

## **Chapter 4: Using Context to Probe for Coherence within Physics Topics**

Introduction	37
Newton's 1 <sup>st</sup> law	39
Impetus force	42
Motion diagrams and the velocity vector	43
Newton's 3 <sup>rd</sup> law	44
Summary	47

## Chapter 4: Using context to probe for coherence within physics topics

### Introduction

In this chapter we examine how students respond to questions about a particular physics topic that are posed in different contexts. The data clearly shows that when students are given a question or problem they often activate schema partially based on the context of the question. This leads to many of our students answering inconsistently on questions that deal with identical physics concepts. The knowledge that is brought to the task is often characterized by a local coherence, which indicates that our students activate schemas of physics knowledge that are isolated from one another.

In addition, the knowledge that is contained in the two schemas may even be contradictory. This causes certain contexts to trigger different types of physics knowledge, which include different procedural rules, and different declarative knowledge. An expert answering the two problems would tend to bring up schemas that depend more on the underlying principles. Even though the context of the question is different the expert will usually solve the two questions in the same way, using a consistent set of knowledge.

In this study we have changed the contexts by changing the format of the questions (i.e. multiple choice vs. open-ended) and by changing the type of questions (i.e. real world vs. physics class.) Multiple choice diagnostics are often used to assess student understanding of physics, but there have been few studies documenting how student responses in a multiple-choice context compares with their responses in other contexts such as interviews or open-ended exam questions.

<b>Question Number (old version)</b>	<b>Topic</b>
18	Newton's 1 <sup>st</sup> and 2 <sup>nd</sup> Laws
9,22	Impetus Force
7	Motion Diagrams
2,11,13,14	Newton's 3 <sup>rd</sup> Law

**Table 4 - 1**

*List of four topics covered on the FCI and their corresponding questions.*

In this study the same group of students were asked questions on a multiple-choice diagnostic and on open-ended problems. The multiple-choice questions we

used were chosen from the Force Concept Inventory (FCI).<sup>1</sup> The FCI, as well as other multiple-choice instruments have become increasingly popular. Because of their popularity and ease of use, they play a large role in evaluating curriculum and also in developing curriculum and instructional strategies. Despite its wide acceptance and use there have been few studies published on how student performance on the FCI correlates with their understanding of the subject matter.<sup>2,3,4</sup>

Students were given the FCI as an ungraded diagnostic, near the end of the physics 161 course at UMd. In order to compare how students answer questions on the FCI to how they answer questions on an open-ended exam problem we developed and administered the two final-exam problems shown in Figure 4 - 1 and Figure 4 - 2. The exam problems dealt with a number of concepts that are tested for on the FCI. In particular, we examined the concepts of Newton's 1<sup>st</sup> Law (NI), the Impetus Force, motion diagrams, and Newton's 3<sup>rd</sup> Law (NIII). Table 4 - 1, shown on the previous page, shows the concepts and the corresponding questions on the FCI. Students had instruction on all the material presented in this chapter prior to the administering of the FCI and final exam.

The Physics Education Research Group at UMd has been administering the FCI for several years both before instruction and after instruction. In this study the

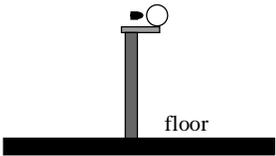
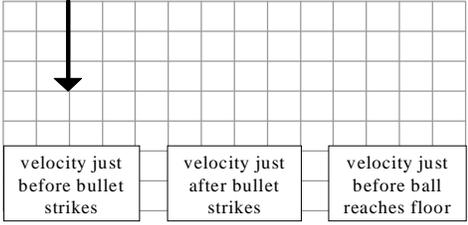
**Ignore all friction and air resistance in this problem.**

A. A steel ball resting on a small platform mounted to a hydraulic lift is being lowered at a constant speed, as shown in the figure at right.

- Draw a free body diagram of the ball. Describe each type of force.
- Compare the magnitudes of the forces you have drawn.  
Explain your reasoning.

B. As the ball is moving down, a bullet moving horizontally hits the exact center of the ball (see figure at right) and then ricochets straight back. This causes the ball to immediately fall off the platform

- Draw a free body diagram of the ball after it is no longer in contact with the bullet or the platform. Describe each type of force.
- A vector that represents the velocity of the ball just before the bullet hits is shown below. Draw vectors that could represent the velocity at each of the 2 other times indicated. The scales of the 3 vectors should be consistent with each other. Explain your reasoning.

**Figure 4 - 1**

*Exam problem testing student understanding of Newton's 1<sup>st</sup> and 2<sup>nd</sup> laws, and motion diagrams.*

original version of the FCI has been used. Although some questions have been changed on the new version, it does not affect the conclusions we can draw from the results. A full copy of the FCI is included in Appendix E.

The work was conducted by Richard Steinberg and myself and has since been published in *The Physics Teacher*.<sup>5</sup>

The problem shown in Figure 4 - 1 was asked in spring 1996, in a single lecture class. Twenty-eight students also completed the FCI at the end of the semester. The problem shown in Figure 4 - 2 was given to three different lecture classes in the fall of 1995. In these classes, 128 students also completed the FCI. The data we discuss are broken down according to concept. Richard Steinberg wrote the problem shown in Figure 4 - 1 and Jeff Saul wrote the problem shown in Figure 4 - 2.

Two carts, A and B ( $Mass_A > Mass_B$ ), are placed on a table then stuck together with Velcro. Using pulleys, two small blocks, C and D ( $mass_C < mass_D$ ), are connected by light strings to the carts as shown below. Initially, the carts are held in place. **Ignore all friction in this problem.**

At  $t = 0$ , the carts are released. At  $t = 3$  seconds, the Velcro pulls apart and the two carts separate. At some later time, cart A returns to its starting point.

- Draw and label two separate free-body diagrams, **one for each cart**, for a time after the carts start moving but before the Velcro pulls apart.
- Rank all the horizontal forces from both your diagrams by magnitude, from largest to smallest. **Explain the reasoning that you used to rank the forces.**

**Figure 4 - 2**

*Exam problem testing student understanding of Newton's 2<sup>nd</sup> and 3<sup>rd</sup> Laws.*

## Newton's 1<sup>st</sup> Law

The first concept we will discuss is NI. Previous research in physics education has shown that students have many profound difficulties with the concept of NI and NII.<sup>6</sup> The FCI uses specific misconceptions that students have as distractors. These distractors as well as the correct response are included as choices for each FCI question. The distractors are based on research into student understanding of the various concepts. Halloun and Hestenes also conducted many interviews with students to validate the FCI.<sup>7</sup>

The FCI question that tests the concept of NI, on the FCI is question 18. The question involves an elevator moving upward at constant speed. Students are asked to compare the force exerted on the elevator by the cable to the force of gravity exerted on the elevator. The UMd PERG has consistently seen about 50% of the students answering this question correctly after instruction, by stating that the two forces are equal. The most common error, given by approximately 40% of the students was that the upward force on the elevator by the cables was larger than the downward force due to gravity. This response is inconsistent with NI and NII but is consistent with the naïve belief that a force is needed to maintain a constant velocity.

The first part of the problem shown in Figure 4 - 1 is very similar to question 18 on the FCI. If student models were consistent we would expect them to answer this question the same as they answered question 18 on the FCI. In the exam problem, instead of an elevator being raised at constant speed, a steel ball is being lowered at constant speed. The students are asked to draw the free-body diagram for the ball and then compare the magnitudes of the forces acting on the ball. The correct answer to the problem would include the students identifying the normal force from the platform on the ball directed upward and the force of gravity on the ball acting downward. The magnitudes of the two forces must be equal since the ball is moving at constant velocity.

A comparison of how the students responded in the two contexts shows that these two questions are not equivalent for our students. Of the twenty-eight students who answered both the FCI question and the exam question we see 54% of the students answering the FCI question correctly and 90% of the students answering the open-ended question correctly. Ten of the thirteen students who answered the FCI question incorrectly selected choice A, stating that the force directed up from the cable was greater than the downward force of gravity. Two of the three students who answered the exam question incorrectly gave responses that were consistent with choice A on the FCI.

Since the students in the study are matched, meaning that they took both the FCI and the open-ended question, we can examine how they answered on each instrument. These correlations are displayed in Table 4 - 2. It is interesting to note the large difference in the way students performed on the two questions, despite the similarity in the items.

One difference between the multiple-choice question and the corresponding open-ended question is that in one situation an object is moving up (elevator) and in the other situation an object is moving down (ball). A physicist will view this difference to be irrelevant in answering the question. More importantly many instructors will see this as irrelevant to the way the students solve the problem. We have investigated whether the direction of the object's motion has an effect on how students answer the questions.

Upon the recommendation of a colleague we undertook an investigation in the Physics 161 class in spring 1999. During the last week of lecture students in one class were given an open-ended diagnostic and students in the other class were given a multiple-choice diagnostic. It was announced to the students that the diagnostics would not count toward their grades but that they should take them seriously.

N=28

Exam Question	FCI	
	Correct:	Incorrect
Correct	14	11
Incorrect	1	2

**Table 4 - 2**

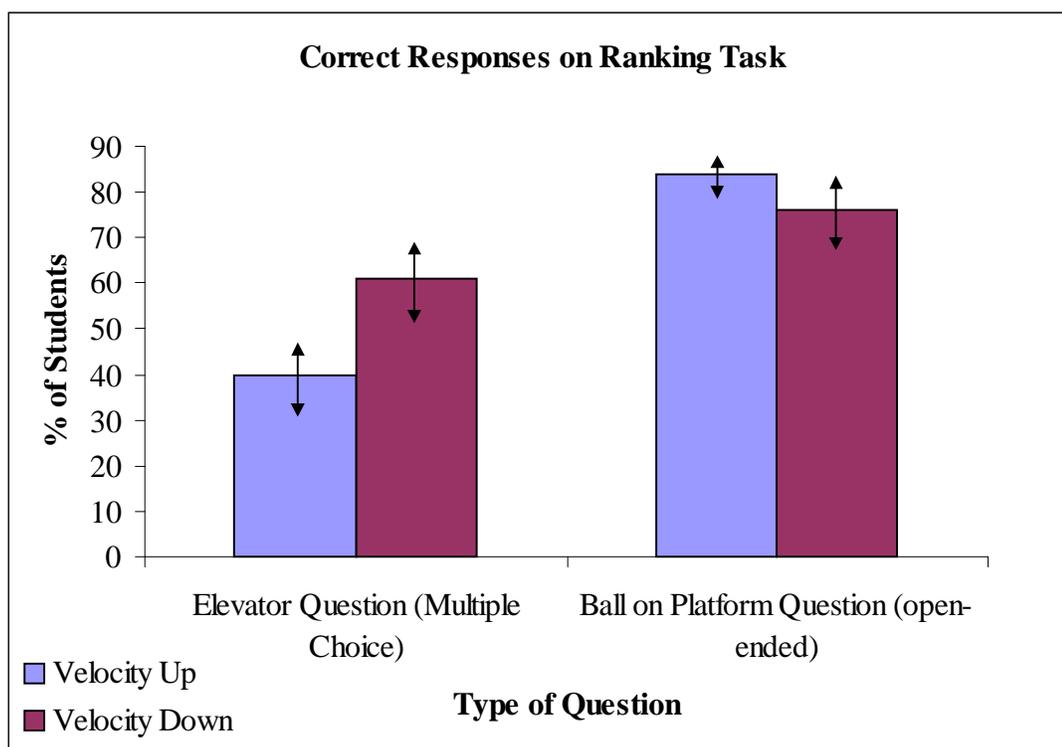
*How students answered the FCI question on Newton's 1<sup>st</sup> Law and how they answered on the corresponding open-ended problem.*

Students in one class, taking the open-ended diagnostic, were given three open-ended questions in addition to two multiple-choice questions. Two versions of the *ball-platform question*, shown in Figure 4 - 1, were included on the diagnostic. Both versions were given in each tutorial section. In one version the ball was moving down at constant velocity, as in Figure 4 - 1, and in the other version the ball was moving up at a constant velocity.

Students in the other class were given the FCI, which included the *elevator-tension question*. Two versions of this question were given in different tutorial sections. One version was identical to the FCI question described above. On the other version the elevator was moving down, instead of up, at constant speed. All other questions on the FCI were identical. It should also be noted that for this study the new version of the FCI was used. The elevator-tension question is essentially the same on both the old and the new version of the FCI.

The student responses indicate that the direction of the motion of the object does have some effect on how the students answered the multiple-choice question, but not the open-ended question. The results are shown in Figure 4 - 3. On the open-ended question, 84% of the students are answering the question correctly on the version with the platform moving down and 76% answered correctly on the version with the ball going up. On the multiple choice question 40% answered correctly with the elevator moving down, while 60% answered correctly with the elevator moving up. The results suggest that on the multiple-choice question some students are answering differently on the two versions. Differences on the open-ended question are not significant. Error bars are provided to show whether the results are indeed different based on an estimate of the error.<sup>8</sup>

The data shows that on the multiple-choice question the direction of the elevator's motion made a difference in the way students answered. Students tended to answer correctly more often when the elevator was moving up instead of down.



**Figure 4 - 3**

*Comparison of how students answered the questions depending on whether the object was moving up or down.*

## Impetus Force

Students often have the naïve belief that an object that receives a brief impulse will experience the force from the impulse even after the impulse is over. For instance, if a ball is thrown up in the air, students will often include the force of the hand on the ball even though the ball is no longer in contact with the hand. This is a common error and is well documented in the literature.<sup>9</sup> This topic is addressed on the FCI by questions 9 and 22. The first question involves a sliding hockey puck getting kicked and the second question involves a golf ball after it has been hit. Students are asked to identify the forces that are acting on the puck and ball after the initial strike. The correct response was given by 46% of the students for the question about the hockey puck. Students answered the question about the golf ball correctly 43% of the time. Although the percentage of students answering correctly is about equal for these two similar FCI questions the students who answered one question correctly did not necessarily answer the other question correctly. Only 29% of the students answered both questions correctly. Therefore 71% of the students stated that there was either a force in the direction of motion or a force of the hit, on at least one of the two questions. These responses are consistent with the idea of an impetus force.<sup>10</sup> The results also indicate that even on the same type of measurement instrument students

can answer differently to very similar questions.

Part B of the problem in Figure 4 - 1 is very similar to these two FCI questions. The steel ball on the platform is hit by a bullet and is knocked off the platform. The students are asked to draw a free-body diagram for the ball when the ball is no longer in contact with the platform and the bullet. A correct answer would state that only the force due to gravity would be acting on the ball.

The relationship between how students answered the three questions is shown in Table 4 - 3. The table indicates that all students who answered both FCI questions correctly answered the open-ended question correctly. It also shows that students who did not answer any of the FCI questions correctly were unable to answer the exam question correctly. Despite this consistency, the table also shows that over one third of the students did not answer all the questions the same way. We expect an expert to answer all three questions in the same way.

**N=28**

<b>Exam Question</b>	<b>FCI</b>		
	0 FCI's correct:	1 FCI correct	2 FCI's correct
Correct	1	7	8
Incorrect	10	2	0

**Table 4 - 3**

*How students answered the FCI questions on the impetus force and how they answered the corresponding open-ended problem.*

## **Motion Diagrams and the Velocity Vector**

The last part of the problem shown in Figure 4 - 1 asks the students to sketch a velocity vector for the ball that is consistent with the velocity vector just before the bullet strikes the ball. FCI question 7 asks a similar question about the speed of a hockey puck right after being kicked in a direction perpendicular to its motion. Although in one case the students are asked to draw a velocity vector and in the other case students are asked to pick the correct description of the velocity vector, the questions are similar in the concepts they are testing. These results will provide us with additional insight into student coherence and point out some of the limitations of multiple-choice questions. Table 4 - 4 shows how students performed on the two parts of Bii and FCI question 7.

The correct choice on the FCI was choice E, which states that the speed after the kick is greater than the speed before the kick and greater than the speed it "acquires" from the kick, but smaller than the arithmetic sum of these two. Choice E was selected by 61% of the class. The class performed almost as well on the first part

of Bii. About half the class drew the velocity vector correctly by drawing a vector with a vertical component that was the same as the vertical component just before the kick and a non-zero horizontal component.

The results also showed that only 12 of the 28 students who answered both the FCI and the exam had answers where the FCI response matched the exam response. In addition, four of the students gave answers on the exam that did not correspond to any of the choices on the FCI. The remaining 12 students had answers on the exam that did not correspond to the answers that were given on the FCI. One observation that is particularly interesting is that three of the students drew velocity vectors on the exam with vertical components that were shorter than the velocity before the ball was struck by the bullet. If the students answered correspondingly on the FCI they would answer the FCI question correctly, despite having this particular error. All three of these students stated that the speed would be independent of the initial speed of the puck on the FCI.

**N=28**

<b>Exam Question</b>	<b>FCI</b>	
	Correct	Incorrect
2 Correct	8	0
1 Correct	4	6
0 Correct	5	5

**Table 4 - 4**

*Performance on the two parts of exam problem Bii (on motion diagrams) and the corresponding FCI question.*

Students also struggled with the second part of Bii, where they were asked to draw the velocity vector just before the steel ball hits the ground. The nature of the difficulties on the exam made comparisons with similar FCI questions difficult.

### **Newton's 3<sup>rd</sup> Law**

Newton's 3<sup>rd</sup> Law is a very difficult concept for our students to grasp and these difficulties are well documented in the literature.<sup>11</sup> Students often make the error that big objects exert larger forces or that the object that is accelerating is providing the bigger force. The concept of NIII has the most questions devoted to it on the FCI. Questions 2, 11, 13, and 14 all involve two objects interacting in some way and students are asked to choose from five different descriptions about the forces the objects are exerting on one another.

The problem shown in Figure 4 - 2 addresses NIII, although it is couched in terms of a larger problem. In the problem two carts are attached together and are

accelerating on a tabletop. The problem asks the students to first draw free-body diagrams for each of the carts and then rank the relative magnitudes of the horizontal forces. Students must recognize the existence and the magnitudes of the forces while describing the behavior of a multifaceted problem. The reason the open-ended question was posed this way was to prevent the knee-jerk response "equal and opposite." A superficial understanding of the relationship between the force that cart A exerts on cart B and the force that cart B exerts on cart A is unlikely to result in a correct response.

Because of the large number of students answering both the FCI and the exam question and the relatively large amount of questions devoted to the concept of NIII on the FCI we were able to perform additional studies. By having four questions on the FCI devoted to the concept of NIII we can get a better idea of the consistency in students' knowledge of NIII. Because 128 students answered both the exam question and the FCI, as opposed to 28 students, we can perform some other types of analysis. In particular, we will compare student responses on the exam to their responses on question 13 of the FCI. Question 13 is the most similar to the exam question.

Table 4 - 5 shows how students performed on the exam question and the FCI questions. The five categories correspond to the number of students who answered 0, 1, 2, 3, or 4 of the FCI questions correctly. The table indicates that 67% of the students who answered all four FCI questions correctly answered the exam question correctly. Similarly 65% answered the exam question correctly with three FCI questions correct, 53% for two FCI's correct, 29% for one FCI correct, and 14% for no FCI questions answered correctly. This data shows that performance on the FCI is correlated with performance on the exam; the more questions students answer correctly on the FCI the better they do on the corresponding exam question. It is a little surprising that even if students answered all four of the FCI questions correctly, which many instructors would say indicates a good conceptual understanding of NIII, only 67% of these students answered correctly on the exam.

**N=128**

<b>Exam Question</b>	<b>FCI</b>				
	0 FCI's correct:	1 FCI correct	2 FCI's correct	3 FCI's correct:	4 FCI's correct
Correct	3	8	11	21	24
Incorrect	11	20	8	10	12

**Table 4 - 5**

*How students answered the FCI questions on Newton's 3<sup>rd</sup> Law and how they answered the corresponding open-ended problem.*

The most interesting comparison to make is the way students answered question 13 on the FCI and the exam question. Question 13 on the FCI is shown in Figure 4 - 4. In it, a car pushes a truck and the entire system accelerates. On the exam question the two carts interact and the system accelerates. Therefore, in both cases two objects remain in contact and accelerate uniformly for the entire motion considered. Table 4 - 6 shows how students responded on the two similar questions. If the students' knowledge about NIII is consistent the off-diagonal elements of the table would be close to zero. The chart shows that only about half of the students fell on the diagonal indicating that these similar questions were answered inconsistently by about half the class. The open-ended question on the exam gives the students the opportunity to explain their reasoning. There were 21 students on the exam that clearly stated their reasoning for their incorrect responses. When we look at how these students answered question 13 on the FCI, we see that only six of them answered FCI question 13 in a way that was consistent with their reasoning used on the exam.

A large truck breaks down out on the road and receives a push back into town by a small compact car as shown in the figure below.



While the car, still pushing the truck, is speeding up to get up to cruising speed

- (A) the amount of force with which the car pushes on the truck is equal to that with which the truck pushes back on the car.
- (B) the amount of force with which the car pushes on the truck is smaller than that with which the truck pushes back on the car.
- (C) the amount of force with which the car pushes on the truck is greater than that with which the truck pushes back on the car.
- (D) the car's engine is running so the car pushes against the truck, but the truck's engine is not running so the truck cannot push back against the car. The truck is pushed forward simply because it is in the way of the car.
- (E) neither the car nor the truck exert any force on the other. The truck is pushed forward simply because it is in the way of the car.

**Figure 4 - 4**

*FCI question 13. This question is the most similar to the exam problem on NIII. In both cases the system of interacting objects is accelerating.*

N=128

Exam Problem Response	FCI Question 13 response			
	Correct	Forces not equal	No contact forces shown	Other
Correct	33%	16%	0%	1%
Forces not equal	12%	20%	0%	0%
No contact forces shown	2%	2%	0%	0%
Other	5%	9%	0%	1%

**Table 4 - 6**

*Comparison of how students responded on FCI question 13 and on the open-ended problem. Approximately half of the students answered inconsistently on the two instruments.*

## Summary

Our results show that students often answered differently on questions from the FCI and our open-ended problems, even though the questions were essentially the same. This provides us with evidence that, even within a particular topic, student content knowledge is fragmented. This study also suggests that student understanding cannot be measured by only one type of evaluation tool.

There are a number of possibilities that might account for some of the differences we have observed in how the students answered the corresponding questions. These possibilities include the fact that student models for physical systems can be ill formed and inconsistent, the FCI looks colloquial while the exam question looks formal, and the nature of the measurement instruments are different.

The different responses indicate that for many of our students the schemas they activate strongly depend on the context of the question. In contrast, an experienced physicist will tend to activate schemas that are more dependent on the concepts and principles involved and not so much on the context of the questions.

One possible reason different schemas get activated is because the exam question looks like a physics question, with carts and strings and free-body diagrams, while the FCI looks more colloquial. Instead of carts colliding on a table the FCI asks questions with objects from the real world. An anecdote given by Eric Mazur epitomizes this distinction. Upon looking at the questions on the FCI, one student asked, "Professor Mazur, how should I answer these questions? According to what

you taught us, or by the way I think about these things?"<sup>12,13</sup> Different questions might cue schemas for physics knowledge or for experiential knowledge, therefore causing students to answer differently. These ideas are related to the way students view physics in that many students do not relate the concepts and principles they learn in their physics course to the real world.<sup>14</sup>

The fact that the exam is open-ended and the diagnostic is multiple choice may also have an effect on how students answer the two types of questions. We found that most of the incorrect responses on the exam question were choices on the FCI but there were a handful that were not. We also noticed that there were a number of answers given on the exam that were incorrect but would be measured as correct on the FCI. In addition we saw students give correct answers on the exam for the wrong reasons. Another important feature that separates the multiple-choice questions from the open-ended questions is that the FCI includes distractors that are very appealing to the student. The distractors may have triggered responses, either right or wrong, that would not have been produced by the students on their own.<sup>15</sup> These choices on the FCI can trigger particular schemas. One of the goals in the physics class is to have students recognize physics principles in a wide variety of real-world contexts in addition to physics-class contexts. The distractors on the FCI measure the robustness of the physics schemas students have learned.

It is important to note that these results do not imply that one type of measurement instrument is better than the other. They both provide the instructor and the physics education researcher with valid, yet different, information about student understanding. When used in conjunction with one another they provide a useful tool in evaluating students coherence about different topics and concepts in physics. The danger is when instructors or physics education researchers use only one type of measurement tool to evaluate student understanding of physics.<sup>16</sup>

Because of their ease of use, many physics educators are solely using multiple-choice diagnostics to evaluate their students' understanding of the subject. They are therefore assuming the correlation between measurement items to be much greater than it is. It also may lead to instructors viewing student learning in a simplified way. This data shows that student understanding is very complex and depends on a number of factors.

This chapter demonstrates that students may use particular sets of knowledge (schemas) to answer a question or solve a problem. We have shown that different schemas often get activated when the context of question changes, even though the underlying physics is the same. In addition the knowledge in the schemas, used in the corresponding exam and FCI questions, often does not overlap.

---

<sup>1</sup> D. Hestenes, M. Wells and, G. Swackhammer, "Force Concept Inventory," Phys. Teach. **30** (3) 141-153 (1992).

<sup>2</sup> For an account of the research that preceded the FCI, see I. A. Halloun and D. Hestenes, "The initial knowledge state of college physics students," Am. J. Phys. **53** (11) 1043-1055 (1985).

<sup>3</sup> For a discussion of a factor analysis on the FCI, see D. Huffman and P. Heller, "What does the Force Concept Inventory actually measure?" Phys. Teach. **33**, 138-143 (1995); D. Hestenes and I. Halloun, "Interpreting the force concept inventory, a response to the March 1995 critique by Huffman and Heller," Phys. Teach. **33**, 502 (1995); P. Heller and D. Huffman, "Interpreting the force concept inventory, a reply to Hestenes and Halloun," Phys. Teach. **33**, 503 (1995).

<sup>4</sup> For an example that includes thorough research on reliability of multiple-choice physics diagnostics, see R.J Beichner, "Testing student interpretation of kinematic graphs," Am. J. Phys. **62**, 750-762 (1994); for further discussion see references therein.

<sup>5</sup> R. S. Steinberg and M. S. Sabella, "Performance on multiple-choice diagnostics and complimentary exam problems," Phys. Teach. **35** (3), 150-155 (1997).

<sup>6</sup> L. C. McDermott, P. S. Shaffer, and M. D. Somers, "Research as a guide for teaching introductory mechanics: An illustration in the context of the Atwood's machine," Am. J. Phys. **62** (1) 46-55 (1994) and J. Clement, "Students' preconceptions in introductory mechanics," Am. J. Phys. **50** (1) 66-71 (1982); I. A. Halloun and D. Hestenes, "Common sense concepts about motion," Am. J. Phys. **53** (11) 1056-1064 (1985); I. A. Halloun and D. Hestenes, "The initial knowledge state of college physics students," Am. J. Phys. **53** (11) 1043-1055 (1985).

<sup>7</sup> See Ref. 2.

<sup>8</sup> A brief discussion of the estimate in the error is presented in chapter 3.

<sup>9</sup> See Ref. 6.

<sup>10</sup> For a discussion on the impetus force see Ref. 2.

<sup>11</sup> See D. P. Maloney, "Rule-governed approaches to physics: Newton's Third Law," Phys. Educ. **19**, 37-42 (1984). and E.F. Redish, J.M. Saul, and R.N. Steinberg, "On the effectiveness of active-engagement microcomputer-based laboratories," Am. J. Phys. **66** (3), 212-224 (1998).

<sup>12</sup> E. Mazur, *Peer Instruction*, (Prentice Hall, NJ, 1997).

<sup>13</sup> This student recognized that she had two different schemas for a particular question.

<sup>14</sup> The connection of physics and reality are discussed in E. F. Redish, J. M. Saul, and R. N. Steinberg, "Student expectations in introductory physics," Am. J. Phys. **66** (3), 212-224 (1998).

<sup>15</sup> M.C. Wittmann, "Making Sense of how students come to an understanding of physics: An example from mechanical waves," Ph.D. dissertation, Department of Physics, University of Maryland, College Park, (1998).

<sup>16</sup> Redish cautions that it rarely pays to build universal theories come from a few good insights, see E.F. Redish, "Millikan Award Lecture (1998): Building a Science of Teaching Physics," Am. J. Phys. **67** (7), 562-573 (1999).