HW#4 —Phys879—Spring 2014
Due before class, Tuesday, March 25, 2014
http://www.physics.umd.edu/rgroups/grt/buonanno/Phys879/

Prof. Alessandra Buonanno
Room PSC 3149, (301)405-1440
buonanno@umd.edu

Recommended readings:

1. P.C. Peters & J. Mathews, Phys. Rev. **131**, 435 (1963).

Exercises:

1. **Mass octupole radiation from a binary system** [10 pts.]

   Let us consider a binary system of masses m_1 and m_2 and reduced mass μ whose center-of-mass coordinate moves along the circular orbit

   $$x(t) = R\cos\omega\, t \qquad y(t) = R\cos\iota\sin\omega\, t \qquad z(t) = R\sin\iota\sin\omega\, t\,,$$

   where $R^2 = x^2 + y^2 + z^2$. Let us set the observer's direction $\hat{n}$ along the z direction. [Henceforth, the (x, y, z) axes are also labeled as $(1, 2, 3)$.] The octupole (oct) gravitational radiation is

   $$(h_{ij}^{\rm TT})_{\rm oct} = \frac{1}{r}\frac{2G}{3c^5}\,\Lambda_{ij,kl}(\hat{n})\,\dddot{M}_{kl3}\,,$$

   where r is the distance of the observer from the binary and

   $$M^{klm} = \mu\,\frac{\delta m}{m}\,x^k\,x^l\,x^m \qquad \Lambda_{ij,kl} = P_{ik}\,P_{jl} - P_{ij}\,P_{kl}/2 \qquad P_{ij} = \delta_{ij} - \hat{n}_i\,\hat{n}_j$$

   with $\delta m = m_1 - m_2$ and $m = m_1 + m_2$. Evaluate the $(h_+)_{\rm oct}$ and $(h_\times)_{\rm oct}$ components. At which frequencies the octupole gravitational radiation is emitted?

2. **Current quadrupole radiation from a binary system** [10 pts.]

   Evaluate the current-quadrupole (cq) radiation, notably the components $(h_+)_{\rm cq}$ and $(h_\times)_{\rm cq}$, for the binary of Problem 1.

3. **Power radiated** [10 pts.]

   Using results of Problem 2 determine the power radiated using the following formula,

   $$P_{\rm oct+cq} = \frac{r^2\,c^3}{16\pi G}\,2\pi\int_{-1}^{1} d\cos\iota < \dot{h}_+^2 + \dot{h}_\times^2 > .$$

   Express the ratios $P_{\rm oct+cq}(\omega)/P_{\rm quad}(2\omega)$ and $P_{\rm oct+cq}(3\omega)/P_{\rm quad}(2\omega)$, where $P_{\rm quad} = 32G\mu^2R^4\omega^6/(5c^5)$, in terms of v/c using the Newtonian relation between R and ω, and estimate those ratios for $v/c = 10^{-2}$.

4. **How double neutron stars form** [5 pts.]

   As you learned from Cole Miller's lecture, to get a double neutron star system out of the evolution of a massive binary, the original stars must both explode as supernovae. When the second star explodes, it is probably common that more than half of the total system mass leaves suddenly. For example, if the first star has already left behind a neutron star with a mass $1.4M_\odot$ and the second star, just prior to the collapse and explosion, has a mass of $10M_\odot$, then an explosion that takes away $8.6M_\odot$ of the total $11.4M_\odot$ is needed to leave behind a 1.4–$1.4M_\odot$ binary.

- Assuming that the binary just prior to the second supernova was in a circular orbit, that the supernova exploded isotropically in its rest frame and delivered no kick to the neutron star, and that the mass that is lost is more than half of the system mass and is lost instantaneously, what happens to the binary?
- How does the above result change if the supernova delivers a kick to the neutron star?

5. **Common-envelope phase in binary evolution** [optional!]

The common-envelope phase is a process that occurs in the evolution of close binary systems from their initial, main-sequence configurations to final compact-body configurations (NS-NS, NS-BH or BH-BH).

Let us assume that a $16M_\odot$ star, late in his life, evolves into a red giant phase, which is characterized by a $4M_\odot$ compact core and a $12M_\odot$ puffed-up envelope. The radius of the envelope is $R \simeq 200R_\odot$, but most of its mass is contained inside $d \simeq 100R_\odot$, which you can idealize as having uniform density.

Suppose that this star has a $3M_\odot$ companion, and when the big star expands into its red giant phase, the outer part of its envelope engulfs this companion. As the companion orbits inside the giant star's envelope, it stirs the envelope up, gradually filling the orbital energy into the envelope's gas, causing gas to get ejected and causing the companion to slowly spiral inward. Ultimately the companion reaches a radius small enough that the energy it has ejeced into the giant's envelope is enough to eject all of the envelope's mass. The result is a $3M_\odot$ companion orbiting a $4M_\odot$ compact star (the red giant's remnant core). Estimate what is the separation between the stars at that stage. [Hints: do the calculation using Newtonian dynamics. Compute the potential energy needed to eject the envelop and the energy the companion feeds into the envelope.]