HW#6 —Phys374—Fall 2013 Prof. Alessandra Buonanno
Due before class, Thursday, Oct. 31, 2013 Room 4212, (301)405-1440
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Fun with complex numbers

1. Express the following in "Cartesian form" $x + iy$, where x and y are real: $1/(2-3i)$, $(1+2i)/(3+4i)$, $5e^{6i}$. [2+2+2=6 pts.]

2. Express the following in "polar form" $re^{i\varphi}$, where r is a real positive number and θ is real: -6, $-5i$, $(1+i)/\sqrt{2}$, $2-3i$, $(2+i)/(1+2i)$. (*Note*: Be careful to get the correct sign for the phase.) [2+2+2+2+2=10 pts.]

3. (i) Find all the cube roots of -1, i.e. $(-1)^{1/3}$, and express them all in both polar form and in Cartesian form. (ii) Plot and label them in the complex plane. [10+5=15 pts.]

4. Show that there are infinitely many values of i^i and they are all real. (*Hint*: Remember the definition of the complex exponential: $w^z = \exp(z \ln w)$.) [5 pts.]

5. Prove the trigonometric identities $\cos(a+b) = \cos a \cos b - \sin a \sin b$ and $\sin(a+b) = \sin a \cos b + \cos a \sin b$ by taking the real and imaginary parts of the identity $\exp(i(a+b)) = \exp(ia)\exp(ib)$. You may of course use the fact that $\exp(i\theta) = \cos\theta + i\sin\theta$. [5pts.]

6. Express the real and imaginary parts of the following functions in terms of $x = Re(z)$ and $y = Im(z)$: z^3, e^z, e^{iz}, $\sin z$, $1/(z^2+1)$. [2+2+2+2+2=10 pts.]

7. Problem 16.1h [10 pts.]