Modeling student thinking: An example from special relativity

Rachel E. Scherr

Department of Physics, University of Maryland, College Park, Maryland 20742

Our understanding of the nature of student ideas informs our instructional and research agendas. In this paper, I characterize student ideas in terms of five observable properties (determinacy, coherence, context-dependence, variability, and malleability) and describe how those observable properties correspond to the "misconceptions" and "pieces" models of student reasoning. I then analyze instructional materials and student thinking in a particular topic area (special relativity) in terms of each of those two models. I show that specific instructional strategies reflect specific theoretical orientations, and explore the extent to which observed student behavior corresponds to predictions made by the theoretical models. The analysis suggests that while both the misconceptions and pieces models are flexible enough to accommodate all of the data, some aspects of student thinking are best described in terms of pieces, and others are better characterized as misconceptions. The purpose of the analysis is to illustrate the effect of theoretical orientation on instruction, instructional research, and curriculum development.

I. INTRODUCTION

Science education research in general, and physics education research in particular, is concerned with the nature of student ideas. The better we understand the properties that student ideas have – whether they are coherent or contradictory, context-dependent or independent, and so on – the better we may explain and predict student thinking. Such explanatory and predictive power can help us function better as instructors and curriculum designers, in addition to potentially being an interesting end in itself.

Historically, much of physics education research literature has modeled student reasoning in terms of "misconceptions," coherent frameworks of ideas that are stably present in students' minds and that present obstacles to instruction.[1-4] More recently, some researchers have proposed a model of student thinking in which student ideas are made up of flexibly combinable "knowledge pieces" that may be activated independently or in networks and whose activation may change sensitively depending on the situation.[5-7] In this paper, I explore the ways in which a particular set of instructional materials in a particular topic area (special relativity) enacts the strategies supported by each of these two theoretical models. I then observe the extent to which student thinking in special relativity has the properties associated with each of these two models. I find that while either model is flexible enough to potentially accommodate all of the data, student thinking is more misconception-like for some subtopics and more piece-like for other subtopics. The purpose of the analysis is to illustrate the effect of theoretical orientation on instruction, instructional research, and curriculum development.

Section II of this paper describes five observable properties of student ideas. Section III describes the misconceptions and pieces models of student thinking in terms of observable properties (that is, in the terms of greatest practical utility for instructors), and shows how the theoretical models inform our research and instructional agendas. Section IV reviews previous research identifying student ideas about special relativity (in particular, the relativity of simultaneity). Section V contains the specific analysis of student ideas about special relativity in terms of the misconceptions and pieces models of student thinking. I conclude with observations about the use and importance of theoretical models of student thinking for physics instructors.

II. OBSERVABLE PROPERTIES OF STUDENT IDEAS

In this section I describe five observable properties of student ideas. The properties that I describe here are those that I have found instructionally useful in characterizing student ideas. I do not expect that they form a complete list. Because we never access student thinking directly, all of the dimensions I propose are based to some degree on inference. My hope, however, is to limit the discussion as much as possible to observationally distinguishable properties of student ideas, in order to maximize the relevance of this analysis to physics instructors.

A. Determinacy

As instructors, we naturally characterize student ideas in terms of whether they are right or wrong. It is certainly desirable in many instances to label student ideas as being either one or the other. However, some student ideas are not specific enough to be classifiable in this way. For example, the idea that "more effort gives more result" is true if used to relate net force to acceleration, but false if used to relate net force to velocity.[8] Until it's used in a particular situation, it has an indeterminate truth value. Other examples of truth-indeterminate statements would be "closer means stronger," or "effects die away."

Student statements tend to be mainly determinate, since students are usually referring to a specific situation when they make them. However, we may infer student statements to be based on underlying ideas that are either truth-determinate, or truth-indeterminate. This choice relies on inference, and different researchers may model student thinking differently in this respect.

B. Coherence

We are often interested in the extent to which student ideas fit together logically. Researchers have observed both cases in which student ideas appear to be coherent and cases in which students appear to have a loose collection of mutually independent, sometimes even contradictory ideas. The latter is claimed to be more typical of novices.[6,7]

Observational difficulties with coherence include the fact that students rarely represent their own ideas as contradictory; either they seem to see coherence where observers see contradictions, or they don't visibly concern themselves with coherence. Another observational difficulty arises if observers prompt students to test their ideas for coherence, potentially inspiring them to make connections on the spot. Even aside from these issues, inferences about coherence rely primarily on the observer's sense of what is logical, and therefore may tell us more about the stance of the observer than any independent properties of the students' ideas.

C. Context-dependence

For our purposes, context is the textual, social, and educational "surround" of a physics question: the wording and representation of questions, the social setting and structure of lectures and recitation section, and so on.[9] Researchers have found that student responses to a question are affected by questions that precede it[10]; by whether the question arises in a classroom or in everyday life[11]; and by whether the question is posed in a physics class or in a clinical interview.[12] Context-dependence may be employed as an instructional resource, as in the use of relatively easy questions that prime students to answer more difficult ones. To the extent that student performance is affected by what surrounds the question of interest, student understanding is judged to be more or less context-dependent.

D. Variability

I will use the term *variability* to describe the extent to which students' ideas change spontaneously within the same situation. I will describe student ideas as *fluctuating* when they change frequently and without much provocation, and *stable* when they don't. Observing variability requires observing students as they think aloud independently or talk to one another. Pre- and post-tests capture "snapshots" of student responses to particular questions, but don't represent their level of intellectual commitment to an idea.

E. Malleability

Malleability is my term for the ease with which student ideas change in response to instruction. I will describe student ideas as *rigid* when they are very difficult to reshape and *pliable* when they are more easily molded. Again, pre- and post-tests don't measure malleability. They provide evidence of the end result of the instructional effort, but they don't describe the ease or difficulty of the change that took place. Observing malleability presumes a relatively high degree of confidence about teaching methods, since an appearance of rigidity may be created by ineffective instruction.

III. MODELS OF STUDENT THINKING

In this section I describe two models of student thinking in terms of the observable properties discussed in Section II, and show how the theoretical models inform our research and instructional agendas.

A. The misconceptions model

1. Properties of student ideas

Many of us have observed, as instructors or researchers, that students express wrong (and therefore truth-determinate) ideas. We have also, unfortunately, observed that many of these ideas are quite rigid (resistant to instruction). It seems natural to many of us that rigid ideas would also be coherent; students are probably more likely to hold on to ideas that form a logical framework. It's also easy to imagine such student ideas being stable and context-independent. In this way of thinking, students "have" an idea (and probably have had it for a long time). An idea lives pretty much permanently in the mind, constituting an intellectually solid, structured framework that students access in order to solve problems.[6]

Such frameworks of ideas – for example, an impetus theory of motion– are referred to in research literature as alternative theories, mental models, or misconceptions, among other terms.[14] As in the example of impetus theory, misconceptions are sometimes observed to parallel historical conceptions,[2] and for this reason some authors prefer the more neutral terms. However, because the focus of the model is typically on students' incorrect ideas, the negative term is appropriate. For the purposes of this paper, I will use the term "misconceptions model" to refer to the model of student thinking in which student ideas are imagined to be determinate, coherent, context-independent, stable, and rigid.[15]

2. Research agenda

Theoretical models set the agenda for investigation and provide a framework for interpretation. The research agenda that accompanies the misconceptions model is to examine student data for a *wrong but coherent* way of thinking. If we imagine that students are responding to our questions by accessing a single conceptual framework, we search a variety of student responses for the unifying pattern that we believe underlies them. Context-independence and stability, in this way of thinking, are evidence of the "robustness" of an idea. They are evidence that the idea is "really there," and can be expected to appear reliably in a variety of classroom situations. Research attention goes mainly to ideas that are rigid, since the instructional focus is on correcting students' false beliefs (and rigid ideas are by definition the most challenging to correct).

3. Instructional agenda

Our model of student thinking also sets expectations for student learning. In the misconceptions model, we tend to imagine that in order for students to change their ideas, incorrect ideas must be destroyed, dismantled, or otherwise discarded, and correct ideas must be inserted or constructed in their place.[16] Instructional sequences based on a misconceptions model tend to favor an "elicit, confront, resolve" strategy: students are first made aware of their initial ideas, then shown evidence that their initial ideas are inadequate, and finally assisted in resolving contradictions.[17] The expectation is thus for student learning to be *difficult* and *permanent.*

B. The pieces model

1. Properties of student ideas

As an alternative to the previous model of student thinking, we may imagine that student ideas have properties complementary to those named above: that is, we may model their ideas as being at least potentially truth-indeterminate, independent of one another, context-dependent, fluctuating, and pliable. In this way of thinking of student thinking, student ideas don't necessarily form a stable, coherent framework. Instead, ideas consist of flexibly combinable "knowledge pieces" that may be activated independently or in networks, and whose activation may change sensitively depending on the situation. Such "pieces" appear variously in research literature as phenomenological primitives,[5] facets of knowledge,[7] resources,[18] and intuitive rules,[19] with each of these having somewhat different properties. I will use the general term *knowledge pieces* to refer to any of these, without intending to support any specific theoretical description.

Although knowledge pieces do not *necessarily* form a coherent framework, most authors with a pieces model of student thinking assert that they *can* do so.[20] Accounts of such coordination have suggested that misconceptions are made up of pieces, while also clarifying what is meant by a "concept."[21] A pieces model is thus more inclusive (and in some cases more theoretically careful[6]) than a misconceptions model.

2. Research agenda

In the pieces model of student thinking, students almost always have a good reason for thinking the way they do in that their thinking is based on an idea that is useful in many situations. The research agenda that accompanies the pieces model is to identify the knowledge elements that might be guiding student reasoning. To do this, one examines student data for *potentially useful, but inappropriately applied* ideas that could account for student responses. Revisiting an example introduced earlier: students who incorrectly treat force as increasing with velocity may be seen as misapplying the knowledge piece "more effort gives more result."

If we hold a pieces model of student reasoning, we will not necessarily expect coherence among a student's responses to various questions. Instead, we are likely to imagine that different situations activate different, independent knowledge pieces, and we will be ready for the possibility that they will be contradictory. We will take particular notice of situations in which student responses are fluctuating and context-dependent – that is, easily disrupted by an interviewer's questions or even the student's spontaneous thought processes. We may observe students change their minds and then apparently forget that they changed them. Students may progress and relapse, revisiting the same ideas many times as if from scratch each time. The pieces model provides a good account of this behavior; the misconceptions model does not.

3. *Instructional agenda*

One account of an instructional agenda consistent with a pieces model of student thinking is given by Hammer and Elby, who suggest a process of "refining intuitions."[8],[22] This approach claims that students' incorrect responses are attributable to the particular knowledge pieces activated by the question. The pieces themselves are not inherently wrong, but are inappropriately applied to the situation at hand. Instruction, in this model, should attempt to illustrate to students that their intuitions are correct, but need refining. Students might be guided to recognize, for example, that their intuitive idea that "bigger objects exert larger forces" (an application of "more effort gives more result") is incorrect regarding Newton's third law. However, a different refinement of "more effort gives more result" is useful: more massive objects cause more acceleration of the target object. Instead of rejecting intuitions, students are guided to find appropriate ways to incorporate them into their thinking.

The pieces model, as stated earlier, is more inclusive than the misconceptions model and makes a wider range of predictions about student learning. In addition to the possibility outlined above (that student ideas will be rigid, truth-determinate, and so on) the pieces model makes a prediction about student learning that is complementary to that offered by the misconceptions model: it predicts that in some cases, gaining new insight will be relatively *easy*, but also potentially *temporary.* Knowledge pieces, in this model, may be sensitively activated or inactivated by many factors. They are not permanently "on" or "off," and student answers to instructional tasks can in some cases be expected to change accordingly.

C. Summary

Neither the misconceptions nor the pieces models of student thinking set clear limitations on the kinds of ideas we may see in students, and neither model is currently well-formed enough to be proven or falsified. However, the models do set up "norms" of student thinking. The misconceptions model does not rule out student thinking that fluctuates and draws on everyday experience, but neither does it do a good job of reminding us that student thinking can have those characteristics. The pieces model allows for student learning in which a new idea replaces a firmly held one after an intellectual struggle, but it does not orient us toward identifying such events as common. Table I summarizes the properties that I associate with the misconceptions and pieces models of student thinking.

IV. STUDENT IDEAS ABOUT THE RELATIVITY OF SIMULTANEITY

In the previous section, I introduced two models of student thinking. In this section I review what is known about student thinking about a specific topic in physics (the relativity of simultaneity), in preparation for discussing the properties of these student ideas in section V. This review draws heavily on Scherr et al,[23] but is greatly abridged.

The special theory of relativity flows from a few key ideas and definitions. In particular, a "reference frame" is defined as an arrangement of observers and measurement devices that determine the position and time of any event. "Intelligent" observers correct for signal travel time to determine the time of events that occur far from their own location. As a result of such corrections, observers at rest relative to one another make identical determinations of the times of events. Such observers are said to be "in the same reference

Table I. Properties of student ideas in the misconceptions and pieces models of student thinking.

	Misconceptions model	**Pieces model**
Determinacy	True or false	Indeterminate
Coherence	Coherent	Potentially mutually independent
Context-dependence	Context-independent	Potentially context-dependent
Variability	Stable	Potentially fluctuating
Malleability	Rigid	Potentially pliable
Research agenda	Find coherent frameworks	Find useful pieces
Instructional agenda	Elicit, confront, resolve	Refine intuitions
Changes in understanding	Difficult, permanent	Potentially easy, temporary

frame." These definitions, in combination with a few other principles such as the isotropy of free space and the invariance of the speed of light, lead to the fact that the time ordering of events depends on observers' relative motion. This idea is the *relativity of simultaneity*.

Student understanding of the relativity of simultaneity has been investigated with a number of performance tasks. The Spacecraft question is prototypical. In that question, two volcanoes (Mt. Rainier and Mt. Hood) erupt simultaneously in the reference frame of an observer at rest on the ground, midway between the volcanoes. A spacecraft moving at a given relativistic velocity from Mt. Rainier to Mt. Hood is directly over Mt. Rainier when it erupts. Event 1 and event 2 are defined to be "Mt. Hood erupts" and "Mt. Rainier erupts," respectively. Students are asked whether, in the spacecraft frame, event 1 occurs before, after, or at the same time as event 2.

A correct answer to the Spacecraft question can be obtained by qualitative reasoning, by quantitative reasoning (such as application of the Lorentz transformations), or from a spacetime diagram. A correct qualitative argument might run as follows: In the spacecraft frame, light from the two eruptions moves outward at the speed of light in spherical wavefronts whose centers are stationary in the spacecraft frame. In that frame, the observer on the ground is moving backward (in the direction of an arrow from the front to the rear of the spacecraft). The observer on the ground receives the two signals simultaneously. However, according to the spacecraft observer, the observer on the ground is closer to the center of the wavefront from Mt. Rainier at the instant that observer receives both signals. The spacecraft observer therefore concludes that Mt. Hood erupted first, since its signal travels farther in order to reach the observer on the ground at the same time as the signal from Mt. Rainier.

In a multi-year project, Shaffer, Vokos, and I gave the Spacecraft question as a written task or in interviews to non-physics students, introductory honors physics students, advanced undergraduates in physics, and physics graduate students. Results were similar for all groups, independent of whether the question was given before or after instruction, in an interview, or on a graduate qualifying exam.[24] Correct responses of any kind (qualitative, quantitative, or diagrammatic) were given by less than one-quarter of the students in any group. Typical responses tended to associate the time of an event with the time at which an observer receives a signal from the event:

> "The spaceship is near Rainier, so he gets the signal about the same time Rainier erupts. So the spacecraft pilot would say Rainier erupts before Hood." (graduate student)
>
> "Mt. Rainier erupts first because the light from Mt. Hood takes time to reach the spaceship." (introductory student)

Responses consistent with this reasoning were offered by 35%-75% of the students in the various groups. In an attempt to insure that students were not hindered by semantic misinterpretations of technical terms such as "reference frame" or "intelligent observer," researchers administered a modified version of the Spacecraft question in which the correction for signal travel time was made explicit. This version stated that all observers correct for signal travel time to determine the time of events in their reference frame. This change elicited a rather different pattern of responses: with signal travel time corrected for, students claimed that simultaneity was absolute. In making this claim students appeared to be regarding the relativity of simultaneity as an artifact of signal travel time.

> "If we are in relative motion we will measure different distances and so on but if we are all intelligent observers we will all figure out that the events were simultaneous in our rods-and-clocks reference frame." (graduate student)

On the basis of student responses to questions such as these, we suggested that students "construct a conceptual framework in which the ideas of absolute simultaneity and the relativity of simultaneity harmoniously co-exist." In particular, we claimed that students held three "beliefs" that fit together into a coherent, but incorrect, understanding of the nature of spacetime. The three beliefs were: (1) the belief that events are simultaneous if an observer receives signals from the events at the same instant, (2) the belief that simultaneity is absolute, and (3) the belief that every observer constitutes a distinct reference frame.

V. MODELS OF STUDENT THINKING ABOUT THE RELATIVITY OF SIMULTANEITY

In this section, I review previous research about student reasoning in special relativity in terms of the misconceptions and pieces models of student thinking. Within the discussion of each model, I present research results and observations of student learning. The research results exhibit how the research agenda associated with each model is carried out. The observations of student learning are taken from classroom observations, and are intended to illustrate the ways in which the instructional agenda associated with each model is promoted by a particular set of instructional materials. The instructional materials are those described in my previous work with collaborators.[25]

The results I describe in this section are preliminary and suggestive. The generalizations offered here are too rough to establish the ontology of student ideas, but are hopefully specific enough to illustrate the effect of our understanding of the nature of student ideas on our instruction and our instruction-focused research.

A. The misconceptions model of student thinking about the relativity of simultaneity

1. Research results

Research on student understanding of special relativity includes both theoretical work and studies of student conceptual difficulties, and has been largely framed by models of student thinking that resemble the misconceptions model I described in section III. The theoretical work explicitly models student ideas as forming coherent conceptions, many of which are wrong and need to be replaced by correct conceptions.[16,26] The studies of conceptual difficulties don't take an explicit theoretical stance, but use language of presentation that strongly suggests a misconceptions model.[23,25,27] The article abridged for section IV states, for example, that "evidence…suggests many students construct a *conceptual framework*" consisting of the three incorrect beliefs cited in that section. In identifying such a conceptual framework, the authors were carrying out the research agenda associated with the misconceptions model: to find a wrong-but-coherent way of thinking that accounts for student responses.

Researchers have given good evidence of both stability and rigidity for student (mis)understanding of special relativity. Students' responses to their probes change little during an interview, as a result of traditional instruction, or in the course of age and experience.[27] However, it is worth recalling that researchers with a misconceptions model of student understanding *seek* stability and rigidity; in that model, you have located student understanding only when you have found a relatively fixed idea that students hold. A misconceptions theoretical orientation probably presupposes stability as an outcome.[28]

2. Instructional activities

In the misconceptions model, learning is expected to be *difficult* and *permanent.* It may seem obvious that learning the relativity of simultaneity is difficult, but most of what we know (from common experience and from prior research) is that students don't learn it at all.[23,25] In the section after this one ("Observations of student learning"), I try to document that when students *do* learn the relativity of simultaneity (using the instructional materials described in previous papers), they do it by a process consistent with that predicted in the misconceptions model. In this section I describe the instructional activities relevant to those observations.

The instructional materials that my colleagues and I have described include a particular scenario that appears to be pivotal to student learning of the relativity of simultaneity. That pivotal scenario, which I will call the *tape player scenario*, is described in detail (and in the context of other instructional activities) in a previous paper.[25] Here, I summarize the main ideas of the scenario, in order to provide context for the discussion below.

The tape player scenario takes place in the context of a modified version of Einstein's train paradox. In that paradox, two sparks occur at either end of a train that moves with relativistic speed relative to an observer (named Alan) who is at rest on the ground. The sparks are simultaneous in Alan's frame. Another observer (named Beth) is standing at the center of the train. Students are asked whether, in Alan's reference frame, Beth receives the wavefront from the front spark (wavefront F) before, after, or at the same time as the wavefront from the rear spark (wavefront R). Most students recognize that in Alan's frame, Beth receives wavefront F before wavefront R since in Alan's frame she is moving toward the center of the front wavefront.

The students' next task is to determine the order of events in the train frame. To assist students with this part of the analysis, we introduce a cassette tape player that sits at Beth's feet and operates as follows: When wavefront F reaches the tape player, it starts to play music at top volume. When wavefront R reaches it, the tape player is silenced. If both wavefronts reach the tape player at the same instant, it remains silent. Students are asked whether the tape player plays (i) in Alan's frame and (ii) in Beth's frame. The analysis in Alan's frame (described above) shows that Beth receives wavefront F before wavefront R, and thus the tape player plays. The fact that if the tape player plays in any frame, it plays in all frames is not immediately obvious to students. Instead, many claim that the music plays in the ground frame but not in the train frame. Subsequent questions in the tutorial ask whether Beth will hear the music and whether Beth will later observe the tape to have advanced from its starting position.

The design of this sequence of questions is consistent with a misconceptions model of learning. The instructional materials are meant to *elicit* the incorrect idea that sparks that jump simultaneously in

Alan's frame also jump simultaneously in Beth's frame. Students are then supposed to be *confronted* with the uncomfortable implication that if the sparks jump simultaneously in both frames, the tape player plays in one frame and not in the other. In the end students are intended to *resolve* the paradox by accepting the relativity of simultaneity.[29]

3. *Observations of student learning*

a. *Evidence of difficulty*

The tape player scenario provides evidence that learning the relativity of simultaneity is difficult for students, as predicted by the misconceptions model. In the classroom, we find three categories of student behavior that constitute evidence of difficulty: *denial, withdrawal,* and *absurdism.*

i. Denial

Students openly refuse to change their minds about simultaneity. When students see the implications of the tape player scenario, they become very agitated. If one member of a group gets close to recognizing the logical necessity of the relativity of simultaneity, the rest of the group may make loud verbal objections to that member's conclusions, as in the following example.

Dennis: We just figured out that the tape player plays in Alan's frame.

Tom: But it can't. In Beth's frame they hit her at the same time. So she won't hear it.

Jane: But look down here, it's asking if she hears it and if the tape will have wound from its starting position. If the tape is going to play, that's it, it's going to play.

Tom: But it can't play for Beth! She's in the middle! They hit her at the same time!

Dennis: But we just figured out that it plays!

Tom: Right! And then a black hole opens up! And God steps out! and he points his finger and says [shouting] "YOU CAN'T DO THAT!"

In my opinion, this level of objection is a healthy response to the fundamental issues raised by the special theory of relativity.

ii. Withdrawal

In an interview situation, students have no peers with whom to conduct a debate, and their behavior tends to be very different than in a classroom setting. Instead of denying or objecting to the implications of the tape player scenario, they fall silent. Typically, they are unresponsive for thirty seconds or more. During this period they tend not to answer questions or make eye contact. When this period ends, they frequently have changed their minds, deciding, typically, that the tape player must play in all frames of reference. They show low awareness of the fact that they were silent for some time.

iii. Absurdism

Another common response to the tape player scenario is for students to apparently give up on logic, or on making their results consistent with one another. They will "resolve" a paradox by ascribing the results to magic, or to quantum mechanics, as in the following example.

Raj: Wait, so Alan hears it and Beth doesn't? That's one awesome tape player.

Marcus: That's so cool!

Teacher: But when you take the tape out, when you stop the train and you look at the tape, has it been wound or has it not been wound?

Raj: This is what you were telling us last week. That in some universe Sara was wearing purple and in another one she was wearing blue or something.

b. *Evidence of permanence*

The misconceptions model predicts that conceptual change will be not only difficult, but also permanent. Examination questions administered long after students complete the special relativity tutorials provide some evidence that this is the case. Perhaps more importantly, the *single* intervention with the tape player scenario seems to be the primary occasion for learning the relativity of simultaneity. The instructional materials include opportunities for practice, but later exercises have little of the dramatic impact of the first one. Hewson's statement that "individuals *replace* conceptions when faced with new anomalous experiences" appears apt.[16]

4. *Commentary*

Of course not all of student learning in special relativity, even within the relativity of simultaneity, is difficult or permanent. There are aspects with a very different character, as I will show in the next section. However, this fact does not threaten the misconceptions characterization of student reasoning. The model doesn't claim that *all* student ideas are misconceptions of the caliber of absolute simultaneity; it encompasses a spectrum of possibilities, of which absolute simultaneity is only an extreme case. But the extreme illustrates the type. The extreme would also be the focus of the misconceptions-oriented instructor or researcher. Stable, rigid, context-independent ideas would be the most crucial obstacles to successful instruction, and therefore would deserve the most attention. Misconceptions that are "not strongly held" might just take care of themselves, and so might never really merit an instructor's concern.

B. The pieces model of student thinking about the relativity of simultaneity

1. Research results

No existing literature characterizes student understanding of the relativity of simultaneity from a pieces theoretical perspective. In this section, I undertake that characterization myself.

The first task, in a pieces model of student thinking, is to identify the knowledge elements that might be guiding student reasoning. Existing data on student understanding of the relativity of simultaneity is fortunately rich enough to make this identification plausible. In particular, the three beliefs identified previously as forming "the misconception" can be recast in terms of two knowledge pieces: *visual reality* and *ultimate reality*.

The first and third beliefs – that events are simultaneous if an observer receives signals from the events at the same instant, and that every observer constitutes a distinct reference frame – both describe the perceptions of an individual observer, and how that observer interprets those perceptions. We may interpret these two perception-based "beliefs" as both being specializations of a single knowledge piece that I call "visual reality:" the idea that *what you see is what there is*. This is certainly true most of the time in our everyday lives, particularly with the order of observed events. If we *see* a red car go by, and later see a blue car go by, the red car really did go by first.

The second belief, that simultaneity is absolute, is not perceptual in nature, and thus might be a specialization of a different knowledge piece: "ultimate reality," the idea that *things "really happen" in only one way*. Crime victims, police officers, and witnesses might report events differently, for example, but by careful deduction we expect to be able to determine what actually took place.

One can interpret some student statements about the relativity of simultaneity as being applications of these knowledge pieces. For example, one graduate student's statement that "if I see [the events] at different times, they occurred at different times in my reference frame" could be an (incorrect) application of the *visual reality* knowledge piece. A different student who stated, "If we are in relative motion we will measure different distances and so on but if we are all intelligent observers we will all figure out that the events were simultaneous in our rods-and-clocks reference frame," might be (incorrectly) applying the *ultimate reality* knowledge piece. According to that student, things might look different to different observers, but they really happen in only one way (in a "shared" reference frame).

Different tasks seem to activate different knowledge pieces in many students. In particular, tasks in which *people see light signals* elicit a very different pattern of responses than tasks that are more disembodied, such as drawing a spacetime diagram.[23] In particular, the first class of tasks appears to activate visual reality, while the second class activates ultimate reality.

One might equally well model student thinking in terms of other knowledge pieces than the two I have suggested here. The aim of this paper is not to demonstrate the existence and use of particular knowledge pieces, but to illustrate that a pieces model of student thinking can account for our observations.

2. Instructional activities

The instructional materials documented in the research literature were designed by researchers with an implicit misconceptions theoretical orientation. Nonetheless, the materials include activities consistent with the pieces instructional agenda ("refining intuitions"). This section describes those activities.[30]

The tape player scenario described in the previous section is preceded, in the instructional materials, by exercises designed to guide students to formulate appropriate procedures for the measurement of the time of an event. In the first exercise, an observer wishes to know the time at which a beeper beeps but is constrained to a location far from the beeper. The observer is equipped with accurate meter sticks and synchronized clocks, and has assistants who can help. The tutorial asks students to describe two procedures by which the observer can determine the time at which the beeper beeps: (i) using knowledge of the speed of sound in air and (ii) without knowing or measuring the speed of sound first. In this way students articulate for themselves two operational definitions for the time of a distant event: (i) an observer may record the time of arrival of the sound from the beeper, measure the distance to the beeper, and correct for the signal travel time, or (ii) an observer may place an assistant at the beeper and have the assistant mark the time at which it makes a sound. The exercise builds on student understanding of the finite nature of signal travel time, which generally appears to be good.

In a subsequent exercise, students generalize their measurement procedure for the time of an event. They are asked to devise an arrangement of observers and equipment for recording the position and time of an arbitrary event. The term "reference frame" is introduced to describe the system of observers. The term "intelligent observer" is defined as an observer who takes into account signal travel time.

After students have constructed the concept of a reference frame, they are asked to apply it. Students are told that a horn is placed between an observer and a distant beeper. The observer hears a honk and a beep at the same instant. Students are asked two questions. The first is to describe a method by which the observer can measure the time separation between the emission of the two sounds in his reference frame without knowing or measuring the speed of sound first. They are also asked whether, in the observer's reference frame, the beeper beeps before, after, or at the same time as the horn honks. Students use the idea of a reference frame and the definition of the time of an event to conclude that, in order for the signals to reach the observer simultaneously, the more distant event must have occurred first.

We can model this sequence of instructional activities as a process of refining intuitions. The intuition that "what you see is what there is," which I termed the *visual reality* knowledge piece, has two competing specializations in this situation: one that the order in which you see events is the order in which they occur, and one that the order in which you see events is the order in which signals generated by the events arrive at your location. The instructional materials are arranged so as to activate students' robust knowledge pieces about the finite nature of signal travel time, which support the second specialization.[31]

3. *Observations of student learning*

a. Evidence of ease

Earlier, I described the "belief in absolute simultaneity" as changing in a misconceptions-like way – in a way that appeared to be both difficult and permanent. The process of change for the other two "beliefs," which correspond to the visual reality knowledge piece, appears to be very different – it is relatively *easy* and *temporary*. In the classroom, categories of student behavior include *common sense, comfort,* and *fluidity.*

i. Common sense

Far from appealing to magic, students dealing with issues of signal travel time use *common sense,* bringing in everyday experience and easily reconciling physics definitions with their own interpretations. They know from their ordinary lives that signals take a finite amount of time to travel from one place to another, and they extend that knowledge readily to light signals.

ii. Comfort

When students consider changing their mind about the idea that every observer constitutes a distinct reference frame, they are quite *comfortable* with the prospect. Although they initially tend to neglect signal travel time, they are willing and able to include it when so prompted. When working through the first section of the instructional materials, in which issues of signal travel time are addressed, students appear relaxed. They answer questions quickly and they agree readily with one another.

iii. Fluidity

If anything, students agree with each other *too* readily in the activities that deal with signal travel time, sometimes seeming to change their minds without actively considering alternatives. Students change their minds *fluidly:* they change, and then change back, progressing and relapsing, revisiting the same ideas many times as if from scratch each time. In developing the described instructional materials, the developers found it necessary to deal with this issue by revisiting questions of observer location and signal travel time repeatedly in the course of the exercises. This need for repetition is in stark contrast with the tape player exercise, for which a single iteration appears to be sufficient.

Further evidence of fluidity is found in observations from individual student interviews. In that setting, we found that students were easily able to shift between responses that we can account for with the visual reality knowledge piece, and responses that appear to draw on the ultimate reality knowledge piece. In one interview protocol, for example, two volcanoes (Mt. Rainier and Mt. Hood) erupt simultaneously in the reference frame of the observer midway between them. In the following sequence, a student initially claims that an observer closer to Mt. Rainier (named Bob) would measure Rainier's eruption to occur first, but then changes his mind unproblematically when reminded to account for signal travel time. [I = interviewer, S = student]

> S: Bob, where is Bob. He's at rest in the lab frame at the base of Mt. Rainier. I would say that event 1 [Rainier's eruption] would happen before event 2 [Hood's eruption] in that case since he's much closer to Mt. Rainier. He would measure that. That's what he'd measure.

I: What would be the measurement that he would do?

S: He could just use that measurement, you know, looks, well if you have very good sight, sees Mt. Rainier erupt and look and the light takes a finite time to travel from Mt. Hood to him and then say, Mt. Hood erupts some time later. So in his frame event 1 happens before event 2.

[Later in the interview]

I: Suppose that – it takes some time for the signal to get from Hood to Bob.

S: Yeah. Bob's at Rainier, yeah.

I: So there's the time that the signal actually gets to Bob, and then there's the time that Hood actually erupted, which was a while ago. However long ago. A thousandth of a second ago. Suppose that all of the observers corrected for that, so that they figured out how long it took the signals to get to them and so on, and then they subtracted that off from the time that they read at their location to figure out when the eruption actually happened according to their measurement.

S: You mean so they could determine the precise time?

I: So that they could determine the time that it erupted correcting for the amount of time it took the signal to get from the mountain to them.

S: I would say that since Bob and Alan are both not moving the time would just be x/c. The time it takes the signal to get there. So their correction would be correct.

I: So suppose they did that correction, then what would they decide about whether event 1 happened before, after, or at the same time as event 2?

S: For Alan [the observer in the middle], I think it's easy, if he corrects for both times the two effects would just, I don't know if cancel is the right word, but he measured them simultaneous anyway and then he corrects for the time it takes to get there since the path lengths are equal he'll still measure them at the same time. For Bob, when he corrects, since he's at rest, I believe he would then say yes they did happen at the same time. (graduate student)

b. Evidence of temporariness

The pieces model predicts that in many cases students have insights relatively easily, but that sometimes those insights do not "stick" – students who have decided on a particular conclusion don't necessarily stay committed to it for very long. This lack of commitment is typical of student responses to questions about the order of events for different observers at rest relative to one another. Such questions recur regularly in the instructional materials, and each time many students revisit the idea as if considering it for the first time. Apparently, student understanding of reference frame issues has the quality of temporariness.

More evidence of temporariness comes from the fact that in some implementations of the instructional materials, students take a diagnostic quiz after completing the activities about reference frame (described in section 0), but before experiencing the tape-player scenario (section V.A.2). On that diagnostic quiz, students as a group do much better than before the reference-frame activities, but not nearly as well as they do after the tape-player scenario.[25] Apparently the understanding they gain in doing the reference-frame activities is not really "fixed" until they learn the relativity of simultaneity.

4. *Commentary*

The three properties of students' interaction with the material outlined above are consistent with one of the possibilities described by a pieces model of conceptual change. They are not well characterized by the misconceptions formulation. Of course, there are aspects of student learning that don't have the properties discussed in this section. There are times, as we've seen, that change is difficult and permanent, instead of easy and temporary. The pieces model easily accommodates these cases as well as the more "exemplary" case described in this section. The pieces model does not *insist* that student responses be fluctuating, context-dependent, and so on; it only *allows* for those possibilities.

C. Summary

The available data about student reasoning in special relativity favors *both* misconceptions and pieces models of student reasoning – sometimes one, and sometimes the other. Regarding the fact that simultaneity is relative, student ideas are more consistent with a misconceptions model (or with the misconceptions-like part of a pieces model). Students learning about the nature of a reference frame, on the other hand, behave more as predicted at the other end of the spectrum of possibilities in the pieces model. My analysis suggests that we may find it useful to explicitly recognize both the misconceptions-like and the pieces-like extremes of student behavior that we observe in instructional and research settings. Researchers should expect to locate both wrong-but-coherent ideas and correct-but-inappropriately-applied ideas in students. Instructors should be prepared to implement either intuition-refinement or elicit-confront-resolve strategies, depending on the nature of student reasoning in particular situations.

VI. CONCLUSION

Many of us who are physics instructors and physics education researchers use either the pieces model or the misconceptions model to inform our instructional and research agendas (perhaps implicitly). We necessarily see our students through a theoretical lens that shapes our interpretations. The data about student thinking in special relativity, then, suggests a challenge for many of us: to keep in our awareness the extremes of student behavior that fall outside the norm suggested by our model of student thinking. For some of us, that means keeping the properties associated with the pieces model in mind, since we tend to default to the misconceptions model.[32] We can keep ourselves open to observing diverse types of student ideas by thinking in terms of individual properties of student ideas, including those outlined in this paper (determinacy, variability, and so on).

ACKNOWLEDGEMENTS

The work described in this paper has been a collaborative effort by members of the Physics Education Research Group and other colleagues. The analysis was inspired by insightful questions posed by David Hammer. I am especially grateful for the intellectual contributions of Michael C. Wittmann, E. F. Redish, Andrew Elby, and two anonymous reviewers. This research is supported in part by the National Science Foundation (REC 008-7519).

---

[1] J. Clement, "Student preconceptions in introductory mechanics," Am. J. Phys. **50**(1), 66-71 (1982).

[2] M. McCloskey, "Naive theories of motion," in *Mental Models*, D. Gentner and A. Stevens, ed. (Lawrence Erlbaum, Hillsdale, NJ, 1983), pp. 299-324.

[2] D. Hestenes, M. Wells, and G. Swackhamer, "Force Concept Inventory," Phys. Teach. **30**, 141-158 (1992).

[3] D. I. Dykstra, C. F. Boyle and I. A. Monarch, "Studying conceptual change in learning physics," Sci. Ed. **76** (6), 615-652 (1992).

[4] S. Vosniadou, "Capturing and modeling the process of conceptual change," Learning and Instr. **4**, 45-69 (1994).

[5] A. diSessa, "Towards an epistemology of physics," Cognit. and Instruct. **10** (2-3), 105-225 (1993).

[6] J. Smith, A. diSessa, and J. Roschelle, "Misconceptions reconceived: A constructivist analysis of knowledge in transition," J. Learning Sciences **3**(2), 115-163 (1993).

[7] J. Minstrell, "Facets of students' knowledge and relevant instruction," in *Research in Physics Learning: Theoretical Issues and Empirical Studies*, R. Duit, F. Goldberg and H. Niedderer, eds. (IPN, Kiel, Germany, 1992), pp. 110-128.

[8] As discussed in A. Elby, "Helping physics students learn about learning," Phys. Educ. Res., Am. J. Phys. Suppl. **69**, S54-S64 (2001).

[9] A more theoretically precise definition of "context" (and references to other definitions) appears in D. Hammer, A. Elby, R. E. Scherr, and E. F. Redish, "Resources, framing, and transfer," to be published in *Transfer of learning: Research and perspectives*, J. Mestre, Ed. (Information Age Publishing, 2004).

[10] M. S. Sabella, "Using the context of physics problem-solving to evaluate the coherence of student knowledge," Ph.D. dissertation, Department of Physics, University of Maryland, 1999 (unpublished).

[11] See Ref. 9.

[12] L. Lising and A. Elby, "The impact of epistemology on learning: A case study from introductory physics," Am. J. Phys. **73** (4), 372-382 (2005).

[14] A review of various terms used in the literature appears in J. H. Wandersee, J. J. Mintzes, and J. D. Novak, "Research on alternative conceptions in science," in *Handbook of Research on Science Teaching and Learning*, D. Gabel. ed. (Simon & Schuster Macmillan, New York, 1994), pp. 177-210.

[15] This is in contrast to the idea that misconceptions are items on an unstructured list of wrong student ideas. An example of such a list appears at http://www.amasci.com/miscon/opphys.html.

[16] Hewson, for example, states that individuals "replace [conceptions] when faced with new anomalous experiences." In P.W. Hewson, "A case study of conceptual change in special relativity: The influence of prior knowledge in learning," Int. J. Sci. Educ. **4**, 61-78 (1982), p. 63.

[17] See, for example, L.C. McDermott, Oersted Medal Lecture 2001: "Physics education research: The key to student learning," Am. J. Phys. **69** (11), 1127-1137 (2001).

[18] D. Hammer, "Student resources for learning introductory physics," Phys. Educ. Res., Am. J. Phys. Suppl. **68**, S52-S59 (2000).

[19] D. Tirosh, R. Staby, and S. Cohen, "Cognitive conflict and intuitive rules," Int. J. Sci. Educ. **20**, 1257–1269 (1998).

[20] See, for example, the discussions of "systematicity" in Ref. 5 and "patterns of association" in E. F. Redish, *Teaching Physics* (John Wiley & Sons, Hoboken, NJ, 2003).

[21] A. diSessa and B. L. Sherin, "What changes in conceptual change?" Int. J. Sci. Educ. **20**(10), 1155-1191 (1998).

[22] For other accounts of intuition refinement as an instructional approach, see A. diSessa, "Momentum flow as an alternative perspective in elementary mechanics," Am. J. Phys. **48**(5), 365-369 (1980); J. Minstrell, "Explaining the 'at rest' condition of an object," Phys. Teach. **20**, 10-20 (1982); J. Clement, D. Brown, and A. Zeitsman, "Not all preconceptions are misconceptions: Finding 'anchoring conceptions' for grounding instruction on students' intuitions," Int. J. Sci. Educ. **11**, 554-565 (1989); D. E. Brown, "Re-focusing core intuitions: A concretizing role for analogy in conceptual change," J. Res. Sci. Teach., **30**(10), 1273-1290 (1993); D. J. Grayson, "Concept substitution: A teaching strategy for helping students disentangle related physics concepts." Am. J. Phys. **72**(8), 1126-1133 (2004).

[23] R. E. Scherr, P. S. Shaffer, and S. Vokos, "Student understanding of time in special relativity: Simultaneity and reference frames," Phys. Educ. Res., Am. J. Phys. Suppl. **69**, S24-S35 (2001).

[24] As I discuss later, I had an implicit misconceptions theoretical orientation at the time that the research was conducted, which may have biased me toward stability in the data.

[25] R. E. Scherr, P. S. Shaffer, and S. Vokos, "The challenge of changing deeply held student beliefs about the relativity of simultaneity," Am. J. Phys. Suppl. **70**(12), 1238-1248 (2002).

[26] G. J. Posner, K. A. Strike, P. W. Hewson, and W. A. Gertzog, ''Accommodation of a scientific conception: Toward a theory of conceptual change,'' Sci. Educ. **66**, 211–227.

[27] S. Panse, J. Ramadas, and A. Kumar, ''Alternative conceptions in Galilean relativity: Frames of reference,'' Int. J. Sci. Educ. **16**, 63–82 (1994); J. Ramadas, S. Barve, and A. Kumar, ''Alternative conceptions in Galilean relativity: Inertial and non-inertial observers,'" *ibid.* **18**, 615–629 (1996); E. Saltiel and J. L. Malgrange, '' 'Spontaneous' ways of reasoning in elementary kinematics,'' Eur. J. Phys. **1**, 73–80 (1980); A. Villani and J. L. A. Pacca, ''Students' spontaneous ideas about the speed of light,'' Int. J. Sci. Educ. **9**, 55–66 (1987); ''Spontaneous reasoning of graduate students,'' *ibid.* **12**, 589–600 (1990).

[28] For more about the effect of researcher agenda on data collection and interpretation, see R. E. Scherr and M. C. Wittmann, "The challenge of listening: The effect of research agenda on data collection and interpretation," and M. C. Wittmann and R. E. Scherr, "Student epistemological stance constraining researcher access to student thinking: An example from an interview on charge flow," in S. Franklin, K. Cummings, and J. Marx (Eds.), Physics Education Research Conference Proceedings 2002.

[29] It is also possible to characterize this instructional sequence as a process of refining intuitions, consistent with the pieces model of student thinking. The relevant knowledge piece is the one termed "ultimate reality" in section V.B.1 below (page 8). The common (inappropriate) refinement of that knowledge piece is "Events that are simultaneous in one frame are simultaneous in all frames;" the correct refinement is "Events that happen in one frame happen in all frames." The classroom observations, however, support the misconceptions characterization of the instructional sequence.

[30] It should come as no surprise that the instructional materials are not perfectly theoretically consistent, especially when the theoretical orientation of the researchers was merely implicit. In any case, the goal of the materials was not to enact a particular instructional agenda, but to teach students particular concepts in physics. The approach was pragmatic.

[31] One may also model this instructional sequence in terms of misconceptions; Ref. 25 does so. The classroom observations, however, support the pieces characterization of the instructional sequence.

[32] A. Elby, "Why teachers should care about cognitive theory," Paper presented at 125th National Meeting of the American Association of Physics Teachers, Boise, ID (2002).