Modeling student reasoning in physics:

An example from special relativity

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Research question

In physics education research, there are two prevalent models of student reasoning.

Does the available data about student reasoning in special relativity favor one of these two models?

Outline of presentation

- **Intro: Two models of student reasoning**
- **Understanding students' ideas about simultaneity**
  - **The *content* of students' ideas**
  - **The *form* of students' ideas: Two accounts**
- **How students change their minds**
  - **Encountering simultaneity with *Tutorials***
  - **Two very different modes of learning**

Two models of student reasoning

Students reason by referring to a structured framework of knowledge.

"Misconceptions"
"Alternative theories"
"Mental models"
"Schemas"

Classic example:
Impetus theory of motion

Students reason by referring to a weakly systematic collection of intuitions.

"Primitives"
"Resources"
"Rules"
"Facets"

Classic examples:
Closer means stronger
Effects die away

Two models of student reasoning

***Misconceptions* model**

Student ideas are
- **False**
- **Coherent**
- **Context-independent**
- **Stable**

Classic example:
Impetus theory of motion

***Resources* model**

Student ideas are
- **Unproblematic**
- **Mutually independent**
- **Context-dependent**
- **Fluid**

Classic examples:
Closer means stronger
Effects die away

Two models of student reasoning

***Misconceptions* model**

***Resources* model**

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Student reasoning in special relativity

Scherr, Shaffer, and Vokos
"Student understanding of time in special relativity: Simultaneity and reference frames," Phys. Educ. Res., Am. J. Phys. Suppl. 69, 2001
"The challenge of changing student beliefs about the physical world: An example from the relativity of simultaneity," accepted for publication in Phys. Educ. Res., Am. J. Phys. Suppl., 2002

Posner, Strike, Hewson, and Gertzog (1980-82)

Panse, Ramadas, and Kumar (1994-96)

Saltiel and Malgrange (1980)

Villani and Pacca (1987-90)

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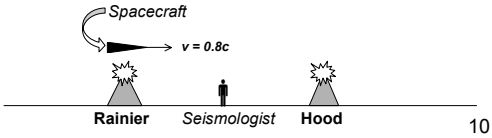
Spacecraft question

Mt. Rainier and Mt. Hood erupt at the same time in the reference frame of a seismologist at rest in a laboratory midway between them. A spacecraft flying past Rainier towards Hood at v=0.8c is directly over Mt. Rainier when it erupts.

Let Event 1 be "Mt. Rainier erupts," and Event 2 be "Mt. Hood erupts."

In the spacecraft frame, does Event 1 occur before, after, or at the same time as Event 2?



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Spacecraft question: a correct response

Application of the Lorentz transformations:

$$\Delta t_{RH}' = \gamma(\Delta t_{RH} - v\Delta x_{RH}/c^2)$$

Time between eruptions in ground frame (zero)

Velocity of spacecraft (positive)

Distance from Rainier's eruption to Hood's eruption (positive)

Negative: ***Rainier erupts after Hood***

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Spacecraft question: a typical response

"The spacecraft is near Rainier, so he gets the signal about the same time Rainier erupts. So the spacecraft pilot would say Rainier erupts before Hood."
(graduate student)

"Mt. Rainier erupts first because the light from Mt. Hood takes time to reach the spaceship."
(introductory student)

Students seem to be associating the time of an event with the time at which an observer receives its signal.

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Results of spacecraft question

Introductory students	**Before traditional instruction** Sp98, Au99 (N=67)	**After traditional instruction** Sp97, Au98, Sp99 (N=73)
Correct answer with correct reasoning *or* incomplete reasoning	5% (3)	10% (8)
Closer event occurs first	65% (46)	75% (55)
Simultaneous eruptions (absolute simultaneity)	20% (12)	5% (5)

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Results of spacecraft question

Graduate students	**Interview task** Sp99 (N=11)	**Qualifying exam** Au00 (N=23)
Correct answer with correct reasoning *or* incomplete reasoning	25% (3)	30% (7)
Closer event occurs first	55% (6)	60% (14)
Simultaneous eruptions (absolute simultaneity)	20% (2)	10% (2)

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Modified spacecraft question

In this problem, all observers are intelligent observers, *i.e.,* they correct for signal travel time to determine the time of events in their reference frame. Each observer has synchronized clocks with all other observers in his or her reference frame.

Mt. Rainier and Mt. Hood erupt at the same time in the frame of a seismologist at rest in a laboratory midway between them. A spacecraft flying past Rainier towards Hood at v=0.8c is directly over Mt. Rainier when it erupts.

Let Event 1 be "Mt. Rainier erupts," and Event 2 be "Mt. Hood erupts."

For the intelligent observer in the spacecraft, does Event 1 occur before, after, or at the same time as Event 2?

During interviews:

Students were reminded of correction for signal travel time

Students who used technical terms such as reference frame were probed for understanding and corrected if necessary

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Reconciling absolute simultaneity with relative simultaneity *(graduate student)*

S: There are really two separate kinds of reference frames. There is the kind of reference frames with all those rods and clocks extending to infinity, like in [the textbook]. But in practice, nothing happens except right where you are. So **really, your reference frame is something you carry around with you.**

I: There is this thing about...events that are simultaneous in my reference frame not necessarily being simultaneous in another reference frame. Which kind of reference frame does that refer to?

S: **Relativity of simultaneity is this local thing.** It's not the rods and clocks thing, because if we are intelligent, we correct for that. It's this thing that if I see them at different times, they occurred at different times in my reference frame.

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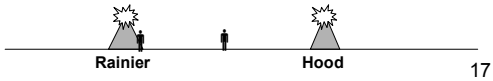
Belief that relativity of simultaneity is due to differences in signal travel time

I: This thing about events that are simultaneous in one reference frame, not being simultaneous in another reference frame? Do you have a sense of where that comes from?

S: Light has a finite speed, so it's going to take some time for the information to travel from point A to point B wherever the observer is. This is a pretty good example actually. **One observer is right between the mountains and he sees them at the same time, the other observer is not and so he sees them at different times.** (graduate student)



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Significance of models

Theoretical models

set the agenda for investigation and provide a framework for interpretation.

Look for a wrong, but coherent, way of thinking.

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The misconception

1. Belief that events are simultaneous if an observer receives signals from the events at the same instant
2. Belief that simultaneity is absolute
3. Belief that every observer constitutes a distinct reference frame

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Misconceptions model of student understanding

(Quotations from Scherr et al)

"Evidence…suggests many students **construct a conceptual framework**"

"Ideas… **harmoniously co-exist**"

three core **"beliefs"**

[determine] "what students **actually thought**"

[attempt to] "obtain a **detailed picture** of student thinking"

Student ideas are described as being a fixed structure of entities, reliably present in students' minds.

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Characterizing the data

Are student ideas

- **false?**
- **coherent?**
- **context-independent?**
- **stable?**

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Characterizing the data

Are student ideas

- **false?** ***Yes***
- coherent?
- context-independent?
- stable?

…if you know special relativity.

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Characterizing the data

Are student ideas

- false? *Yes*
- **coherent?** ***Yes – to the students***
- context-independent?
- stable?

They make explicit, simultaneous use of all three "beliefs."

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Characterizing the data

Are student ideas

- false? *Yes*
- coherent? *Yes – to the students*
- **context-independent? *Yes***
- stable?

Responses are unaffected by changes in vehicles, wording, and social setting.

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Characterizing the data

Are student ideas

- false? *Yes*
- coherent? *Yes – to the students*
- context-independent? *Yes*
- **stable? *Yes***

Answers don't change during interviews, with instruction, etc.

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Characterizing the data

Are student ideas

- **false? *Yes***
- **coherent? *Yes – to the students***
- **context-independent? *Yes***
- **stable? *Yes***

As far as we can tell, the data are consistent with a misconceptions model.

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Significance of models

Theoretical models
set the agenda for investigation and provide a framework for interpretation.

Look for resources that would account for responses.

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Resources model of student understanding

Student ideas are described in terms of intuitive ideas that are obviously correct in some situations.

1. Belief that events are simultaneous if an observer receives signals from the events at the same instant
2. Belief that simultaneity is absolute
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"Visual reality:" *What you see is what there is.*

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"Visual reality:" *What you see is what there is.*

"Ultimate reality:" *Things "really happen" in only one way.*

Resources model of student understanding

"If I see [the events] at different times, they occurred at different times in my reference frame." (graduate student)

"Visual reality:" *What you see is what there is.*

"If we are in relative motion we will measure different distances and so on but if we are all intelligent observers we will all figure out that the events were simultaneous in our rods-and-clocks reference frame." (graduate student)

"Ultimate reality:" *Things "really happen" in only one way.*

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Characterizing the data

Are student ideas

- **unproblematic?**
- **mutually independent?**
- **context-dependent?**
- **fluid?**

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Characterizing the data

Are student ideas

- **unproblematic?** ***Yes***
- **mutually independent?**
- **context-dependent?**
- **fluid?**

Anyone in their right mind would think this way.

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Characterizing the data

Are student ideas

- **unproblematic?** ***Yes***
- **mutually independent?** ***Yes – to experts***
- **context-dependent?**
- **fluid?**

Some find these resources logically incompatible.

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Characterizing the data

Are student ideas

- **unproblematic?** ***Yes***
- **mutually independent?** ***Yes – to experts***
- **context-dependent?** ***Yes***
- **fluid?**

Tasks in which people see light signals elicit distinctive responses.

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Characterizing the data

Are student ideas

- unproblematic? *Yes*
- mutually independent? *Yes – to experts*
- context-dependent? *Yes*
- **fluid?** ***No…***

…but the original research wasn't looking for fluidity.

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Characterizing the data

Are student ideas

- **unproblematic?** ***Yes***
- **mutually independent?** ***Yes – to experts***
- **context-dependent?** ***Yes***
- **fluid?** ***No…***

The data do not rule out a resources model, and perhaps we need more information.

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Tutorials in Physics: Special relativity

Topics include:

- **Events and reference frames**
- **Relativity of simultaneity**
- **Spatial measurements**
- **Length contraction and time dilation**
- **Applications of the Lorentz transformations**
- **Lorentz invariance**
- **Energy, momentum, and relativistic collisions**
- **The Schwarzschild metric**
- **Geodesics in curved spacetime**

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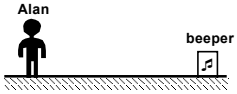
Excerpt: Construction of a reference frame



Alan, equipped with meter sticks, clocks, and assistants, hears a beep.

How can he determine the time at which the beep was emitted:

- Using his knowledge of v_{sound}?
- *Without* knowing v_{sound}?

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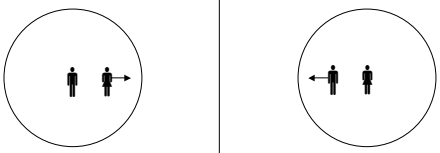
Excerpt: Applying invariance of speed of light

At the instant Alan and Beth pass, a spark jumps between them, emitting light.

Sketch the light wavefront:

in Alan's frame | ***in Beth's frame***



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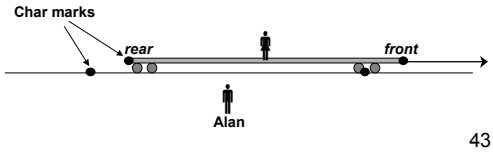
Excerpt:
Relating ideas to physical context

Alan stands at a train track; Beth stands on the train. Sparks jump at front and rear of train, simultaneously in Alan's frame, leaving char marks.

- **Draw the light wavefronts in Alan's frame.**
- **In Alan's frame, do the wavefronts hit Beth at the same instant?**



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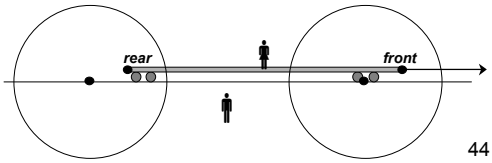

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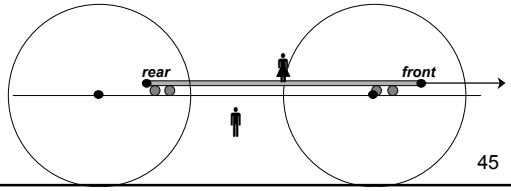

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Excerpt:
Determining order of events in train frame

A cassette player sits at Beth's feet. If the front wavefront hits it first, it plays Beethoven's Fifth Symphony at top volume; if the wavefronts hit simultaneously, it does not play.

- **Does the player play in Alan's frame? In *Beth's* frame?**

Later in the day, Beth ejects the tape from the player. She descends from the train, and she and Alan examine the tape together.

- **Will the tape have wound at all from its starting position?**

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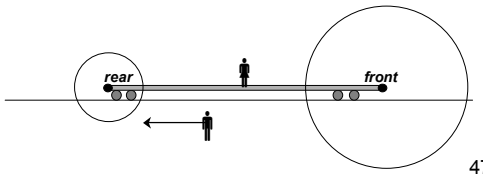
Excerpt:
Determining order of events in train frame

The tape plays in Alan's frame.
The associated events are frame-independent; the tape also plays in Beth's frame.
Hence, the front wavefront reaches Beth first.



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Significance of models

Theoretical models
set the agenda for investigation and provide a framework for interpretation.

Look for change that is <u>difficult</u> and <u>permanent</u>.

"Elicit, **confront, resolve**"
[Scherr et al]

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Response to tape player task

Dennis: We just figured out that the tape player plays in Alan's frame.

Tony: But it can't. In Beth's frame they hit her at the same time. So she won't hear it.

Jana: But look down here, it's asking if she hears it and if the tape will have wound from its starting position. If the tape is going to play, that's it, it's going to play.

Tony: But it can't play for Beth! She's in the middle! They hit her at the same time!

Dennis: But we just figured out that it plays!

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Tony: But it can't play for Beth! She's in the middle! They hit her at the same time!

Dennis: But we just figured out that it plays!

Tony: Right! And then a black hole opens up! And God steps out! and he points his finger and says [shouting] "YOU CAN'T DO THAT!"

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Evidence that change is *difficult*

Denial

Students make loud verbal objections to a partner's conclusions.

Withdrawal

Interviewees silent or unresponsive for thirty seconds or more.

Absurdism

Students ascribe results to magic, or quantum mechanics.

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Response to tape player task

Jamal: Wait, so Alan hears it and Beth doesn't? That's one awesome tape player!

Ravi: That's so cool! This is what you were telling us last week! That in some universe Sara was wearing purple and in another one she was wearing blue or something,

Jamal: Oh oh oh, **parallel universes!** Yeah!

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Evidence that change is *permanent*

Student performance on research tasks is greatly improved.

[See "The challenge of changing student beliefs…" accepted for publication in Phys. Educ. Res., Am. J. Phys. Suppl., 2002]

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Classroom observations

Change is difficult and permanent

1. Belief that events are simultaneous if an observer receives signals from the events at the same instant
2. Belief that simultaneity is absolute
3. Belief that every observer constitutes a distinct reference frame

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Classroom observations

Change is easy and fleeting

1. Belief that events are simultaneous if an observer receives signals from the events at the same instant
2. Belief that simultaneity is absolute
3. Belief that every observer constitutes a distinct reference frame

55

Evidence that change is *easy* and *fleeting*

Comfort

Students answer questions quickly and agree readily with one another.

Fluidity

Students change their minds frequently, progressing and relapsing, revisiting the same ideas many times.

Common sense

Students appeal to everyday experience.

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Evidence that change is *easy* and *fleeting*

Original Spacecraft question

"Visual reality:" *What you see is what there is.*

Painless transition

Modified Spacecraft question

"Ultimate reality:" *Things "really happen" in only one way.*

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Sometimes.

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Does the available data about student reasoning in special relativity favor one of these two models?

Misconceptions:	*Resources:*
• False	• Unproblematic
• Coherent	• Mutually independent
• Context-independent	• Context-dependent
• Stable	• Fluid

Sometimes.

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Possible refinements or hybridizations

"Pure" models

Perhaps some misconceptions (or resources) are stable, and some are fragile.

"Mixed" model

Perhaps some student ideas are misconceptions and others are resources.

"Structural" model

Perhaps misconceptions are made up of resources.

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Summary

- **We see our students through a theoretical lens that shapes our interpretations.**
  We may not even be aware of it.
- **Explicit consideration of theoretical alternatives expands our repertoire for teaching and research.**

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