Chapter 5: Using Complex Problems to Evaluate Coherence in Physics Understanding

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Introduction

In this chapter we examine how students approach complex problems that require the integration of multiple schemas. In order for students to solve complex problems, it is important that our students integrate the knowledge from different physics topics into a coherent package. When given a problem-solving task students will activate a schema that they will use to solve the problem. Unfortunately, the schema that is activated is often not sufficient to solve the problem and it is difficult for many of our students to activate the relevant schema. We also begin discussing isolated schemas in students' qualitative and quantitative knowledge.

The first study we present involves one-on-one interviews with advanced students studying physics. These students were given a problem involving the concepts of dynamics and the work-energy theorem. The data shows a clear distinction in students who are able to go back and forth between knowledge for related topics and students who do not exhibit coherence between different physics topics. We present interview transcripts along with *reasoning maps* that represent the students' solutions to the problem. The transcripts and maps can help researchers and instructors look for coherence between different physics topics.

After looking at the case studies with advanced students we look at the way classes of undergraduate students in the Physics 161 course perform on two variations of the *dynamics-work energy problem*. We present responses to an ungraded quiz, a bridging problem, and a one-on-one interview protocol. We find that many of the undergraduate students exhibit schemas that are characterized by local coherence, but not by the global coherence that would characterize an expert problem solver. The data also shows that qualitative force-based questions cause the students to activate a force schema, which was isolated from the concepts of work and energy. Students who were not presented with the qualitative questions were more likely to apply their knowledge of work and energy to solve the problem.

To show the pervasiveness of these difficulties, we also present data from the physics 262 class. Student responses to two versions of a problem involving the work done by a piston in a thermodynamic process are analyzed. The data shows that qualitative questions, asked on one version of the problem, caused students to perform worse on the question. When students were not presented with the qualitative questions they were more likely to activate a quantitative schema they could use to solve the problem.

Dynamics and Work-Energy Problem

Introduction

Two versions of a dynamics-work energy problem were administered to advanced physics students and undergraduate engineering students. The original version was given to undergraduate students in a bridging problem format and in a one-on-one interview format in the Spring '97 semester. A revised version was written in the Fall '97 semester. It has been given as a one-on-one problem-solving interview with advanced students and as an ungraded quiz with undergraduate engineering majors. All students involved in these studies had completed instruction on the material.

Graduate Student Interviews

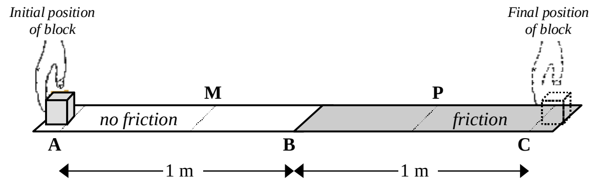
We presented the dynamics-work energy problem to six students enrolled in graduate classes at the University of Maryland, as a problem-solving interview.[1] The problem was written by the author and is shown in Figure 5 - 1 along with a model solution. The graduate students were first given a short version of the question, consisting of only parts a and d. The long version, shown in the figure, was given to one of the students (after he was having difficulty solving the short version) to help him solve the problem.

In a problem-solving interview the researcher provides the student with a problem and asks the student to solve the problem explaining what he or she is thinking and writing.[2] It was usually not difficult to get the students to talk about the physics. In addition to getting the students to explain their work out loud, the researcher must be careful not to guide the students. Instead, the researcher must ask questions to get the students to provide a clear record of their understanding and reasoning. The transcripts presented in the dissertation include the code name of the student and the gender of the student. The transcripts contain the following short-hand notation: [ ] indicate comments about the interview added after the fact, {—} is a short pause, [pause] is a long pause, {…} indicates that unimportant words were purposely omitted from the transcript, and (IA) indicates that the words were inaudible.

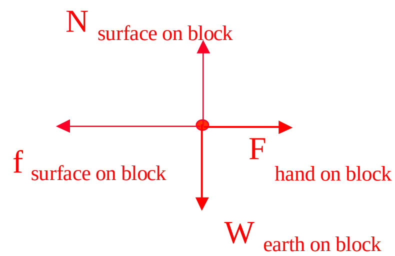
The pool of volunteers contained one upper-level undergraduate student who was enrolled in graduate level classes, three first-year graduate students, and two second-year graduate students. The complete transcriptions of the advanced student interviews presented in this dissertation are included in Appendix C.

The three first-year graduate students had many conceptual difficulties with the problem. The two-second year and the upper-level undergraduate student answered the question correctly, with little or no prompting. The undergraduate student seemed to exhibit the most coherent knowledge. He would continuously go back and forth between the concepts of work-energy and forces. These excerpts of the interview transcripts provide insight into how advanced students solve this problem. (The names presented are code names chosen by the students.)

A hand applies a force to a small 1 kg block from "A" to "C." The block starts at rest at point "A" and then comes to rest at point "C." The block moves along a frictionless surface from "A" to "B" and then travels an **equal** distance along a surface with friction from "B" to "C" with the force of the hand **remaining constant**. The force of the hand is 2 N to the right and the distance from "A" to "C" is 2 m. (See figure above.)



a) **Draw a free body diagram for the block when it is at "P."**



b) Is the magnitude of the net force acting on the block at "M" greater than, less than, or equal to the magnitude of the net force acting at "P"? Explain your reasoning.

Since the change in kinetic energy from A to B and from B to C are equal, the magnitudes of the net works are equal therefore the magnitudes of the net forces are equal.

c) i. Draw a vector representing the acceleration of the block at "P." If the acceleration is zero state that explicitly.

Since the block is coming to rest at C and the force of the hand and the force from the friction are constant the acceleration must be toward the left.

ii. Does the magnitude of the acceleration increase, decrease, or remain the same as the block moves from "B" to "C"? Explain.

The magnitude of the acceleration vector remains the same since the two forces acting on the block are constant.

d) **Calculate the coefficient of kinetic friction μ.**

From (b) we know that $T - f = -T$ *and* $f = \mu mg$ *therefore* $\mu = .41$

Figure 5 - 1

Revised version of the original bridging problem with a model solution. This problem was asked as an ungraded quiz in the Physics 161 class.

Eagle

A model solution to this problem would involve the application of multiple physics principles and concepts. In particular we would like to see the students tying the ideas of the work-energy theorem to the ideas of force. This particular section of a transcript is from the interview with Eagle, the upper level undergraduate student. Eagle integrates different physics concepts into a schema that he uses to solve the problem. In this section of the interview, Eagle is looking for the coefficient of kinetic friction. He has already drawn a correct free-body diagram.

> *E: Let's see— the block travels an equal distance (IA) with the force remaining constant — Let's see — (IA) — let me think — does it say anything about the speed — it doesn't — oh okay I see — I suppose the force is being applied until the end of the trajectory and the block stops due to the friction and not that the hand stops.*
>
> *I: The hand keeps applying from A to C.*
>
> *E: Okay — So I'm going to calculate the kinetic energy that the block has until point B —*
>
> *I: How come you're doing that?*
>
> *E: To find out what the total — what the energy it loses on the friction surface is — which should tell me —yes of course — what the force acting against it was. So that is going to be 2 Newtons times 1 meter, which is 1 Joule and that is equal to ½ m v squared — … v being the velocity of the block — and that is exactly what it is going to lose which means the force — the friction force should be equal to 4 Newtons in the other direction — meaning … in the direction C to A, or to the left — So that the net force being applied on the block is 2 Newtons in the other direction so that the loss of energy is equal to the gain of energy in the first half of the trajectory.*

At this point Eagle is connecting the ideas of force and work-energy and using the two concepts to solve the problem. He makes these connections throughout.

> *I: How did you know the loss in energy was the same as the gain in energy?*
>
> *E: Because it started at rest and it ends at rest — I suppose. I assume that is what it means when it travels an equal distance — …* [Rereads part of the question.] *So the force is 4 Newtons, which is equal to the magnitude of the normal force times μ — the kinetic friction coefficient— … so the normal force is equal to the weight of the block which is 1 kilogram times 9.8 so — μ is equal to 4 over 9.8 which is about .4.*

I: … Can you compare the magnitude of the net force at M to the net force at P — how would they compare?

E: The magnitude of the net forces? — well they should be equal and opposite —

I: And how did you know that?

E: By the same argument — because I assume that the force due to friction — which is constant along the whole surface since the weight of the block doesn't change — I assumed that it was equal and opposite to the force — I'm sorry — I mean the sum of the force being applied by the hand and the friction should be totally equal and opposite to just the force applied by the hand on the block so that the loss of energy is equal along the same distance traveled — so they will be equal and opposite.

The question concerning the magnitude of the net forces on the two regions was particularly difficult. Eagle answers correctly without hesitation by applying the integrated knowledge in his schema for this problem. Even some of the graduate students who solved for μ correctly answered the question about the net force incorrectly (at least at first.)

Peter

Peter is a 2^{nd} year graduate student. Like Eagle, he solved the problem correctly in a short amount of time. Unlike Eagle, he used a dynamics schema to solve the problem, instead of a single schema containing knowledge of dynamics and work-energy. It is interesting that even though Peter has correctly solved the problem he first states that the magnitude of the net force at P is smaller than the magnitude of the net force at M. He does correct himself soon after. The excerpt below is taken from the interview.

P: … so it will be minus 4 over 2 — which is the same acceleration — but opposite sign … and now we can plug it back into this equation (points to $F - \mu_k N = —ma$) for the force and the coefficient of friction and so we have 2 Newtons — we are going to put numbers immediately — 2 minus μ_k will be equal to F plus ma over N — μ_k is 2 Newtons plus … I'm using here the absolute values of the acceleration over the 10 … so we are going to have 0.4.

I: How does the net force at M compare to the net force at P?

P: Umm — the net force at P is smaller than at M — by the amount of the kinetic friction. The net force is smaller by this amount because — the y components of the two forces are canceled out. So the only difference — they will be cancelled out at point A too — and the only difference comes at point P *because of the introduction of the force of friction, which is directed opposite to the applied force.*

I: So can you draw me a vector for the net force at point P — how would that look?

P: It would look — almost caught me there — yeah the force of friction is bigger than the force — this is net force — since the acceleration is negative — so negative y direction — net force — according to the famous Newton's second Law should … I'll put it here to support my statement — the net force should be in the same direction as acceleration —

Granola

Granola, a 1st year graduate student, exhibited many conceptual difficulties while solving this problem. In addition we see a lack of coherence in his knowledge, evident from his inability to go back and forth between the two topics and his contradictory statements in the interview.

Granola identifies all the forces in the free-body diagram correctly. He later incorrectly describes the force of the hand to be greater at point P because the block is still moving toward the right. This indicates that Granola has the common misconception that force is proportional to velocity. In addition, Granola states that at point C the force of the hand would be equal to the force of friction. (Although this is true after the block has stopped, it seems that Granola makes the statement because the velocity is zero at C and not because it remains at zero.) When solving for the coefficient of friction Granola sets the two forces equal and solves for μ. But this solution does not feel right to him. In the interview Granola tries to draw from different physics principles when his analysis using dynamics does not seem correct, but these alternate principles lead to dead ends and Granola goes back to thinking about the forces.

G: Yeah. So wait maybe that should be right. [pause] *That's not right at all — … it doesn't seem right to me — just give me a second — I just started to solve it assuming it would be easy — then realized maybe it wasn't as easy as I thought. Could also do the work — the work from here* [points to A] *to there* [points to B] *— no that's got nothing to do with it — no — friction — 2 Newtons — the force of the hand remains constant — because according to this — this wouldn't be stopped there — it would stop there if this were the case — … if this were the case if it was 9.8 Newtons and 2 Newtons because that is the same circumstance that we have at B — so it would stop at B the way I have it set up — why would it stop at C? There is something about the 1 meter that I'm not getting — I'm not thinking very well. There has to be something to do with … the velocity — with the hand force I'm thinking — so I think — there is something to do with friction apparently —*

This excerpt shows that Granola is using pieces of his knowledge to try to trigger some procedure that he can use to solve the problem. In this small excerpt he brings up work, the force of the hand, the force of friction, the 1 meter, and the velocity. Unfortunately, these items lead to dead ends even though these ideas are all closely related to the work energy theorem and the definition of work, stating that $\Delta KE = W = \sum F \cdot d$. Granola is therefore unable to activate a schema containing knowledge about work and energy. Without links between these pieces of knowledge Granola cannot get to the work-energy theorem. Eventually Granola obtains the correct solution to the problem after some assistance from the interviewer.

Erica

Another student who had serious conceptual difficulties with the material was Erica. Like Granola, Erica drew a correct free-body diagram for the block but seemed to be confusing velocity with acceleration throughout the interview. One way to look at this type of error is in terms of diSessa's p-prims. P-prims are small logical building blocks that can be applied in many different situations.[3] The particular p-prim that seems to come up here is the maintaining agent p-prim, which states that a force is required to keep an object moving.[4] A section of the transcript follows.

> *I: How does this force* [friction] *compare to that force* [hand]*?*
>
> *E: Well if it's still moving forward then this* [points to force of hand] *is bigger then this* [points to friction force] *— it's not enough to stop it.*

This statement is particularly interesting because Erica states NII correctly during the interview. Erica also believes that once the block is stopped the net force has to be zero, despite the fact that she states NII correctly. The excerpt below shows that Erica is very unwilling to give this up, even though she has already written down the algebraic form of NII. Here we see that her qualitative responses and her quantitative responses are contradicting each other even though they are being presented very closely in time. Erica seems to have a conceptual form of NII (force implies motion), which directly contradicts the quantitative form of NII ($\sum F = ma$).

> *I: So what happens at point C?*
>
> *E: It stops — zero — velocity equals zero — …Oh — I see what your saying — but that is because the force is not acting anymore — the hand is not pushing anymore —*
>
> *I: The hand acts all the way to point C.*
>
> *E: Oh — so it just stops at point C — for no reason? — …*
>
> *I: The force of the hand is remaining constant from point A to point C*

E: Then if there is no impediment there then friction is greater than the hand pushing it.

I: How did you know that? How did you know friction was greater?

E: How? Because the force of the hand is the same — so friction must have dominated that.

I: How did you know it dominated?

E: Because it came to a stop —

I: Before, you said this force [points to force of hand] *was greater than the force of friction.*

E: Cause it was moving that way … — The coefficient must have been — No — Not the coefficient — I don't know — if the force of the hand is the same then the friction must have been different.

Advanced students' reasoning maps

These interviews show that even advanced students exhibit fragmentation between related physics topics. Three of the six students had many contradictory remarks in their interviews. These contradictions were often hard for the students to resolve. This was partially due to a weak conceptual understanding of the material and not attaching conceptual meaning to the equations. These students were also unable to go back and forth between the ideas of force and work and energy.

We present *reasoning maps* to represent some of the interview data. Each main statement from the interview is presented in the map, with a link to the next statement and each statement is coded. Statements are shaded lightly (yellow in the color version) if they are based on ideas that come from dynamics knowledge, dark gray (blue in the color version) if they come from work and energy, speckled if it is unclear where they come from, and shaded darkly (red in the color version) with words written in italics if the statement was made by the interviewer.

The maps show that the three students who had difficulty with the problem had distinct force and work-energy schemas and had difficulty going between them. The maps of two of the three students who had difficulty with this problem are shown in Figure 5 - 2 and Figure 5 - 3. Figure 5 - 2 is a map of Granola's interview. We can see that he primarily uses the ideas of force and dynamics in the interview. Although he mentions statements about work and energy they lead to dead ends and he goes back to thinking about the forces and the motion of the block.[5] The map also shows that Granola makes many contradictory remarks in his interview. Some of these inconsistencies would not exist if the appropriate links were made between different knowledge elements.

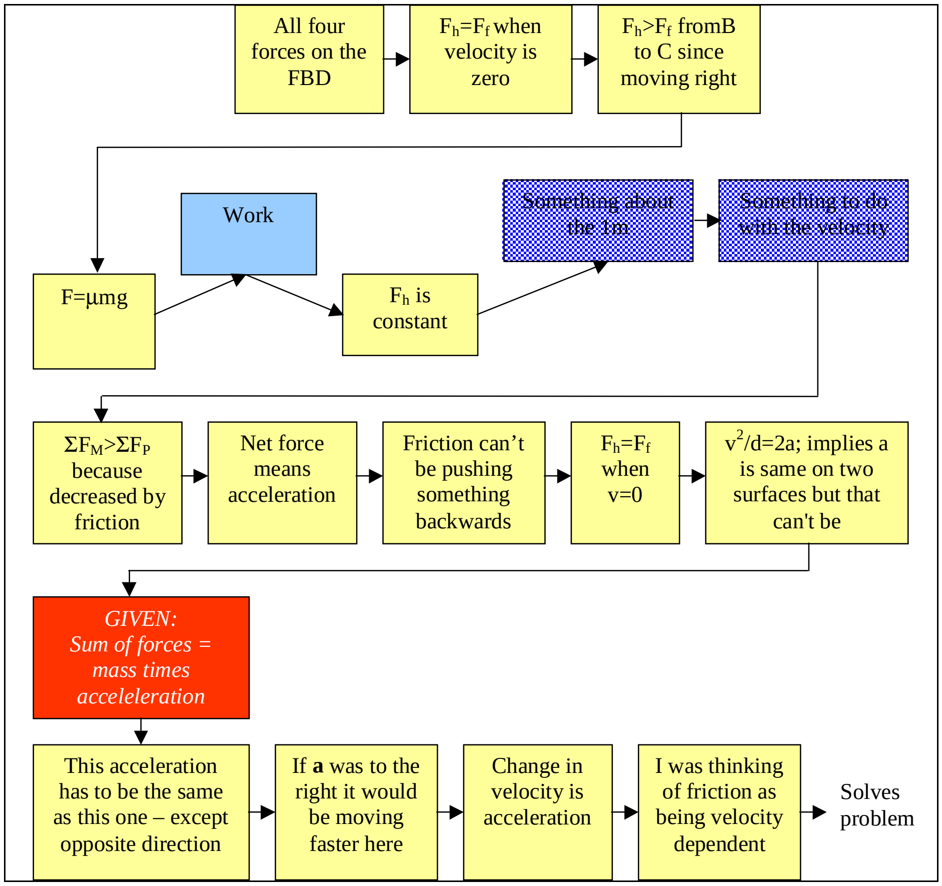


Figure 5 - 2

Reasoning map showing the main statements from Granola's interview. The map shows that although he tried to bring up the ideas of work and energy they lead to dead ends.

Erica's map is shown in Figure 5 - 3. It shows that she also tries to solve the problems by thinking about the forces involved in the situation through most of the interview. I asked her three questions relating to the work-energy theorem but she had difficulty tying these ideas into her analysis using forces.

In contrast, the other three students performed very well on the problem. These three students provide us with a picture of how experts might solve the problem. Connections between content knowledge are made more easily for these students. In addition the experts had more correct conceptual knowledge that they attached to the equations.

Two maps are shown in Figure 5 - 4 and Figure 5 - 5 for the students who solved the problem correctly. Even though both students did not have difficulty with the problem and each students' statements were consistent and correct, the two solutions were quite different. Eagle went back and forth between his knowledge of dynamics and his knowledge for work and energy. Peter primarily used the ideas that would be associated with a dynamics schema. When Peter needed some additional information he used a formula from kinematics. (This formula could also be interpreted as coming from the work-energy theorem but Peter did not explicitly make this connection.) We have shaded the statement lightly (yellow) because he uses the form of the equation usually introduced in dynamics. Peter's reasoning map shows that he has a coherent force schema but we cannot say anything about his work-energy schema or how well these topics are integrated. Eagle's map is shown in Figure 5 - 4 and Peter's map is shown in Figure 5 - 5.

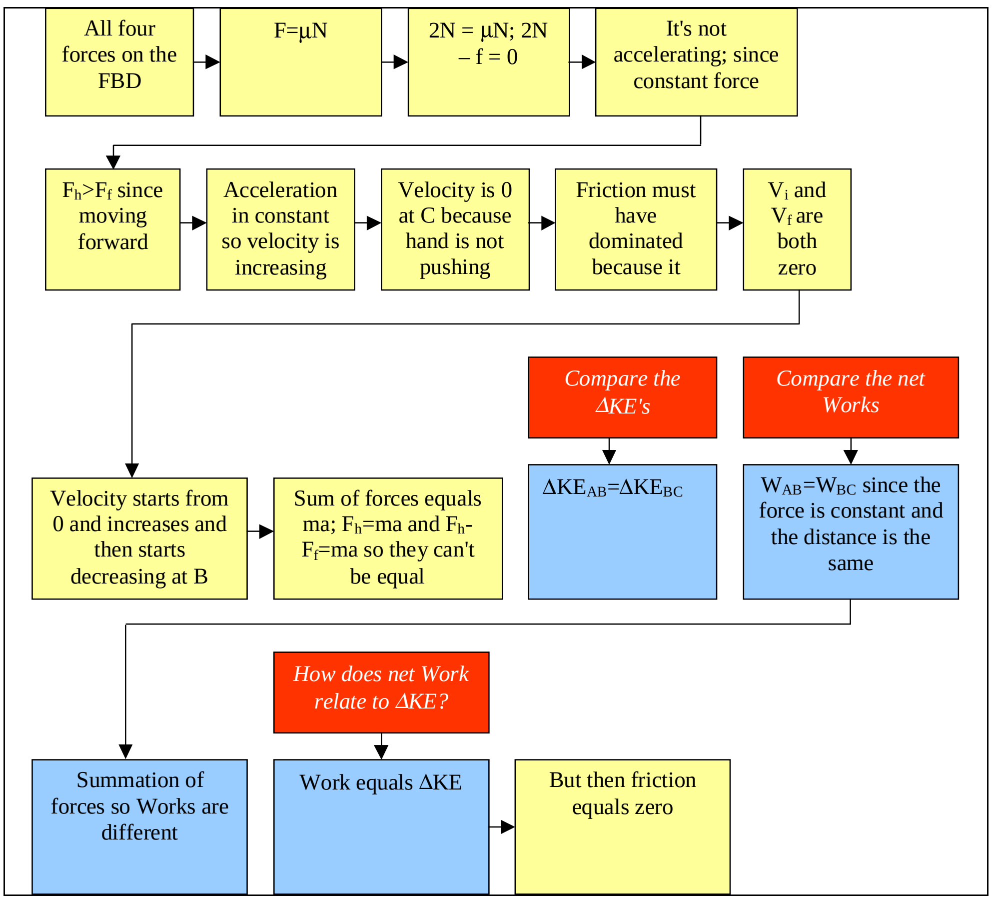


Figure 5 - 3

Erica's reasoning map showing that even with hints from the interviewer it was difficult for her to reconcile the dynamics information with the work and energy information.

The interviews and the reasoning maps show what we mean by a schema. They are strong patterns of association between particular knowledge elements in response to a given context. They show that for different individuals the same knowledge can be connected in different ways and for different individuals connections can be strong or weak.

In addition, we observed that Granola had a similar cognitive style (i. e. he could also be described as a lateral thinker.) In his interview he attempted to trigger other types of knowledge, from different physics topics; recall the excerpt where he brings up work, velocity, and the distance the block travels. In that section of the interview he is thinking laterally by attempting to activate knowledge from different topics. But because his schemas are isolated, these attempts lead to dead ends.

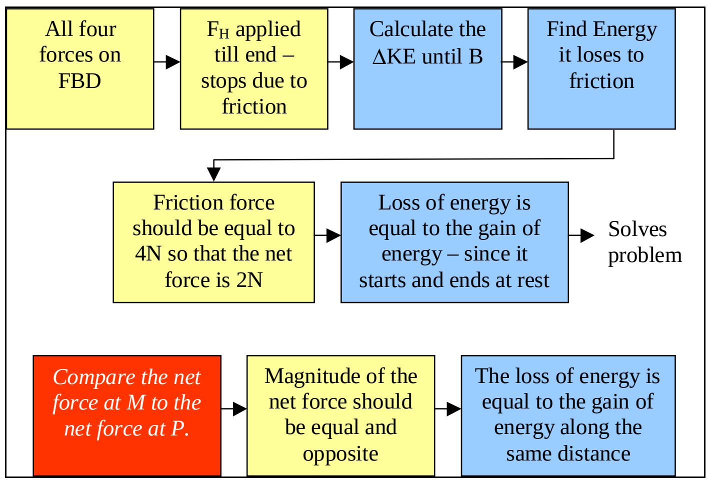


Figure 5 - 5

Eagle's reasoning map showing that he was able to go back and forth between the schemas for force and work and energy.

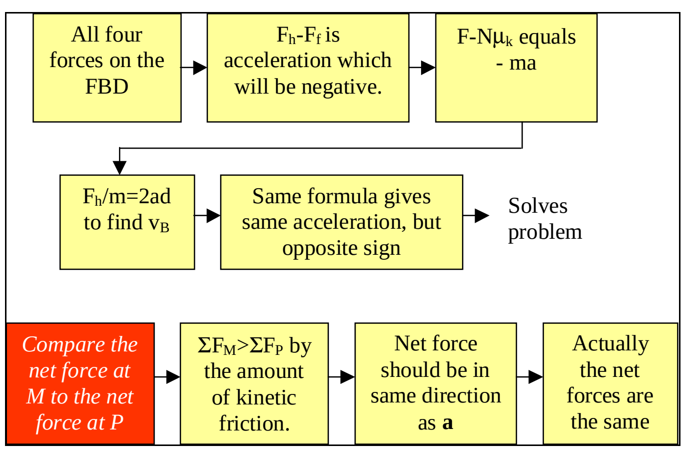


Figure 5 - 4

Peter's reasoning map showing that even though he used the knowledge from the dynamics schema almost exclusively he was able to solve the problem in very few steps.

Analysis of Ungraded Quiz

A short version and a long version of the question shown in Figure 5 - 1 was given to students in the physics 161 course without tutorials. Only the questions in boldface appeared on both the short and the long version of the problem.

Physics 161 was divided into three classes and each class was taught by a different instructor. The question was asked in the recitation sections of the class in the format of an ungraded quiz. The short version of the question was administered to 40 students and the long version was administered to 69 students. The short version of the question was asked in the two smaller classes and the long version was asked in the large class. Because the quiz was asked in the recitation sections, and not all students attend recitations, not all students in the class participated in the study. The quiz was allotted 15 minutes of the recitation.

The results on the ungraded quiz indicate that, although many instructors would like to believe that conceptual questions help the students trigger the correct answer, it can actually hurt their performance. These results are both surprising and disturbing. Many instructors believe that our students think, learn, and organize their knowledge the way we (physicists) do, except that they have less physics content knowledge.[6] This leads some instructors to a number of assumptions as they teach their course. They often ignore student epistemologies and ignore the way students organize their knowledge. We, as instructors, often assume that if we can get students to learn a set of items, they will organize their knowledge of this set of items the same way we have them organized. In reality our students often do not make the same links between different items that we do. Cues that help us activate a set of different interrelated schemas do not necessarily help our students activate those schemas. In addition, some of these cues, or triggers, can cause students to activate a particular schema that may be isolated from the relevant schema for a given task.

We will first discuss the student responses on the long version of the question and then show the results of a comparison on the responses to the last part of the question, which was answered by students on both versions of the problem.

Part a of the question asked students to draw a free-body diagram for the block when it was on the surface with friction. Almost all students correctly identified all four forces on the block. In most cases it was difficult to check the relative magnitudes of the forces, and in practice students are usually not expected to make their forces consistent with one another unless the forces are equal.

On part b of the problem, only 12% of the students answered correctly. The results are shown in Table 5 - 1. The most common error, given by 56% of the students, was that the magnitude of the net force on the non-friction surface was greater than the magnitude of the net force on the friction surface. We can explain this error in terms of diSessa's Ohm's P-prim.[7] The Ohm's primitive comes from the compensating type of reasoning that is associated with Ohm's Law.[8] A part of the Ohm's primitive states that an "increased resistance leads to less result."[9] Because the block first travels over a non-friction surface and then over a surface with friction the resistance on the block increases thereby decreasing the result, which in this case can be interpreted as the net force. This type of response is common with both the undergraduate engineering students and the advanced physics students.

N=69

Compare magnitude of net force at **M** to **P**	Correct: The net force is equal	Incorrect: F_{net} greater on non-friction surface	Incorrect: F_{net} greater on friction surface
	12% ± 4%	56% ± 6%	26% ± 5%

Table 5 - 1

Performance on the question asking students to compare the magnitudes of the net forces in the two regions. This question proved to be extremely difficult for the students.

There were also a large number of students stating that the net force on the frictionless surface would be less. One way to explain this result is that students were not considering the vector nature of the forces and were just thinking of the net force as the number of forces acting on the block. This is evident from some of the students' responses. Two examples of student responses showing each type of incorrect response are shown in Figure 5 - 6. They have been typed from the student papers.

In part c the students are asked two conceptual questions about the acceleration vector on the surface with friction. The results, shown in Table 5 - 2, indicate that 41% of the students answered correctly, that the direction of the acceleration vector was to the left. Only 32% of the students stated that the magnitude of the acceleration vector does not change as the block moves from point B to point C, where it comes to rest. These results are consistent with previous work indicating that students often treat acceleration as if it were proportional to velocity.[10] There were also a significant percentage of students stating that the acceleration of the block at point P was zero.

Two sample student responses	Case 1: "It is greater at M because there is no frictional force working against the F_{hand}."	Case 2: "[It is] less than [at M] because friction is being invoked at point P in addition to the 2N."

Figure 5 - 6

Sample student responses comparing the net force on the friction surface to the non-friction surface.

The results on the qualitative questions indicate that students still have many conceptual difficulties with NII, even though instruction on Newton's laws were completed a few weeks before this study was conducted. We are now in a position to examine the student responses on the final part of the problem, where students are asked to calculate the coefficient of kinetic friction.

Figure 5 - 7 shows the results on the quantitative aspect of this problem for the students taking both the long and the short version. It shows the percentage of correct responses as well as the percentage of students setting the net force on the block equal to zero on the region with friction. We should also note that only students who had enough time to attempt the final part were included in the analysis. Therefore, all the percentages listed include only the students who answered the final question. Even though students had covered this material in lecture and had homework assignments on the material fewer than 30% of the students answered this question correctly.

Student performance was significantly different on the two versions of the problem. The results show that students performed better on the short version of the problem.

N=69

Acceleration vector at point **P**	Correct: Vector directed to the left	Incorrect: Zero	Incorrect: Vector in the direction of motion
	41% ± 6%	20% ± 5%	19% ± 5%

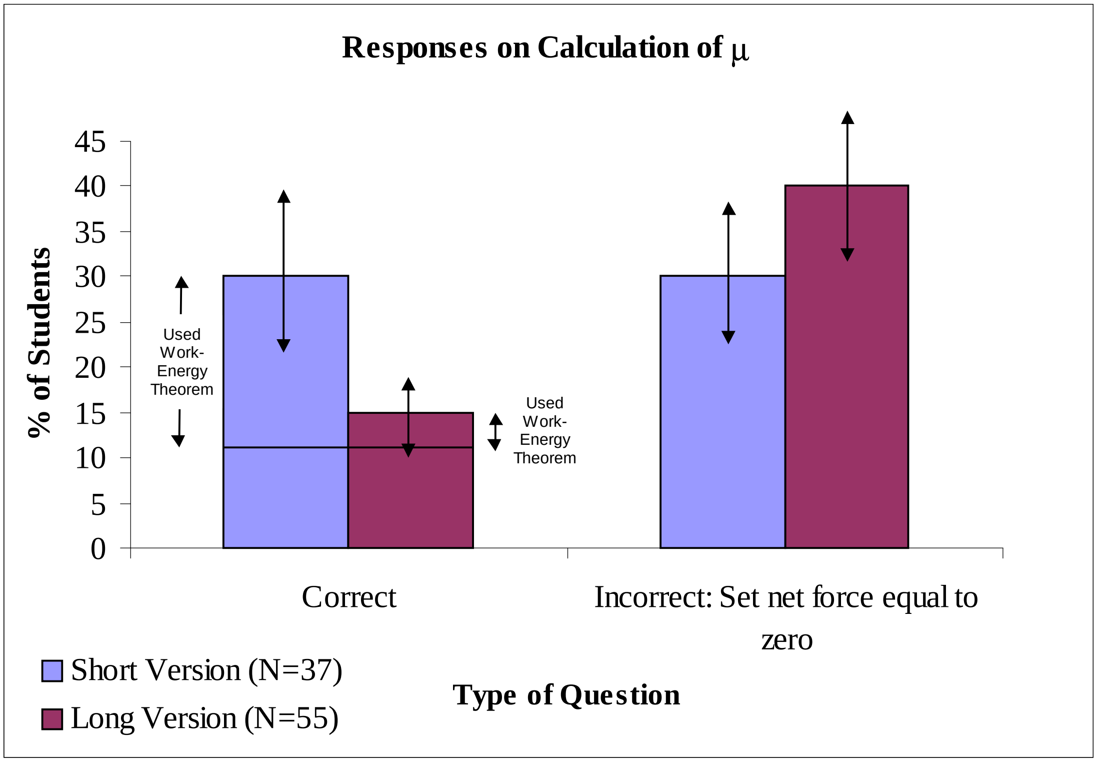
How does the vector change?	Correct: Acceleration is constant	Incorrect: Acceleration is decreasing
	32% ± 6%	55% ± 6%

Table 5 - 2

Performance on the questions concerning the acceleration vector of the block on the surface with friction.

We would also like to examine the methods of solution used by the students who answered correctly. We can see from Figure 5 - 7 that more students used the ideas of work-energy to solve the question on the short version of the problem. Most of the students who did not solve the problem using work-energy used the ideas from kinematics to try to solve for the acceleration of the block from point B to point C.

Larkin talks specifically about schemas for force and work-energy, but does not talk about how these schemas are linked.[11] If the conceptual questions lead students into a dynamics or force schema and that schema is not linked to the work or work and energy schema, the students may try to solve the problem using only the force schema. Previous research has also shown that novice problem solvers tend to focus on the surface features of a problem.[12] Since all the conceptual questions focus on force it is possible that our students responded by triggering a dynamics schema. This was actually the intent of providing the conceptual questions. But, we hoped that our students would use the dynamics schema to activate the work-energy schema. Based on our results this is not the case. In contrast, experts will tend to bring a larger



Data does not include students that did not complete the problem.
Unfinished: 8% on short version, 20% on long version

Figure 5 - 7

Performance on the quantitative part of the problem showing that students performed better on the short version. The graph also indicates that the methods of solution on the long and short version are different.

set of knowledge that is integrated to the problem-solving task. Experts will also be more able to activate additional schema if needed.

In addition to schemas for organizing specific physics content, the data also suggests that our students often have isolated schemas for their qualitative knowledge and their quantitative knowledge within a given physics topic. The qualitative and quantitative questions provide information on the coherence between qualitative and quantitative knowledge. There are two main types of errors students can make. Students may be able to solve for the coefficient of kinetic friction correctly yet have serious conceptual errors, or they may have qualitative ideas that they do not apply when answering the final quantitative question.

Table 5 - 3 shows examples of student inconsistencies on this problem. Even though some of the students could solve the quantitative question correctly many of those answering correctly had serious conceptual difficulties. Answering the quantitative question correctly requires that the student use the fact that the

magnitudes of the accelerations are equal in the two regions. (Note that only 8 students answered part d correctly on the long version.) We observe that five of the eight students, who answered the final part correctly, stated incorrectly that the magnitude of the net force was different in the two regions. In addition, three of the students who solved for the coefficient of friction correctly stated that the acceleration vector was decreasing from point B to point C. Perhaps an even more surprising result is that seven of the students who drew a non-zero acceleration vector in part c set the net force equal to zero when solving the quantitative question.

Of the students who could calculate μ correctly many made serious conceptual errors.	Incorrect: Stated that the magnitude of the net force in the two regions were different on part B	Incorrect: Stated that the acceleration vector decreases as the block moves from B to C	
	5 students	3 students	**N=8**

Of the students who had a non-zero acceleration vector some answered inconsistently on the quantitative part.	Incorrect: Stated that the net force was equal to zero when solving for μ.	
	7 students	**N=41**

Table 5 - 3

Inconsistencies in the student responses on the hand-block problem.

In Figure 5 - 8 we represent the possible paths of solution to the two versions with *solution maps*. The solution maps indicate strong paths and weak paths by the thickness of the lines that link the different responses. This allows us to map the number of students going from one response to the next response and allows us to see what types of responses to a certain part of the question tend to trigger certain responses on the next part. To make the maps easier to read, links that are sufficiently weak are excluded. It is important to note that the solution maps show the paths of solution for the entire class; they do not tell us about individual students. Therefore we cannot say that most of the students followed the thick lines to the final part of the question. Among other things, the maps do show that the conceptual questions seem to scatter the responses away from a correct analysis using the work-energy theorem.

The data from the ungraded quiz indicates that, for our students, physics content knowledge may be organized by schemas that are only weakly linked. If our students were developing coherence in their content knowledge, qualitative questions would tend to help in solving these problems, instead of hurt. In the problem discussed in this chapter, qualitative questions that lead students to a force schema tended to isolate them from other pieces of knowledge that could have been helpful in solving the problem. We have also begun to see that many of our students form isolated schema for their qualitative and quantitative knowledge.

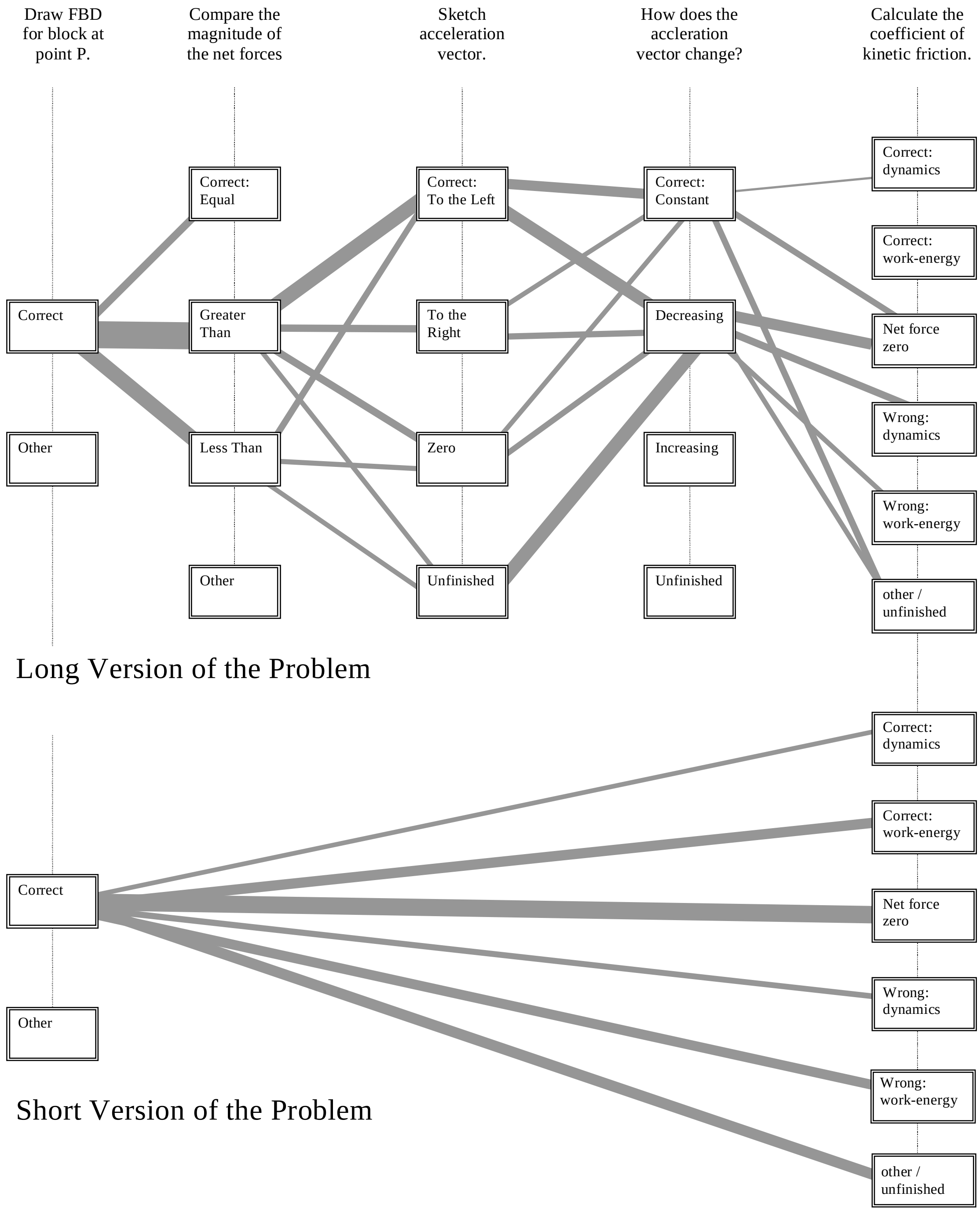


Figure 5 - 8

Paths of solutions on the long version and the short version of the problem. The intermediate qualitative questions seem to lead students away from using the ideas of work and energy.

Bridging Problem Analysis

The results presented so far in this chapter have described isolated student schemas for different physics topics. The data also indicates that many of our students have isolated schema for their qualitative and their quantitative knowledge in Newtonian dynamics. We have observed that many of our students answer inconsistently on the qualitative and quantitative parts of a single question. The following results concentrate on the qualitative and the quantitative schemas our students use in answering a similar question involving dynamics and work and energy.

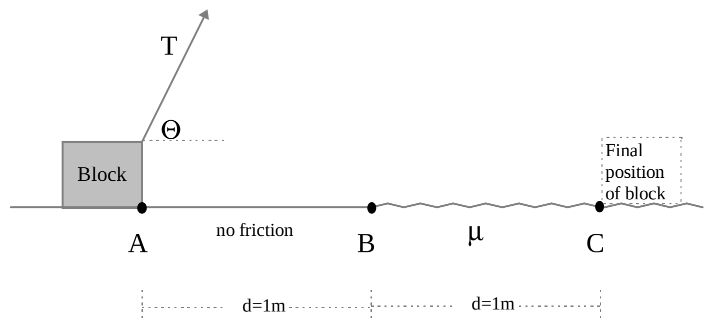
An older version of the problem shown in Figure 5 - 1 was asked as a bridging problem[13] in Spring '97. We observe that in their solutions many of the students directly contradicted one of the statements in the exposition of the problem. This result prompted us to perform more studies in order to understand whether students were making a careless error or a more profound error.

Bridging problems attempt to help students incorporate the qualitative knowledge they were developing in tutorial with the quantitative problems they were solving on exams and textbook homework assignments. The bridging problem, asked in the physics 161 class at the University of Maryland with tutorials, is shown in Figure 5 - 9 with a model solution. The students are asked two qualitative questions and then a final quantitative question about the coefficient of kinetic friction. They are first asked to draw a free-body diagram and then asked to compare the magnitudes of the net forces in the two regions. The question about the net forces acting on the two blocks was intended to help the students make the connection between work, forces, and changes in kinetic energy.

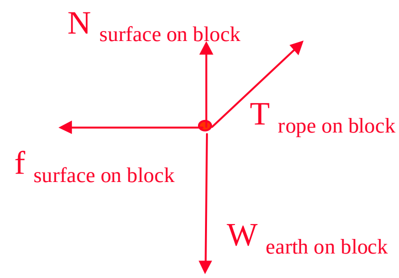
We will concentrate on the responses to part (c) of this question. The most common error involved the students setting the net force equal to zero on the friction surface in order to calculate the coefficient of friction. This response seemed odd since the problem explicitly states that the block slows down from point B to point C. Despite this statement many students set the net force equal to zero implying that the acceleration of the block in the region from B to C was zero.

The results, along with a sample student response, are shown in Figure 5 - 10. The figure shows that only about 35% of the students answered this question correctly after lecture and tutorial instruction on the work-energy theorem. The response shown in Figure 5 - 10 is an incorrect response in which the student sets the sum of the forces in the *x* direction equal to zero. Unfortunately the student does not explain his or her reasoning so it is difficult to extract much more information from this response. One of the difficulties with quantitative questions is that it is often hard to probe deeply into the students reasoning based on a collection of formulas. Open-ended conceptual questions are sometimes more helpful but can be limited in the same way.

A block is pulled from point A to point C by a tension force, T, as shown. The block starts off at rest at point A and speeds up to point B. The block then slows down finally stopping at point C (as shown). The magnitude and direction of the tension are constant throughout the motion. Note that the block is first pulled across a surface with no friction and is then pulled an equal distance across a surface with friction.



a) Draw a free-body diagram for the block when it's between B and C.



b) Is the magnitude of the net force acting on the block from A to B greater than, less than, or equal to the magnitude of the net force acting on the block from B to C? Explain your reasoning.

Since the change in kinetic energy is equal the magnitudes of the net works are equal therefore the magnitudes of the net forces are equal.

c) Calculate the coefficient of kinetic friction between the surface and the block between B and C if $\Theta=60^{o}$, $m_{block}=1.5kg$, and T=5N.

From (b) we know that $T\cos\theta - f = -T\cos\theta$ *and* $f = \mu(mg - T\sin\theta)$ *therefore* $\mu = .41$

d) Calculate the work done by friction to move the block from A to C. Show all work.

Since $f = 2T\cos\theta = 5N$ *the work is* $W = f\,d = 5\ J$

Figure 5 - 9

Bridging problem asked as part of the tutorial homework assignment and as a one-on-one interview with undergraduate students.

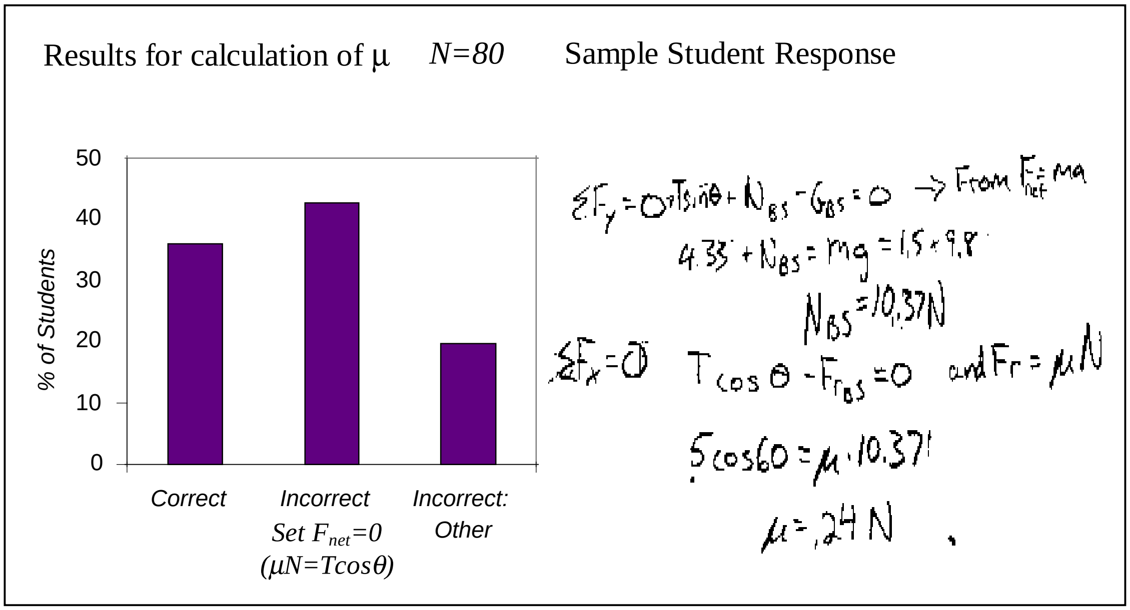


Figure 5 - 10

Performance on the quantitative part of the bridging problem and a sample student response. The sample shows the most common error that students made on this problem.

Interview Analysis

We followed the bridging problem analysis with one-on-one interviews, allowing us to further probe student understanding. Results of the interview analysis showed that the most common mistake, of setting the net force equal to zero, could be explained by the students not attaching the correct qualitative knowledge to the quantitative statement of NII. In particular, one student (out of the six interviewed) applied NII over a time interval rather than at an instant. Although these ideas are not consistent with NII they do show that our students are capable of rather sophisticated reasoning. Without using interviews as a research tool it is difficult to show the types of sophisticated reasoning that we, as instructors, can build upon to help students achieve the correct understanding.

We conducted one-on-one problem-solving interviews with six students from the Physics 161 class. The six students were chosen at random from a pool of eighteen volunteers.[14] Most students in the class were given the opportunity to volunteer for the interviews.

Interviews were conducted with students who solved the bridging problem discussed earlier in the chapter. Because of this, all students being interviewed were seeing the problem for the second time. Complete transcripts for the interviews presented in this section are found in Appendix D.

Pink

We will first look at Pink's[15] response in answering the final part of the problem about the coefficient of kinetic friction. On the bridging problem homework assignment Pink made the error of setting the net force equal to zero on the friction surface.

In this interview, Pink began the final part of the question by examining the forces on the block. Eventually she set the net force in the *x* direction equal to *ma*, unlike what she did on the homework assignment. She did however state, in the interview, that the acceleration is zero when the block reaches point C.[16] As Pink starts to get stuck when trying to calculate the acceleration, I suggested that she think about the work-energy theorem, which states that the net work equals the change in kinetic energy. The following is an excerpt of Pink's interview with some commentary.

> *P: Force times the distance is 1/2 m v squared.* [pause] *… force equals mass times acceleration times distance which is the work and that equals 1/2 times the mass times the final velocity squared. — so between A and B the final velocity is —*[pause]
>
> *I: How does the change in kinetic energy from A to B compare to the change in kinetic energy from B to C?*
>
> *P: … The change in kinetic energy from A to B is going to be 1/2 times the mass times the velocity and from B to C — its going to be — well you're staring out with a velocity so — They will be the same because you are starting from rest and you get to a final v and that is the initial velocity from here to here and you're stopping. So the velocity is the same and the mass is the same so the change in kinetic energy is the same.*

At this point Pink seemed to be having difficulty applying the work-energy theorem to this situation. The reason that the hint to use the change in kinetic energy was given was to provide a cue that might help her make the connection, and activate a schema for work and energy. Although she was able to correctly identify the changes in kinetic energy the work-energy theorem was not brought up, which indicates a very weak link between these two topics. At a later stage in the interview Pink begins to focus on the acceleration of the block.

> *P: So the acceleration is zero.*
>
> *I: So how do you get that?*
>
> *P: Because it is not acceleration … because it is not accelerating because the velocity is not changing.*
>
> *I: What do you mean the velocity is not —*

P: Well, it is starting and ending at the same velocity so the change in the velocity is zero. And the acceleration is change in velocity over change in time so its zero.

I: So you said acceleration is zero because it starts off at rest and it ends at rest … so the change in velocity from A to …

P: A to C is zero —

[Pink then proceeds to solve the problem, setting the sum of forces equal to zero. She therefore has $T \cos\theta - \mu_k n = 0$ and gets $\mu_k = .24$.]

In this section Pink states that the acceleration of the block is zero because it starts at rest and ends at rest. One explanation for this type of response is that Pink is not breaking up the problem into local and global parts.[17] She correctly identifies the average acceleration from the beginning to the end of the motion but then uses NII, which is valid at a single instant in time. The time issue associated with equations is an interesting subject that warrants more research.[18] A more general way to classify this error is to notice that Pink does not seem to be attaching appropriate conceptual meaning to the equation for NII. This is an error we often see when students solve quantitative questions. To get a better idea of Pink's misinterpretation of NII, I ask her about the equation she has just written.

I: This equation here — this F cosine 60 plus the force of friction equals ma — when does this equation apply?

P: Between B and C.

I: Between B and C— so between B and C —

P: And between A and B it is just T cosine theta.

I: T cosine equals —

P: Mass times acceleration.

I: … Can you write the sum of the forces in the x direction between A and B?

P: … It's just T cosine theta cause that is the only force in the x direction.

I: And that's equal to ma?

P: Yeah —

I: And the acceleration?

P: Well, it's going to be … cause it starts from rest and its got a final velocity at B so its just change in velocity over change in time.

I: So it is accelerating? —

P: Between A and B — and its decelerating between B and C.

I: … so when you wrote this equation here [points to T cosθ – F_r= ma] — *this is for — you said this is for the part of the motion between B and C?*

P: Yeah.

Here Pink sees a contradiction in her reasoning after I intervene. Pink states explicitly that the equation she wrote down applies in the region from B to C and she also stated the correct definition of acceleration and that the block is decelerating from B to C. Earlier, she wrote down that the net force (from B to C) is equal to zero. To justify it she stated that the acceleration from A to C is zero. These statements can be attributed in part to a lack of coherence in her schema for dynamics. Some of her statements directly contradict each other. By the end of the previous exchange she starts to believe that the net force from B to C is non-zero. Different pieces of knowledge get brought out depending on the cues presented. In the section above, the questions I present cue a different set of knowledge. In the next section Pink resolves the discrepancy.

P: So if you have … I guess if you add T cosine theta then it would be for the whole thing

I: Say that again.

P: If you add T cosine theta again that would add all the forces in the x direction from here to that [points to A and C]. *So this would be* [writes a 2 in front of the tension force — she has therefore changed T cos θ – μ_k n = ma into 2 T cos θ – μ_k n = ma]

I: So you would get 2 T … so what is this equation again?

P: If we add T cosine theta — that's the forces between A and B plus the forces between B and C it would be the whole thing.

I: So what do you mean the whole thing?

P: I mean the sum of the forces … from A to C.

I: The sum of the forces from A through C is this guy [points to 2Tcosθ – μ_k = ma.]

P: Right.

I: Now why do you say that?

P: Its just the sum of the force from A to B plus the forces from B to C.

I: So when you say this [points to $2T\cos\theta - \mu_k = ma$] *what's the acceleration?*

P: Well, there it is zero.

I: There its zero because …

P: Because the change in velocity over the change in time is zero —

I: … So, is it zero here? [Points to old sum of forces from B to C which is $T\cos\theta - F_r = ma$.]

P: Umm — That's between B and C — well it's decelerating so it can't be zero.

In this section Pink resolves her dilemma but instead of switching from global (or average acceleration) to the instantaneous acceleration which would be correct, she reinterprets NII as a global quantity. Pink applies NII across time summing the forces throughout the entire motion. She is therefore exhibiting some profound conceptual difficulties with Newton's Laws.

These difficulties seem to come from Pink not being able to see the role of time in the equation for NII. Redish discusses Newton's 0^{th} (N0) Law in his Millikan paper. N0 states that "at a time t, an object responds only to forces that are exerted on itself at time t."[19] This is not trivial for our students. Each equation a student uses has information about time that is not explicit in the equation. For instance, when using NII the quantities on either side of the equal sign are evaluated at the same instant. In contrast, when using conservation laws, such as conservation of momentum the quantities on either side of the equal sign are evaluated before and after a collision. Bruce Sherin discusses ways to put extra information into equations to help students attach more conceptual information to equations.[20] Attaching time information to different formulas may help students make this connection. The time issue is an interesting area of research. There are very few studies examining this difficulty.

It is also interesting that Pink's last statement would yield the correct answer to the problem. Because the accelerations are equal and opposite on the frictionless surface and on the surface with friction, summing the forces on each region and adding the two NII equations would give you zero. In the next chapter we present results showing that students can get the correct answer to a quantitative problem even though they may have serious conceptual difficulties. Again, we see why physics education researchers must support analysis on written responses with the analysis of interviews.

It is also important to note that Pink is using some very sophisticated reasoning to solve the problem and resolve her inconsistency. She constructs her own theoretical interpretation for the algebraic equation of NII and this interpretation does not come from a book or an instructor. Even though it is incorrect it allows her statements to be consistent with each other. The interview allows us to observe the expert-like reasoning skills that our students may possess, such as Pink's interpretation of NII. It is easy to view wrong answers (and correct answers) in a simplified way when we are simply presented with written responses to homework and exam questions because we are unable to see the types of reasoning our students are capable of. Interviews can provide us with information on the resources students have, which we can build upon, to help students come to a correct solution to the problem. Although Pink's time averaged use of NII is a sophisticated piece of reasoning and it allows Pink to obtain the correct answer for the coefficient of kinetic friction it may lead to more difficulties in the future.

Michelle

In another interview we saw the same mathematical error (i.e. solving for μ by setting the force of the hand equal to the force of friction), although in this interview it is done for a different reason. This excerpt comes from an interview with Michelle, who was one of the top students in the class. In this excerpt Michelle begins solving for the coefficient of kinetic friction.

> *M: Friction — I usually start with any definition or formula I can think of — so that is μ N and in this case it is not moving off the table or surface so you are going to have the normal plus the Tension times sine theta will equal m of the block times gravity. And we can solve for N so we can put in this equation* [points to μ N = F_r] *— They give us mass of block …1.5 times 9.8 meters per second squared minus the tension which is 5 N times sine sixty degrees. … I'd get a number for this and plug that in here* [points to μ N = F_r] *— Friction — because it is going to come to a stop at point C you are going to take friction at rest will be equal to tension times cosine of theta. … Because at the end it is going to be at rest so it is not going to be moving in the horizontal direction — I took that at point C.*
>
> *I: So you are saying that at point C the force of friction has to be equal to that* [points to T cos 60] *because it is at rest?*
>
> *M: So they are equal — 5 N times cosine sixty — that will give me another number which can go into there* [points to μN = F_r] *and I can solve for μ*
>
> *I: … So at point C you said it was at rest so these two forces are equal?*
>
> *M: Yes — I said it's got a velocity coming this way and the friction is eventually going to slow it to a stop and the μ isn't changing at all so —*

I: So the acceleration at point C is what?

M: The acceleration at point C is zero because it is not moving.

Michelle is making a different type of error. She is summing the forces in the region from B to C but only using point C to evaluate the acceleration of the block. This error is similar since we again see that the quantity on the left side of the equation is being evaluated at a different time than the quantity on the right side of the equation in NII. Again the time issue seems to be a factor in the way the students respond to the question. Later in the interview, after some intervention, Michelle resolves some of these issues.

These interviews show why it is important to probe student understanding in a number of different ways. The interview provides a detailed look into students' reasoning. It is impossible to say that each student who made the error of setting the horizontal component of tension equal to the force of friction went through the same type of reasoning Pink and Michelle went through. But these results do give us an idea of some of the difficulties we must look out for. They also tell us about some of the resources students have that we can build upon.

One possible explanation for the incorrect response is a lack of conceptual information attached to the quantitative form of NII. In order for our students to apply equations correctly at appropriate times it is necessary that we attach conceptual meaning to the equations they use. As stated earlier, when our students view an equation, they often observe it as simply a tool to obtain an unknown variable. This seemed to be the case with Michelle and Pink. Although they had strong reasoning skills and often had the correct concepts, they did not attach the concepts to the equations and they did not check that there was consistency between the concepts and the equations.

Thermodynamics Question

Introduction

The question discussed in this section was posed as a pretest for the physics 262 course with tutorials at UMd. The pretest preceded a problem-solving tutorial that was developed by the UMd PERG.[21] Problem-solving tutorials are given after the students hear lectures on the material and after they go through tutorials based on the subject. The students taking this pretest have therefore had lecture and tutorial instruction on the material.

Pretest Analysis

Two versions of the thermodynamics problem were given to the students. Figure 5 - 11 shows the problem with a model solution. Each version was asked in a different lecture class with a different instructor. The first version had many qualitative parts before a final quantitative part, while the second version had only the final quantitative part. The problem involved a piston that undergoes an isothermal expansion and then an isovolumetric process. Only the bold faced question appeared on both the short version and the long version of the question. This problem probes a schema for work. Some nodes that an individual may activate in a work schema are:

- the work is equal to the force dotted with the displacement,
- the work is the integral of *Pdv*,
- the work is equal to the area under a *PV* diagram, etc.

A thermodynamics problem involving a piston would most likely activate the last two nodes in order to solve the problem.

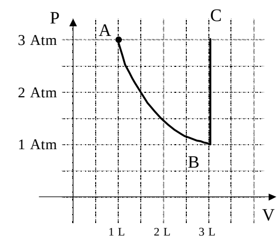
The most interesting data comes from looking at how the students answer the final question where they are asked to calculate the work done by the gas in the process A⇒B⇒C. First we will concern ourselves with looking at the general categories of correct and incorrect. Table 5 - 4 shows the results on the quantitative part for the two classes. (Only the students who answered the final part are included in the analysis.)

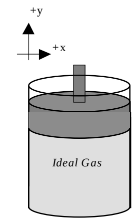
The data shows that the students performed better on the short version where they are simply asked to calculate the work done in the process. This result was surprising because one would think that the conceptual questions on the long version would help the students solve the problem. In particular, questions a and b, where the students are asked to draw a *PV* diagram, should help them calculate the work done by the piston.

In order to determine why the performance on the short version is better, we break down the responses into more specific categories. Table 5 - 5 shows that on the long version of the question a large percentage of the students incorrectly calculated the area under a triangle and a square. Most of these students (16 out of 18) drew the isotherm as a straight line. The students who drew the *PV* diagram have therefore cued into the node that work is the area under the curve, whereas the students who were given the short version cued into the node that $\int Pdv = Work$. Because of a lack of conceptual understanding in the isothermal processes the piston goes through (only 61 out of 114 graphed the isotherm correctly), the students on the long version actually perform worse. We expect the students taking the short version of the pretest to have the same conceptual difficulties with graphing the isothermal process we observed with the students on the long version.[22] Sketching the *PV* curve seems to have triggered a geometrical solution for the problem, while the students who were not given the qualitative questions solved the problem using calculus. The two versions of the questions have therefore cued different schemas with different procedural rules.

An expert problem solver would tend to relate the geometrical and calculus based knowledge so that performance on the long and short versions would be similar. When experts attempt to solve problems, they will tend to draw upon both their qualitative and their quantitative knowledge in most situations. Occasionally experts are presented with problems that they may have solved many times, in which case they may not need to use both their qualitative and quantitative knowledge. For the most part this will only happen when the expert is reasonably sure of the answer. Any doubt will usually cause experts to check for consistency between their qualitative and quantitative knowledge.

An ideal gas in a container with a piston starts at a pressure of 3 Atm and a volume of 1 L. The gas first goes through an isothermal process ending up with a pressure of 1 Atm and a volume of 3 L. The gas then undergoes a process at constant volume ending up with a final pressure of 3 Atm.





a) Sketch the isothermal process on the PV diagram and label the resulting state "B."

b) Sketch the constant volume process on the PV diagram and label the final state "C."

c) What is the direction of the force exerted on the piston by the gas in the container in the process A⇒B? Is the work done by the gas positive, negative, or zero? Explain.

The direction of the force from the gas on the piston is in the +y direction. Since the volume of the gas is increasing the displacement of the piston is in the +y direction. The work done be the gas is positive since the force and the displacement are in the same direction.

d) What is the direction of the force exerted on the piston by the gas in the container in the process B⇒C? Is the work done by the gas positive, negative, or zero? Explain.

Again the force of the gas on the piston is in the +y direction. Since the displacement of the piston is zero the work done by the gas on the piston is also zero.

e) Calculate the work done by the gas in the process A⇒B⇒C. Show all work.

The work done by the gas is equal to the area under the isotherm. With the temperature remaining constant we have

$$\int_A^B PdV = \int_A^B {nRT}/{V}\, dV = 3.3 Atm \cdot L$$

Figure 5 - 11

Two versions of a thermodynamics question asked as a pretest with a model solution. The first version contained all five parts and the second version consisted of only the last part.

Calculate the work done.	Correct: $\int Pdv = Work$	Correct: Area under the curve (approx)	Incorrect	
Long version	20% ± 5%	7% ± 3%	73% ± 5%	**N=74**
Short version	44% ± 7%	0%	56% ± 7%	**N=55**

Table 5 - 4

Performance on the final part of the question.

Calculate the work done.	Incorrect: area of Δ + □	Incorrect: area of Δ	Incorrect: PΔV, ΔPΔV, ΔPV	Incorrect: other	
Long version	24% ± 5%	5% ± 3%	23% ± 5%	21% ± 5%	**N=74**
Short version	2% ± 2%	11% ± 4%	33% ± 6%	10% ± 4%	**N=55**

Table 5 - 5

A more detailed description of how students answered the final part of the problem.

Summary

This weak link between qualitative knowledge and quantitative knowledge can actually cause our students to perform worse when they are presented with qualitative questions before the final quantitative part. This may be due to students activating a particular schema because of the qualitative questions and then getting trapped in that particular schema. If the schema that is activated does not contain all the information needed for the problem, the student will not be able to solve the problem. Evidence from the hand-block ungraded quiz and the thermodynamics pretest supports these conclusions. In the hand-block problem we observed that some students tended to get cued into a dynamics schema where they could not make the link to the work and energy schema. In the thermodynamics problem we saw that students activated a schema for work which involved the area under the *PV* curve and could not make the link to the integral of *PdV* when conceptual questions were presented.

This chapter provides researchers and instructors with methods that can be used to evaluate student coherence. The methods we used involved problem-solving interviews, and open-ended questions. By asking questions with parts that are qualitative and parts that are quantitative we were able to identify contradictions and inconsistencies in student knowledge and reasoning. We are also able to probe for coherence between the schemas students have for different physics topics.

We have introduced two new representations that can help instructors and researchers look for coherence: reasoning maps and solution maps. We can also see how students are using concepts in answering the quantitative questions by examining performance on different versions of a problem. We believe that forming coherence and evaluating coherence should be an explicit goal of the physics course.

The responses students gave on the qualitative and the quantitative parts of a problem also indicate that students have isolated schemas for qualitative and quantitative knowledge. The conceptual responses on the hand-block problem were not always consistent with the quantitative expression for NII. On the thermodynamics problem, the quantitative expression for the work done by the piston involving the integral of PdV was used less often for students who answered qualitative questions first. In the following chapters we provide more detail on these issues.

[1] Problem-solving interviews are discussed in chapter 3.

[2] Another option for a problem-solving interview would involve two or more students. Bruce Sherin discusses the advantages of using two students. One particularly relevant benefit is that when two students are present they must talk to each other and explain what they are doing. In contrast, the one-on-one interview is a more artificial setting for the student. The researcher often has to remind the student to explain their thoughts and what they are writing out loud. Because we do not want any external cues or aids for the student, we decided to use the one-on-one interview setting.

[3] For more information see chapter 2 pages 10-12 and A. A. diSessa, "Knowledge in Pieces," In *Constructivism in the Computer Age,* G. Forman and P. Pufall (Eds.) (Lawrence Erlbaum, NJ, 1988), pp. 1-24; and D. Hammer, "More than misconceptions: Multiple perspectives on student knowledge, and an appropriate role for education research," Am. J. Phys. **64** (10) 1316-1325 (1996).

[4] Maintaining agent is a term used by Hammer to describe diSessa's continuous force p-prim. See D. Hammer, "More than misconceptions: Multiple perspectives on student knowledge, and an appropriate role for education research," Am. J. Phys. **64** (10) 1316-1325 (1996).

[5] We believe that the graduate students who could not use work-energy in this problem could have done a straightforward work-energy problem.

[6] E.F. Redish, "Millikan Award Lecture (1998): Building a Science of Teaching Physics," Am. J. Phys. **67** (7), 562-573 (1999).

[7] For more information on P-prims see Ref. 3.

[8] P-prims were discussed in chapter 2 on pages 10-12.

[9] See Ref. 3.

[10] D. E. Trowbridge and L.C. McDermott, "Investigation of student understanding of the concept of acceleration in one dimension," Am. J. Phys. **49** (3), 242-253 (1981).

[11] J. H. Larkin, "The role of problem representation in physics." In *Mental models*, D. Gentner and A. L. Stevens (Eds.) (Lawrence Erlbaum, NJ, 1983), pp. 75-98.

[12] M.T.H. Chi, P.S. Feltovich and R. Glaser, "Categorization and representation of physics problems by experts and novices," Cognitive Science, **5**, 121-152 (1981).

[13] Bridging problems are discussed in detail in chapter 3.

[14] Problem-solving interviews are discussed in detail in chapter 3.

[15] Students use code names they choose in the interviews.

[16] Pink may be making the common error that the acceleration at point C is zero because the velocity is zero at point C. She may also be answering correctly by recognizing that the block remains at rest at C. It is difficult to tell from the transcript.

[17] L.C. McDermott and P.S. Shaffer, "Research as a guide for curriculum development: An example from introductory electricity, Part I: Investigation of student understanding." Am. J. Phys. **60** (11), 994-1002 (1992); Erratum to Part I, Am. J. Phys. **61** (1), 81 (1993).

[18] Bruce Sherin discusses some of these issues in B. Sherin, "The Symbolic Basis of Physical Intuition: A study of two symbol systems in physics instruction," Ph.D. dissertation, School of Education, University of California, Berkley, (1996).

[19] See Ref. 6.

[20] For more info see Ref. 18.
[21] Problem-solving tutorials are described in chapter 3.
[22] We believe that the two student populations we investigated are similar. Both populations went through the same curriculum and both instructors for the course were well liked by their students.