Chapter 2: Review of Previous Research

Introduction

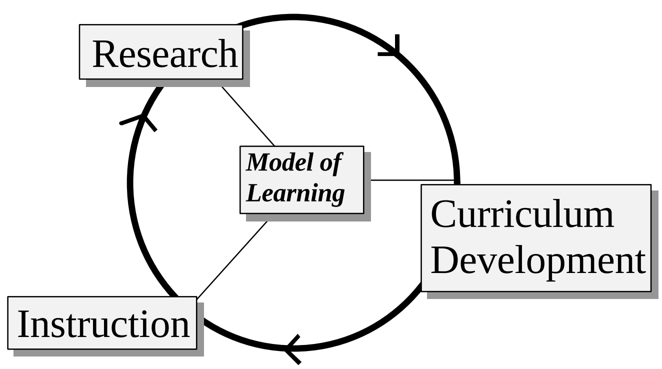
Work in physics education research (PER) seeks to understand how students come to understand physics and, as a result, how better to teach them. This work includes the development of relevant measurement techniques that investigate student reasoning in physics, the design of effective curriculum materials that address student difficulties, and the development of techniques and statistical methods to describe and organize our understanding of student difficulties. Research into student understanding forms a required basis for curriculum development. Instruction consists of the implementation of curriculum materials. The cycle of research used by the Physics Education Research Group (PERG) at the University of Maryland (UMd) is shown in Figure 2-1.[1] We believe that a model of learning can inform the research cycle at all of its stages.

Because a knowledge of the research methods of PER is important for an understanding of the later discussion, I begin this chapter with a brief overview of the different methods used to investigate student knowledge in physics. I then describe previous PER results which are relevant to my work, including previous investigations into how students make sense of introductory physics materials, student difficulties with wave physics, and the development of research-based curriculum materials.

Research Methods

Research results in PER depend on a rigorous and repeatable methodology that effectively probes student understanding of physics. A variety of methods has been developed to investigate student ideas, abilities, and concepts. These include: individual demonstration interviews, written questions on quizzes or exams, and specially designed diagnostic tests. When multiple research methods are used in conjunction, the researcher is able to gain deeper insight into students' reasoning patterns, providing detailed knowledge that can be used to develop more effective curriculum.

Figure 2-1



The iterative cycle of research, development, and instruction, centered around an understanding of student models of learning.

Observations of students often start through informal observations during lecture, office hours, help sessions, or discussion sections. Student comments may raise the interest of the researcher, or show where many students are having common problems with the material. Most often, informal observations are made in a setting where the goal is to help students arrive at the right answer or, in a sense, certify that they have stated the correct answer.

To describe how students approach physics, we must approach them with a different investigative method that does not attempt to teach them but rather gives us insight into their understanding. This requires that we go beyond trying to help students immediately and instead listen to what they are saying and doing. Observations of their understanding can come from listening to their descriptions of physical situations, asking them to explain their reasoning in solving problems, or continuing a series of questions that follow the thread of a concept that students seem to have difficulty with.

We call these investigations interviews. We ask for student volunteers who are then videotaped while answering questions in a one-on-one setting for approximately 45 minutes. The students are usually getting A's and B's in their physics classes. (We have found that weaker students are usually shyer about presenting their understanding of the physics.) In demonstration interviews, a researcher presents questions about a demonstration apparatus or situation to a single student. The researcher probes the student's understanding by following up on the student's predictions of the physical behavior of the system or by asking for clarifications of the student's descriptions of the physics. In problem interviews, a student solves (one or more) problems while the researcher asks questions that help elaborate the student's understanding of physics, reasoning methods, and the manner in which the student approaches the problem.

What sets interviews apart from informal observations is that the researcher can ask further, unscripted questions that probe deeper into student responses while flexibly adapting to the student's responses and not certifying or teaching the correct response. The power of the individual demonstration or problem solving interview lies in the fact that the researcher has chosen the context which the student must describe. By listening to many students describing the same physical situation, it is possible to compare their results and gain deeper insight into the common difficulties they are having. Interview data are used as the basis for coming to an understanding of students' reasoning processes and knowledge, forming a "state space" of possible student responses for other research investigations.

Interview videotapes must be transcribed, a time-consuming process. The transcript and the actual video of the interview are used to analyze student understanding, reasoning, and performance in the interview. Transcripts should be read and analyzed by multiple researchers so that personal bias of a single researcher does not skew the results.[2] Due to the amount of time required to carry out a large number of interviews (and often, the lack of students willing to volunteer to take part in this aspect of the research), interviews are rarely used as the sole source of data. The detailed student interview responses are used to help make sense of other data that is more easily collected.

Using written questions allows data to be gathered from many more students. Questions can be asked on examinations, in quizzes, in homework sets, or on pretests (an aspect of tutorials, which will be described below). Students are asked to answer a question or solve a problem and explain how they arrived at their answer. Student explanations on written questions are not as detailed as those which can be found through the use of interviews, but they help the researcher make a connection between student solutions to the written question and other students' explanations in interviews. The state space of student difficulties that was developed through interviews can help in interpreting student written responses.

I have used two types of written questions in my research. The first and most common is the free response (FR) question. Students are given an open-ended question and asked to give a response they believe is correct. We have found that students often do not give *all* answers they believe are correct, leading us to believe that the responses they give are filtered in some fashion. The second type of question is the multiple-choice, multiple-response (MCMR) question. Students are given a multiple-choice question with a long list of possible responses and asked to give *all* responses that they believe are correct. Individual students tend to use more explanations and give more responses on MCMR questions than on FR questions. It seems that the many offered responses trigger students into giving more of the explanations that they believe are correct than the free response leads them to. (I will address the issue of filtering and triggering in more detail in Chapters 3 and 5 of this dissertation.)

Some student learning takes place when answering a physics question. Students who have participated in interviews may do better on written questions than those students who have not participated in interviews. As a result, we usually try to use different students in the same class or students from separate but identical classes (or sections) for interviews and for written questioning. At times, we have carried out interviews on students who have previously answered written questions on a physical topic. Most commonly, we do this to see if they are answering the question consistently in both settings. We use these interviews to better understand the links between the common written questions and common interview explanations.

In summary, using both written and interview questioning of students on a single topic gives a rich understanding of how students approach the physics of that topic. The purpose of interviews in PER is to gain deeper insight into student responses by providing the opportunity for deeper questioning through follow-up questions asked in response to student comments. The purpose of written questions such as the diagnostic test is to help researchers gain a better statistical overview of the distribution of student responses as understood from interviews.

Common Sense Physics

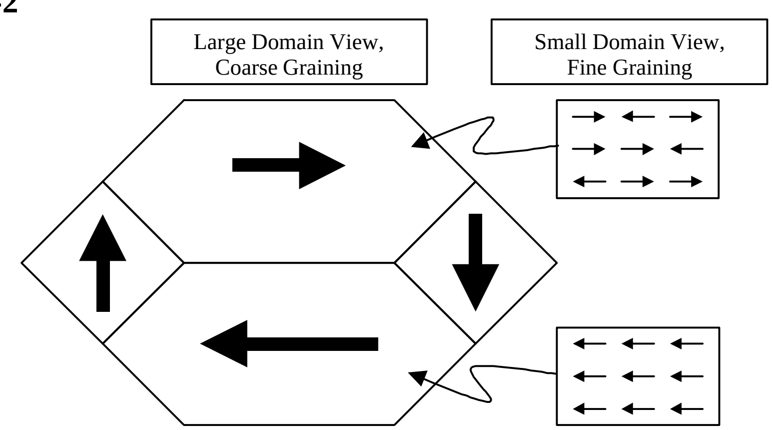
Previous PER has shown that students bring a common sense understanding of the world around them to their study of physics.[3] Many studies have investigated student understanding of specific topics to illustrate how common sense reasoning plays a role in how students come to understand physics. The studies summarized below take two different positions about the manner in which we should pay attention to student difficulties. The first position states that PER should pay attention to specific difficulties that students have and try to address these specific and profound difficulties such that students improve their understanding of fundamental physics ideas. The second position states that PER should focus on a broader understanding of student difficulties in terms of the general types of reasoning that they use to describe a large set of phenomena. In other words, the two positions differ on the issue of the domain size of the analysis.

By domain size, we mean the realm in which it is fruitful and meaningful to study student understanding of the physics. We use the term in an analogy with the physics of ferromagnetic materials. When considering a model of ferromagnets in terms of aligned atomic spins within the material, it is often found that regions of the material may have, on average, aligned spins (and therefore be magnetic) while the whole system is, on average, only weakly aligned (and barely magnetic). Two possible domain sizes with which one can study ferromagnets are at the level of the individual spins (fine graining) or at the level of the larger, aligned domains (coarse graining) (see Figure 2-2). Presently in PER, the investigation of student difficulties focuses primarily on individual difficulties and not on sets of difficulties that involve more general reasoning patterns.[4] The domain size of these investigations is at the "spin" (fine grain) level in the sense that only specific pieces of student knowledge are investigated, while the self-organized, aligned domains (course grain) are not investigated.

One goal of this dissertation is to show that an analysis at a larger domain size is productive and relevant to a study of student understanding of physics. Furthermore, there are connections between the different grain sizes used to investigate and describe student difficulties with physics.

As an example of a small domain investigation in PER, consider early work done by Clement.[5] As part of a larger investigation of student's understanding of force and motion, Clement investigated student descriptions of the forces acting on a coin tossed vertically in the air (see Figure 2-3). Students were given a brief description of

Figure 2-2



Blocking of spin states in a ferromagnet as an analogy to describe levels of analysis possible in a system. In the large domain view, a coarse graining creates an average over the system, while in the small domain view, each individual element of the system is considered and described.

each system and shown a diagram. They were asked to sketch vectors to show the direction of the force acting on the coin at different points in its trajectory.

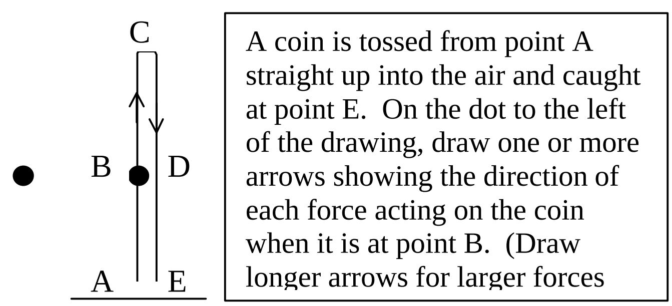
Clement studied the responses of a calculus-based engineering physics class on interviews and on a diagnostic test, both given before and after instruction. Before instruction, 34 students (group 1) participated in the research, while 43 different students (group 2) participated after they had received physics instruction. Clement points out that group 2 consisted of paid volunteers whose grades were all far above the course mean. A further 37 students (group 3) from another institution answered the question after having taken two semesters of physics. Eleven members of group 1 were interviewed. No members of the other groups were interviewed.

Before instruction, only 12% of the students described the forces on the coin correctly. After instruction, only 28% of group 2 and 30% of group 3 answered correctly. Clement states that, before instruction, "virtually all (90%) of the errors … involved showing an arrow labeled as a force pointing upwards" when the coin was on its upward path. Similar results were found in the post-instruction data.

Clement found that many students have what he called the "'motion implies a force' misconception." Evidence of this was found in student interview comments about the coin problem, done before instruction. Clement quotes students using phrases such as: "the 'force of the throw,' the 'upward original force,' the 'applied force,' the 'force that I'm giving it,' 'velocity is pulling upwards, so you have a net force in this direction [points upwards],' 'the force up from velocity,' and 'the force of throwing the coin up.'" Students had difficulty separating the motion of the coin in one direction from a force acting in another direction. Students had similar difficulties even after instruction. Clement states that "most errors are not due to random mistakes but rather are based on a stable misconception that is shared by many individuals." He adds that "the data support the hypothesis that for the majority of … students, the 'motion implies a force' preconception was highly resistant to change."

In focusing on the "motion implies force" misconception, Clement illustrates the small domain size of his research. He summarizes his findings in three comments. First, "continuing motion, even at a constant velocity, can trigger an assumption of the presence of a force in the direction of motion that acts on the object to cause the

Figure 2-3



The Clement coin toss problem. A correct answer would show only one force acting on the coin when at point B (the force of gravity, pointing downward). The most common student response was to include a force that pointed in the direction of the motion, often described as the "force of the throw."

motion." In other words, students will invent forces that point in the direction of motion. Second, "such invented forces are especially common in explanations of motion that continues in the face of an obvious opposing force. In this case the object is assumed to continue to move because the invented force is greater than the opposing force." Finally, students "may believe that such a force 'dies out' or 'builds up' to account for changes in an object's speed."

Whereas Clement and others focus on a small domain size of descriptions of student difficulties, Halloun and Hestenes[6] choose a larger domain size with which to analyze student difficulties. They speak of the need for "a more systematic and complete taxonomy of CS (common sense) beliefs" that goes beyond an identification of "specific CS beliefs" (i.e. specific student difficulties). Halloun and Hestenes investigate student understanding of Newtonian particle mechanics. Their study used data gathered from 478 university physics (calculus-based) students. These students answered a pre-instruction and post-instruction diagnostic test. From this population, 22 students were interviewed within a month of having taken the pre-instruction test. The analysis that follows is based on pre-instruction results. In a separate paper, the authors show that overall student scores on the diagnostic (as measured by correct responses) do not change very much over the course of a semester (from 51% to 64%).

Halloun and Hestenes state that "each student entering a first course in physics possesses a system of beliefs and intuitions about physical phenomena derived from extensive personal experience. This system functions as a *common sense theory* of the physical world which the student uses to interpret his experience." The authors use three descriptions for the most common student responses: Newtonian physics, Aristotelian physics, and impetus physics.

A single physical situation, such as Clement's coin toss (used in modified form by Halloun and Hestenes), can be described using all three theories. A Newtonian response would describe a constant force being exerted downward on the coin, causing an acceleration which causes a change in velocity such that the coin slows and reverses direction. As an example of an Aristotelian response, Halloun and Hestenes describe the idea that a force must act in the direction of motion to keep an object moving, and that the "force does not move an object unless it overcomes (exceeds) the object's *inertia*, an intrinsic resistance (mass) which is not distinguished from weight." An example of an impetus response would be "when an object is thrown, the active agent imparts to the object a certain immaterial motive power which sustains the body's motion until it has been dissipated due to resistance by the medium." Thus, the motive power eventually dies away, so that the object no longer moves. The impetus theory has been described in detail by McCloskey. He states, "the act of setting an object in motion imparts to the object an internal force or 'impetus' that serves to maintain the motion … [A] moving object's impetus gradually dissipates."[7]

Most students entering the course are not consistent in their use of theories. Halloun and Hestenes use their observations of student responses to describe students as predominantly Aristotelian (18%), predominantly impetus type (65%) or predominantly Newtonian (18%). Most of the students using theories inconsistently have predominantly non-Newtonian ideas. As the authors state, "no doubt much of the incoherence in the student CS systems is the result of vague and undifferentiated concepts." Common incorrect responses given during interviews were: "every motion has a cause," elaborating with statements such as "a force of inertia," "the force of velocity," or "it's still got some force inside" (this force is seemingly in the process of getting used up as time passes, showing evidence of the impetus theory). On the Halloun and Hestenes pretest, 65% of the students gave answers which Clement would describe as "motion implies force." Other students state "the speed is equal to the force of pull," or "the energy of blast has to be greater than the force" (indicative of an Aristotelian response). Many of these students do not distinguish between force and acceleration, think of force as a quantity that gets used up, and have difficulties distinguishing between related quantities whose distinctions help build a detailed understanding of physics.[8]

One weakness of the classification scheme used by Halloun and Hestenes is that they import "a ready-made classification scheme" taken from Newtonian mechanics. Student difficulties are interpreted in terms of the correct response, which we hope students will learn in our classrooms. But, Clement's research, described above, shows that students bring their own level of understanding to the classroom, and they are willing and able to invent forces to help analyze motion in their own framework. Thus, it may be that the description of student common sense beliefs according to a Newtonian taxonomy may not provide the most insight into student understanding or give the best guidance for our curriculum development.

Still, Halloun and Hestenes show that there is value in analyzing student performance on many questions in order to gain a more complete understanding of how students approach physics. The "coarse grain" analysis emphasizes both individual and specific difficulties while trying to analyze the whole system of reasoning that the student uses. Students answering pre-instruction questions have very little understanding of Newtonian mechanics. Individual students use many different theories to describe the same physics. Also, based on a comparison of pre- and post-instruction results, student reasoning does not seem to change very much during the course of instruction.

From the two summarized papers, one can conclude that the analysis of student difficulties with physics can lead to a meaningful and rich discussion of how students make sense of the physical world and the description of the physical world which they must learn in our classrooms. Both papers dealt with student understanding of mechanics. To improve student learning at all levels of instruction, issues of common sense physics and multiple theory use must be investigated in other areas of physics. In the next section, I will discuss the physics of waves and the previous research into student difficulties that forms the basis of the physics of this dissertation.

Wave Physics: Basic Concepts

A wave is a propagating disturbance to a medium. At the level of wave physics taught in introductory classes, mechanical wave phenomena occur via local interactions between neighbors and the propagation of disturbances can be described simply through spatial translations. The discussion below will use one-dimensional waves propagating on a taut string as an example (though also referring to sound waves when appropriate). One fundamental assumption of the introductory model of wave physics is that disturbances have only small effects on the system. In the case of transverse waves, this means that disturbances cause only small angle deviations from equilibrium. In the case of sound waves, it means that the deviations of pressure and density from equilibrium are very small compared to the unperturbed values.

Consequences of and problems with this assumption will be discussed below. The purpose of this section is to discuss the way that the professional physics community understands and describes wave physics. Assumptions, mathematical tools, and insights commonly used to think about wave physics are described below. Issues that might play a role in student understanding of wave physics will be raised.

Deriving the wave equation for mechanical waves

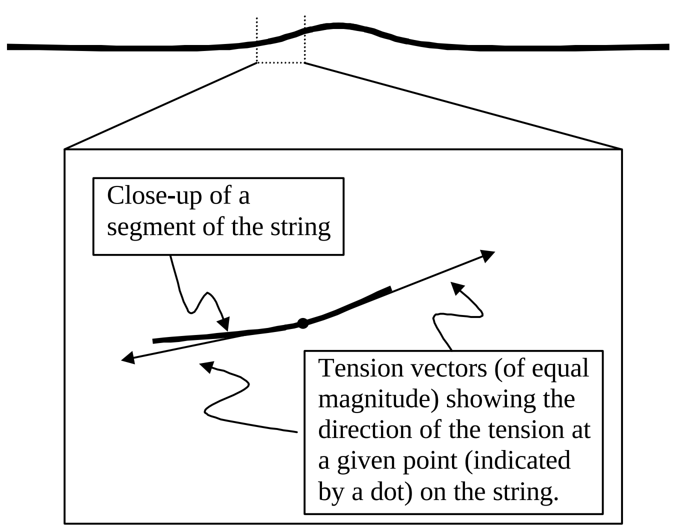
Consider a disturbance to a long, taut, ideal string consisting of a transverse displacement of the string from equilibrium (see Figure 2-4). In the case that the deviation from equilibrium is small, one can assume that the tension, *T*, in the string is constant. The sum of the forces exerted in the transverse direction, *y*, on a string element of mass density *m* and length *dx* arises from the different angles at which the tension is being exerted on the element. Using Newton's Second Law, ΣF = ma, in the one transverse direction gives

$$(T\sin\theta_1 - T\sin\theta_2) = (m\,dx)\frac{\partial^2 y}{\partial t^2}. \qquad (2\text{-}1)$$

Because of the small angle approximation, the sine terms can be approximated by saying

$$\sin\theta \approx \theta \approx \tan\theta = \frac{dy}{dx}. \qquad (2\text{-}2)$$

Figure 2-4



A small amplitude wave propagating along the length of a long, taut string. The vertical displacement in this diagram has been exaggerated to emphasize the string's displacement.

Rewriting equation 2-1 in terms of this approximation gives

$$T\left[\left(\frac{\partial y}{\partial x}\right)_1 - \left(\frac{\partial y}{\partial x}\right)_2\right] = (m dx)\frac{\partial^2 y}{\partial t^2}. \qquad (2\text{-}3)$$

Noting that the change in *dy/dx* on the left side of the equation is the change in the slope of the line on either side of the string element, we get the wave equation in its usual form,

$$T\frac{\partial^2 y}{\partial x^2} = m\frac{\partial^2 y}{\partial t^2} \qquad (2\text{-}4)$$

which can be rewritten in a form analogous to Newton's Second Law,

$$Tdx\frac{\partial^2 y}{\partial x^2} = (m dx)\frac{\partial^2 y}{\partial t^2}. \qquad (2\text{-}5)$$

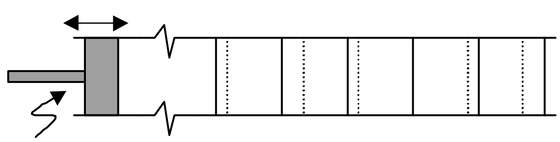
of the forces acting on the string element, while the right side of the equation contains mass and acceleration terms of the displacement from equilibrium.

Deriving the wave equation for sound waves

Equations like 2-4 can be constructed for other systems that show wave behavior. A detailed derivation of the wave equation for sound requires the use of fundamental concepts that may present some difficulty for students. Consider a tube filled with air extending infinitely in one direction with a movable piston at one end (see Figure 2-5). In the figure, the average equilibrium location of a plane of molecules and the average displacement of these molecules from equilibrium is shown. Note that not all planes are displaced an equal amount. Due to conservation of mass in the region between the planes, the density of the air inside the tube is no longer uniform when a sound wave is propagating through it. Students may have difficulty with the idea of mass conservation, since it is a fundamental concept that is rarely used explicitly in introductory physics.

Furthermore, the description of the motion of the air molecules may present difficulties for students. The air molecules are never motionless. The intrinsic motion of the medium is due to the temperature of the system (which is proportional to the average kinetic energy of the molecules in it). We can only describe the average equilibrium location of a plane of molecules and the average displacement from equilibrium of these molecules. Students may not recognize the distinction between

Figure 2-5



Piston (moving back and forth)

A sound wave propagating through a long air-filled cylinder. The average equilibrium position of a plane of air molecules is shown by a solid line, the average longitudinal displacement from equilibrium of a plane of air molecules by a dashed line intrinsic motion described by the temperature of the system and induced motion caused by the sound wave.

To derive the appropriate wave equation for sound waves, we can again use Newton's second law, as we did when deriving the wave equation for waves on a taut string. Consider a plane of air located (on average) at position x along the tube. The displacement due to a sound wave is described by $y(x,t)$, where the variable y describes a longitudinal and not transverse displacement from equilibrium. In the same way that we described the tension on a taut string, we can describe the equilibrium pressure on the plane of air by P (a constant). The change in pressure at that location at a given time will then be $DP(x,t)$. In the same way that we described the linear mass density of a string, we can describe the equilibrium volume density of the air by a constant, r, and the density at a given location and time as $r(x,t) = r + Dr(x,t)$.

In this situation, Newton's second law states that the force exerted on a plane of air located at x consists of two parts. At a given time t, the magnitude of the force exerted by the air to the right of the plane is $F_{left} = A(P+DP(x))$ and the magnitude of the force exerted by the air to the right of the plane is $F_{right} = A(P+DP(x+dx))$. The net force is then equal to

$$F_{net} = A[\Delta P(x) - \Delta P(x+dx)] = -Adx \frac{\partial(\Delta P)}{\partial x}. \qquad (2\text{-}6)$$

By Newton's second law, we know that the net force equals the mass times the acceleration of the gas. Since the mass of gas in a region dx is $Ardx$ and its acceleration is given by $\frac{d^2 y}{dt^2}$, we have

$$\frac{\partial(\Delta P)}{\partial x} = -r\,\frac{\partial^2 y}{\partial t^2}. \qquad (2\text{-}7)$$

To develop this equation further, we must apply concepts from thermodynamics. In a sound wave, we assume that the oscillation of the system is such that the temperature of the gas does not remain constant. Instead, we can state that the heat exchange of a region of gas with another region is zero, since all processes happen too quickly for heat exchange to occur. We can write an equation for the differential change in heat, dQ,

$$dQ = \left(\frac{\partial Q}{\partial V}\right) dV + \left(\frac{\partial Q}{\partial P}\right) dP = TC_V \frac{dP}{P} + TC_P \frac{dV}{V} \qquad (2\text{-}8)$$

where the last part of the equation includes the definition of specific heat of an ideal gas

at constant volume and at constant pressure.

In a tube with cross-sectional area A, the volume of air between two planes of air molecules separated by a distance dx will equal Adx. When a sound wave is propagating through the system, each plane will be displaced a different amount from equilibrium. The first plane will be displaced $y(x)$ and the second $y(x+dx)$. Thus, the volume of air between the two planes is equal to.

$$A[dx + y(x+dx) - y(x)] = A\left[dx + dx\frac{dy}{dx}\right] = V + dV \ . \tag{2-9}$$

Setting $dQ = 0$ in equation 2-8, we can use equation 2-9 to write

$$dP = -P\frac{C_P}{C_V}\frac{dy}{dx} = -P\gamma\frac{dy}{dx} \tag{2-10}$$

where the term γ has been introduced to describe the ratio of the specific heats.

Using equation 2-10 in equation 2-7 gives the wave equation for the propagating sound wave. We find

$$\frac{P\gamma}{\rho}\left(\frac{\partial^2 y}{\partial x^2}\right) = \frac{\partial^2 y}{\partial t^2} \ . \tag{2-11}$$

Note that the only difference between equation 2-11 and equation 2-4 is in the variables that describe medium properties. Otherwise, the mathematical form is identical for waves on a taut string and sound waves, meaning that an analysis of the mathematics and physics for the two types of waves should be very similar.

Physical meaning of the wave equation

Local interactions on a global scale

Because the wave equation for a wave on a taut string arises from Newton's Second Law, we can interpret the physics in terms of concepts that students have learned in their previous mechanics course. For example, a correct interpretation of Newton's Second Law requires that only those forces acting directly on an object influence that object (this concept has been referred to as Newton's "Zeroth Law"[9]). Though this seems obvious, students have great difficulty with the idea when applied to free body diagrams of point particles.[10] The difficulties students have with this concept when applied to point particles should exist when applied to continuous systems, also.

In continuous systems, the additional difficulty exists that Newton's "Zeroth Law" must be applied to every point in the medium. Local interactions at all locations in the medium must be considered. The conceptual distinction between interactions on a local level and the analysis of these interactions everywhere (i.e. globally) requires an understanding of the relevant size of analysis of the system. Since this is often a new concept for students, we can expect them to have difficulties making the distinction between local and global analyses of the physics.

For a sound wave, the interpretation of the wave equation uncovers a subtlety with which many students may have problems. The fundamental idea is that one considers the pressure gradient across the region of air through which the wave propagates. For compression (high density) to be followed by rarefaction (low density), the pressure gradient across the region of air must be both positive and negative. A model describing sound waves in terms of the transfer of impulses from one region to another, *only* in the direction of wave propagation, would not account for the rarefaction process. An effective force in the direction opposite to the propagation direction must also be exerted. This effective force can only be described by considering the pressure gradient and not the pressure in the direction of wave propagation.

Solutions of the wave equation as propagating waves

Solutions to the partial differential wave equation depend in a conceptually difficult manner on two variables. When describing the one-dimensional spatial translation of particles whose location at time $t = 0$ is the origin, we can write $x = \pm vt$ or $x \pm vt = 0$. More generally, we can say that the function,

$$g(x,t) = x \pm vt \tag{2-12}$$

describes the location of a particle that starts out at any position x and moves either to the left or the right with a velocity v. Waves are also spatially translated at constant speed, but they are spread out over a large region in space. As such, the displacement from equilibrium in the medium can be thought of as being translated independently. We can expect solutions to the wave equation to have some similar form.

If we carry out a coordinate transformation on the variables in the wave equation, we can find this form. By substituting $\eta = x+ct$ and $\xi = x-ct$ into equation 2-4 (where c is an as-yet undefined variable), we can rewrite equation 2-4 to say

$$\frac{\partial^2 y}{\partial \xi \, \partial \eta} = 0$$

In the process of simplifying the equation, the variable c has been defined as $c^2 = T/m$ and represents the speed with which the wave propagates through the medium. Thus, the solutions to the wave equation have the form of $x \pm ct$. The displacement of the medium from equilibrium is then a function of a single function which is a function of two variables,

$$y(g(x,t)) = y(x \pm ct)\,. \tag{2-14}$$

Since the quantity y describes the displacement of the medium from equilibrium at all points (i.e. the shape of the wave), equation 2-14 describes the translation of this shape through the medium.

Plugging equation 2-14 into the wave equation shows that this functional dependence leads to correct solutions of the wave equation, as long as the velocity of the wave is set equal to physical quantities in the wave equation. (This will be discussed in more detail below.) Thus, the spatial translation of waves through a medium arises as a consequence of the local interactions between elements of the medium. Spatial translation is a consequence of the local interactions of the physical system.

The issue of the distinction between local and global descriptions of the system again may cause difficulties for students. The displacement of the medium is described by $y(x_0,t_0)$, where x_0 and t_0 are specific values of position and time. Thus, an equation of the form $y(x_0,t)$ describes the motion of a single point in the medium as a function of time, while $y(x,t_0)$ describes the displacement from equilibrium of the entire medium at one instant in time. But, the shape of the entire medium at all times is described by $y(x,t)$, where the specific functional dependence describes the translation of the entire shape to the left or the right (in one dimension). The global translation of the entire system is the most visible phenomena of visible wave systems (ocean waves, waves on a string or spring, etc.). Thus, we can expect students to focus on the global descriptions of waves rather than the local phenomena within the system that cause the spatial translation.

Wave velocity depends on medium properties

One consequence of equation 2-14 being a solution to equation 2-4 is that the wave velocity, *c*, is a function of properties of the medium. We find that

$$c = \sqrt{T/m} \qquad (2\text{-}15)$$

for waves on a string or tightly coiled spring, while

$$c = \sqrt{pg/r} \qquad (2\text{-}16)$$

for the propagation of sound through air.

If we assume for the moment that the gas in which the sound is propagating is an ideal gas, we can relate *P* to the temperature, *T*, of the system using the ideal gas law, *PV=nRT* (*R* is the Rydberg constant). Since $r=M/V$ (*M* is the mass of the gas in a volume *V*), the speed of sound in the system is equal to

$$c = \sqrt{g\,RT/M}\,. \qquad (2\text{-}17)$$

Further analysis is possible, but the basic conceptual meaning of equations 2-15 and 2-16 is that the speed of propagation of a wave through a medium depends on the properties of the medium and nothing else.

This concept may be difficult for students to understand. In all other instances, students have learned that some sort of force was necessary to cause a change in motion. (Note also that many students have felt that some sort of force was necessary to continue a motion, see chapter 2). In this situation, no external force is necessary for the propagation of a wave through a system (where there are obvious changes to the motion of elements of the system). We can again expect students to have difficulty distinguishing between the internal forces and the external observable elements of the system.

In the case of waves, ignoring the internal forces of the system and focusing only on the spatial translation of the waveshape may create a dilemma for some students. They may revert to the impetus physics or Aristotelian physics which Halloun and Hestenes describe. The description of wave propagation through a system can be thought of as a large domain description. An analysis of the internal forces that allow the wave to propagate can be thought of as a small domain description of waves. In other words, a failure to understand the relevance of different domain sizes of wave physics may push students toward incorrect reasoning.

Superposition

Since the wave equation is linear, any linear superposition of solutions of the form *y(x±ct)* will also be a solution to the wave equation. Thus, a wave described by

$$y(x,t) = y_1(x-ct) + y_2(x+ct) \tag{2-18}$$

leads to two separate wave equations, one for y_1 and one for y_2.

Buried within the summation of these individual waves is the concept that the waves described by y_1 and y_2 can only add when their values of x and t are the same. Though this seems obvious from a mathematical point of view, it may not be so for students. The dependence on two variables again plays a role here, and the fact that both variables must be equal may be difficult to interpret physically and mathematically.

In addition, values of y_1 and y_2 are only added at a specific time, t, when the values of x are equal, but they are added for all x. The issue of local summation done on a global level (i.e. everywhere) shows that the fundamental conceptual distinction between local and global phenomena also plays a role in superposition.

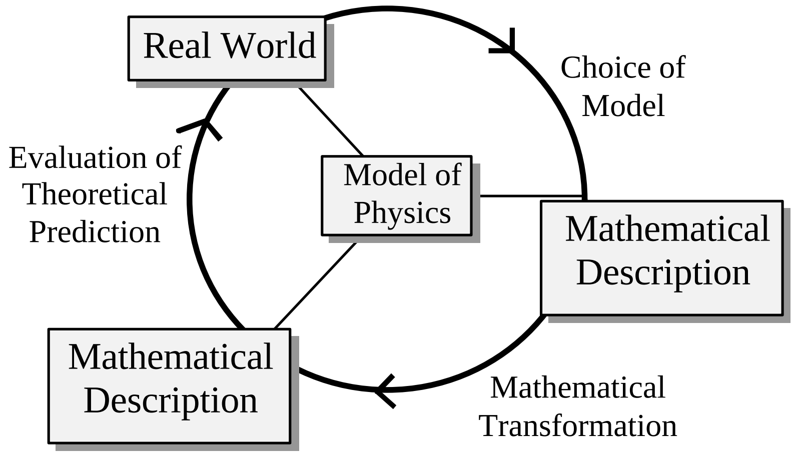
It is possible, through superposition, to have the model which led to the linear wave equation break down in certain situations. For example, two waves individually may still be within the small angle regime, but added together may fall outside the regime. Recall that there was a simplification in the derivation such that the sine terms describing the vertical components of the tension on the string element could be replaced by the slope of the string on either side of the string element. If two waves add in such a way that their sums no longer hold to the model because of large angle deviations from equilibrium, then an inconsistency of the model is uncovered.

The role of modeling

The possibility of the linear model breaking down raises an issue with respect to the way in which physical models are used in science. Based on observations, we develop or choose mathematical models to describe the physical world. These mathematical models can then be modified through mathematical transformations to either account for other observations or make predictions. Any predictions made by the model must then be compared to the physical world. A representation of the cycle that describes the relationship between observations of the real world, the choice of mathematical model and analysis, and the interpretation and comparison of the predictions with the real world is shown in Figure 2-6.

In the case of linear superposition breaking down due to the inapplicability of the small angle approximation, the difficulty lies in the use of the mathematical transformation to interpret the new physical situation. The choice of model for the system seems appropriate because each wave satisfies the small angle approximation. But, the choice of model is shown to be incomplete because it cannot adequately describe the phenomena that it claims to. The inconsistency between prediction and model choice is not found until mathematical predictions are compared to physical reality. In chapter 6, I describe an instructional setting where exactly this breakdown in the model of superposition occurs.

Figure 2-6



The interaction between the real world and a theoretical model which describes it and predicts its behavior. The choice of a model of physics affects the choice of mathematical model and how the model is mathematically transformed.

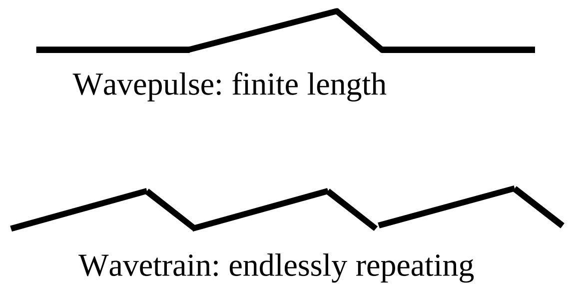
Initial conditions and boundary conditions

The wave equation describes only the manner in which the wave propagates through a system and how waves interact with each other but does not describe how the wave was created nor its behavior at boundaries to the medium. The speed of wave propagation (which enters into the wave equation) depends on medium properties. Linear superposition is a consequence of the wave equation. But the manner in which waves are created is determined by the initial conditions of the system (i.e. in terms of time dependent events at a specific location or possibly many locations in space). The manner in which waves interact with the boundaries of the medium are determined by the boundary conditions of the system.

To describe how a wave is created, we can discuss boundary or initial conditions that are either continuous disturbances to the medium at one location or disturbances that last a finite amount of time. Because the disturbance to the system propagates through the system, the former leads to a disturbance of finite length and duration (such as a wavepacket) while the latter leads to a continuous disturbance (such as a sine curve or sawtooth wave). In this dissertation, I will describe the the finite length waves as *wavepulses* and continuous waves as *wavetrains*. An example of each is shown in Figure 2-7. I will use the term waves to mean both wavepulses and wavetrains, i.e. all propagating disturbances to a system.

The boundary condition plays a role by driving the shape of the string at a given location in space. This type of boundary condition is a time dependent function for a point in space.[11] For example, the boundary condition can be given at some location x_0 by a function depending on time. For a sinusoidal wavetrain on a string which stretches from $x=0$ in the positive x direction, this equation may be of the form $y(x=0,t)=A\sin(2pct/l)$, where λ is the wavelength of the propagating sinusoidal wavetrain of amplitude A. For a wavepulse with a Gaussian shape, the boundary

Figure 2-7



The difference between a wavepulse and a wavetrain, illustrated with a finite length sawtooth shape and a repeating sawtooth pattern. Both shapes represent propagating disturbances to the equilibrium state of the system, but, for example, the propagation of the wave is more easily visible with a wavepulse than a wavetrain.

condition may be of the form $y(x=0,t) = Ae^{-(ct/b)^2}$, where *b* describes the width of the wavepulse.

Note that the creation of the wave does not determine the speed with which the wave moves. The velocity is determined by the medium through which the wave propagates. In both examples of initial conditions above, the velocity of the wave is therefore a given that determines the relationship between the duration of the motion and the width or wavelength of the propagating wave. For sinusoidal waves, this relationship is given by $l=cT$, where *T* is the period of the wave. For a wavepulse created on a taut string by moving one's hand quickly back and forth, one can describe the width of the wave (at its base) by $W=cT$, where *T* is the amount of time the hand was in motion. Of course, the creation of the wave may affect the validity of the approximations we use to describe the system (for example, a large amplitude wave may lead to large angles which may make the linear wave equation inadequate as a description of the physical situation).

Students may have difficulty understanding wave motion without additional discussion of how waves are created. The interpretation of boundary conditions as the source of wave motion is rarely emphasized in physics textbooks.[12] Most commonly, portions of the medium (either a string or air, for sound) are shown with a propagating wave, without discussion of how that wave was created. We know, from previous PER, that students often have difficulty separating the cause of motion from the motion itself (for example, the impetus model described above shows this confusion). We also know that students often invent forces to account for motion (for example, Clement's results described above). We can expect students to invent causes or forces for the wave motion that they see.

Furthermore, it may be difficult for students to distinguish between the velocity as determined by the medium and the motion (described by boundary conditions) which causes the wave. Consider a person holding a long, taut spring lying on the ground and shaking it regularly back and forth (this is a common demonstration done in classrooms). The time it takes for the demonstrator to complete either a full period of a wavetrain or to create a wavepulse is determined by the speed with which the hand moves back and forth. If the hand moves faster over the same distance as in a previous demonstration, the effect is to create wavetrains and wavepulses that are narrower. The effect is not to make the wave move faster. The distinction between transverse velocity and propagation velocity may cause difficulties for students.

In order to describe the physical behavior of waves at the edge of the system in which they are propagating, we again must use boundary conditions. For example, a string on which a wave propagates can either be attached or free to move. In the case of sound waves, similar distinctions exist between regions where displacement from equilibrium is possible and where none is possible. The boundary conditions then describe the properties of reflection and transmission. They whether or not there can be a displacement and what sort of displacement can exist at the location of the boundary. For example, for a string fixed to a wall at location x_0, the boundary condition might be $y(x_0,t)=0$ for all times t.

Students might have problems with this idea for a variety of reasons. Rather than showing a distinction between spatially local or global domains, the issue of boundary conditions involves the distinction between constant situations (the boundary condition) and instantaneous events (the shape of the string at an instant in time). Previous PER has shown that students often have difficulties distinguishing between two events that occur at different times, and that students often integrate all times into a single description.[13] We can expect to find the same types of difficulties in wave physics.

Previous Research Into Student Difficulties with Waves

Very little previous research has been published on student difficulties with mechanical waves. Maurines[14] and Snir[15] studied student understanding of wave propagation, Grayson[16] (also with McDermott[17]) and Snir studied student understanding of the mathematical description of waves and the superposition of waves, and Linder[18] (also with Erickson[19]) studied student descriptions of sound.

In the discussion below, I will first describe the research setting and methods of each of these researchers. This will include a more complete description of the issues and the student populations they investigated. This brief discussion will be followed by descriptions of the observed student difficulties with the wave physics topics outlined above.

Research context and setting of previous research

The student populations investigated in previous research include pre-service teachers, engineering students, physics majors, high school students, and physics graduate students.

Maurines[14] asked 1300 French students questions which dealt with the topic of wave propagation and simple mathematical reasoning about waves. Of these, 700 students had no previous instruction on waves and were in secondary school (the age equivalent to American high schools) and 600 had previous instruction on wave physics. The latter group was a mixture of secondary school and university students.

The investigation consisted of eight written free response questions. The questions addressed the topic of wave motion through a medium, the relationship between the creation of the wave and its subsequent propagation, and the motion of an element of the medium due to the propagating wave.

Maurines points out that results within each of the two groups were so similar that "no distinction can be made between the different subgroups." Thus, Maurines uses representative data from subgroups of her study to describe student difficulties. There were differences between the students who had received instruction on waves and those who had not. Specific questions that Maurines asked will be discussed in more detail below.

Linder and Erickson's[18] work on student understanding of sound waves took place with ten Canadian physics majors who had graduated from college in the previous year and were enrolled in an education program to get certification in teaching physics. The ten interviewed students were enrolled in a one year course for teacher certification to teach at the high school (secondary) school level. Students were interviewed for 40 to 80 minutes. During this time, they answered a variety of questions dealing with their personal experiences with sound, descriptions of simple phenomena, interpretations of typical representations of sound waves, and predictions of how the speed of sound can be changed in a medium. Examples of student comments and reasoning will be given below. Data were gathered from an extensive analysis of interview transcripts. Data were analyzed by categorizing student interview explanations in terms of elements common to other explanations given by the same student and elements common to explanations given by other students.

Grayson and McDermott's work was done at the University of Washington, Seattle (UW), and Grayson continued this work at the University of Natal, South Africa (UNSA). The work done at UW consisted of investigations of the kinematics of the string elements for propagating and superposing waves. Student understanding of two-dimensional kinematics was investigated to help develop a computer program that would address student difficulties with the material. At UW, individual interviews were conducted with 18 students after they had instruction on waves and kinematics. (The questions will be described in more detail below.) Grayson continued this research at UNSA with two different student populations. The first consisted of in-service teachers taking a six week summer program that focused on the teaching of kinematics. Most teachers were not physics instructors, so this was their introduction to kinematics. They were asked the same types of questions as the UW students before, immediately after instruction, and then again on the final examination. In a third study, Grayson investigated the understanding of twelve introductory physics students who had studied kinematics but not waves. They were also asked the same types of questions as the other students before and after instruction. Instruction in both instances at UNSA consisted of students using a program designed to help students view the motion of string elements as waves travel along the string. Grayson made additional observations as the students used the programs, noting both difficulties with the program and conceptual difficulties with the material.

Like Grayson, Snir developed a computer program to help students develop their reasoning skills with waves. In the development of the program, he investigated the difficulties of Israeli students with wave propagation and superposition after they had completed instruction on waves. Studies were conducted with tenth grade students who were interviewed before and after instruction. The complete research protocol and results were never published.[20] The number of students and the types of questions asked are thus not known. Snir's results will be mentioned but not elaborated upon below, since they are consistent with those of Grayson and Maurines.

Student difficulties with the propagation of waves

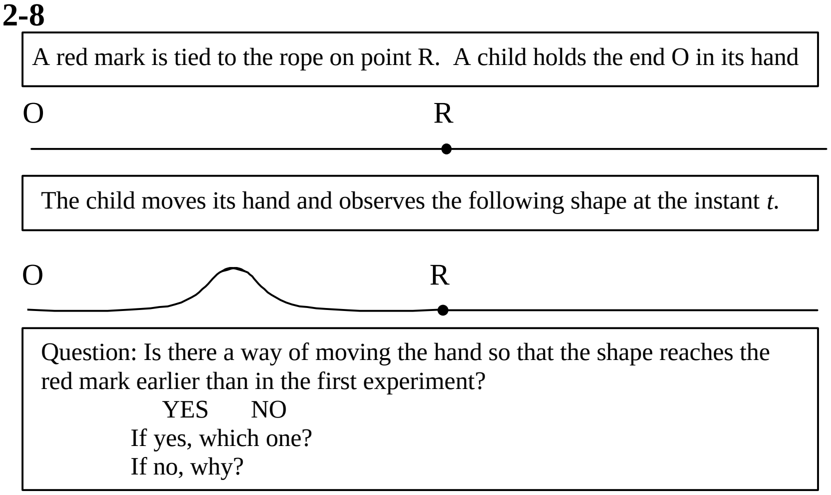
Maurines and Snir focus on the reasoning students use when describing wave propagation on a taut string. Linder (and Erickson) focus on student explanations of sound wave propagation. The similarities between some of the explanations indicate that students have similar difficulties with the material.

Propagation on a taut string or spring system

Two questions by Maurines show student difficulties with the relationship between wave creation and wave propagation. In the first, students were asked if it was possible to change the speed of a wavepulse by changing the motion of the hand that creates it (see Figure 2-8). In the second, Maurines describes the realistic scenario that the wavepulse amplitude decreases over time, and students are asked if the speed of the wavepulse changes as this occurs (see Figure 2-9).

Common student responses indicated that a majority of the students thought of wave propagation in terms of the forces exerted by the hand to create the wavepulse on the rope. For example, students stated, "the speed depends on the force given by the hand," or "the bump will move faster if the shake is sharp" (i.e. if the movement of the hand is faster). Maurines gives results from subsets of the secondary school and the

Figure 2-8



Question asked by Maurines to investigate how students viewed the relationship between the creation of the wave and the motion of the wave through the medium. A correct answer would be "no," because only medium properties affect wave speed. See reference 14 for further discussion.

university student population. (Recall that she said that results within each group were similar, implying that the statistics she gives for the subgroup are consistent with the statistics for the whole group.)

Very few students who had completed instruction gave the correct answer to the question in Figure 2-8, which states that there is no way to move the hand to create a faster wave. Of 42 secondary school students who had no instruction in waves (and 16 university students who did), 36% (25%) gave the correct answer. Of the students who gave incorrect responses, 60% (75%) stated that it was possible to change the wave speed through a different hand motion. For these students, 84% (67%) gave justifications that mentioned force, as indicated with the first quote above. Students seem to have profound difficulties separating the creation of a wavepulse (i.e. the initial conditions of the system) from its propagation through the system. The quotes given above, though brief, indicate that students are using an impetus-like model to describe the movement of a wavepulse through a medium. The wavepulse propagates due to the motion of the hand and a change in hand motion will affect wave speed.

Student responses to the question shown in Figure 2-9 also indicated that many students did not separate the initial conditions from the propagation properties of the wavepulse. A correct answer to the question would state that the speed of the wavepulse would not change while the amplitude decreased. Maurines quotes a student saying “The height decreases as the action of the hand gets weaker. The speed decreases also. If the bump disappears, it is because the force which caused it disappears as well. During that time, the speed decreases.” This student’s reasoning is indicative of the impetus model of mechanics, described above. The “force which caused” the wavepulse disappears as the amplitude disappears, and as the force is used up, “the speed decreases.” Maurines states that of 56 secondary school students who had not received instruction in waves (and 42 university students who had), 30% (45%) gave the correct answer and 68% (55%) gave incorrect answers. Of the students giving incorrect answers, 58% (35%) used reasoning force-based similar to the student quote above. Again, the evidence indicates that students misinterpret the physics of the creation of the wave with its propagation.

Maurines interprets student descriptions in terms of students’ notions of force and a quantity she calls “signal supply.” This signal supply is a “mixture of force, speed, [and] energy.” The impetus model often guides student reasoning with respect to the signal supply. Thus, the higher the signal (the more force is used to create the

Figure 2-9

This bump disappears before reaching the other end of the rope. Does the speed of the bump vary on the way?

YES NO

Why?

Question asked by Maurines to investigate how students interpreted damping in a wave system. A correct answer would be that the damping affects only the amplitude but not the propagation speed. See reference 14 for further discussion.

wave, the wavepulse), the faster the wave. Student comments are consistent with this interpretation. For example, some students state that the propagating wavepulse "is losing its initial power," and others state that "there is a [wavepulse] which is moving because of the force F" exerted by the hand. The latter student is confusing the force needed to create the wave with the forces internal to the medium that allow the original force to propagate through the medium. Thus, we see that students are unable to separate the creation of the wave from its propagation. Maurines states that many do not make the distinction between force and velocity.

Snir's[15] interpretation of student difficulties with the relationship between wave creation and propagation is similar to Maurines's, but he does not cite evidence for his result. He describes finding that students speak of a wave's "strength," or "energy," or "intensity," much like Maurines describes "signal supply." He also implies that students use impetus-like reasoning to say that waves with larger intensity (higher amplitude or frequency) have more strength and therefore move faster. Because he does not provide evidence for his interpretation (as described above), it is difficult to interpret his findings, but they seem to be consistent with Maurines's. In Chapter 3, I discuss similar results have found at UMd. In Chapter 5, I propose a more detailed explanation for student reasoning than the one used by Maurines or Snir.

Sound wave propagation

Linder[18] (also with Erickson)[19] found that students who had completed their undergraduate studies of physics (including wave physics and sound) have great difficulties understanding the propagation of sound waves through air. In one question, they asked students to describe what would happen to a candle flame located near the end of a tube when one clapped two pieces of wood together at the other end of the tube (see Figure 2-10). Student descriptions of the effect on the candle flame of the sound wave caused by the clap showed that students thought of sound using incorrect models and inapplicable analogies. Similar questions involved sound due to the popping of a balloon and sound caused by the vibration of a tuning fork.

Common student descriptions of sound waves in these settings involve the incorrect descriptions of the motion of air or air molecules to account for sound. For example, one student states that "sound creates a wave that is emitted and is focused on the tube - and so the wave travels down." The interviewer asks "Pushing air in front of it?" as a provocative question to elicit possible difficulties the student may have with

Figure 2-10

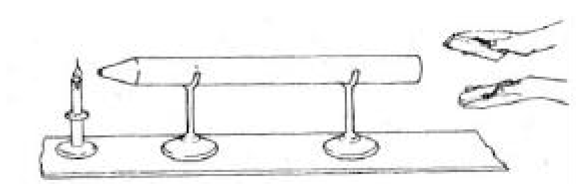


Figure given students in the Linder and Erickson interviews. Students were asked to describe how clapping two pieces of wood together would affect the candle flame located on the other end of a long tube from the location of the clap. See reference 19 for further discussion.

source to the ear that hears it. Linder and Erickson observe that students describe the motion of air as either the flow of large blocks of air from one point to another or as the motion of specific air molecules that transmit sound while all other molecules continue in their usual random motion.

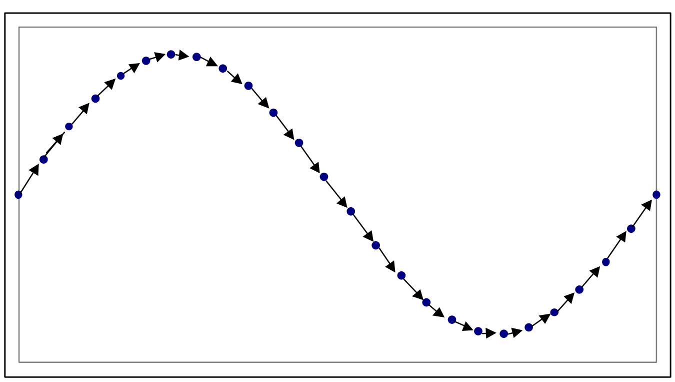
Another common model that Linder and Erickson describe involves the impulse transfer model, as if sound were transmitted linearly along a path of adjacent beads. Rather than describe sound waves in terms of a pressure gradient, one student speaks of forces only in the direction of wave propagation. He states,

> *[J]ust consider a row of beads sitting on the table. And you tap a bead at one end and you knock all the beads along and at the other end you have your finger and you can feel the tap. That would be analogous to a book dropping and creating the motion of all these smaller things in the air we call molecules which act the same as the beads and move this disturbance around until your finger at the other ends can feel it; in this case with the ear at the other end that is feeling it.*

This student is thinking of sound waves on a microscopic level of individual colliding air molecules, but avoids the very difficult idea that density and pressure propagation through air forms a sound wave. The problematic physics of the impulse transfer model of sound has been discussed above.

Linder observed an interesting variation of the impulse transfer model that allowed a student to account for the sinusoidal path of sound waves that is commonly drawn in textbooks. In textbooks, the sinusoidal path describes the longitudinal displacement of a region of air from its equilibrium position. Linder observes that a student who sketches a sinusoidal curve made up of colliding air molecules (see Figure 2-11). Linder summarizes the student's model as: a sound wave consists of "molecules in the air colliding with each other in such a way that a transverse pathway" is created. As Linder states, "The molecular collisions are generally not 'head-on' but rather tend

Figure 2-11



Student sketch to show how sound propagates. Sound consists of glancing collisions between adjacent particles such that the recognizable sinusoidal shape is created. See reference 18 for further discussion.

to be ‘glancing’ in such a manner as to give rise to the ‘correct’ changes in direction to form a sinusoidally shaped collision-wave.” The student giving this response is mistaking the graph of displacement from equilibrium as a function of position for a picture that describes the interaction between elements of the medium through which the wave travels. This confusion of graphs and pictures has been investigated in more detail with respect to student interpretations of graphs in the kinematics.[21]

Linder and Erickson observe that some students think of sound as the motion of a quantity (like energy or impetus) that is transferred from molecule to molecule. This is similar to the idea of “signal supply” described by Maurines and the “strength” described by Snir. Linder has observed that many students believe “changing particle displacement, changing sound pressure, and changing molecular velocity all to be in phase with one another.” Thus, students do not distinguish between different variables that describe the system, much like the students observed by Maurines and Snir do not distinguish between velocity, frequency, power, and energy.

Other similarities also exist between Linder (and Erickson’s) findings and Maurines’s results in the overall confusion students have about propagation speed. Some students state that the speed of sound is determined by the physical obstruction of the medium (thus, a denser medium should have slower sound waves, the opposite of what actually occurs). This idea seems related to the concept that Maurines discusses, where students describe a wave exerting a force on the medium. The less force is exerted, the slower the wave. Similarly, the less resistance from the medium, the less force is needed to create a fast wave, and the faster a wave created with great force will move. The relationship to Maurines’s “signal supply” and Snir’s “strength” is supported by Linder’s comment that some students state that wave speed is a function of inertia reduction. Thus, we see that students seem to use the same descriptions of waves when describing mechanical waves on strings or springs and sound waves.

Linder presents an interesting result which has not been discussed by others who have investigated student understanding of wave physics. He observed that students have great difficulty with the idea of the equilibrium state of the air through which sound waves travel. As one student states (when describing the problem shown in Figure 2-10), “Equilibrium position will be a position of rest. Before you clap, all the [air] particles are in a position of rest and as you clap you are causing particles to move so particles start jumping all over the place; then they all return back because they try and return to equilibrium. Everything always tries to go to equilibrium.” The student is having difficulty distinguishing between the different scales of the system, air molecules or regions that are, on average, at equilibrium. If students have difficulty with understanding the equilibrium condition of a system through which waves propagate, their understanding of wave propagation may be much less robust than we would like. Similar difficulties have been observed and discussed in introductory mechanics by Minstrell with respect to force and motion and the at rest condition.[22]

The study of student understanding of sound waves is rich because it shows evidence of many of difficulties found in other areas of PER. Some students have difficulties with the representations used to describe sound and misinterpret graphs as pictures (as in the example of colliding air molecules traveling along a sinusoidal path).

Many students have difficulty with the equilibrium state of the system (as in the inability to distinguish between air molecules and a description of the medium based on density of a region of air). One should note that many of the student difficulties are specific to sound waves but also related to difficulties students have in other areas. This suggests that common descriptions can be found to account for a large variety of student difficulties with physics.

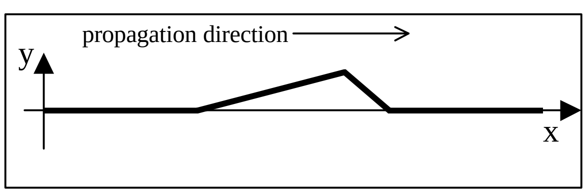
Student difficulties with the mathematical description of waves

Grayson's work investigates student use of two-dimensional kinematics to describe the propagation of waves and the motion of the medium through which the waves propagate. Students were asked questions of the following type: given a graph of *y* vs. *x* (vertical and horizontal position, respectively), of an asymmetrically shaped pulse,[23] graph *y* vs. *t* and *x* vs. *t* for a point, and *v* vs. *x* for the string (see Figure 2-12). The discussion below uses results from written responses and comments and quotes gathered by Grayson while observing students using a program to help them develop their conceptual understanding of the topic.

A correct understanding of kinematics and physics in these questions would include the idea that solutions of the form $y(x \pm ct)$ propagate without changing their shape (in an ideal, dispersionless medium). The motion of the medium is transverse to the motion of the wave. To describe the velocity of a piece of the medium (a small section of the string) over time, one can sketch the string at regular time intervals and use the definition of average velocity ($v=Dx/Dt$) to describe the velocity at different instants in time. To describe the velocity of the entire string at some instant in time, one can use the same method and find the velocity of each element of the string at one time.

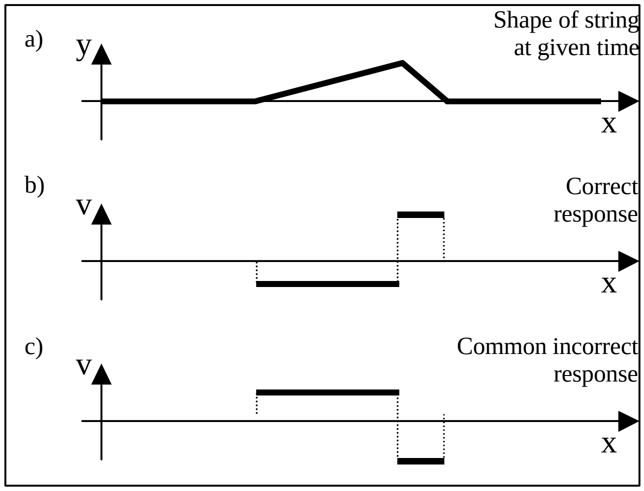
Grayson describes how students (both before and after instruction) approach certain ideas algorithmically when she describes how students attempt to find the shape of a *v* vs. *x* graph. She found that many took the slope of the *y* vs. *x* graph rather than thinking of the time development of the *y* vs. *x* graph and using a relevant procedure to find the velocity at each point along the string (see Figure 2-13). In other words, the students did not have an operational understanding of how to find *v* vs. *x* and used an incorrect algorithmic method instead.

Figure 2-12



Grayson presented students with a diagram like this one, indicating an asymmetric wavepulse propagating to the right on a long, taut string. Grayson then asked students to sketch graphs of the following quantities: *y* vs. *t* and *v* vs. *t* for a string element as the wave passes that string element, and *v* vs. *x* for the entire string at the instant in time shown in the diagram. See references 16 and 17 for further discussion.

Figure 2-13



Common student difficulty with a *v* vs. *x* graph of a string on which a wave is propagating. a) Given sketch of an asymmetric wavepulse propagating to the right, b) correct response, showing velocity of each string element based on the motion of the string, c) most common incorrect response, showing velocity of each string element based on the slope of the given *y* vs. *x* graph. See references 16 and 17 for further discussion.

Table 2-1

	Pretest	Posttest	Final Exam
UW phyiscs students (N=18)	--	22%	--
UNSA introductory physics students (N=12)	33%	75%	--
In-service teachers (N=19)	53%	84%	79%
In-service teachers (N=23)	26%	65%	

Percentage of correct responses for students sketching a *v* vs. *x* graph of an asymmetric wavepulse propagating along a string (see Figure 2-13). The UW students were interviewed, the other students answered written questions. Two different populations of in-service teachers were investigated in successive years. None of the incorrect responses by in-service teachers used the slope of the *y* vs. *x* graph to answer the question. See reference 16 for further discussion.

Data for the three student populations which Grayson investigated are shown in Table 2-1. The most common mistake students made was to take the slope of the *y* vs. *x* graph incorrectly, as described above. Grayson attributes the improved post-instruction performance of the in-service teachers and UNSA students (in comparison to the UW students) to the use of the computer program to help address student difficulties. She also notes that none of the in-service teachers took the slope of the *y* vs. *x* graph when answering the question after instruction.

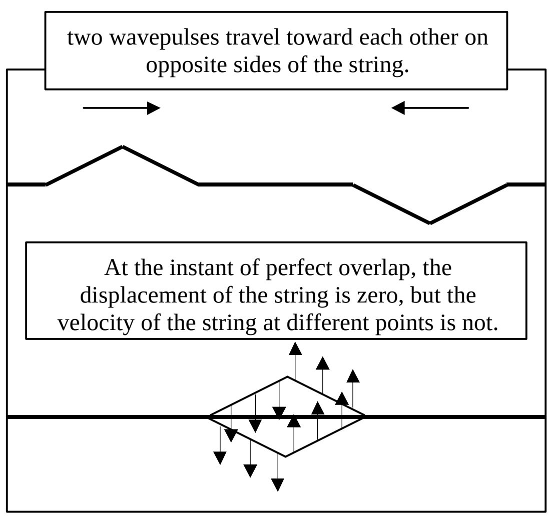
Student use of an inappropriate algorithmic method for finding answers to questions they are otherwise unable to answer suggests that students do not imagine the motion of the medium when the wave passes through it. A consideration of the motion of the medium would show that the incorrect response found by taking the slope of the *y* vs. *x* graph (shown in Figure 2-13c) is inconsistent with the motion of the medium. The leading edge of the wave is moving up, not down, as indicated on the graph. Thus, by investigating student understanding of the mathematics of wave motion through a medium, we find results similar to Linder's. Both Linder and Grayson observe that students have profound difficulties describing the motion of the medium as a result of the wave.

Student difficulties with superposition

Grayson and Snir address the issue of student understanding of wave superposition. Since Snir does not give data to support his conclusions, I will focus on Grayson's work in the following discussion. Grayson asked students to describe the shape of a string on which two identically shaped wavepulses were traveling toward each other on opposite sides of the string. She asked specifically for the shape of the string and the velocity of different elements of the string at the moment of maximum overlap. The situation and a correct response are shown in Figure 2-14.

Grayson finds that students consistently give the same incorrect responses. Students state that waves will collide, and either bounce off each other or cancel each other out and disappear permanently. Grayson states that "some students did not realize that pulses pass through each other. Instead, several students said that two pulses would bounce off each other and travel back towards where they came from."

Figure 2-14



Superposing wavepulses on opposite sides of a long, taut string. In the lower sketch, the individual wavepulses are shown together with velocity vectors indicating the direction of the motion of the string due to the wavepulse. Note that the string has zero displacement but that the velocity of the string is non-zero where the pulses overlap (except at the exact middle point). See references 16 and 17 for further discussion.

Grayson gives a possible explanation for the permanent cancellation of waves when describing student difficulties in distinguishing between the displacement and velocity of the medium through which the wave travels. Students are often unable to distinguish the two, causing problems in their description of continuing wave motion after waves have interacted. For example, students see a flat shape when two symmetric wavepulses of identical amplitude on opposite sides of a string add completely destructively (see the sketch of the correct response in Figure 2-14). Those who interpret the lack of displacement such that the velocity of the string is zero will then state that nothing will move anymore. Thus, an incorrect interpretation of the kinematics (i.e. the difference between displacement and velocity) may be used by students and lead to an incorrect physical interpretation of the situation. This interpretation may guide students to say that the wavepulses are permanently canceled in this situation.

Snir describes similar results. He also discusses how students will speak of waves that bounce, collide, or cancel each other permanently. Snir states that the idea is borrowed from mechanical collisions, but he does not elaborate how this may be the case.

Research as a Guide to Curriculum Development

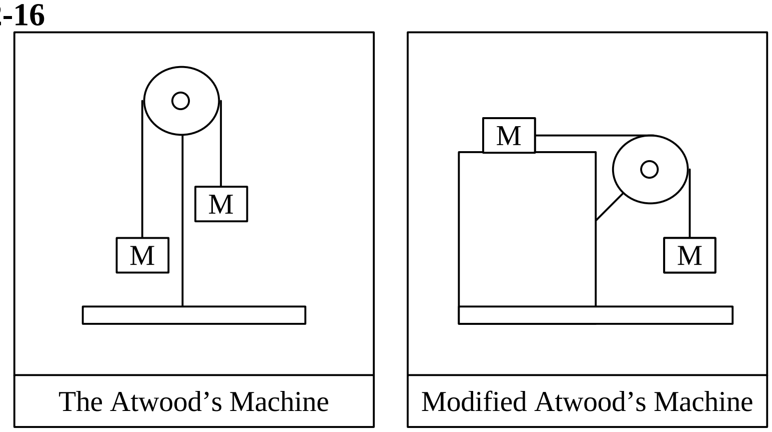
The discussion of previous research into student difficulties with waves serves as an example of PER done to come to a deeper understanding of how students approach physics and build a functional understanding of the material. Another aspect of PER involves the building of curriculum materials that address student needs as effectively as possible. The paradigm of instructional design used by PERG at UMd is based on that of the University of Washington, Seattle (UW) (see Figure 2-1). To show the background of the research-curriculum design paradigm, I will describe one example from UW in detail. Interested readers can find more information about the UW methods by following references in summarized papers and in other sources.[24] Research by UMd PERG has also shown that tutorials are more effective in helping students develop a deeper understanding of the physics.[25]

The development of instructional materials begins with the investigation of student difficulties. Researchers at UW investigated student understanding of tension in the context of the Atwood's and modified Atwood's machines (see Figure 2-16).[26] The apparatus consists of weights attached over an (ideally, frictionless) support by a string. Students often encounter this example in the classroom, solving problems from the textbook or seeing a demonstration done by a professor.

McDermott *et al.* found that a similar situation elicited nearly identical difficulties with the fundamental ideas of acceleration, force, and tension as the original Atwood's machine. Rather than having the force of gravity play a role in the physics, the UW question used an explicit external force to move two blocks (of mass m_A and m_B ($m_A<m_B$)) connected by strings (see Figure 2-15). One hundred students were asked the question in Figure 2-15. These students had previously had instruction on tension and the course included a laboratory that dealt explicitly with the Atwood's machine.

Students had fundamental problems with the concept of tension in this setting. They had the most difficulties when they were asked to compare the force exerted by string #1 on block A with the force exerted by string #2 on block B. (To have the same acceleration, the force of string #1 on block A must be greater, since the force exerted by string #2 on B is equal to the force exerted by string #2 on A, but in the opposite direction, and the sum of the forces must still be to the right for block A.) Only 40% stated that the force exerted by string #1 was greater than that exerted by string #2. The other two most common responses were to say that the tensions were equal and that the tension on string #2 was greater. Students who gave the latter response used the reasoning that the accelerations were equal, F = ma, and $m_A<m_B$ to say that the force exerted by string #2 was greater. As McDermott *et al*. state, “these students seemed to believe that the force exerted by each string depended only on the mass of the block to which it was directly attached and which it was pulling forward.” Students who stated that the tensions were equal (20%) are quoted as saying “it is the same force,” and “the force exerted on string 1 goes through [block A] onto string 2.” This implies that students believed that the force exerted by string 1 was transmitted through block A to string 2. Further analysis of a similar question, not discussed here, showed that this thinking was robust in more advanced situations. Furthermore, graduate students asked the same question had similar difficulties (though only 40% were incorrect).

Figure 2-16



Atwood’s machine and Modified Atwood’s machine apparatus. In both cases, a string is stretched between two masses and the string hangs over a pulley. The UW research project involved an investigation of both apparatuses. See reference 26 for further discussion.

Figure 2-15

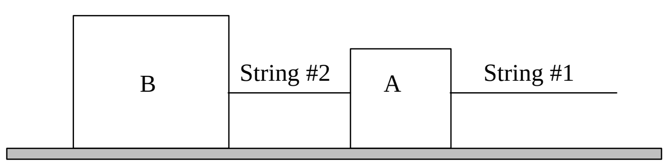


Diagram from the UW pretest. A hand was pulling to the right on string #1. Students were told to assume the strings were massless. They were asked to compare the acceleration of Blocks A and B and to compare the forces exerted on Blocks A and B. See reference 26 for further discussion.

In summary, the students answering the question incorrectly failed to isolate each block and identify the forces acting on it. Also, many failed to correctly analyze that string #2 was pulling on both blocks, not just block B. Thus, in applying Newton's second law to a situation like the Atwood's machine, they were unable to adequately describe which "F" and which "m" to use, even when most knew the "a" was the same for both masses.

To address these difficulties, McDermott *et al*. designed a tutorial to address student difficulties. *Tutorials*[27] are a research-based instructional method developed at UW which place students in small groups and get the students to actively think through the physics content of the worksheets they are completing. Tutorials replace traditional TA-led recitations. The worksheets are designed to challenge students and their understanding of a physical situation and the model they use to understand the situation. Students without a functional understanding of the material (i.e. unable to apply the conceptual ideas relevant to the situation to new and novel topics) will have difficulty with the material and will be helped to develop a functional understanding.

The premise of tutorials is *elicit-confront-resolve*.[26] First, tutorials are designed to *elicit* from students any difficulties they might have with the material by asking for a prediction of a physical situation that has been shown through research to be difficult for students. Then, questions asked in the worksheet or by the facilitator-TA *confront* students with observations or reasoning which contradict students' incorrect predictions. Finally, once students have been confronted with inadequacies (if any) in their understanding, they are led to a *resolution* that helps them gain a deeper understanding of the physics involved.

For the student, the tutorial cycle consists of four aspects. Students take a brief pretest during lecture every week. Pretests are conceptually based, non-graded quizzes which usually follow lecture discussion of a topic. Most commonly, pretests are given after students have completed homework problems dealing with the physical topic addressed in the pretest. After the pretest, students participate in tutorials (attendance is not mandatory, but at UMd, 85% to 100% of the students attends tutorial section). Students have tutorial-based homework which give them the opportunity to apply and develop the ideas they have learned in tutorial in order to further build their functional understanding of the material. Finally, on each examination, one question is based on tutorial materials. These examination questions also help evaluate student performance based on tutorial instruction.

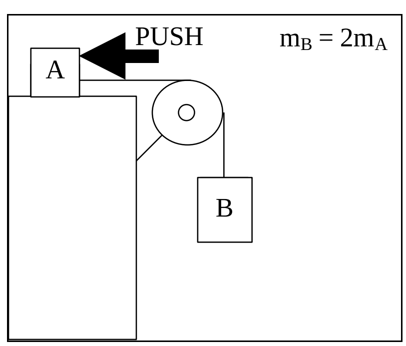
To provide students with an opportunity to develop their understanding of tension, the UW researchers developed a set of activities related to the question in Figure 2-15 and the Atwood's machine apparatuses shown in Figure 2-16. Students are asked to analyze situations where two blocks on a table are in contact with each other (a hand pushes one block which pushes another), where two blocks are connected by a massive string (a hand pushes the first which then pulls the second), and where two blocks are connected by a massless string (and a hand pushes the first block which pulls the second).

In the tutorial,[28] students are required to apply the concepts and skills they have learned in class, such as Newton's second law, free body diagrams, and Newton's third law to analyze the situation. Questions are designed to elicit difficulties that have been found through the analysis described above. Students analyze each situation in detail before moving on to the next, getting help from the TAs in the classroom as needed. For example, many students have difficulty making correct free body diagrams of the strings (both massive and massless strings). Also, many students have difficulty isolating each of the masses in their analyses. After a series of exercises, students extend their understanding by applying the concepts they have worked on to new situations. For example, they repeat the above analyses with friction between the blocks and the table. They also apply their developed reasoning to the actual Atwood's machine.

To investigate whether students who participated in tutorial instruction came to a deeper understanding of the material than students who did not, McDermott *et al*. asked identical examination questions of two different student populations. In one lecture-only class, students had four lectures a week, while in two tutorial classes, students had three lectures and one tutorial a week. As the authors state, "none of the tutorials had dealt with the particular systems involved." Also, all classes used identical textbooks.

Student understanding of tension, as measured by their performance on an examination problem (shown in Figure 2-17) was significantly better than before, though not as good as an instructor would hope. In the examination question, students consider a modified Atwood's machine. They are asked to compare the tension in a string when a force holding a mass in place is removed. The most common incorrect response students gave was to say that the tension would not change since only block A was affected by the removal of the force. In other words, the students were looking only at the local information about block A and not the entire system. In the non-tutorial class, only 25% of the students gave the correct response (that the tension was now less than the weight of block B, since the block would accelerate downward). In the tutorial classes, more than 50% gave this response. McDermott *et al*. point out that far fewer students treated the blocks and string as independent systems. Thus, students who had participated in tutorial were able to think of the global system more

Figure 2-17



Examination question asked at UW to investigate student understanding of tension after instruction. Students are told that masses A and B are originally at rest. Students were asked how the tension in the string would change when the force holding mass A in place was withdrawn. The question was answered by both tutorial and non-tutorial students. See reference 26 for further discussion.

clearly than students who had received traditional lecture instruction on the same material.

In addition, tutorial students were better able to use skills not specific to the situation but important for a detailed understanding of physics, such as "drawing free body diagrams… identifying third law force pairs, and … analyzing dynamical systems qualitatively." Also, while non-tutorial students gave primarily justifications based on algebraic formulas, the tutorial students applied dynamical arguments to the questions. The evidence suggests that tutorials, though only replacing one hour of instruction a week, give students the opportunity to develop their reasoning and skills in ways that traditional instruction does not.

The authors find that the Atwood's machine tutorial addresses student difficulties with tension in such a way that students gain the basic and fundamental skills they need in their study of physics. As they point out, "the emphasis on concept development that characterizes the tutorial materials is not intended to undermine the need for instruction on problem-solving procedures." Instead, the success of the tutorial lies in part with the idea that they do not teach by telling, but provide an opportunity for students to "integrate the counterintuitive ideas that they encounter in physics into a coherent framework" by giving students "multiple opportunities to apply the same concepts and reasoning in different contexts, to reflect upon these experiences, and to generalize from them."

Summary

In this chapter, I have presented evidence that PER can play an important role in helping instructors gain an understanding of student difficulties with physics. PER can also help instructors develop effective instructional materials that provide students with the opportunity to improve their understanding of physics. These materials can be investigated to measure their effectiveness, such that a recurring cycle of research, curriculum development, instruction, and research is put in place. The curriculum development described in this chapter dealt with issues in mechanics, but other areas of physics have also been investigated.

For example, investigations have shown that students have difficulties with some of the fundamental concepts of wave physics. Some of these concepts, such as the mathematics, the distinction between local and global phenomena, and the role of initial conditions, provide physics education researchers with an opportunity to investigate ideas that are important to an overall understanding of physics. To investigate student understanding of waves, one must first summarize the model that we would like our students to learn. By emphasizing the conceptual background in the model of wave physics taught in the introductory courses, we are able to focus our attention on the most fundamental ideas that we would like our students to learn in our courses. Published PER results on student difficulties with waves suggest that students have profound problems that hinder them from developing as deep an understanding of physics as we would like. Furthermore, many of the difficulties that have been described seem related to one another and to other PER results in areas such as kinematics and mechanics. This suggests that a detailed investigation of many areas of wave physics will give researchers a window into how students develop their understanding of physics.

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[1] For a detailed review of the needs and goals of PER, the reader is referred to the UMd dissertation of Jeffery M. Saul. Saul focused on student beliefs and attitudes toward physics and the role of these beliefs on student performance on conceptual and quantitative problems.

[2] The method described for the analysis of transcripts generally falls under the description of phenomenography. For more details, see Marton, F., "Phenomenography – A Research Approach to Investigating Different Understandings **21**:3 28-49 (1986).

[3] See reference 1 for a detailed discussion.

[4] For example, the work done here at UMd has focused on student difficulties with Newtonian physics with respect to the relationship between Force and velocity or Newton's third law; see Redish E. F., J. M. Saul, and R. N. Steinberg, "On the effectiveness of active-engagement microcomputer-based laboratories," Am. J. Phys. **65** 45-54 (1997).

[5] Clement, J., "Students' preconceptions in introductory mechanics," Am. J. Phys. **50**, 66-71 (1982).

[6] See both Halloun, I. A, and Hestenes, D. "The initial knowledge state of college **53**, 1043-1055 (1985); and Halloun, I. A, and Hestenes, D. "Common sense concepts about motion," Am. J. Phys. **53**, 1056-1065 (1985).

[7] McCloskey, M, "Naïve theories of motion," in *Mental Models*, edited by D. Gentner and A. Stevens (Lawrence Erlbaum, NJ 1983) 299-324.

[8] Similar results have been discussed in another context by Trowbridge and McDermott. See Trowbridge, D. E. and L. C. McDermott, "Investigations of student understanding of the concept of velocity in one dimension," Am. J. Phys. **48**, 1020 (1980); "Investigation of students' understanding of the concept of acceleration in one **49**, 242 (1981).

[9] Hestenes, D. "Modeling instruction in mechanics," Am. J. Phys. **55**, 440-454 (1987).

[10] For example, in my classroom experience, I find that students often include inappropriate forces, such as Third Law force pairs and forces exerted by the object rather than those exerted on the object. This result has been investigated in more detail by many researchers; see, reference 9 and references cited therein.

[11] A different possible initial condition may also describe the shape of the string at all locations for a specific instant in time, though the creation of a wave using this method is quite difficult. (But, it is a simple way to use the shape of a string at a given instant in time as an initial condition for all future events).

[12] For example, Alonso and Finn, *Physics*, Tipler, *Physics*, Wolfson and Pasachoff, *Physics Extended with Modern Physics*, and others…

[13] More details can be found in the Mel Sabella's dissertation research at the University of Maryland, College Park. Sabella has found that students often treat an extended period of time as if all events occurred at the same time. Sabella, Mel, Edward F. Redish, and Richard N. Steinberg, "Failing to Connect: Fragmented Knowledge in Student Understanding of Physics," *The Announcer* **28**:2 115 (1998).

[14] Maurines, L., "Spontaneous reasoning on the propagation of visible mechanical *Int. J. Sci. Ed.*, **14**:3, 279 (1992).

[15] Snir, J., "Making waves: A Simulation and Modeling Computer-Tool for Studying Wave Phenomena," Journal of Computers in Mathematics and Science Teaching, Summer 1989, 48 - 53.

[16] See Grayson, D. J., "Using education research to develop waves courseware," Comput. Phys. **10**:1, 30-37 (1996). Also, see Grayson, D. J., "Use of the Computer for Research on Instruction and Student Understanding in Physics," dissertation, University of Washington, Seattle, 1990.

[17] See both McDermott, L. C. "Research and computer-based instruction: Opportunity **58**, 452-462 (1990) and Grayson, D. J. and L. C. McDermott, "Use of the computer for research on student thinking," Am. J. Phys. **64**, 557-565 (1996).

[18] An overview of student conceptions of sound waves can be found in Linder, C. J., "Understanding sound: so what is the problem," Phys. Educ. **27**, 258-264 (1992).

[19] The original research is described in two papers: Linder, C. J., "University physics students' conceptualizations of factors affecting the speed of sound propagation," Int. J. Sci. Ed. **15**:6, 655-662 (1993) and Linder, C. J. and Erickson, G. L., "A study of tertiary physics students' conceptualizations of sound," Int. J. Sci. Ed. **11**, 491-501 (1989).

[20] Personal communication from J. Snir. The graduate student who had been conducting the research did not complete the project and no further findings were published.

[21] See reference 8, reference 17, and also Beichner, R. J. "Testing student interpretation of kinematics graphs," Am. J. Phys. **62** 750-762 (1994).

[22] Minstrell, Jim "Explaining the 'at rest' condition of an object," Phys. Teach. **20** 10-14 (1982).

[23] For a discussion on the usefulness of asymmetric pulses in studying student difficulties with waves, see the discussion in chapter 9 (specifically, p. 202) of Arons, A. B., *A Guide to Introductory Physics Teaching* (John Wiley & Sons Inc., New York NY, 1990).

[24] McDermott, L. C., P. S. Shaffer, and the Physics Education Group at the University of Washington, *Instructor's Guide for Tutorials in Introductory Physics* (Prentice Hall, New York NY, 1998).

[25] See reference 4 for more details. Also, see Steinberg, R. N., M. C. Wittmann, and E. F. Redish, "Mathematical Tutorials in Introductory Physics," AIP Conf. Proc. **399**, 1075-1092 (1997), for a description of materials discussed in more detail in chapter 6.

[26] McDermott, L. C., P. S. Shaffer, and M. D. Somers, "Research as a guide for teaching introductory mechanics: An illustration in the context of the Atwood's machine," Am. J. Phys. **62**, 46-55 (1994).

[27] McDermott, L. C., P. S. Shaffer, and the Physics Education Group at the University of Washington, *Tutorials in Introductory Physics* (Prentice Hall, New York NY, 1998).

[28] Available as part of the materials in reference 27.