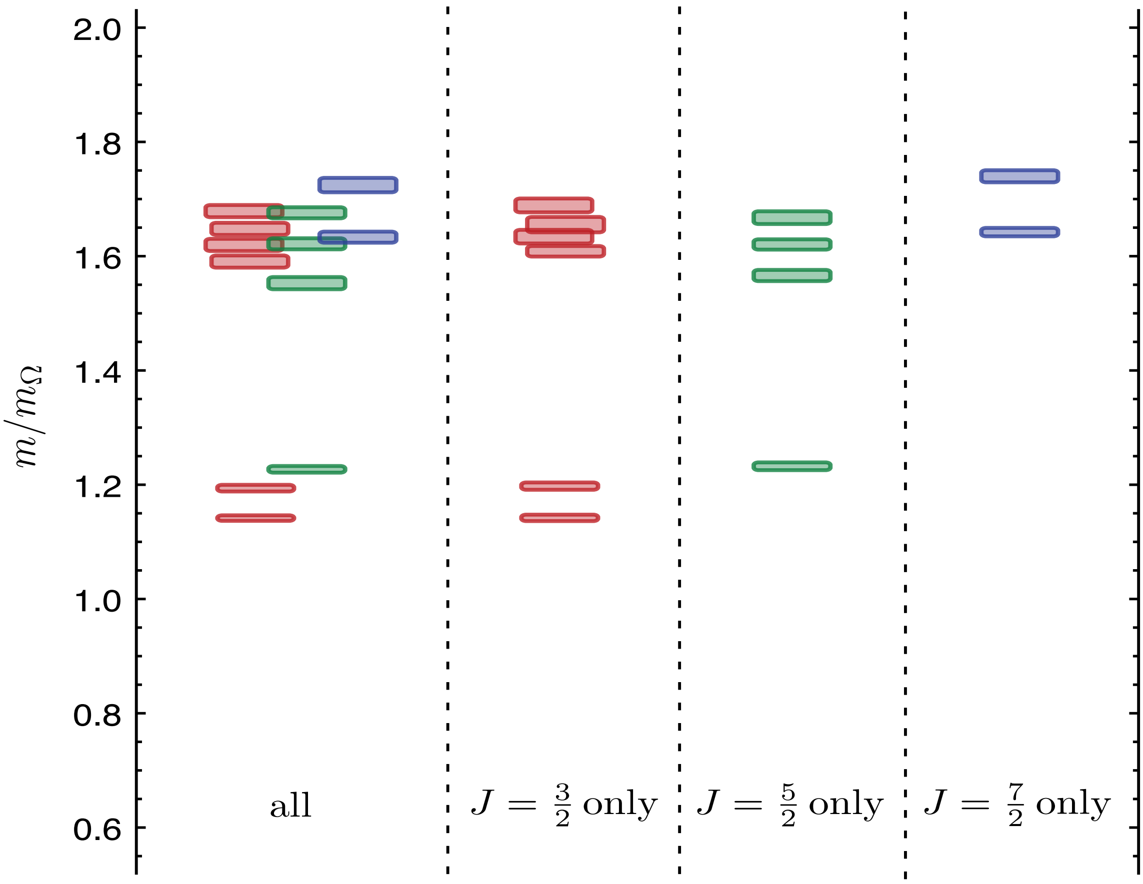

FIG. 4: Spectral test of the smallness of $J \neq J'$ couplings.

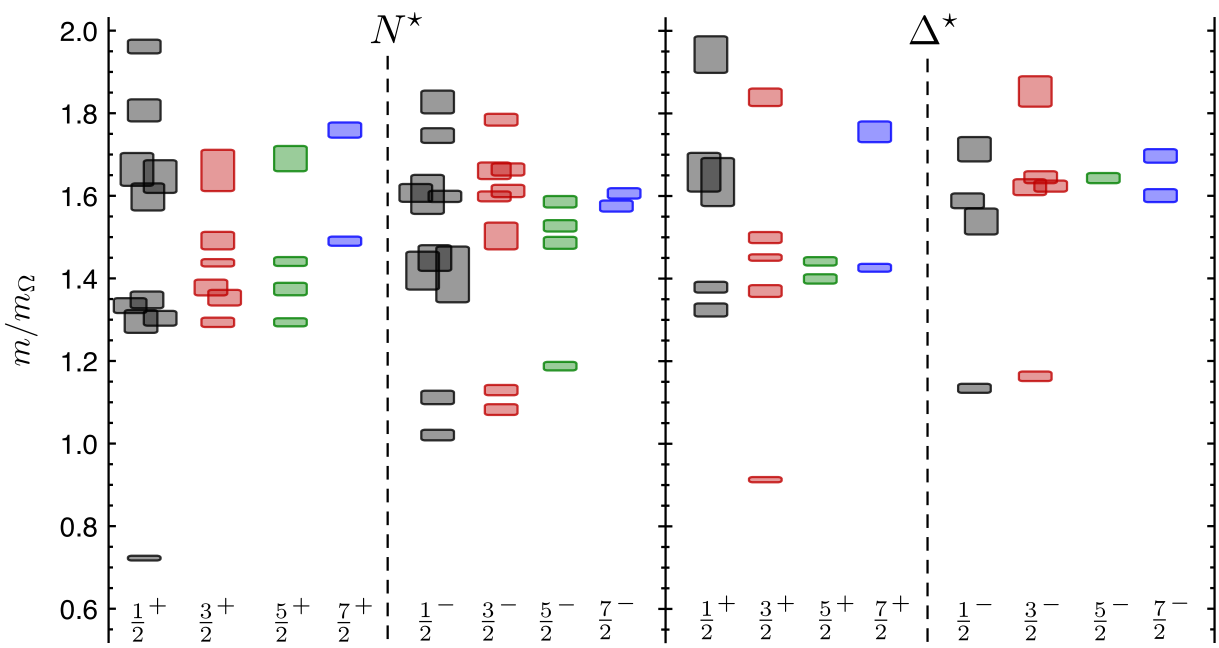

FIG. 5: Spin-identified spectra of nucleon and Δ excited states.

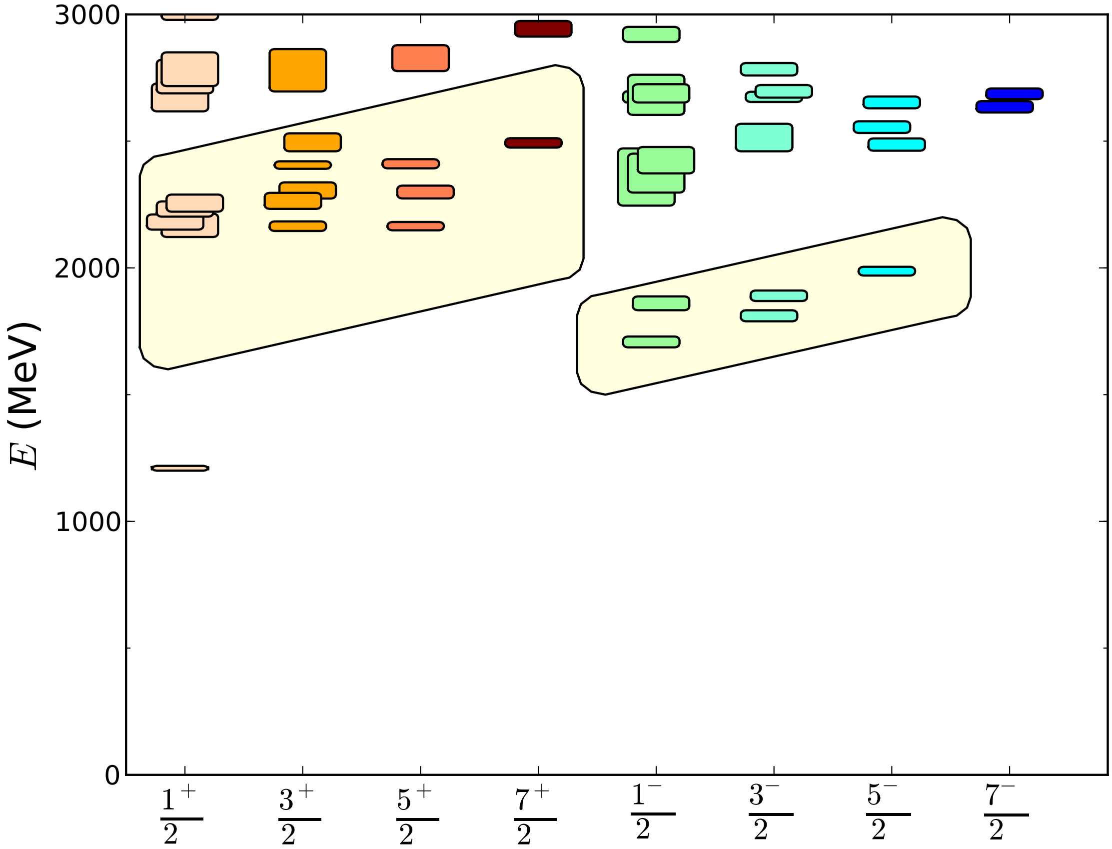

FIG. 6: Nucleon spectra showing bands of states.

in a one-to-one correspondence with those of the $[\mathbf{70}, \mathbf{1}^-]$ multiplet of $SU(6) \times O(3)$, i.e., $L = 1$ excitations of $S = \frac{1}{2}$ and $S = \frac{3}{2}$. The same pattern is seen in experimental spectra. The lowest band of 13 positive-parity states on the left side of Fig. 6 includes four $N\frac{1}{2}^+$ levels, five $N\frac{3}{2}^+$ levels, three $N\frac{5}{2}^+$ levels and one $N\frac{7}{2}^+$ level, which are the same numbers of levels for each J^P as in the $[\mathbf{56}, \mathbf{0}^+]$, $[\mathbf{56}, \mathbf{2}^+]$, $[\mathbf{70}, \mathbf{0}^+]$, $[\mathbf{70}, \mathbf{2}^+]$ and $[\mathbf{20}, \mathbf{1}^+]$ multiplets. Because the quark model of baryons is built on a $SU(6) \times O(3)$ basis of states, the lattice results are similar to the quark-model results with respect to the numbers of levels of each J^P. However, higher excited states generally have more complicated structures in lattice QCD. For example, they can have substantial contributions from operators that are constructed from lower-component Dirac spinors. Although there are many excited states, a noteworthy subset corresponds to the $[\mathbf{20}, \mathbf{1}^+]$ multiplet that is not present in quark-diquark models of the baryon excited states. Thus, there is no evidence in the lattice spectra for “freezing” of degrees of freedom.

E. 8th International Workshop on the Physics of Excited Nucleons - NSTAR2011, Jefferson Laboratory, May 17-20, 2011

The Hadron Spectrum Collaboration results for baryon spectroscopy also were presented at the NSTAR2011 International Workshop by Prof. Wallace. The talk will be written up for the workshop proceedings.

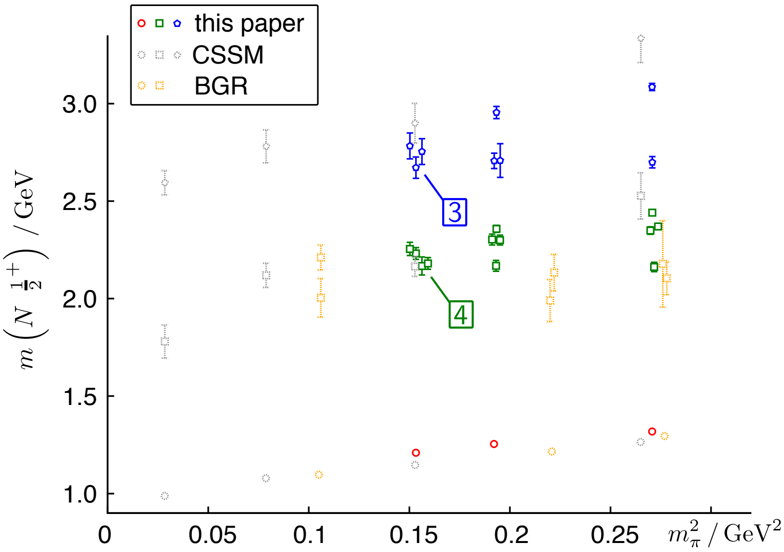

FIG. 7: Comparison of results for the Nucleon $J = \frac{1}{2}^+$ excited states. The results shown in grey are from Ref. [10], while those in orange are from Ref. [11]. Results from our work are shown in red (ground state), green (lowest group of excited states) and blue (next group of excited states). At the lightest pion mass, we obtain a cluster of four states as indicated by [4] near 2 GeV, while there are three nearly degenerate states indicated by [3] close to 2.7 GeV. Operators featuring the derivative constructions feature prominently in these excited states.

One of the goals of the experimental program at Jefferson Laboratory is to explore the "missing" resonances, i.e., resonances that are predicted by the quark model but have not been discovered. The lattice results at the current values of pion mass provide support for a quark-model-like spectrum of excited states, however, the multiparticle states that should be present have generally not been found using three-quark baryon operators. That means that multiparticle operators will have to be included explicitly and coupled-channels analyses will be required. There is much yet to be learned from such lattice calculations.

One of the goals of lattice QCD has been to understand the lowest $N\frac{1}{2}^+$ (Roper) resonance, which is anomalous in the quark model. Although multiparticle states are expected to be essential to a proper description of the Roper, a number of lattice calculations based on three-quark operators have explored the $N\frac{1}{2}^+$ excited states. Figure 7 shows a comparison of recent results of the Hadron Spectrum Collaboration, the CSSM Collaboration [10] and the BGR Collaboration [11]. The CSSM and BGR results have been obtained using small operator sets with no derivatives but with lower pion masses. There is a qualitative difference from our results based on 28 operators, namely, one or two excited states are found where we find a group of four nearly degenerate states. The operators containing derivatives play an important role in our spectrum. The structure of the $N\frac{1}{2}^+$ spectrum evidently is more complex than has been appreciated and it requires operators with $L \neq 0$ in order to be determined. These results suggest that the claims of others that the Roper resonance has been obtained in lattice QCD are likely to be premature.

F. Hamiltonian Method for Determination of Energies of Excited States in Lattice QCD

The standard method used to determine energies in lattice calculations is to fit the time dependence of a correlation function by an exponential function, e^{-Et}. When a matrix of correlation functions is used to get excited states, the generalized eigenvalue problem is solved to get the diagonal elements, from which energies can be extracted by such fits. In order to provide a more direct and transparent method, a Hamiltonian method has been developed. A paper is in preparation.

Calculations of the spectrum of lattice QCD are based upon $N \times N$ matrices of correlation functions that are obtained from a set of N operators $\mathcal{O}_i$,

$$C_{ij}(t) = \langle \mathcal{O}_i(t)\overline{\mathcal{O}}_j(0)\rangle. \tag{6}$$

When the lattice formulation has a transfer matrix, there is in effect a Hamiltonian that governs the Euclidean time dependence. Omitting matrix indices,

$$\overline{C}(t) = Pe^{-Ht}P, \tag{7}$$

where H is the effective Hamiltonian in lattice units, P is a projection operator to the N states created by the operators and a line over a quantity here and in what follows denotes an average over gauge-field configurations. Thus the effective Hamiltonian within the space of N states is defined in terms of the average over gauge-field configurations of the matrix of correlation functions.

A time derivative gives

$$\overline{K}(t) \equiv -\frac{d}{dt}\overline{C}(t) = PHe^{-Ht}P. \tag{8}$$

The time derivative can be realized accurately by the following construction,

$$\begin{aligned}\overline{K}(t) &= \frac{3}{4}[\overline{C}(t-1) - \overline{C}(t+1)] - \frac{3}{20}[\overline{C}(t-2) - \overline{C}(t+2)] + \\ &\quad \frac{1}{60}[\overline{C}(t-3) - \overline{C}(t+3)] \\ &= PHe^{-Ht}P + P\frac{H^7}{140}e^{-Ht}P + \cdots . \end{aligned} \tag{9}$$

Therefore the ratio of diagonal matrix elements calculated in eigenstates of H,

$$\begin{aligned}\tilde{E}_n &\equiv \frac{\overline{K}_{nn}(t)}{\overline{C}_{nn}(t)} \\ &= E_n + \frac{E_n^7}{140} + \cdots , \end{aligned} \tag{10}$$

gives the energy eigenvalues of H with an error term that can be made small. As an example, for $E_n = \frac{1}{2}$ in lattice units, the error term is $\frac{1}{17920}$. On an anisotropic lattice, with $a_t^{-1} \approx 6$ GeV, all states up to 3 GeV in energy would have small errors. However, the cancellation of exponential factors in Eq. (10) is subject to errors when the signal-to-noise deteriorates at large t.

In practice, one does not have the effective Hamiltonian nor its eigenvectors. Matrices $\overline{C}(t)$ and $\overline{K}(t)$ can be calculated based on the projections to a finite number N of states and the issue is how to extract approximately the energy eigenvalues of H from them.

The N eigenpairs of $\overline{K}(t)$ are obtained by solving the generalized eigenvalue problem as follows,

$$\overline{K}(t)v_n(t) = \lambda_n(t)\overline{C}(t)v_n(t), \tag{11}$$

which gives eigenvalues at large t,

$$\lambda_n(t) \approx \tilde{E}_n, \tag{12}$$

where $\tilde{E}_n$ is the energy including the correction term of Eq. (10) and $\lambda_n(t)$ depends on time because of the contributions of states above the ones that can be obtained diagonalizing N operators.

There are contributions to matrices $K(t)$ and $C(t)$ from states that are outside the space that is diagonalized using an $N \times N$ matrix of correlation functions. The omitted contributions are denoted by $\overline{K}'(t)$ and $\overline{C}'(t)$. They are suppressed by exponential factors $e^{-E_{N+1}t}$ because E_{N+1} is the lowest outside energy. A perturbation analysis similar to that of Ref. [12] shows that the contributions from the lowest omitted state give

$$\begin{aligned}\lambda_n(t) &= \tilde{E}_n + a_n\big[\tilde{E}_{N+1} - \tilde{E}_n\big]e^{-(E_{N+1}-E_n)t} \\ &+ \sum_{m\neq n}^{N} \frac{a_n a_m \big[\tilde{E}_{N+1} - \tilde{E}_n\big]^2}{\tilde{E}_n - \tilde{E}_m} e^{-(2E_{N+1}-E_n-E_m)t} \\ &- a_n^2\big[E_{N+1} - E_n\big]e^{-(2E_{N+1}-2E_n)t} + \cdots \end{aligned} \tag{13}$$

where a_n is constant. When there are nearly degenerate energies the second-order term with denominator involving $\tilde{E}_n - \tilde{E}_m$ can provide a significant correction.

The perturbation analysis shows that the eigenvalue $\lambda_n(t)$ consists of the desired constant term, $\tilde{E}_n$, plus corrections that decrease exponentially with t. It can be fit as follows,

$$\lambda_n(t) = E + \delta E e^{-E't}, \tag{14}$$

where E, δE and E' are constants. The exponential term describes the corrections owing to higher energy states.

We have solved for energies using 10 operators in the nucleon G_{1g} irrep ($N\frac{1}{2}^+$) states using the Hamiltonian analysis outlined above and the standard analysis using the same lattices as in Ref. [1]. A comparison of the results is shown in Fig. 8 for the ground state and 5 excited states. The energies agree well up to the third excited state. There are significant differences for the fourth excited state and more substantial differences for the fifth excited state, indicating that a larger basis of operators is needed for the determination of the highly-excited states. The Hamiltonian analysis is expected to be useful as a complement to the standard analysis in that it provides a test of how well the energies are determined.

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[1] J. Bulava, R. G. Edwards, E. Engelson, B. Jo, H-W. Lin, C. Morningstar, D. G. Richards, S. J. Wallace, Phys. Rev. D**82**, 014507 (2010). [arXiv:1004.5072]

[2] S. Basak *et al.* [Hadron Spectrum Collaboration], Phys. Rev. D **72**, 094506 (2005). [arXiv:hep-lat/0506029].

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[10] M. S. Mahbub, W. Kamleh, D. B. Leinweber, P. J. Moran and A. G. Williams [CSSM Collaboration], [arXiv:1011.5724]

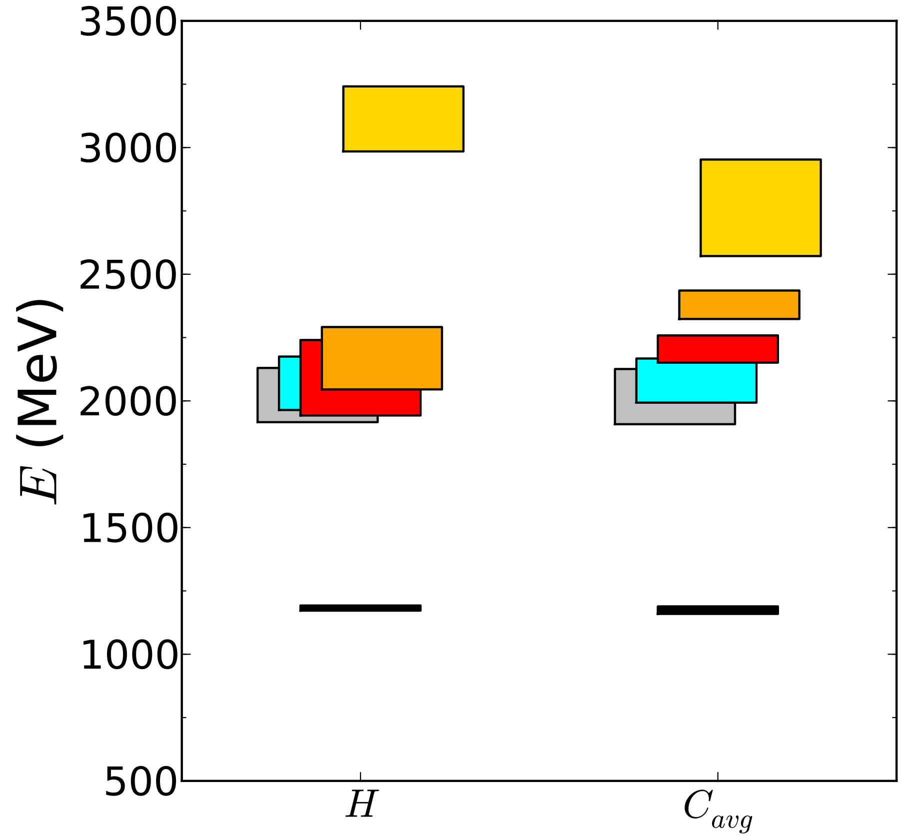

FIG. 8: Six lowest-energy nucleon G_{1g} effective energies are shown for the Hamiltonian analysis in the column labeled H. The energies from the standard analysis are shown in the column labeled C_{avg}, based on an average of jackknife ensembles over $t_0 = 6, 7, 8$ and 9.

[11] G. P. Engel, C. B. Lang, M. Limmer, D. Mohler and A. Schafer [BGR Collaboration], Phys. Rev. D **82**, 034505 (2010). [arXiv:1005.1748]

[12] B. Blossier *et al.*, JHEP 0904:094 (2009). [arXiv:0902.1265]