① x^i_{rel} relative coordinate: $x^i_{rel} \equiv (x_1)^i - (x_2)^i$, where $\begin{cases} x^1_{rel} = R\cos\omega t \\ x^2_{rel} = R\cos i \sin\omega t \\ x^3_{rel} = R\sin i \sin\omega t \end{cases}$

$$(x_{CM})^i \equiv \frac{m_1 (x_1)^i + m_2 (x_2)^i}{m_1 + m_2}$$

$$T^{00} = m_1 c^2 \delta^{(3)}(\vec{x} - \vec{x}_1(t)) + m_2 c^2 \delta^{(3)}(\vec{x} - \vec{x}_2(t))$$

$$M^{klm} = \frac{1}{c^2}\int d^3x\, x^k x^l x^m T^{00} \underset{\vec{x}_{CM}=0}{=} \mu \frac{\delta m}{M} x^k_{rel} x^l_{rel} x^m_{rel}$$

$\mu \equiv m_1 m_2 / M$

where: $\delta m \equiv m_2 - m_1$

$M \equiv m_1 + m_2$

From Maggiore (3.141)

$$(h^{TT}_{ij})_{oct} = \frac{2G}{3c^5 r} \Lambda_{ij,kl}(\hat{n}) \hat{n}_m \dddot{O}^{klm}$$

but we can use M^{klm} interchangably thanks to the contraction with the projectors $\Lambda_{ij,kl} n_m$

$$\Lambda_{ij,kl}(\hat{n}) = P_{ik} P_{jl} - \frac{1}{2} P_{ij} P_{kl}, \quad P_{ij} = \delta_{ij} - \hat{n}_i \hat{n}_j$$

Here $\hat{n} = \hat{z} \Rightarrow \hat{n}_i = \delta^3_i$

$$(h_+)_{oct} = (h^{TT}_{xx})_{oct} = (h^{TT}_{11})_{oct}$$

$$(h_\times)_{oct} = (h^{TT}_{xy})_{oct} = (h^{TT}_{12})_{oct}$$

$$(h_+)_{oct} = \frac{2G}{3c^5 r} \Lambda_{11,kl} \delta^3_m \dddot{M}^{klm} = \frac{2G}{3c^5 r} \Lambda_{11,kl} \dddot{M}^{kl3}$$

$$\Lambda_{11,kl} = P_{1k} P_{1l} - \frac{1}{2} \underbrace{P_{11}}_{1} P_{kl} = (\delta_{1k} - \cancel{\delta^3_1 \delta^3_k})(\delta_{1l} - \cancel{\delta^3_1 \delta^3_l}) - \frac{1}{2}(\delta_{kl} - \delta^3_k \delta^3_l)$$

$$\Lambda_{11,kl} \dddot{M}^{kl3} = \left[\delta_{1k}\delta_{1l} - \frac{1}{2}(\delta_{kl} - \delta^3_k \delta^3_l)\right] \dddot{M}^{kl3} =$$

$$= \dddot{M}^{113} - \frac{1}{2}\left(\dddot{M}^{113} + \dddot{M}^{223} + \cancel{\dddot{M}^{333}} - \cancel{\dddot{M}^{333}}\right) =$$

$$= \frac{1}{2}\left(\dddot{M}^{113} - \dddot{M}^{223}\right)$$

$$\dddot{M}^{113} = \frac{d^3}{dt^3}\left[\mu \frac{\delta m}{M} R^3 \cos^2\omega t \sin\omega t \sin i\right] =$$

$$= \mu \frac{\delta m}{M} R^3 \sin i \frac{d^3}{dt^3}\left[\sin\omega t - \sin^3\omega t\right] =$$

$$= \mu \frac{\delta m}{M} R^3 \sin i \left[-\omega^3 \cos\omega t - 6\omega^3 \cos^3\omega t + 21\omega^3 \cos\omega t \sin^2\omega t\right] =$$

$$= -\frac{1}{4}\omega^3 \mu \frac{\delta m}{M} R^3 \sin i \left(\cos\omega t + 27\cos 3\omega t\right)$$

$$\dddot{M}^{223} = \frac{d^3}{dt^3}\left[\mu \frac{\delta m}{M} R^3 \sin i \cos^2 i \sin^3\omega t\right] =$$

$$= \frac{3}{2}\omega^3 \mu \frac{\delta m}{M} R^3 \sin i \cos^2 i \,(9\cos 2\omega t - 5)\cos\omega t$$

$$\dddot{M}^{113} - \dddot{M}^{223} = R^3 \frac{\mu}{2} \frac{\delta m}{M}\omega^3 \sin i\Big[-\frac{1}{2}\cos\omega t - \frac{27}{2}\cos 3\omega t - 27\cos^2 i\,\underbrace{\cos 2\omega t\cos\omega t}_{\frac{1}{2}(\cos\omega t+\cos 3\omega t)}$$

$$+ 15\cos^2 i\cos\omega t\Big] =$$

$$= R^3\frac{\mu}{2}\frac{\delta m}{M}\omega^3 \sin i\Big[-\frac{1}{2}\cos\omega t\,(1+\overbrace{27\cos^2 i - 30\cos^2 i}^{1-3\cos^2 i}) - \frac{27}{2}\cos 3\omega t$$

$$(1+\cos^2 i)\Big] = \qquad \text{use } \cos^2 i = \frac{1}{2}(1+\cos 2i)$$

$$= R^3\frac{\mu}{8}\frac{\delta m}{M}\omega^3 \sin i\,\big[\cos\omega t\,(1+3\cos 2i) - 27\cos 3\omega t\,(3+\cos 2i)\big]$$

$$(h_+)_{oct} = \frac{\cancel{2}}{3}\frac{G}{c^5 r} R^3 \frac{\mu}{\cancel{8}4}\frac{\delta m}{M}\omega^3 \frac{\sin i}{2}\Big[\quad\Big] =$$

$$= \frac{1}{r}\frac{G R^3 \mu\omega^3}{12 c^5}\frac{\delta m}{M}\sin i\left[\underbrace{\left(\frac{1+3\cos 2i}{2}\right)}_{3\cos^2 i - 1}\cos\omega t - 27\underbrace{\left(\frac{3+\cos 2i}{2}\right)}_{1+\cos^2 i}\cos 3\omega t\right]$$

$$\Lambda_{12,kl} = P_{1k}P_{2l} - \frac{1}{2}P_{12}P_{kl} = (\delta_{1k} - \cancel{\delta^3_1\delta^3_k})(\delta_{2l} - \cancel{\delta^3_2\delta^3_l}) - \underbrace{\frac{1}{2}(\cancel{\delta_{12}} - \cancel{\delta^3_1\delta^3_2})P_{kl}}_{0} =$$

$$= \delta_{1k}\delta_{2l}$$

$$\Lambda_{12,kl}\dddot{M}^{kl3} = \delta_{1k}\delta_{2l}\dddot{M}^{kl3} = \dddot{M}^{123}$$

$$\dddot{M}^{123} = \frac{d^3}{dt^3}\Big[\mu\frac{\delta m}{M}R^3 \sin i\cos i\,\sin^2\omega t\cos\omega t\Big] =$$

$$= +\frac{1}{4}\mu\frac{\delta m}{M}R^3 \sin i\cos i\,\omega^3(\sin\omega t - 27\sin 3\omega t)$$

$$(h_\times)_{oct} = \frac{2}{3}\frac{G}{c^5 r}\frac{1}{4}\mu\frac{\delta m}{M}R^3\sin i\cos i\,\omega^3(\sin\omega t - 27\sin 3\omega t) =$$

$$= \frac{1}{r}\frac{G R^3\mu\omega^3}{6c^5}\frac{\delta m}{M}\sin i\cos i\,(\sin\omega t - 27\sin 3\omega t)$$

The mass-octupole radiation is emitted at $\omega_{GW} = \omega,\ 3\omega$.

② $S^{kl,m} = \int d^3x\, T^{kl} x^m$

$$T^{kl} = m_1 (\dot{x}_1)^k (\dot{x}_1)^l \delta^{(3)}(\vec{x}-\vec{x}_1) + m_2 (\dot{x}_2)^k (\dot{x}_2)^l \delta^{(3)}(\vec{x}-\vec{x}_2)$$

$$\Rightarrow S^{kl,m} = m_1 (\dot{x}_1)^k (\dot{x}_1)^l (x_1)^m + m_2 (\dot{x}_2)^k (\dot{x}_2)^l (x_2)^m = \text{ move to C.O.M.}$$

$$= \mu \frac{\delta m}{M} \dot{x}^k_{rel} \dot{x}^l_{rel} x^m_{rel} \quad \text{where} \quad \begin{cases} \dot{x}^1_{rel} = -\omega R \sin\omega t \\ \dot{x}^2_{rel} = \omega R \cos i \cos\omega t \\ \dot{x}^3_{rel} = \omega R \sin i \cos\omega t \end{cases}$$

$$(h^{TT}_{ij})_{oct+cq} = \frac{1}{r} \frac{4G}{c^5} \Lambda_{ij,kl}(\hat{n}) \hat{n}_m \dot{S}^{kl,m} \quad \text{from Maggiore (3.138)}$$

Again $(h_+)_{oct+cq} = (h^{TT}_{11})_{oct+cq}$ and $(h_\times)_{oct+cq} = (h^{TT}_{12})_{oct+cq}$.

Just like in problem 1, the components we need are only $\dot{S}^{11,3}$, $\dot{S}^{22,3}$ and $\dot{S}^{12,3}$

$$\dot{S}^{11,3} = \frac{d}{dt}\left[\mu \frac{\delta m}{M} (\dot{x}^1_{rel})^2 x^3_{rel}\right] = \mu \frac{\delta m}{M} \frac{d}{dt}\left[\omega^2 R^3 \sin^2\omega t \sin\omega t \sin i\right] =$$

$$= 3\mu \frac{\delta m}{M} \omega^3 R^3 \sin^2\omega t \cos\omega t \sin i = \frac{3}{4}\mu \frac{\delta m}{M} \omega^3 R^3 (\cos\omega t - \cos 3\omega t)\sin i$$

$$\dot{S}^{22,3} = \mu \frac{\delta m}{M} \frac{d}{dt}\left[\omega^2 R^3 \cos^2 i \sin i \cos^2\omega t \sin\omega t\right] =$$

$$= \frac{1}{4}\mu \frac{\delta m}{M} \omega^3 R^3 \sin i \cos^2 i (\cos\omega t + 3\cos 3\omega t)$$

$$\dot{S}^{12,3} = \mu \frac{\delta m}{M} \frac{d}{dt}\left[-\omega^2 R^3 \sin i \cos i \sin^2\omega t \cos\omega t\right] =$$

$$= -\frac{1}{2}\mu \frac{\delta m}{M} \omega^3 R^3 \sin i \cos i (\sin\omega t + 3\sin\omega t \cos 2\omega t) =$$

$$= \frac{1}{4}\mu \frac{\delta m}{M} \omega^3 R^3 \sin i \cos i (\sin\omega t - 3\sin 3\omega t)$$

$$(h_+)_{oct+cq} = \frac{1}{r} \frac{4G}{c^5} \frac{1}{2}\left(\dot{S}^{11,3} - \dot{S}^{22,3}\right) =$$

$$= \frac{1}{r} \frac{4G}{c^5} \frac{1}{8}\mu \frac{\delta m}{M} \omega^3 R^3 \sin i \left[3\cos\omega t - 3\cos 3\omega t - \cos^2 i (\cos\omega t + 3\cos 3\omega t)\right] =$$

$$= +\frac{1}{r} \frac{G\mu\omega^3 R^3}{2c^5} \frac{\delta m}{M} \sin i \left[(3-\cos^2 i)\cos\omega t - 3(1+\cos^2 i)\cos 3\omega t\right]$$

$$(h_\times)_{oct+cq} = \frac{1}{r} \frac{4G}{c^5} \dot{S}^{12,3} = \frac{1}{r} \frac{\cancel{4}G}{c^5} \frac{1}{\cancel{4}} \mu \frac{\delta m}{M} \omega^3 R^3 \sin i \cos i (\sin\omega t - 3\sin 3\omega t) =$$

$$= \frac{1}{r} \frac{G\mu\omega^3 R^3}{c^5} \frac{\delta m}{M} \sin i \cos i (\sin\omega t - 3\sin 3\omega t)$$

Finally

$$(h_+)_{cq} = (h_+)_{oct+cq} - (h_+)_{oct} =$$

$$= \frac{1}{r}\frac{G}{2c^5}\mu\frac{\delta m}{M}(R\omega)^3 \sin i\Big[\cos\omega t\left(3-\cos^2 i-\frac{1}{2}\cos^2 i+\frac{1}{6}\right)$$

$$-3\cos 3\omega t\left(1+\cos^2 i-\frac{3}{2}-\frac{3}{2}\cos^2 i\right)\Big] =$$

$$= \frac{1}{r}\frac{G}{24c^5}\mu\frac{\delta m}{M}(R\omega)^3 \sin i\Big[(29-9\cos 2i)\cos\omega t+9(3+\cos 2i)$$

$$\cos 3\omega t\Big]$$

$$(h_\times)_{cq} = (h_\times)_{oct+cq} - (h_\times)_{oct} =$$

$$= \frac{1}{r}\frac{G}{c^5}\mu\frac{\delta m}{M}(R\omega)^3 \sin i\cos i\left[\sin\omega t - 3\sin 3\omega t - \frac{1}{6}(\sin\omega t - 27\sin 3\omega t)\right]$$

$$= \frac{1}{r}\frac{G}{6c^5}\mu\frac{\delta m}{M}(R\omega)^3 \sin i\cos i\,(5\sin\omega t+9\sin 3\omega t)$$

③ $$P_{oct+cq} = \frac{r^2 c^3}{8G} \int_0^{\pi} di \, \sin i \; \frac{\omega}{2\pi} \int_0^{2\pi/\omega} dt \left[(\dot h_+)^2_{oct+cq} + (\dot h_\times)^2_{oct+cq} \right]$$

Let $h_0 \equiv \frac{1}{r} \frac{G \mu R^3 \omega^3 \delta m}{2 c^5 M}$

$$(h_+)_{oct+cq} = h_0 \sin i \left[(3 - \cos^2 i) \cos \omega t - 3 (1 + \cos^2 i) \cos 3\omega t \right]$$

$$(h_\times)_{oct+cq} = h_0 \sin 2i \left[\sin \omega t - 3 \sin 3\omega t \right]$$

$$(\dot h_+)_{oct+cq} = \omega h_0 \sin i \left[-(3 - \cos^2 i) \sin \omega t + 9 (1 + \cos^2 i) \sin 3\omega t \right]$$

$$(\dot h_\times)_{oct+cq} = \omega h_0 \sin 2i \left[\cos \omega t - 9 \cos 3\omega t \right]$$

$$(\dot h_+)^2_{oct+cq} + (\dot h_\times)^2_{oct+cq} = \omega^2 h_0^2 \sin^2 i \left[(3 - \cos^2 i)^2 \sin^2 \omega t + \right.$$

$$\left. + 81 (1 + \cos^2 i)^2 \sin^2 3\omega t - 18 (3 - \cos^2 i)(1 + \cos^2 i) \sin \omega t \sin 3\omega t \right] +$$

$$+ \omega^2 h_0^2 \underbrace{\sin^2 2i}_{4 \sin^2 i \cos^2 i} \left[\cos^2 \omega t + 81 \cos^2 3\omega t - 18 \cos \omega t \cos 3\omega t \right] =$$

$$= \omega^2 h_0^2 \sin^2 i \left\{ (3 - \cos^2 i)^2 \frac{1 - \cos 2\omega t}{2} + 81 (1 + \cos^2 i)^2 \frac{1 - \cos 6\omega t}{2} \right.$$

$$- 18 (3 - \cos^2 i)(1 + \cos^2 i) \sin \omega t \sin 3\omega t \; +$$

$$\left. + 4 \cos^2 i \left[\frac{1 + \cos 2\omega t}{2} + 81 \frac{1 + \cos 6\omega t}{2} - 18 \cos \omega t \cos 3\omega t \right] \right\} =$$

$$= \omega^2 h_0^2 \sin^2 i \left\{ \frac{1}{2} (3 - \cos^2 i)^2 + \frac{81}{2} (1 + \cos^2 i)^2 - \frac{1}{2} \cos 2\omega t \left[(3 - \cos^2 i)^2 - 4 \cos^2 i \right] \right.$$

$$- \frac{1}{2} \cos 6\omega t \left[81 (1 + \cos^2 i)^2 - 81 \cdot 4 \cos^2 i \right] + 164 \cos^2 i$$

$$\left. - 18 (3 - \cos^2 i)(1 + \cos^2 i) \sin \omega t \sin 3\omega t - 18 \cdot 4 \cos^2 i \cos \omega t \cos 3\omega t \right\}$$

$$\langle \cdots \rangle = \frac{\omega}{2\pi} \int_0^{2\pi/\omega} dt \, (\cdots)$$

$\langle \cos 2\omega t \rangle = 0$, $\langle \sin \omega t \sin 3\omega t \rangle = 0$ $\quad$ only the terms w/out

$\Rightarrow$ t-dependence

$\langle \cos 6\omega t \rangle = 0$, $\langle \cos \omega t \cos 3\omega t \rangle = 0$ $\quad$ survive the $\langle \ldots \rangle$

$$\langle (\dot h_+)^2_{oct+cq} + (\dot h_\times)^2_{oct+cq} \rangle = \frac{1}{2} \omega^2 h_0^2 \sin^2 i \left[(3 - \cos^2 i)^2 + 81 (1 + \cos^2 i)^2 + 328 \cos^2 i \right]$$

$$\int_{-1}^{1} d(\cos i) \, \langle (\dot h_+)^2_{oct+cq} + (\dot h_\times)^2_{oct+cq} \rangle = \frac{\omega^2 h_0^2}{2} \underbrace{\int_{-1}^{1} dx \, (1 - x^2) \left[(3 - x^2)^2 + 81 (1 + x^2)^2 + 328 x^2 \right]}_{\frac{27136}{105}}$$

$$P_{oct+cq} = \frac{r^2 c^3}{8G} \frac{\omega^2 h_0^2}{2} \frac{27136}{35} = \frac{\cancel{r^2} c^3}{\cancel{G}} \frac{1696}{105} \omega^2 \frac{1}{\cancel{r^2}} \frac{G^{\cancel{2}} \mu^2 R^6 \omega^6 (\delta m)^2}{4 c^{10} M^2} =$$

$$= \frac{424}{105}\frac{G}{c^7}\mu^2\left(\frac{\delta m}{M}\right)^2 R^6\omega^8$$

The fraction of P emitted at freq. ω is given by the terms:

$$h_+\big|_\omega = h_0 \sin i\,(3-\cos^2 i)\cos\omega t$$

$$h_\times\big|_\omega = h_0 \sin 2i \sin\omega t$$

$$\dot h_+\big|_\omega = -\omega h_0 \sin i\,(3-\cos^2 i)\sin\omega t$$

$$\dot h_+\big|_\omega = \omega h_0 \sin 2i \cos\omega t$$

$$(\dot h_+\big|_\omega)^2 + (\dot h_+\big|_\omega)^2 = \omega^2 h_0^2\left[\sin^2 i\,(3-\cos^2 i)^2\overbrace{\frac{1-\cos 2\omega t}{2}}^{\sin^2\omega t} + \sin^2 2i\,\overbrace{\frac{1+\cos 2\omega t}{2}}^{\cos^2\omega t}\right]$$

Take the $\langle\ldots\rangle$ and the terms that survive are:

$$\frac{1}{2}\omega^2 h_0^2\left[\sin^2 i\,(3-\cos^2 i)^2 + \sin^2 2i\right] = \frac{1}{2}\omega^2 h_0^2 \sin^2 i\left[(3-\cos^2 i)^2 + 4\cos^2 i\right] =$$

$$= \frac{1}{2}\omega^2 h_0^2 (1-\cos^2 i)\left[(3-\cos^2 i)^2 + 4\cos^2 i\right]$$

$$\int_{-1}^{1} dx\,(1-x^2)\left[(3-x^2)^2 + 4x^2\right] = \frac{1216}{105}$$

$$P_{oct+cq}\big|_\omega = \frac{r^2 c^3}{8G}\frac{1216}{105}\frac{1}{2}\omega^2 h_0^2 = \frac{19}{105}\frac{G}{c^7}\mu^2\left(\frac{\delta m}{M}\right)^2 R^6\omega^8$$

$$\Rightarrow \frac{P_{oct+cq}(\omega)}{P_{quad}(2\omega)} = \frac{\frac{19}{105}\frac{\cancel{G}}{c^7}\cancel{\mu^2}\left(\frac{\delta m}{M}\right)^2 R^6\omega^8}{\frac{32}{5}\frac{\cancel{G}}{c^5}\cancel{\mu^2}R^4\omega^6} = \frac{19}{672}\left(\frac{\delta m}{M}\right)^2\frac{1}{c^2}R^2\omega^2$$

Use $\omega^2 R^2 = \frac{GM}{R} = v^2$ then:

$$\frac{P_{oct+cq}(\omega)}{P_{quad}(2\omega)} = \frac{19}{672}\left(\frac{\delta m}{M}\right)^2\left(\frac{v}{c}\right)^2$$

The fraction of P_{oct+cq} emitted at freq. 3ω is given by the terms:

$$h_+\big|_{3\omega} = h_0 \sin i\left[-3(1+\cos^2 i)\cos 3\omega t\right] \Rightarrow \dot h_+\big|_{3\omega} = \omega 9 h_0 \sin i\,(1+\cos^2 i)\sin 3\omega t$$

$$h_\times\big|_{3\omega} = -3h_0 \sin 2i \sin 3\omega t \Rightarrow \dot h_\times\big|_{3\omega} = -9\omega h_0 \sin 2i \cos 3\omega t$$

$$(\dot h_+\big|_{3\omega})^2 + (\dot h_\times\big|_{3\omega})^2 = \omega^2 h_0^2\, 81\left[\sin^2 i\,(1+\cos^2 i)^2\frac{1-\cos 6\omega t}{2} + \sin^2 2i\,\frac{1+\cos 6\omega t}{2}\right.$$

Take $\langle \cdots \rangle$ and we're left with

$$\frac{81}{2} \omega^2 h_0^2 \sin^2 i \left[(1+\cos^2 i)^2 + 4\cos^2 i \right] =$$

$$= \frac{81}{2} \omega^2 h_0^2 (1-\cos^2 i) \left[(1+\cos^2 i)^2 + 4\cos^2 i \right]$$

$$\int_{-1}^{1} dx\, (1-x^2)\left[(1+x^2)^2 + 4x^2\right] = \frac{64}{21}$$

$$P_{oct+cq}\Big|_{3\omega} = \frac{r^2 c^3}{8G} \frac{64}{21} \frac{81}{2} \omega^2 h_0^2 = \frac{27}{7} \frac{G}{c^7} \mu^2 \left(\frac{\delta m}{M}\right)^2 R^6 \omega^8$$

$$\Rightarrow \frac{P_{oct+cq}(3\omega)}{P_{quad}(2\omega)} = \frac{\frac{27}{7} \frac{\cancel{G}}{c^7} \cancel{\mu^2} \left(\frac{\delta m}{M}\right)^2 R^6 \omega^8}{\frac{32}{5} \frac{\cancel{G}}{c^5} \cancel{\mu^2} R^4 \omega^6} = \frac{135}{224} \left(\frac{\delta m}{M}\right)^2 \frac{R^2 \omega^2}{c^2} =$$

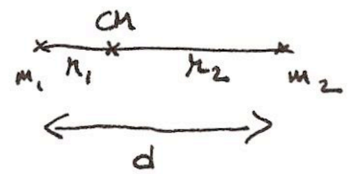
$$= \frac{135}{224} \left(\frac{\delta m}{M}\right)^2 \left(\frac{v}{c}\right)^2$$

Let $v/c = 10^{-2}$, then

$$\frac{P_{oct+cq}(\omega)}{P_{quad}(2\omega)} \cong 2.8 \times 10^{-6} \left(\frac{\delta m}{M}\right)^2$$

$$\frac{P_{oct+cq}(3\omega)}{P_{quad}(2\omega)} \cong 61 \times 10^{-6} \left(\frac{\delta m}{M}\right)^2$$

(4)



$$\omega^2 = \frac{G(m_1+m_2)}{d^3}, \quad m_1 r_1 = m_2 r_2$$

$$d = \frac{m_1+m_2}{m_2} r_1$$

$$V_1 = \omega r_1 = \frac{d m_2 \omega}{m_1+m_2}, \quad V_2 = \frac{d\, m_1 \omega}{m_1+m_2}$$

- Explosion w/out kick $\Rightarrow$ velocities are unaffected.

Only change is $m_2 \longrightarrow m_2 - \Delta \quad (\Delta > 0)$

Escape velocity for the new system for m_1 is:

$$\frac{1}{2} m_1 V_{\infty,1}^2 = \frac{G m_1 (m_2-\Delta)}{d} \Rightarrow V_{\infty,1}^2 = \frac{2G(m_2-\Delta)}{d}$$

But m_1 still has velocity $V_1 = \dfrac{m_2 d \omega}{m_1+m_2}$

$$V_1^2 = \frac{m_2^2 d^2 \omega^2}{(m_1+m_2)^2} = \frac{m_2^2 \cancel{d^2}}{(m_1+m_2)^{\cancel{2}}} \frac{G\cancel{(m_1+m_2)}}{\cancel{d^3}} =$$

$$= \frac{G m_2^2}{d(m_1+m_2)}$$

The binary disrupts if $V_1 > V_{\infty,1}$. If $m_1 = 1.4\, M_\odot$, $m_2 = 10\, M_\odot$, $\Delta = 8.6\, M_\odot$

then

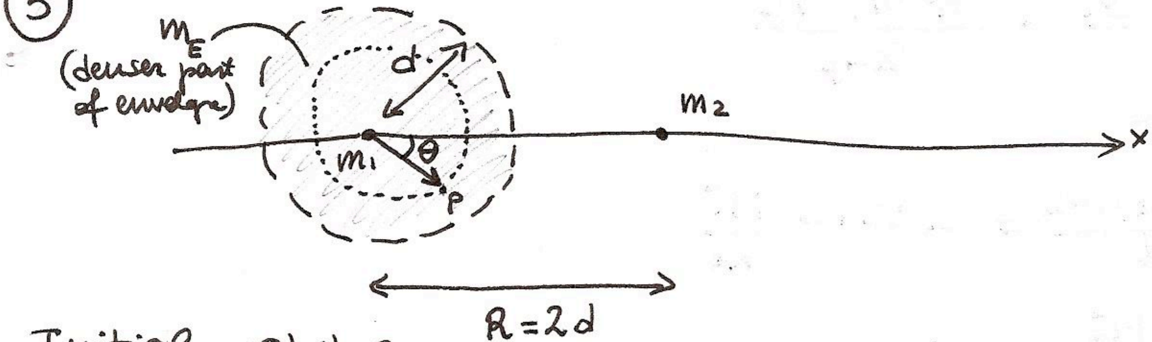
$$\left.\begin{aligned} V_{\infty,1}^2 &= \frac{2G}{d} \times 1.4\, M_\odot = 2.8\, \frac{GM_\odot}{d} \\ V_1^2 &= \frac{G}{d}\, \frac{100}{11.4}\, M_\odot = 8.8\, \frac{GM_\odot}{d} \end{aligned}\right\} V_1 > V_{\infty,1}\text{: the binary disrupts}$$

- Explosion w/ kick: the velocities will be affected. In the CM frame we can work w/ an effective particle $\mu = \dfrac{m_1 m_2}{m_1+m_2}$ orbiting a particle $M = m_1+m_2$ at a distance d, with velocity v_i

Before the SN: $\omega^2 = \dfrac{GM}{d^3}$, $V_i = \omega d$

After the SN: $V_\infty^2 = \dfrac{2G(M-\Delta)}{d}$. If $V_f = |\vec{V}_i + \vec{V}_{kick}| > V_\infty$ then the binary disrupts, otherwise it remains bound on eccentric orbit.

5



$m_1 = 4M_\odot$
$m_E = 12 M_\odot$
$d = 100 R_\odot$
$R = 2d$
$m_2 = 3 M_\odot$

Initial orbital energy : $E_i = -\dfrac{G(m_1+m_E)m_2}{2R}$

A infinitesimal volume element of the denser part of the envelope located at $\vec{r}$ has potential energy :

$$dU = -G\rho\, dV\left[\frac{m_1}{|\vec{r}|} + \frac{m_E}{|\vec{r}|}\left(\frac{|\vec{r}|}{d}\right)^3 + \frac{m_2}{|\vec{r}-R\hat{x}|}\right]$$

Where: $\rho = \dfrac{m_E}{\frac{4}{3}\pi d^3}$ density assumed uniform w/in sphere of radius d

$m_E\left(\dfrac{|\vec{r}|}{d}\right)^3$ is the mass w/in a sphere of radius $|\vec{r}|$

$R\hat{x}$ is the initial position of m_2

The tot potential energy of the envelope is

$$U = \int_{|\vec{r}|\le d} dU = -G\rho\int_{|\vec{r}|\le d} d^3r\left[\frac{m_1}{|\vec{r}|} + \frac{m_E}{d^3}|\vec{r}|^2 + \frac{m_2}{|\vec{r}-R\hat{x}|}\right]$$

$$\int_{|\vec{r}|\le d} d^3r\,\frac{1}{|\vec{r}|} = 4\pi\int_0^d dr\, r = 2\pi d^2$$

$$\int_{|\vec{r}|\le d} d^3r\,|\vec{r}|^2 = 4\pi\int_0^d dr\, r^4 = \frac{4}{5}\pi d^5$$

$$\int_{|\vec{r}|\le d} d^3r\,\frac{1}{|\vec{r}-R\hat{x}|} = 2\pi\int_0^d dr\, r^2\int_0^\pi d\theta\,\sin\theta\,\frac{1}{[r^2+R^2-2rR\cos\theta]^{1/2}} = \text{use Mathematica}$$

$$= 2\pi\int_0^d dr\, r^2\,\frac{4}{R-\cancel{r}+\cancel{r}+R} = 2\pi\int_0^d dr\,\frac{2}{R}r^2 = \frac{4}{3}\pi\frac{d^3}{R}$$

$$U = -G\rho\left[2\pi d^2 m_1 + \frac{m_E}{d^3}\frac{4}{5}\pi d^5 + m_2\,\frac{4}{3}\pi\frac{d^3}{2d}\right] =$$

$$= -\frac{G\, m_E\,(15m_1 + 5m_2 + 6m_E)}{10\, d}$$

$$E_f = E_i - |U| \stackrel{\downarrow}{=} -\frac{G m_2}{2 d_f}\left(m_1 + \frac{4}{3}\pi d_f^3 \rho\right) = -\frac{G m_2}{2 d_f}\left[m_1 + m_E\left(\frac{d_f}{d}\right)^3\right] \simeq -\frac{G m_1 m_2}{2 d_f}$$

(since $d_f \ll d$)

Let $G = 1$, then

$$E_i = -\frac{(m_1 + m_E) m_2}{4 d} = -0.12 \frac{M_\odot^2}{R_\odot}$$

$$U = -1.764 \frac{M_\odot^2}{R_\odot}$$

$$E_f = -(0.12 + 1.764)\frac{M_\odot^2}{R_\odot} \stackrel{\downarrow}{=} E_f \simeq -\frac{6}{d_f} M_\odot^2 \Rightarrow d_f \cong 3.2\ R_\odot$$