Resonant-Mass Gravitational Wave Detector

Gravitational waves (GWs) are ripples in spacetime fabric. The principle of a resonant-mass GW detector is shown in Figure 1. The hyperbolas represent force patterns on test masses resulting from a GW traveling into the screen. If the masses A and B are connected by a spring, the spring will be stretched and compressed as a GW passes by, absorbing energy from the GW [Weber, 1960].

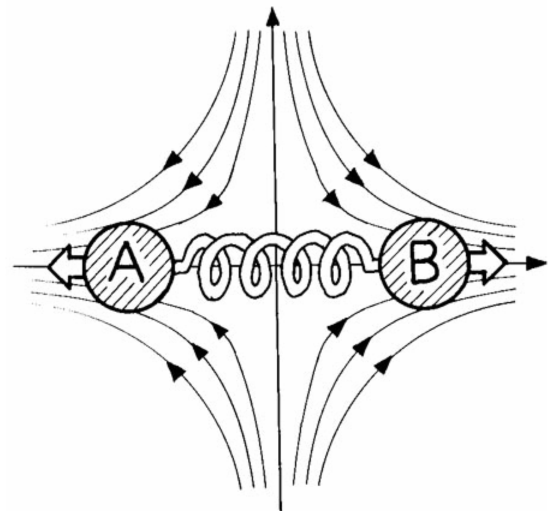

Figure 1. A simple GW detector. Two masses connected by a spring can absorb energy from a GW.

A GW pulse is characterized by a dimensionless metric perturbation *h* with a dominant frequency ω_S and duration τ_S. For a short pulse with $\tau_S \approx 2\pi\omega_S^{-1}$, the energy that the GW deposits into a favorably oriented, noiseless, cylindrical antenna becomes

$$E_S \approx \frac{2}{\pi^2} M\omega_S^2 (Dh)^2, \tag{1}$$

where *M* and *D* are the effective mass and effective length of the antenna.

The total intrinsic noise of the detector, referred to the input of the noiseless antenna, can be shown [Giffard, 1976] to be

$$E_N \approx k_B T_a \frac{\omega_a \tau}{Q_a} + k_B T_N \left[\frac{2(\zeta + \zeta^{-1})}{\beta_{21}\omega_S \tau} + \frac{\beta_{12}\omega_S \zeta \tau}{2} \right], \tag{2}$$

where ω_a, Q_a, and T_a are the (angular) resonance frequency, quality factor, and temperature of the antenna; β_{21} and β_{12} are the *forward* and *reverse* energy coupling coefficients of the transducer; T_N, τ, and ζ are the noise temperature, integration time, and dimensionless impedance matching parameter of the ampifier. For a passive transducer, $\beta_{21} = \beta_{12} \equiv \beta_S$, where β_S is the *signal* coupling coefficient to the amplifier.

The GW pulse is detectable if $E_S > E_N$. According to Eq. (2), E_N comes from three terms: the Brownian motion noise and the *forward* and *reverse action* noise of the amplifier. The amplifier noise contribution can be minimized by choosing

$$\tau = \frac{2}{\beta_S \omega_S}\left(1+\frac{1}{\zeta^2}\right)^{1/2}. \tag{3}$$

Substituting this into Eq. (2) leads to

$$E_N \approx \frac{2k_B T_a}{\beta_S Q_a}\left(1+\frac{1}{\zeta^2}\right)^{1/2} + 2k_B T_N\left(1+\zeta^2\right)^{1/2}. \tag{4}$$

From Eqs. (3) and (4), three optimization conditions follow:

$$\zeta \le 1, \qquad \Delta\omega_S / \omega_S \approx \beta_S, \qquad T_a / Q_a << \beta_S T_N, \tag{5}$$

where $\Delta\omega_S \approx \pi\tau^{-1}$ is the bandwidth of the detector. Thus a large β_S allows for a large fractional bandwidth, which reduces the Brownian motion noise of the antenna. If a near-unity β_S could be achieved *without* restricting the bandwidth of the transducer, a completely wideband detector ($\Delta\omega_S \approx \omega_S$) could be realized without compromising the signal-to-noise ratio.

When conditions (5) are satisfied, the total detector noise becomes amplifier-limited at $2k_B T_N$. A more rigorous theory with application of the optimal filter [Price, 1987] leads to the true amplifier limit of $k_B T_N$. Combining this with Eq. (1) gives

$$h_{\min} \approx \left(\frac{5k_B T_N}{M\omega_S^2 D^2}\right)^{1/2}. \tag{6}$$

For *linear* (phase-preserving) amplifiers, T_N has a quantum limit:

$$T_{N,QL} = \hbar\omega_S / k_B. \tag{7}$$

Back-action-evasion techniques allow, in principle, to beat this "standard quantum limit". For a cylindrical antenna with M = 1200 kg, D = 3 m and $\omega_S / 2\pi$ = 900 Hz at the quantum limit, we find $T_N \approx 0.04$ μK and $h_{\min} \approx 3 \times 10^{-21}$.

Giffard, R. P. (1976), *Phys. Rev. D* **14**, 2478-2486.
Price, J. C. (1987), *Phys. Rev. D* **36**, 3555-3570.
Weber, J. (1960), *Phys. Rev*. **117**, 306-313.