HW#11 —Phys374—Fall 2013 Prof. Alessandra Buonanno
Due before class, Tuesday, Dec . 10, 2013 Room 4212, (301)405-1440
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1. Consider a mass m suspended by a spring with spring constant k from another mass m which is suspended from a fixed support by another spring with spring constant k. (This is the system demonstrated and discussed in class.) Let y_1 and y_2 denote the displacements from their equilibrium positions of the top and bottom masses respectively, with the downward direction taken as positive. [2+2+3+3=10pts]

   (a) Write out Newton's second law governing the displacement of each of the two masses from their equilibrium positions. (Note the equilibrium spring forces balance the gravitational forces, which therefore drop out of the problem.)

   (b) Combine the two Newton's law equations into a single 2×2 matrix equation $\ddot{\mathbf{y}} = A\mathbf{y}$, and specify the components of the matrix A in units with $k = m = 1$.

   (c) Determine the normal mode frequencies and displacement ratios y_2/y_1 by finding the eigenvectors and eigenvalues of A *by hand*. (*To check your result*: The squared frequencies are $(3 \pm \sqrt{5})/2$ and the displacement ratios are $y_2/y_1 = (1 \mp \sqrt{5})/2$.)

   (d) Describe and indicate with arrows the nature of the two normal mode motions, showing both direction and approximate relative displacement of each mass. Label with the frequency of each mode. Which is the higher one?

2. Re-do all parts of the previous problem in the case that there are *three* equal masses hanging in a chain from three springs. In this case the characteristic equation is a cubic and the frequencies and amplitudes cannot be found in a simple closed form, so instead use a computer to get them.[1] [2+2+3+3=10pts]

3. Find the ratio of the two nonzero normal mode frequencies for longitudinal vibrations of the model of the carbon dioxide molecule discussed in class: two oxygen mass points with mass m_O each connected by a spring of spring constant k to a carbon atom in the center with mass $m_C = (3/4)m_O$. (The equilibrium length of the springs is nonzero.) Section 13.6 of the texbook shows that if all masses were equal the frequencies would be $\omega_1 = \sqrt{k/m}$ and $\omega_2 = \sqrt{3k/m}$, so $\omega_2/\omega_1 = \sqrt{3}$. The point of this problem is to find the ratio taking into account that $m_O/m_C = 4/3 \neq 1$. Use units with $k = 1$ and $m_O = 1$, and write your equations using the parameter $a = m_O/m_C$. Put $a = 4/3$ only at the end of your calculation to evaluate the result. [4+4+2=10pts]

   (a) Use the characteristic polynomial to show *by hand* (no computer or calculator) that the frequencies are given by 0, 1, and $\sqrt{1+2a}$. For each mode find the

[1] Use the help function to learn the right command. In Mathematica, the command would be, for example, "Eigensystem[{{1., 2, 3},{0, 1, 0},{1, 0, 1}}]". Note the period after the first entry 1. That tells Mathematica to treat the entries as real numbers rather than integers. This is necessary, when the result has no nice closed form, to obtain a numerical result rather than an abstract "root" expression. Try it—you'll see what I mean.

displacement ratios x_2/x_1 and x_3/x_1, where $x_{1,3}$ are the displacements of the oxygen atoms and x_2 is the displacement of the carbon atom. Describe and indicate with arrows the nature of the three normal mode motions, showing both direction and approximate relative replacements. Label with the frequency of each mode. Which is the highest one?

(b) Look at www.phy.davidson.edu/StuHome/jimn/CO2/Pages/CO2Theory.htm for information on carbon dioxide vibrations. Compare the ratio $\sqrt{11/3}$ of the frequencies in our simple model[2] to the measured value[3] of the ratio for the corresponding modes of carbon dioxide. Show that the measured ratio is smaller, and determine by what fraction it differs from the simple model.

(c) Try to give a physical reason (or reasons) why the observed ratio is smaller than that in our simple model. I'm not sure, but I don't think the difference between quantum and classical mechanics is really the important issue in this case. *Hint*: Think about the physical nature of the "springs" in the molecule and how they differ from those in our model. I think one can explain along these lines the dominant reason why the measured ratio is smaller.

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[2]Note you don't need to know the spring constant or the magnitude of each mass to determine this ratio! This is because it is dimensionless, so can depend only on the mass ratio.

[3]The frequencies are given on the above web page in units of cm^{-1}. This refers to the inverse wavelength λ^{-1} of the radiation that is in resonance with the transitions between the vibrational levels, which is proportional to the frequency ω since $\omega = ck = 2\pi c/\lambda$, where c is the speed of light.