**Exam 1**—Phys374—Fall 2013 Prof. Alessandra Buonanno
Due before class, Thursday, Oct. 10, 2013 Room 4212, (301)405-1440
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1. Consider a binary system composed of two orbiting black holes each of mass $M$. The system emits gravitational waves and the black holes spiral inwards until they finally coalesce and become one black hole. The last burst of gravitational radiation is emitted with some maximum power $P$ (energy per unit time). The only quantities $P$ may depend on are $M$, Newton's constant $G$, and the speed of light $c$. (The speed of light enters in Einstein's field equation of general relativity.) Determine the dependence of $P$ on these quantities.

2. Consider the quadratic equation $x^2 + 2ax - 3 = 0$, with $a$ a small positive number. (a) Find the positive root exactly using the quadratic formula. (b) Expand your exact root in a series in powers of $a$, keeping terms up through $O(a^2)$. (c) Now go back to the beginning and solve for the positive root using the method of series solutions, up through $O(a^2)$. (You should recover the same result as found in part (b)!)

3. A particle of mass $m$ moves in a one dimensional potential that takes the form $V(x) = x^3 - 3x$ in some units. (a) Sketch $V(x)$. Label with their $x$ value the points where $V(x) = 0$, and the points where $V'(x) = 0$. Label the stable and unstable equilibrium points. (b) Find the Taylor series approximation of $V(x)$ expanded around the stable equilibrium point, dropping any terms higher than quadratic order. (c) Find the frequency of small oscillations around the stable equilibrium point. (*Hint*: compare to a harmonic oscillator.)

4. (a) Let $f(x, y) = x^3 + 2x^2 y$, and find $\nabla f$.
   (b) Find the rate of change of $f$ with respect to distance in the direction of the vector $3\hat{\mathbf{x}} + 4\hat{\mathbf{y}}$ at the point $(x, y) = (1, 1)$.
   (c) Find the rate of change of $f$ with respect to distance in the direction of maximum *decrease* at the point $(x, y) = (1, 1)$.
   (d) What is the angle between the vector $3\hat{\mathbf{x}} + 4\hat{\mathbf{y}}$ and the direction of maximum *derease* at the point $(x, y) = (1, 1)$?

5. Let $\mathbf{F} = r^n \mathbf{r}$, where $r$ is the radial distance from the origin, $\mathbf{r}$ is the radial position vector, and $n$ is a constant.
   (a) Evaluate $\nabla \cdot \mathbf{F}$. (*Useful check*: The value of $n$ for which your result vanishes should agree with what you know about the electric field of a point charge.)
   (b) Using only your result from part (5a), evaluate the integral $\int_{\mathcal{V}} \nabla \cdot \mathbf{F}\, dV$, where $\mathcal{V}$ is the spherical shell between the two radii $R_1$ and $R_2 > R_1$.
   (c) Evaluate $\oint_{\mathcal{S}} \mathbf{F} \cdot \mathbf{dS}$, where $\mathcal{S}$ is the surface of a sphere of radius $R$ surrounding the origin.
   (d) Using only Gauss' theorem and your result from part (5c), re-evaluate the integral you computed in part (5b). (You should get the same result, but you must derive it here using Gauss' theorem.)