Optical Nanofibers; some experiments in optomechanics.

Williams College, March 1, 2024

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Work supported by: National Science Foundation of the USA, The Joint Quantum Institute, The University of Shanxi, China, The Natural Science Foundation of China, and CONYCIT Chile.

Travel supported by the Division of Laser Science of the American Physical Society Travelling Lecture program.

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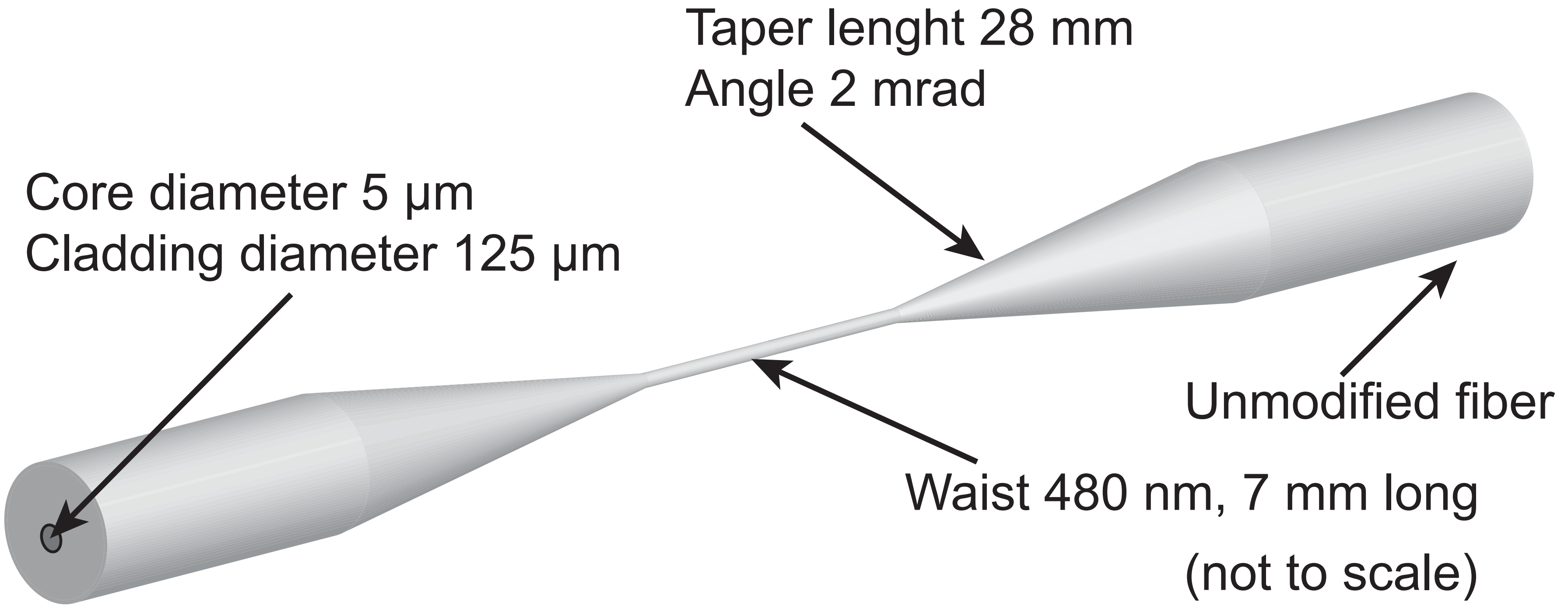
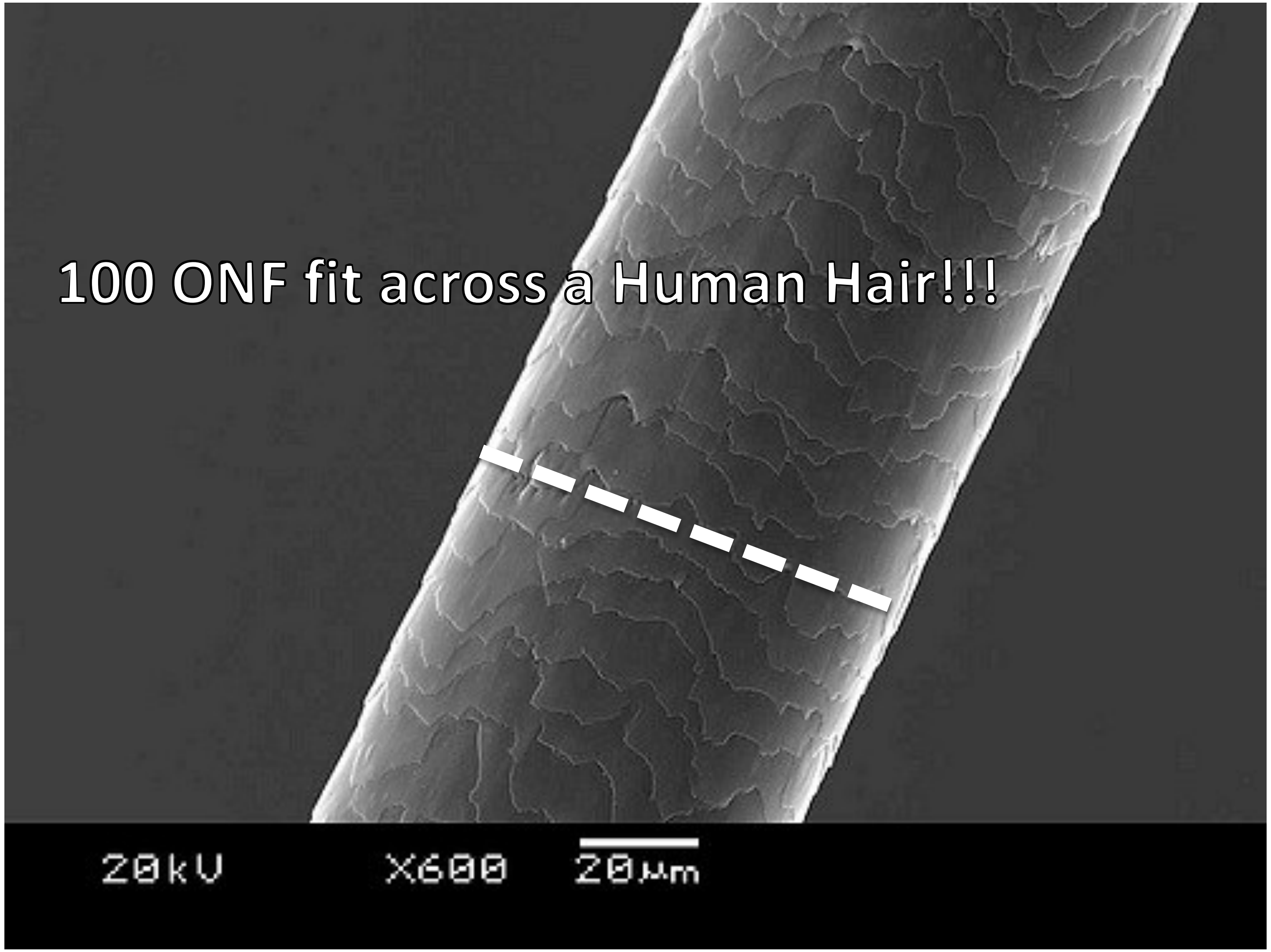
Shanxi University, Taiyuan, China
NIST, Gaithersburg MD. USA

Optical Nanofibers

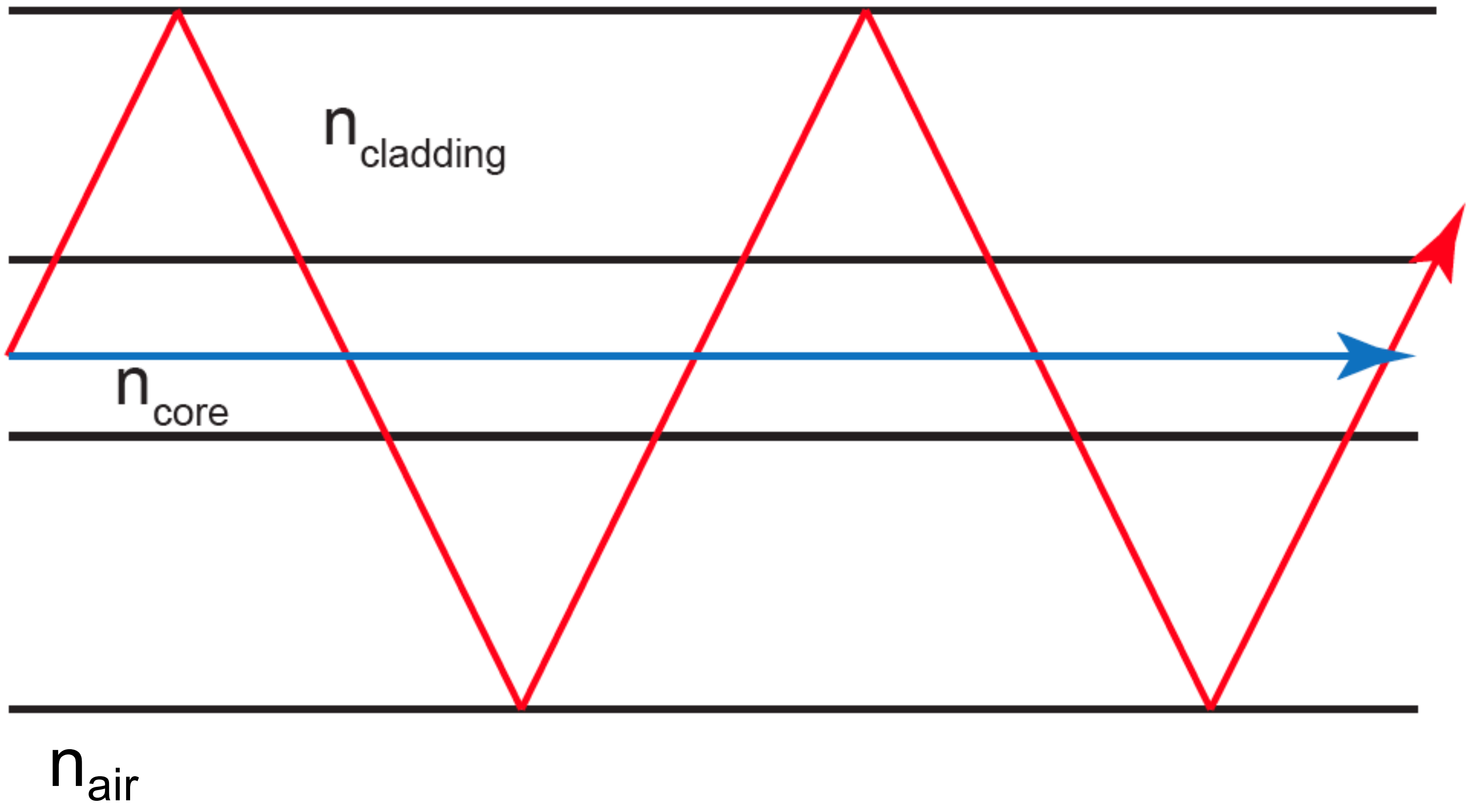
Optical Nanofibers



The scale



Light rays in a fiber with total internal reflection

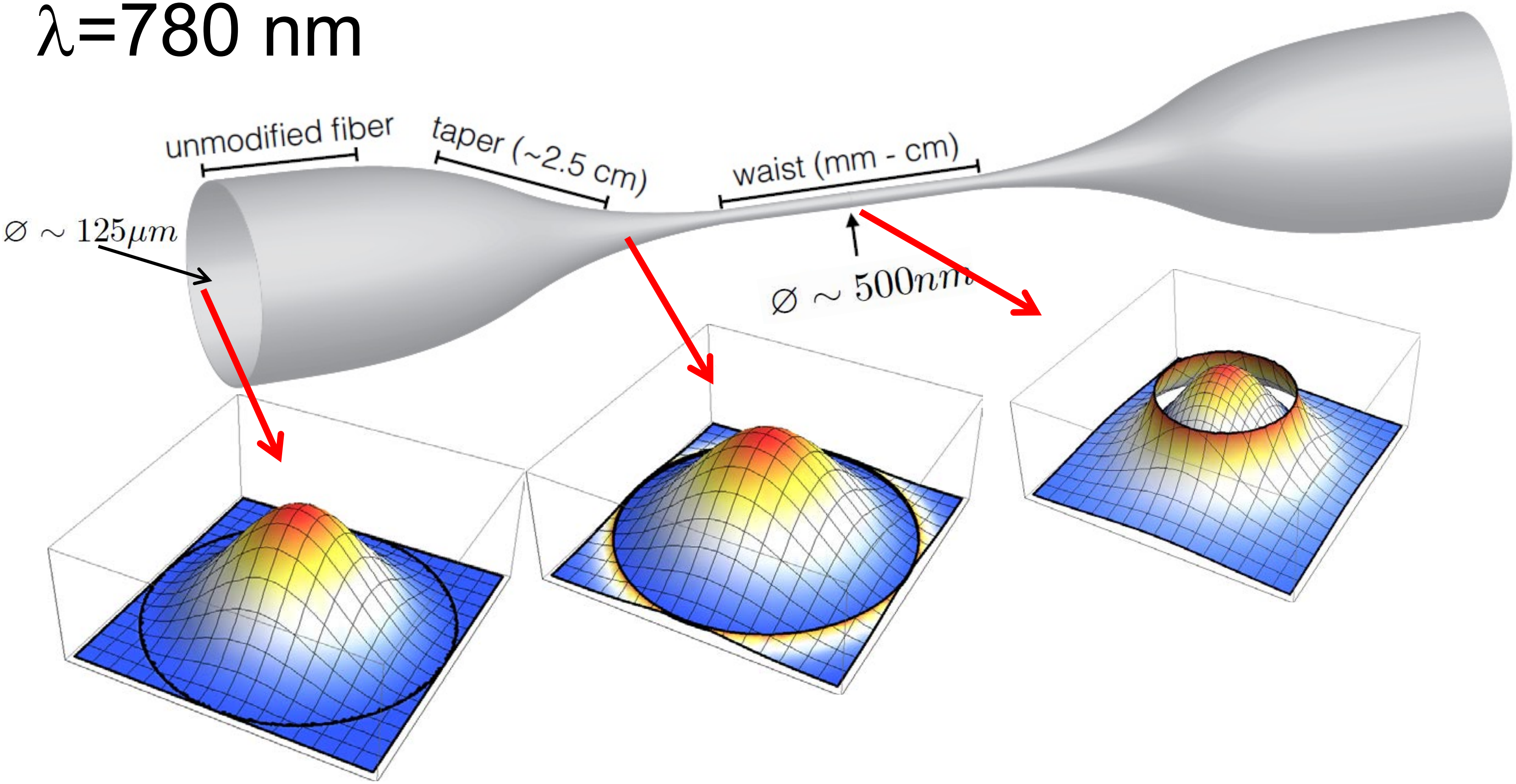


$n_{core} > n_{cladding}$

$n_{cladding} > n_{air}$

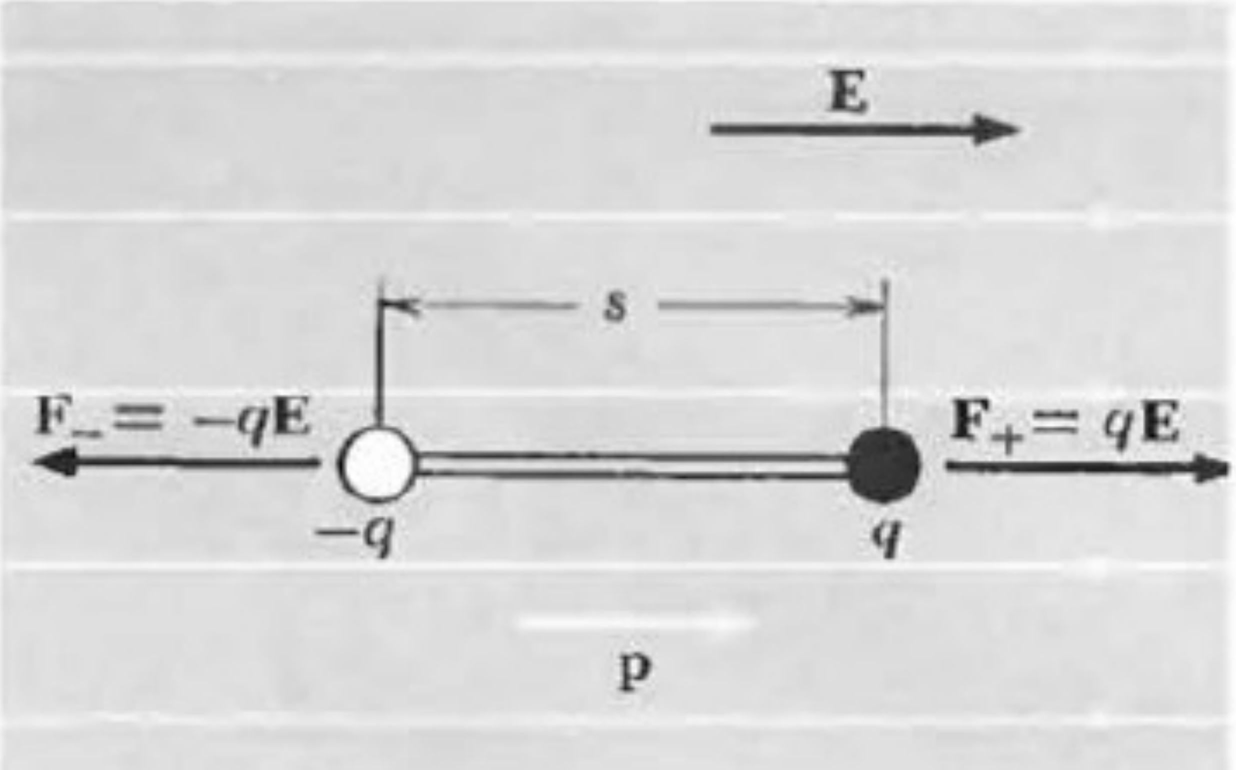
Optical Nanofibers

λ=780 nm



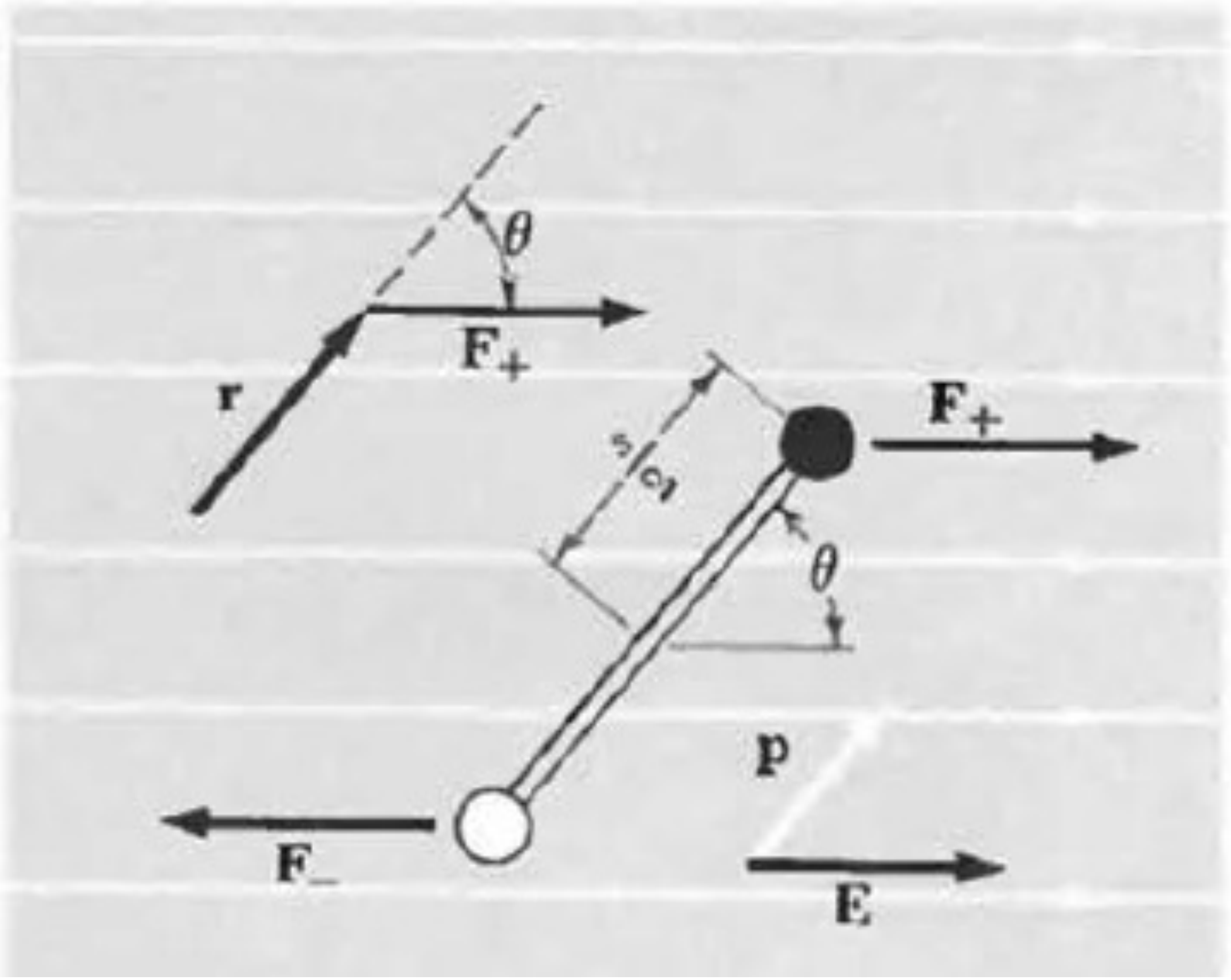
Review of forces and torque on dipoles and Temperature

Force and Energy on an electric dipole



$$U = -\vec{p} \cdot \vec{E}$$

Torque on an electric dipole



$$\vec{N} = \vec{p} \times \vec{E}$$

If there is birefringence (the index of refraction in one direction is different from the perpendicular one) in the nanofiber, there will be a torque.

Can change sign depending on the angle between the light polarization and the birefringence axis.

Quick review of Temperature:

Average of the kinetic energy:

$$kT = \frac{1}{2} m \langle v^2 \rangle$$

the average of the velocity can be zero $\langle v \rangle = 0$,
but not its variance: $\langle v^2 \rangle - \langle v \rangle^2$.

Cooling:

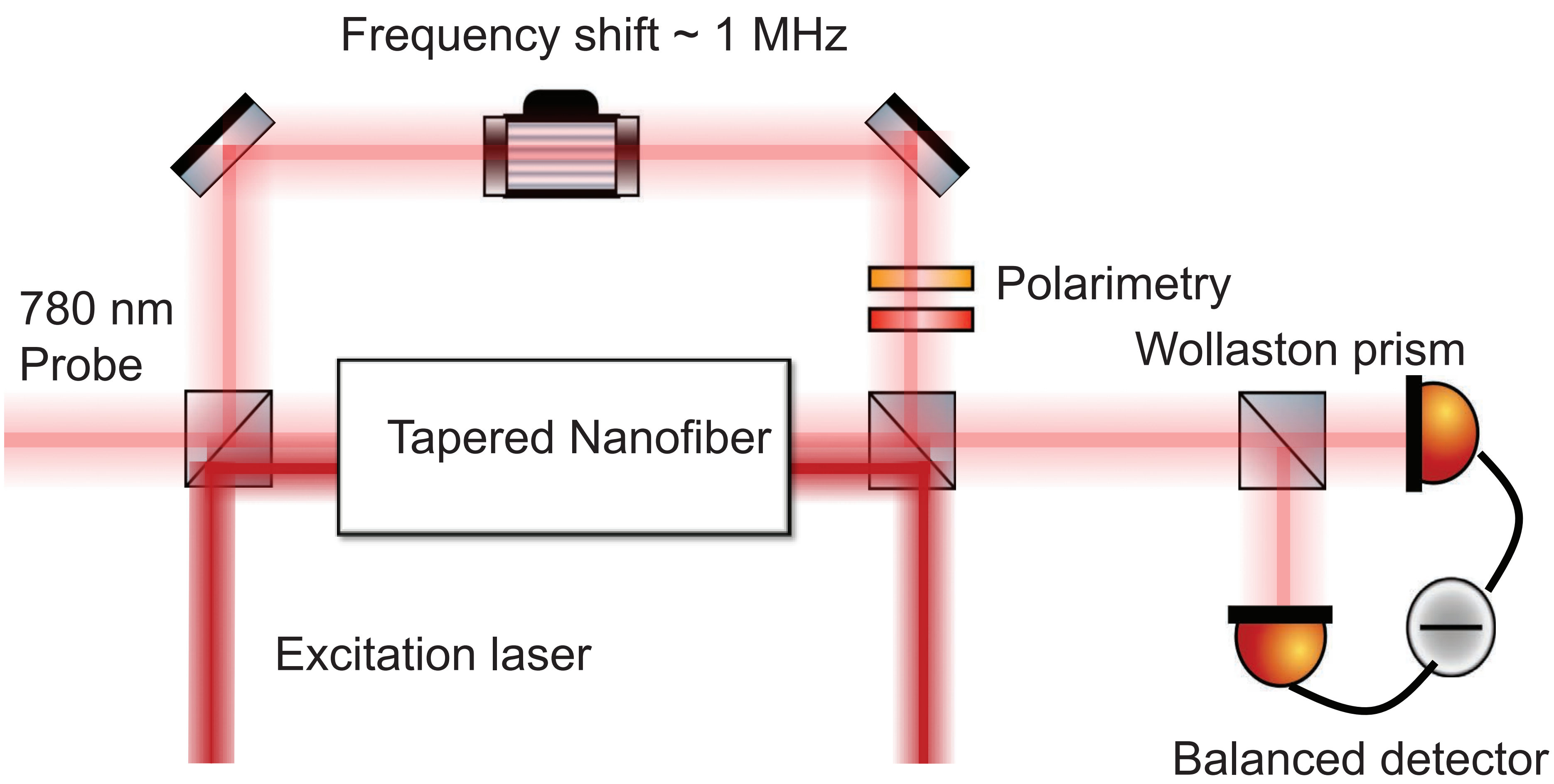
Reduction of the kinetic energy from random fluctuations.

We can look at the power spectral density, (Fourier transform of the autocorrelation of the fluctuations). The amplitude should decrease and the width increase, while the integral decreases. The integral is the total power in the fluctuations

Mechanical modes

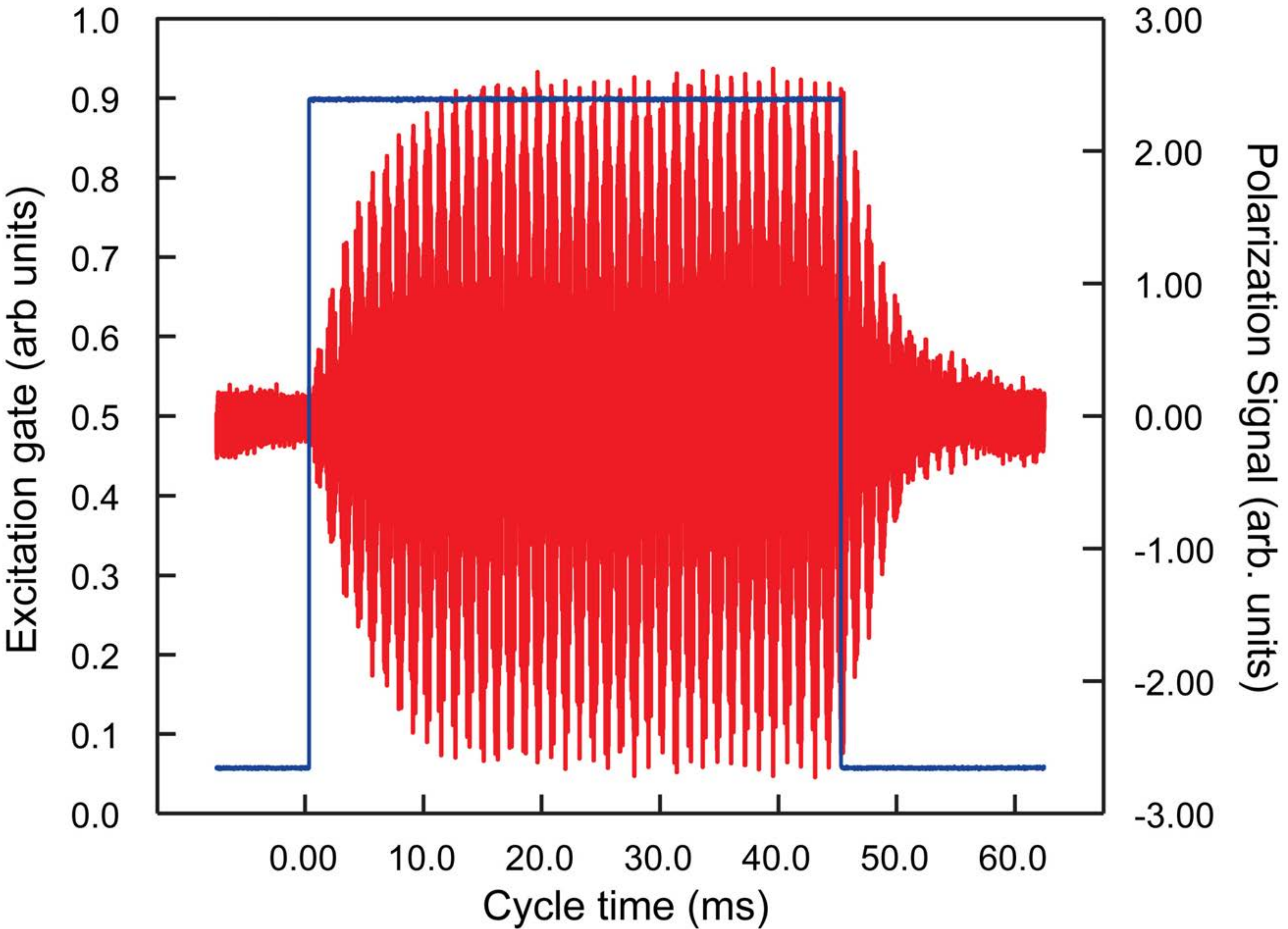
- Vibration (not discussed here)
- Torsion
- Compression (not discussed here)

Transfer the intrinsic angular momentum (circular polarization) from the light to torsional modes of the ONF.

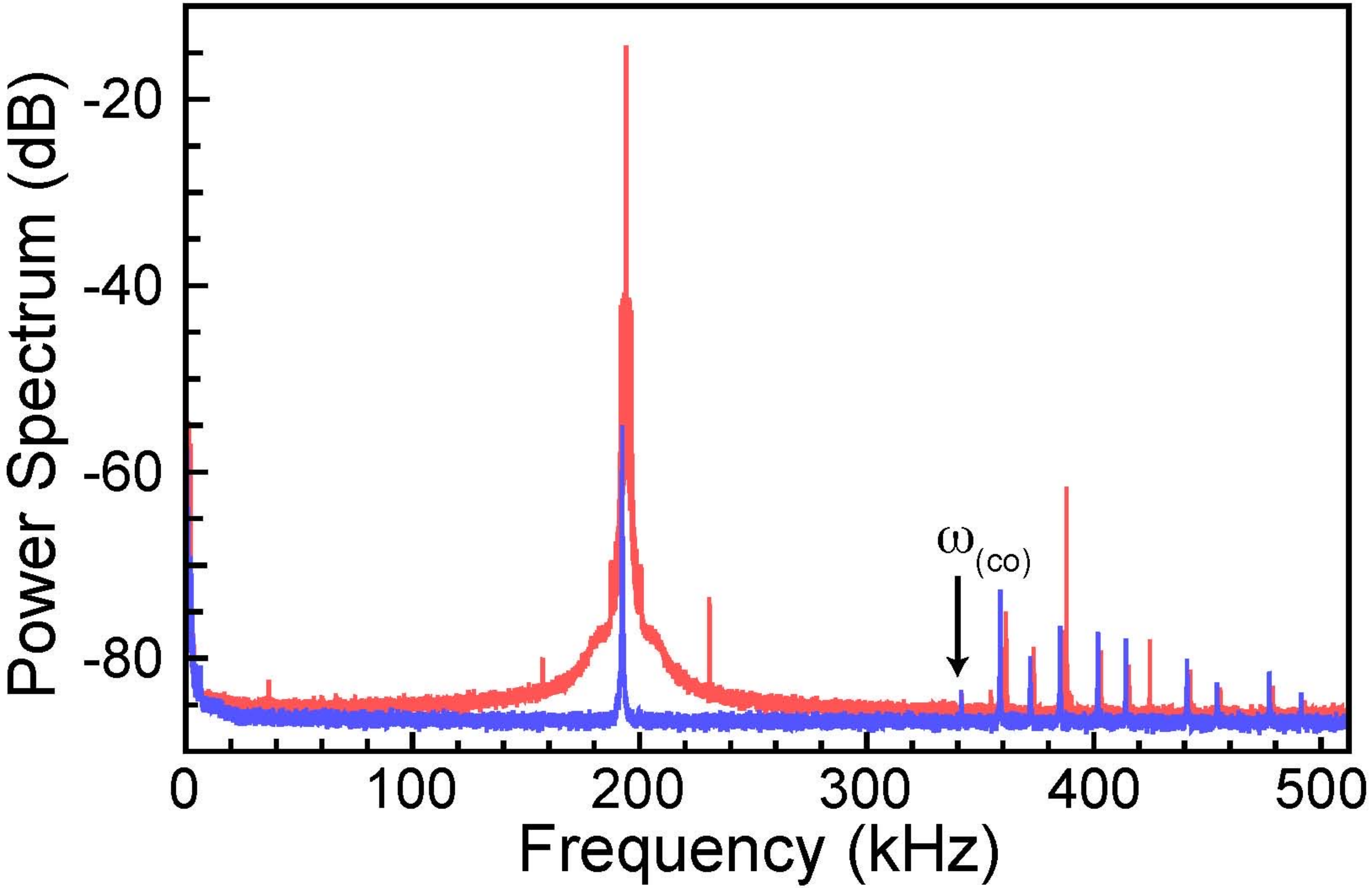


Excitation 1060 nm: circular polarized light (modulated at the mode frequency). Excitation with linear or circular polarization. Probe linear and weak.

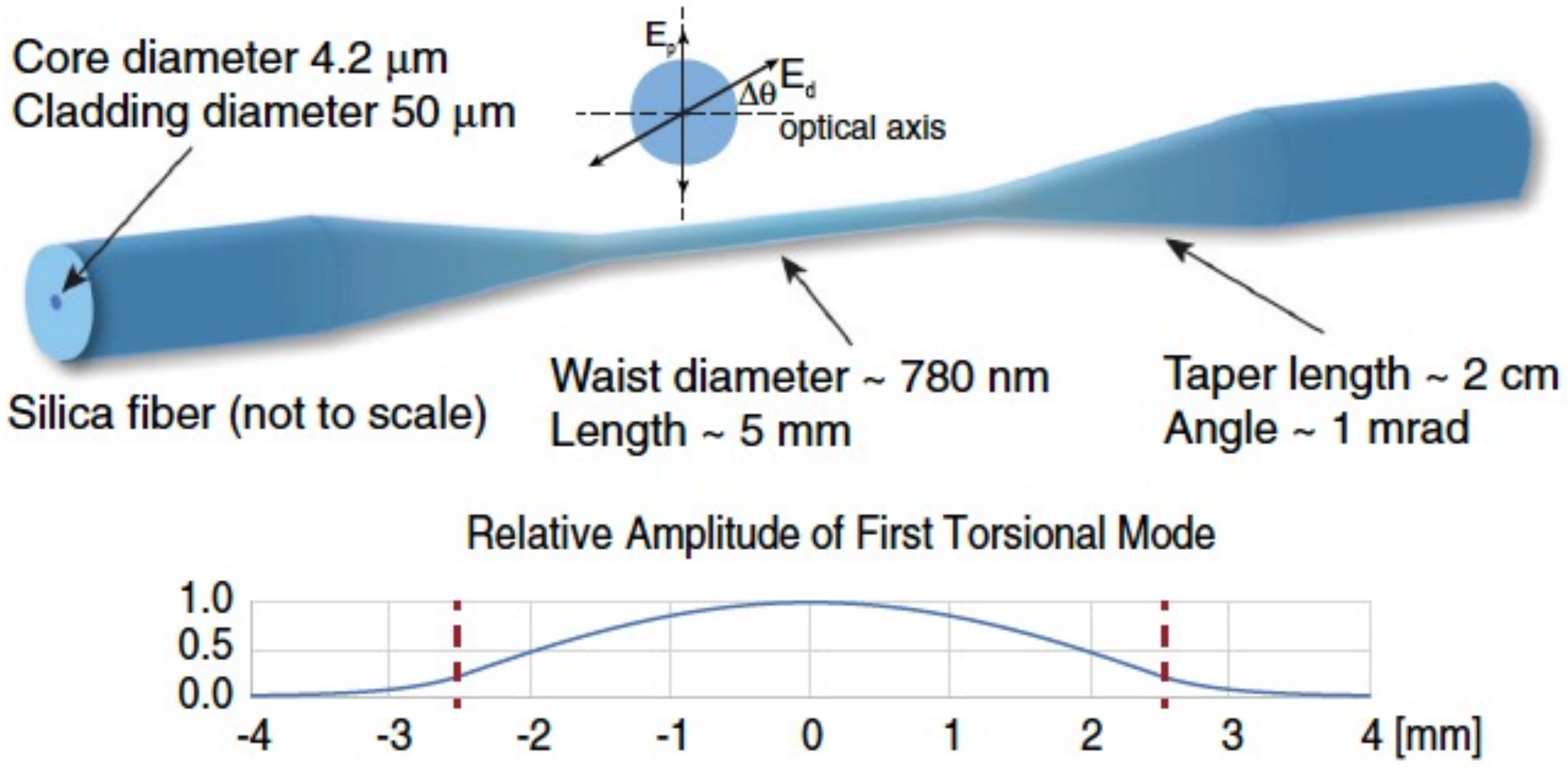
Excitation and response.

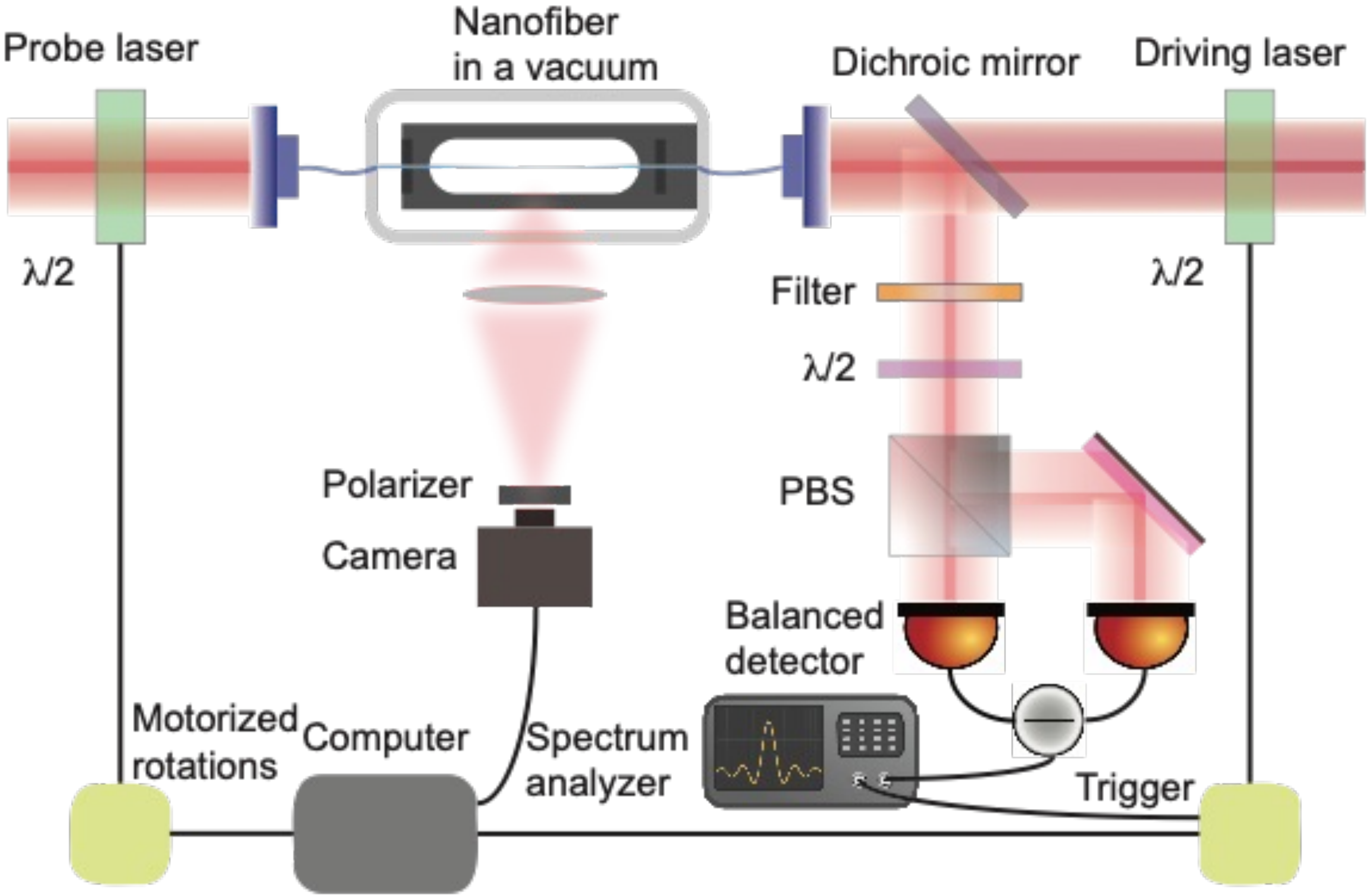


Thermal excitation (blue), Resonant excitation with circular polarization of the first mode (red).



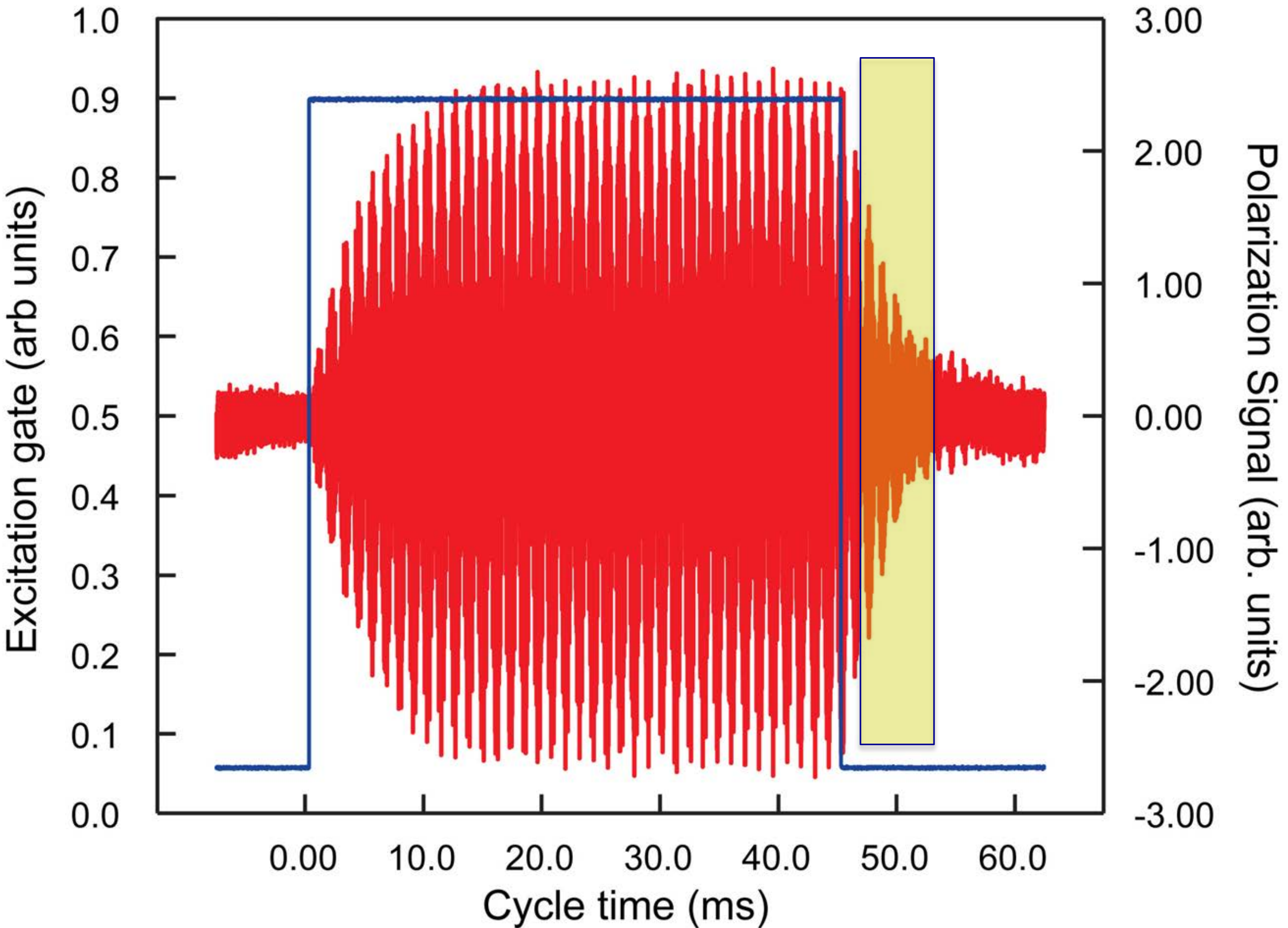
A surprise


Core diameter 4.2 µm
Cladding diameter 50 µm
E_p
E_d
Δθ
optical axis
Silica fiber (not to scale)
Waist diameter ~ 780 nm
Length ~ 5 mm
Taper length ~ 2 cm
Angle ~ 1 mrad
Relative Amplitude of First Torsional Mode
1.0
0.5
0.0
-4
-3
-2
-1
0
1
2
3
4 [mm]

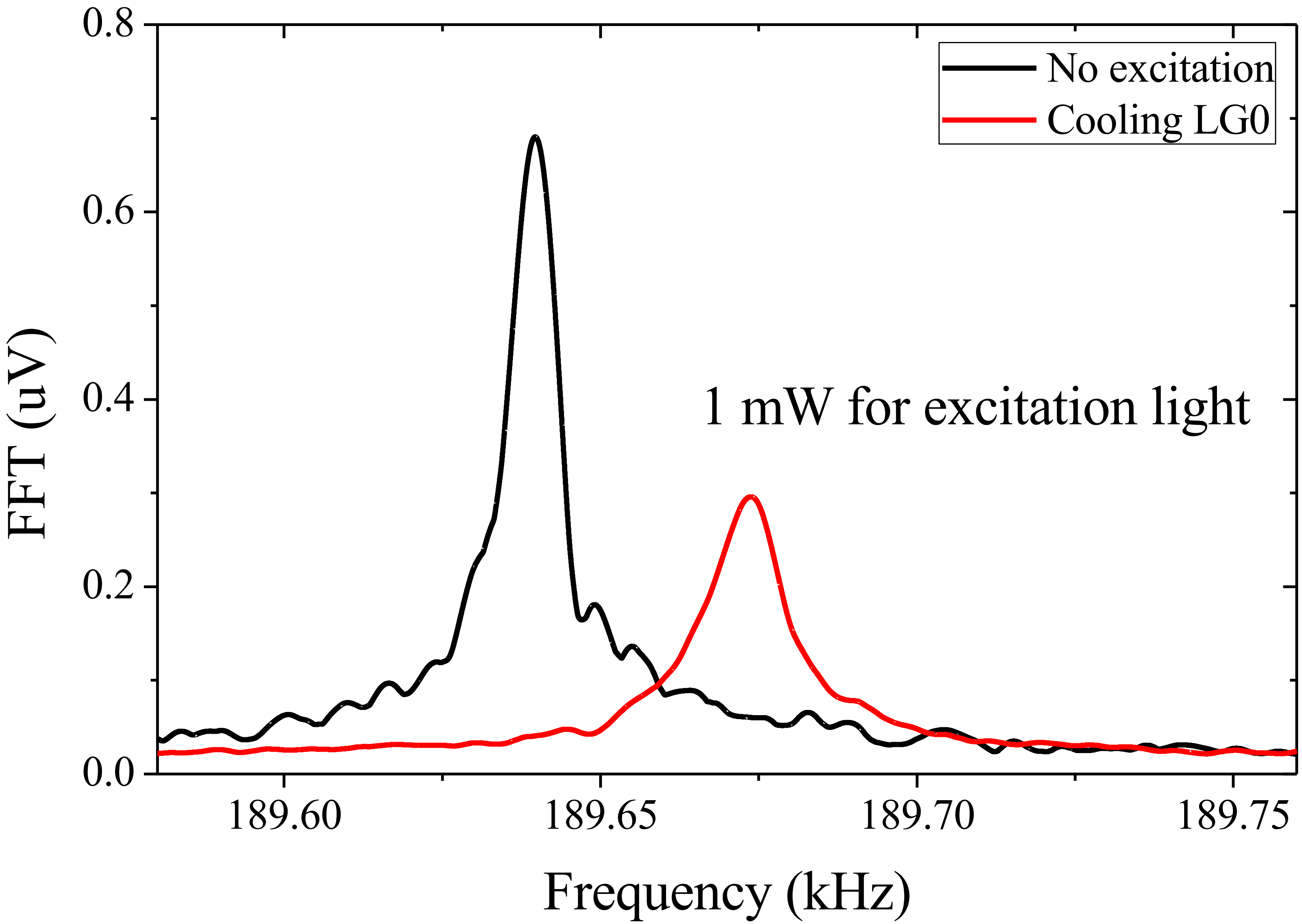


Probe laser
Nanofiber
in a vacuum
Dichroic mirror
Driving laser
λ/2
Filter
λ/2
λ/2
PBS
Polarizer
Camera
Balanced
detector
Motorized
rotations
Computer
Spectrum
analyzer
Trigger

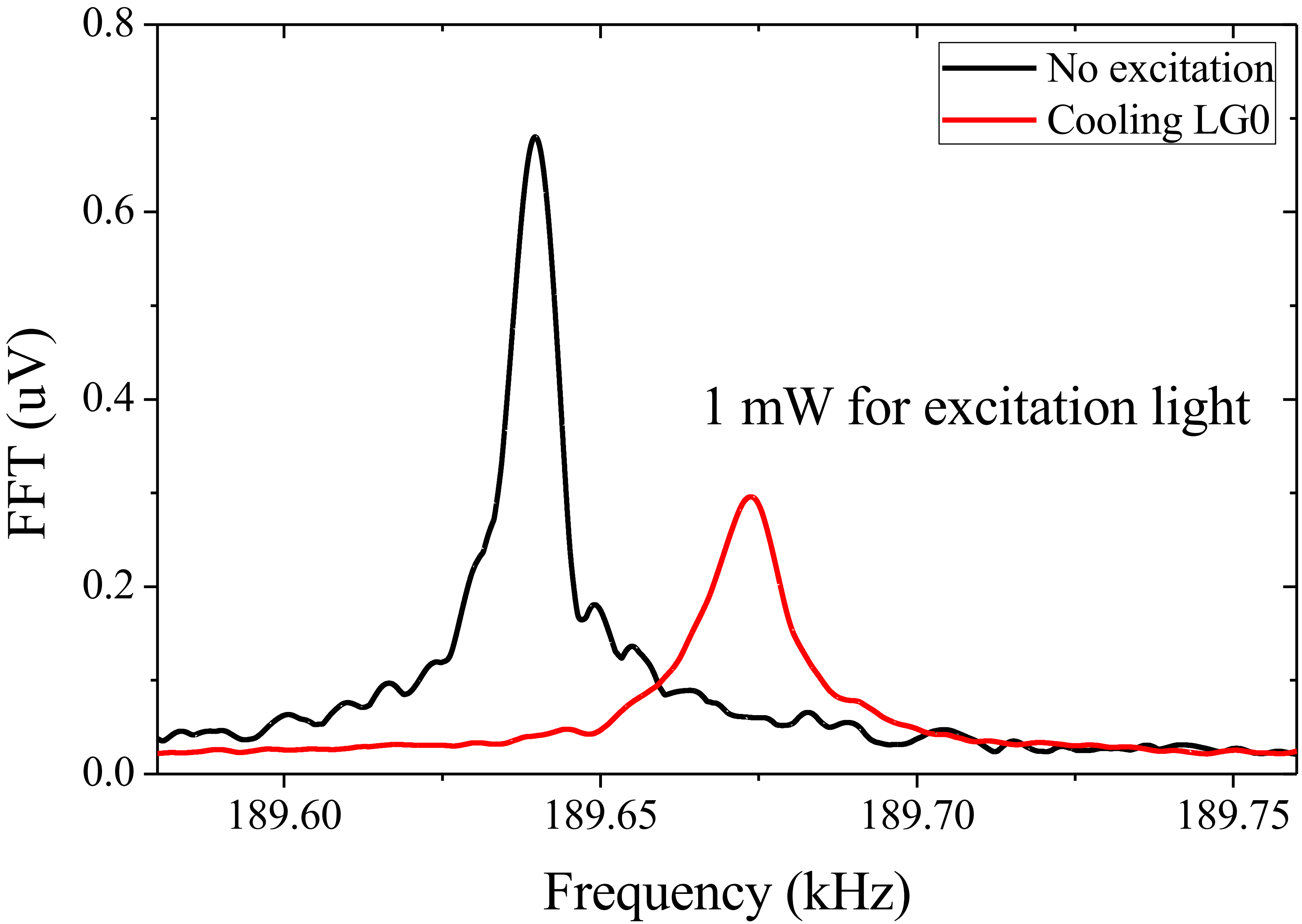
Excitation and response.



The surprise



The surprise



Amplitude decreases

Linewidth increases

What happens in a birefringent material when a beam of light passes?

Richard Beth, based on a suggestion from Poynting, made the "Mechanical detection and measurement of the angular momentum of light.” Using a torsional pendulum mounting a birefringent disc he measured rotation due to the change in polarization.

Torsion of a cylinder (model for the 1st mode)

$$I\ddot{\theta}_{\mathrm{F}}(t) + \gamma\dot{\theta}_{\mathrm{F}}(t) + \kappa\theta_{\mathrm{F}}(t) = \tau$$

Torque due to electric field ~ (P × E)

$$\tau(t) = \frac{c\epsilon}{2\omega_1}|E|^2\pi r_0^2 \sin\Gamma \sin\left(2\Delta\theta(t)\right) \qquad \Gamma = kd(n_o - n_e)$$

thermal torque

$$I\ddot{\theta}_{\mathrm{F}}(t) + \gamma\dot{\theta}_{\mathrm{F}}(t) + \kappa\theta_{\mathrm{F}}(t) - \tau_0 \sin\left(2\Delta\theta(t)\right) = \tau_{\mathrm{th}}$$

Harmonic oscillator — modified potential

I is the moment of inertia; γ is the damping constant;
κ is the restoring (spring constant)

$$\tau(t) = \frac{c\epsilon}{2\omega_1}|E|^2 \pi r_0^2 \sin\Gamma \sin\left(2\Delta\theta(t)\right)$$

Depends on the intensity $|E|^2$ and angle between the birefringence axis and the polarization of the electric field $\Delta\theta$

expanding to first order in the angle:

$$I\ddot{\theta}_{\mathrm{F}}(t) + \widetilde{\gamma}\dot{\theta}_{\mathrm{F}}(t) + \theta_{\mathrm{F}}(t)\left(\kappa - \kappa_{\mathrm{L}}\right) = \tau_{th}$$

The change from κ to (κ-κ_L), can be positive or negative. Softening or stiffening.

Change from γ to $\widetilde{\gamma}$

Torque has a part with a delay, due to the difference in light speed (optical response) and sound speed. This manifests itself in a change in the decay constant, which ensures the possibility of cooling.

$$T_f = T_i \frac{\gamma}{\tilde{\gamma}}$$

Disturbance of the steady state

$$I\delta\ddot{\theta}_{\mathrm{F}}(t) + \gamma\delta\dot{\theta}_{\mathrm{F}}(t) + \delta\theta_{\mathrm{F}}(t)\left(\kappa - \kappa_{\mathrm{L}}\right) = \tau_{th}$$

$$\kappa_{\mathrm{L}} = 2\tau_0 \cos(2\Delta\theta^{(ss)})$$

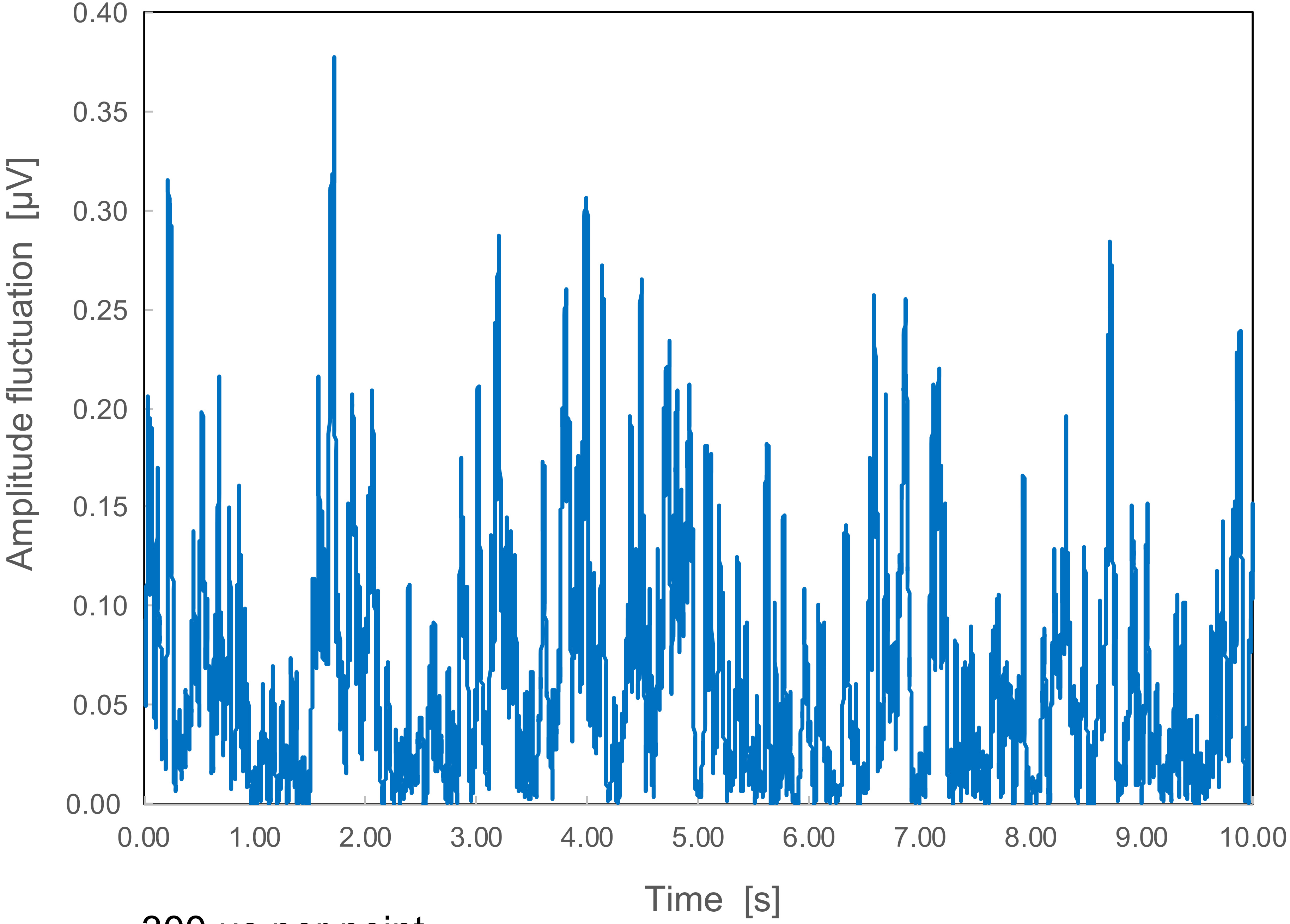
Do the fluctuations follow a Maxwell Boltzmann Distribution?

$$p(\theta_{\mathrm{F}}) = \frac{\kappa\theta_{\mathrm{F}}}{k_{\mathrm{B}}T_{\mathrm{eff}}} e^{-\kappa\theta_{\mathrm{F}}^2/2k_{\mathrm{B}}T_{\mathrm{eff}}}$$

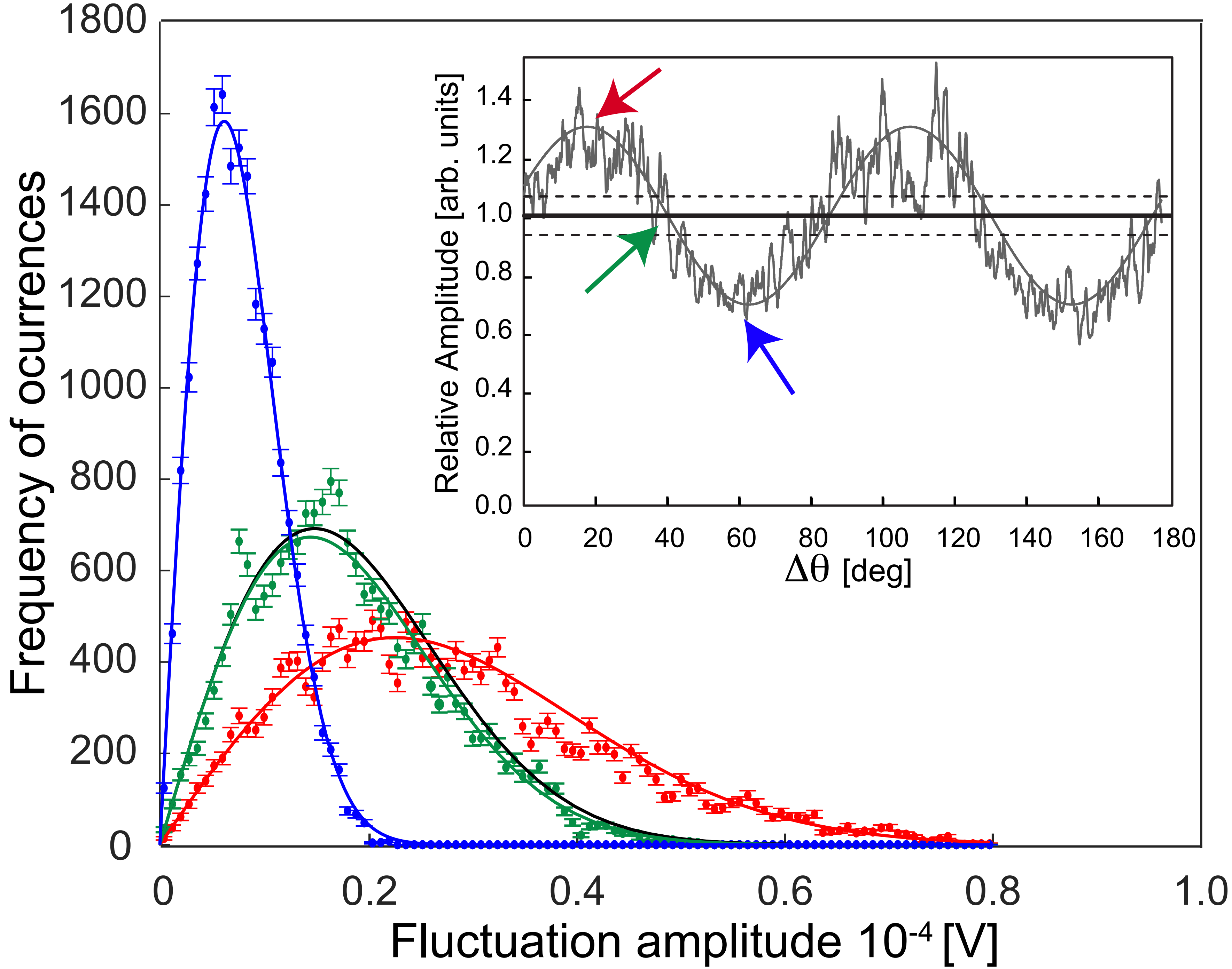
$$\langle \delta\theta_F^2 \rangle = \frac{k_B T}{\kappa - \kappa_L} = \frac{k_B T}{I\omega_m^2}$$

$$\frac{1}{2} I\omega_m^2 \langle \delta\theta_F^2 \rangle = \frac{1}{2} k_B T$$

The variance is proportional to the temperature.



300 µs per point



1800
1600
1400
1200
1000
800
600
400
200
0
Frequency of ocurrences
0
0.2
0.4
0.6
0.8
1.0
Fluctuation amplitude 10-4 [V]
Relative Amplitude [arb. units]
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
20
40
60
80
100
120
140
160
180
Δθ [deg]

Angle	T_{eff}/T_0	Uncertainty
min. (blue)	0.168	±0.004
med. (green)	0.910	±0.004
max. (red)	2.320	±0.014
no drive (black)	1.000	

Decrease in temperature: 1/0.168=5.95

Spectral density (Power and Amplitude)

$$S_{\tau_{\mathrm{th}}} = 4k_B T\gamma \qquad\qquad \Gamma = \gamma/I$$

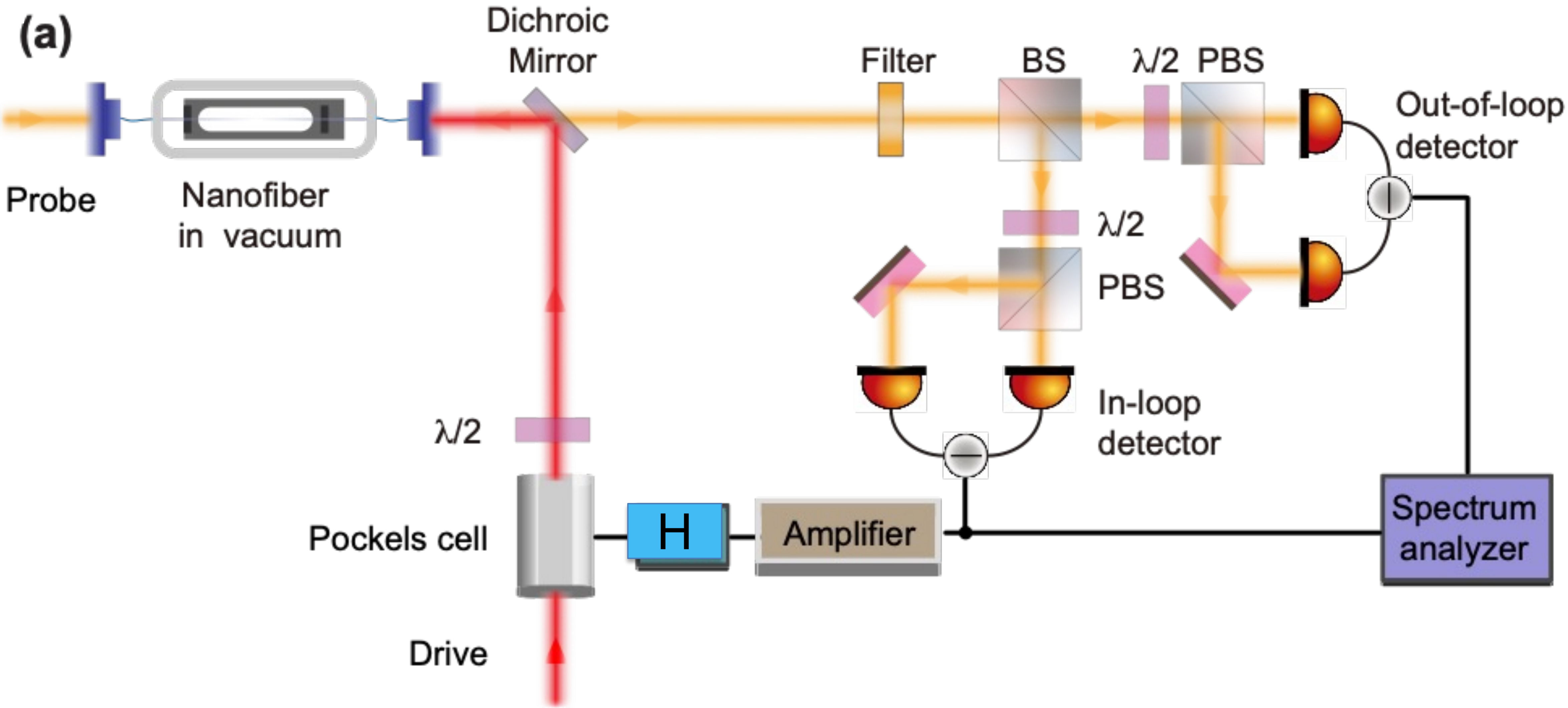
$$S_{\delta\theta_{\mathrm{F}}}(\omega) = \frac{4k_B T \;\; \Gamma\omega_0^2/\kappa}{\left((\omega_0^2 - \kappa_{\mathrm{L}}/I) - \omega^2\right)^2 + (\Gamma + \Gamma_{\mathrm{L}})^2\omega^2}$$

$$T_{\mathrm{eff}} = T\frac{\Gamma}{\Gamma + \Gamma_{\mathrm{L}}}.$$

Dianqiang Su, Pablo Solano, Jeffrey D. Wack, Luis A. Orozco, and Yanting Zhao, "Torsional optomechanical cooling of a nanofiber," Photonics Research Journal **10** 601, (2022). https://doi.org/10.1364/PRJ.440991

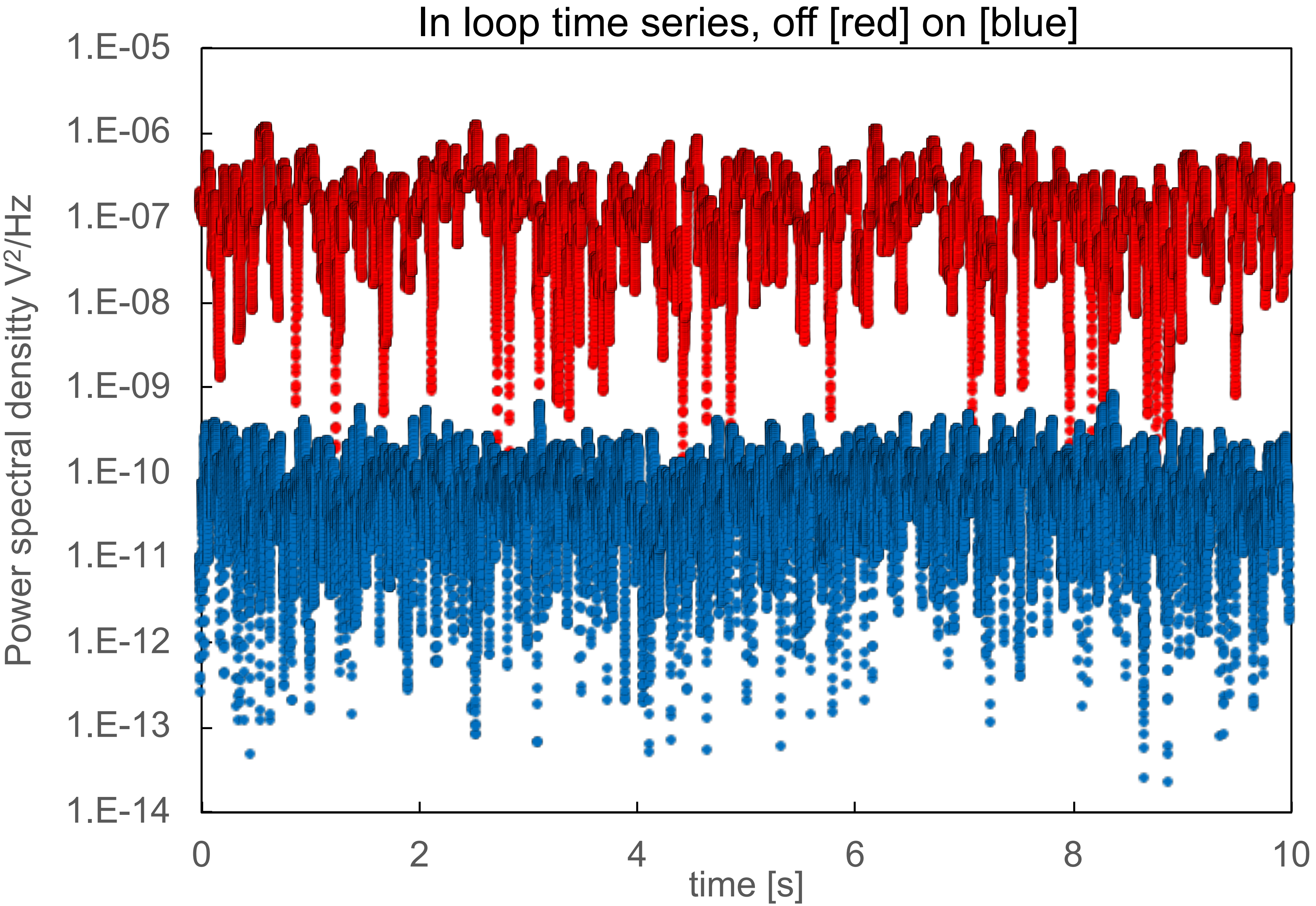
Apply feedback

$$I\ddot{\theta}_{\mathrm{F}}(t) + \widetilde{\gamma}\dot{\theta}_{\mathrm{F}}(t) + \theta_{\mathrm{F}}(t)\left(\kappa - \kappa_{\mathrm{L}}\right) = \tau_{th} + \tau_{\mathrm{fb}}$$

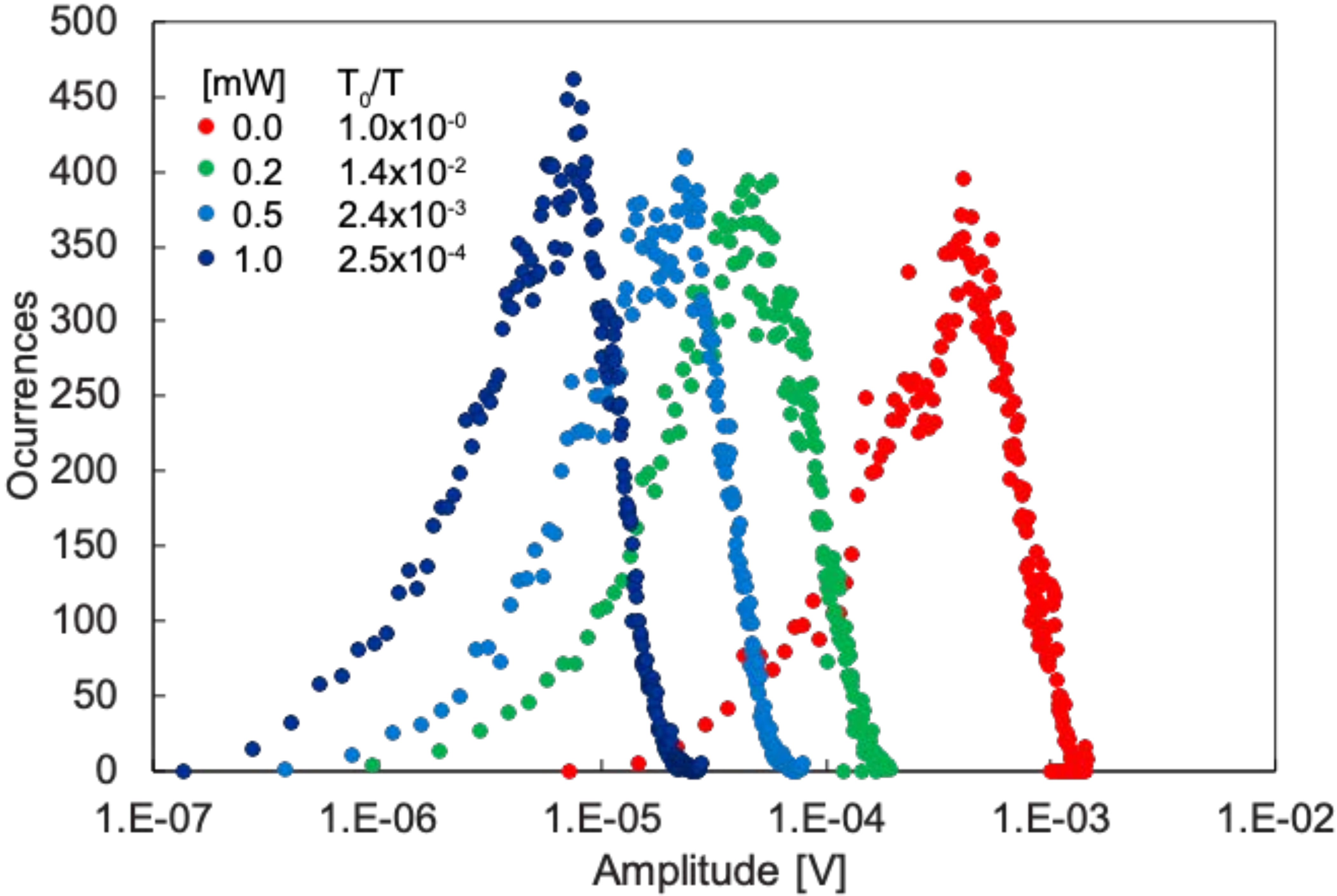


(a)
Dichroic
Mirror
Filter
BS
λ/2
PBS
Out-of-loop
detector
Probe
Nanofiber
in vacuum
λ/2
PBS
In-loop
detector
λ/2
Pockels cell
H
Amplifier
Spectrum
analyzer
Drive

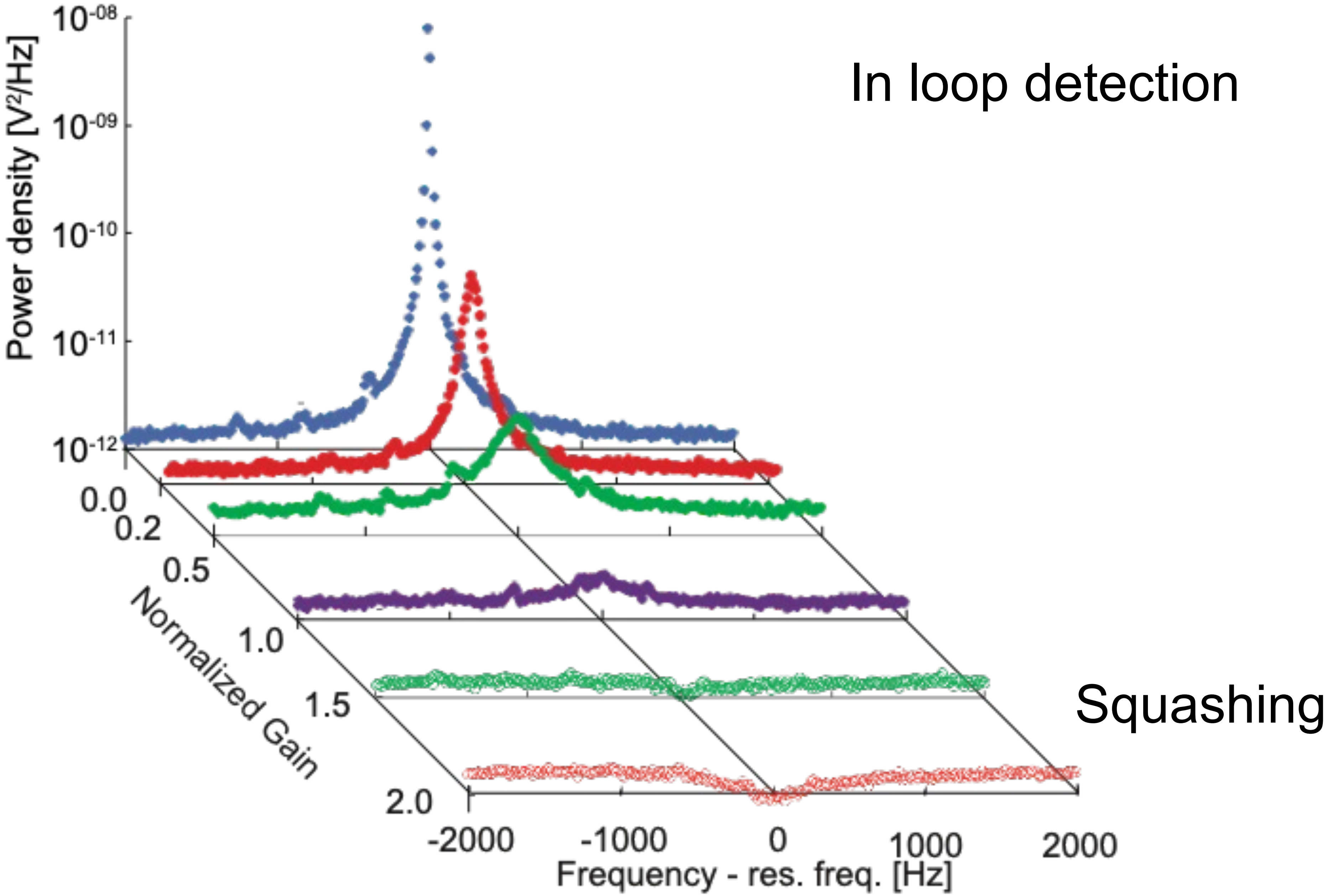
Optimization of Feedback



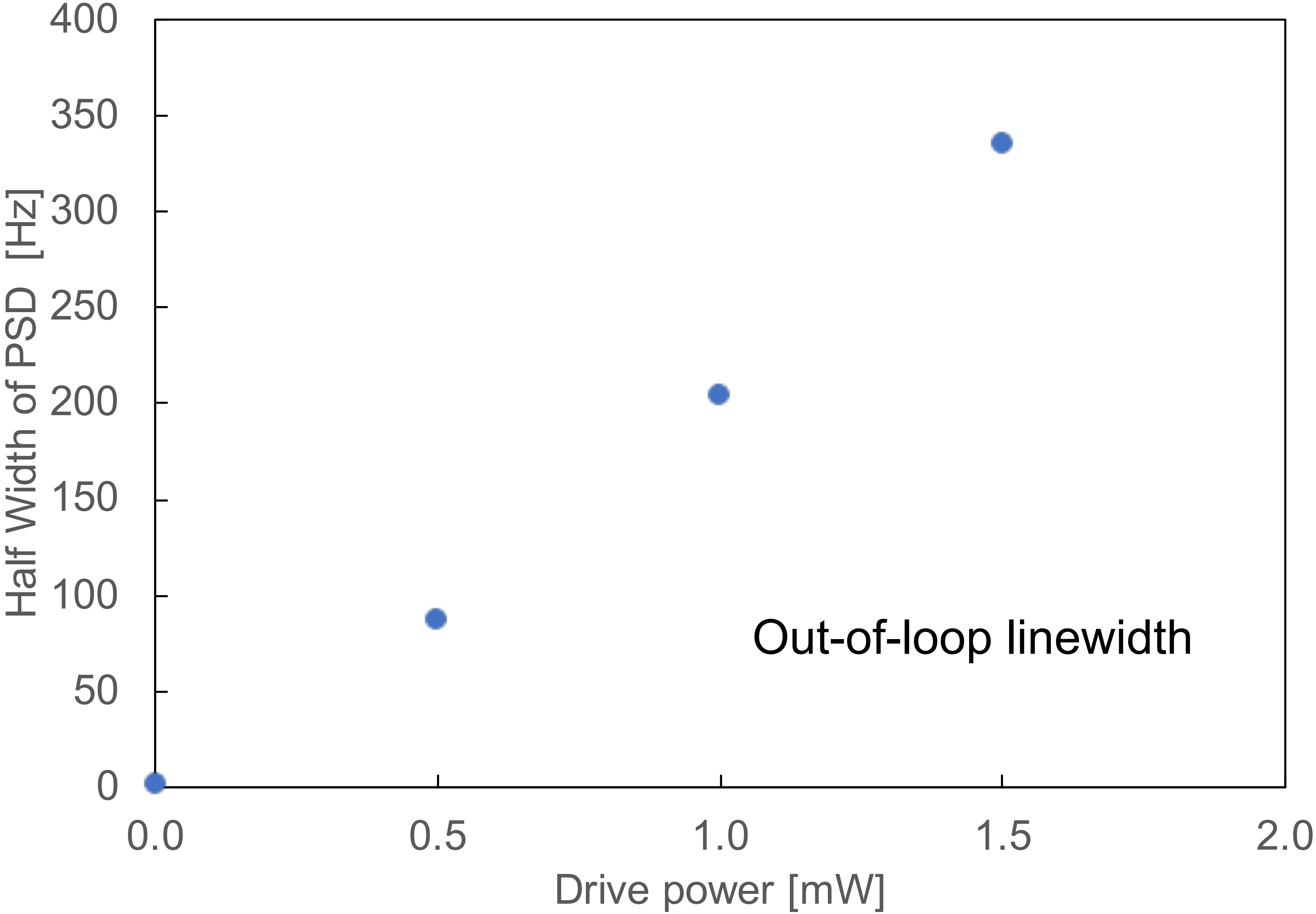
In loop time series, off [red] on [blue]
Power spectral density V²/Hz
1.E-05
1.E-06
1.E-07
1.E-08
1.E-09
1.E-10
1.E-11
1.E-12
1.E-13
1.E-14
0
2
4
6
8
10
time [s]



[mW] T$_0$/T
0.0 1.0x10^{-0}
0.2 1.4x10^{-2}
0.5 2.4x10^{-3}
1.0 2.5x10^{-4}
Ocurrences
Amplitude [V]
0
50
100
150
200
250
300
350
400
450
500
1.E-07
1.E-06
1.E-05
1.E-04
1.E-03
1.E-02



In loop detection
Squashing
Power density [V²/Hz]
10-08
10-09
10-10
10-11
10-12
0.0
0.2
0.5
1.0
1.5
2.0
Normalized Gain
-2000
-1000
0
1000
2000
Frequency - res. freq. [Hz]



400
350
300
250
200
150
100
50
0
Half Width of PSD [Hz]
0.0
0.5
1.0
1.5
2.0
Drive power [mW]
Out-of-loop linewidth

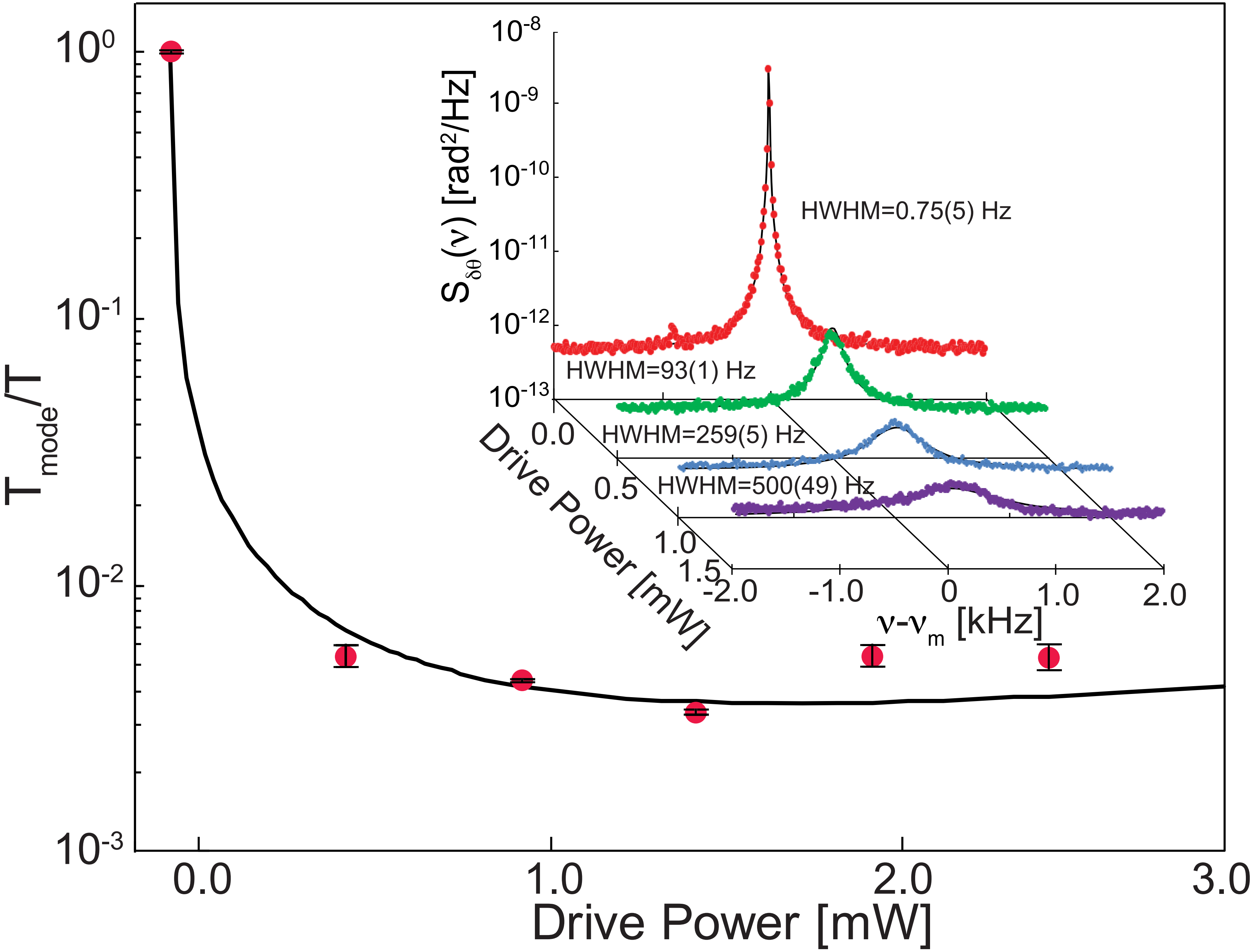
There is a limit on how cold we can get (depends on the noise S_n and the signal S_s).

$$\frac{T_{\text{mode}}}{T} = \frac{1}{1+g}\left(1 + g^2 \frac{S_{\theta_n}}{S_s}\right) \qquad g_{\text{opt}} = \sqrt{1 + S_s/S_{\theta_n}} - 1.$$

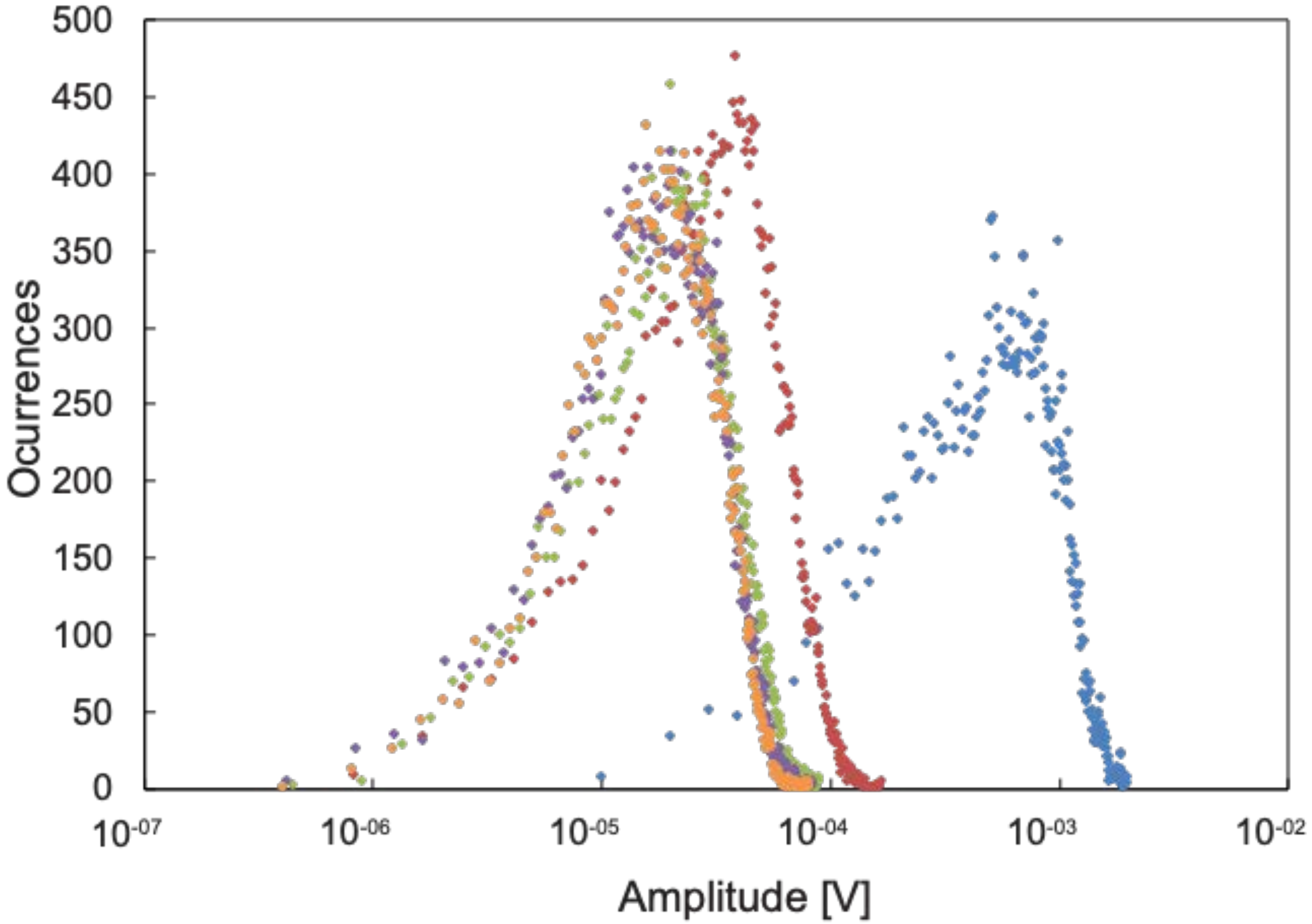
g: dimensionless gain that depends on the PID and drive power, in the limit of large signal to noise ratio

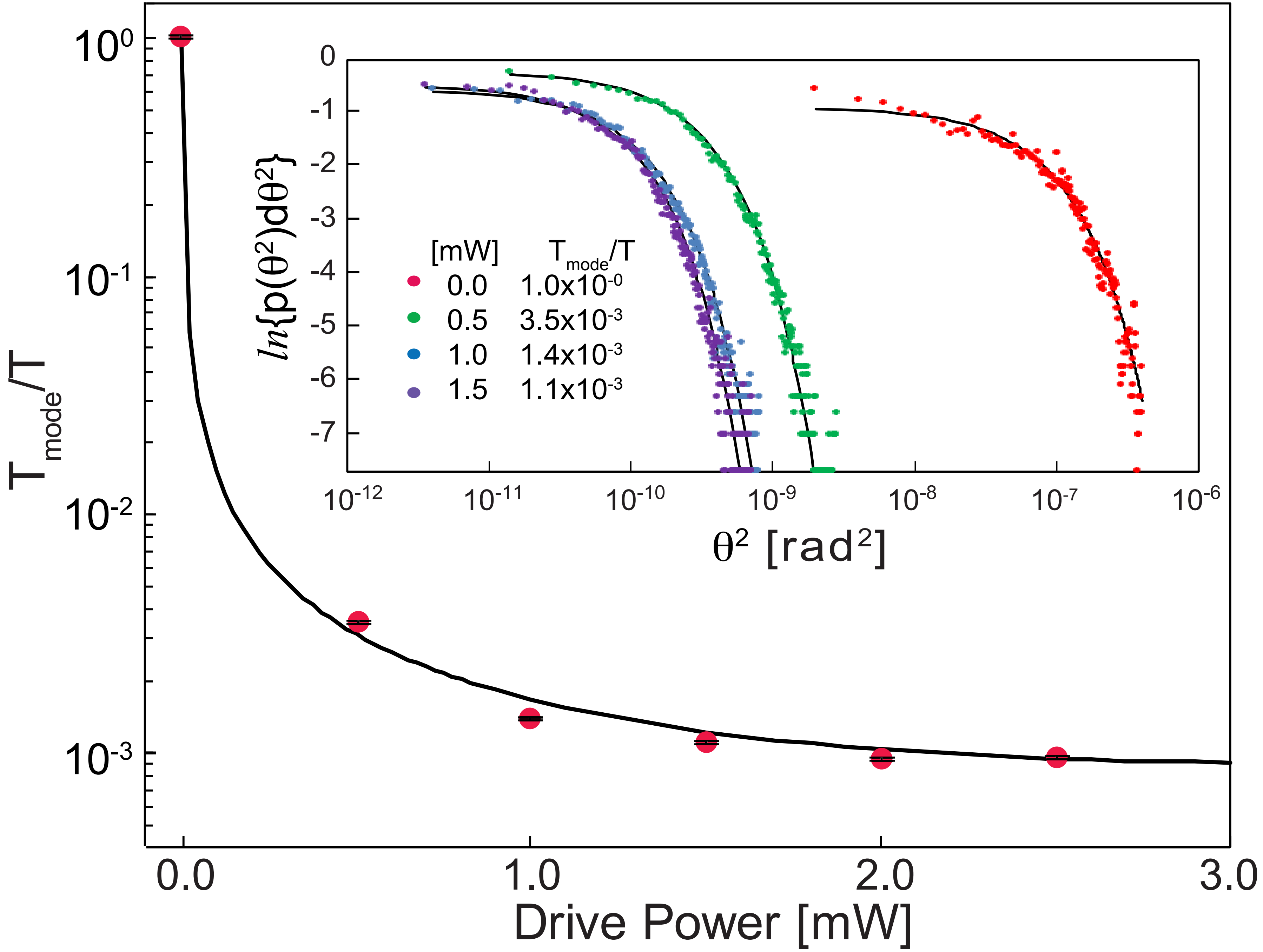
$$g_{\text{opt}} \to \sqrt{\frac{S_s}{S_{\theta_n}}} \qquad \frac{T_{\text{mode}}}{T} \to \frac{2}{\sqrt{S_s/S_{\theta_n}}}$$

M. Poggio, C. L. Degen, H. J. Mamin, and D. Rugar, "Feedback Cooling of a Cantilever's Fundamental Mode below 5 mK" Phys. Rev. Lett. **99**, 017201 (2007).



10^0
10^{-1}
10^{-2}
10^{-3}
T_{mode}/T
0.0
1.0
2.0
3.0
Drive Power [mW]
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
10^{-13}
$S_{\delta\theta}(\nu)$ [rad^2/Hz]
HWHM=0.75(5) Hz
HWHM=93(1) Hz
HWHM=259(5) Hz
HWHM=500(49) Hz
Drive Power [mW]
0.0
0.5
1.0
1.5
-2.0
-1.0
0
1.0
2.0
$\nu-\nu_m$ [kHz]


500
450
400
350
300
250
200
150
100
50
0
Ocurrences
10^{-07}
10^{-06}
10^{-05}
10^{-04}
10^{-03}
10^{-02}
Amplitude [V]



T_{mode}/T
Drive Power [mW]
10^{0}
10^{-1}
10^{-2}
10^{-3}
0.0
1.0
2.0
3.0
$ln\{p(\theta^2)d\theta^2\}$
θ^2 [rad^2]
0
-1
-2
-3
-4
-5
-6
-7
10^{-12}
10^{-11}
10^{-10}
10^{-9}
10^{-8}
10^{-7}
10^{-6}
[mW] T_{mode}/T
0.0 1.0x10^{-0}
0.5 3.5x10^{-3}
1.0 1.4x10^{-3}
1.5 1.1x10^{-3}

Summary:

We have cooled (x1000) the torsional mode of an optical nanofiber by feedback.Torque sensitivity: $2.9 \times 10^{-26}\ \mathrm{NmHz}^{-1/2}$

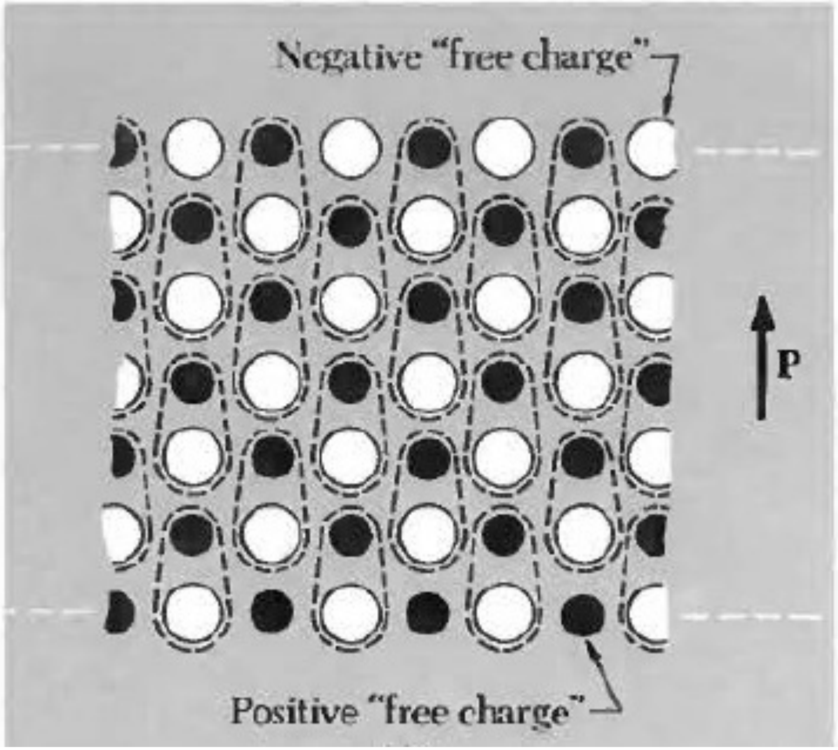
Open question:
Explain the origin of the birefringence.

Quantum limit? $\frac{h\nu}{k_B} = 10^{-5}$K, we are at 3×10^{-1} K

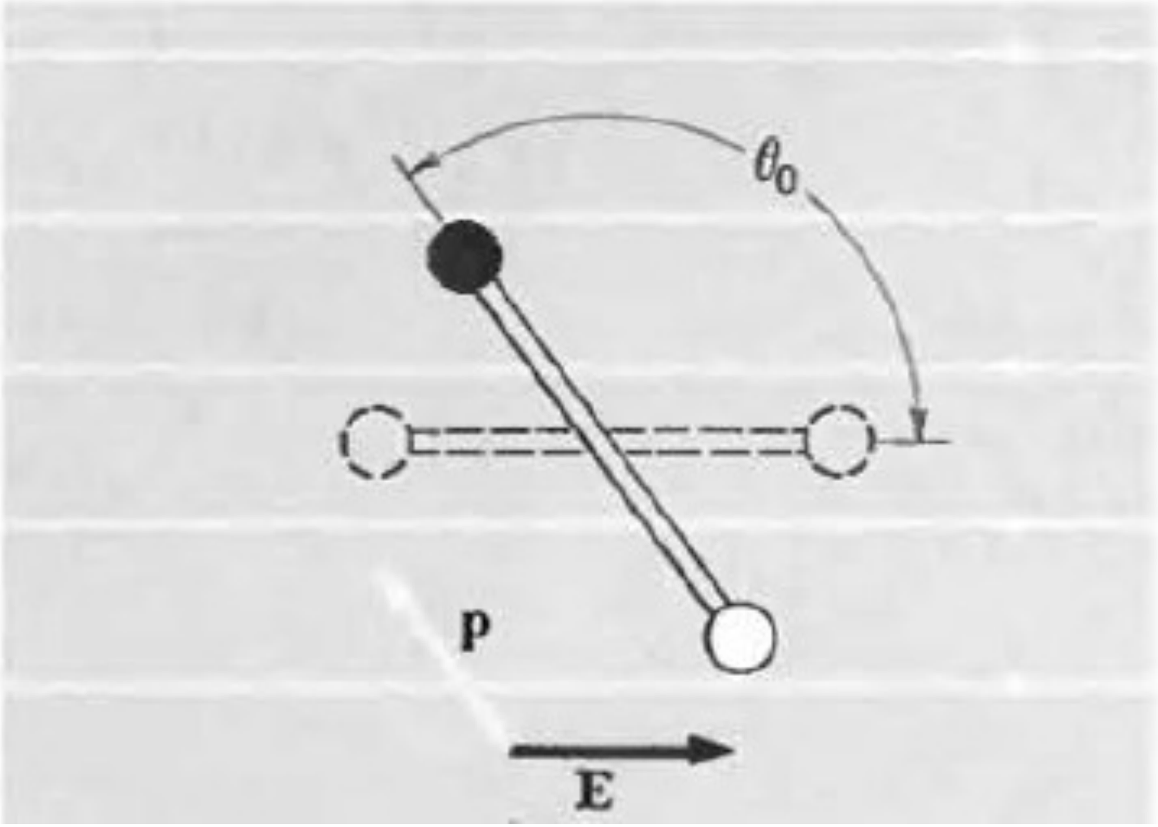
Increase the SNR:. Increase Q and decrease noise.

Thanks

Glass will polarize when an electromagnetic field passes through it. The E field induces the dipole.



If there is stress in the glass, there can be different index of refraction in the orthogonal directions: Birefringence. There is an optical axis.



Work done to orient the dipole from aligned with E to θ_0

$$\int_0^{\theta_0} N\, d\theta = \int_0^{\theta_0} pE \sin\theta\, d\theta = pE(1 - \cos\theta_0)$$

Torsional Modes

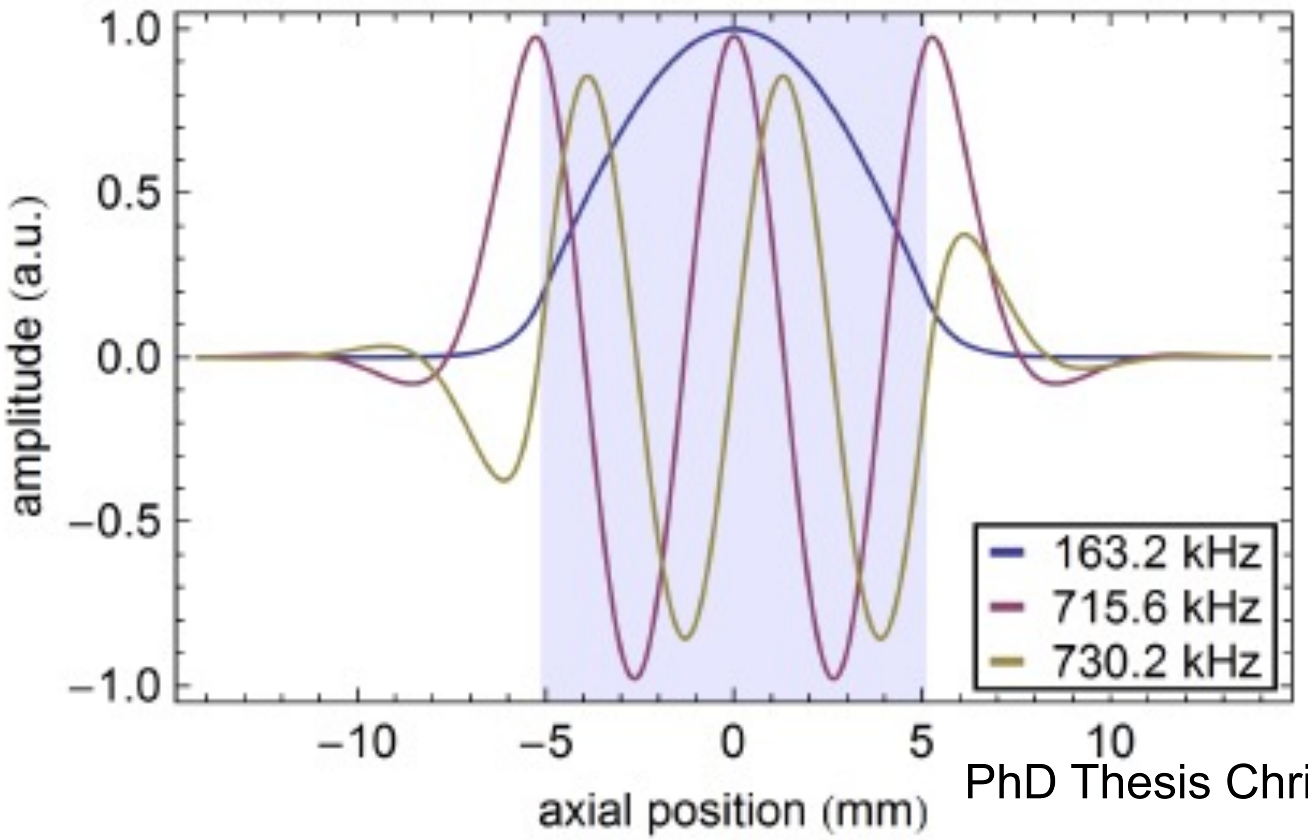
Torsion of a plane of a cylinder by an angle ϕ

$$c_t^{-2}\,\partial_t^2\,\phi(t,z) - \partial_z^2\,\phi(t,z) = 0$$

$$\phi(t,z) = \phi(z)\cos(\omega\,t)$$

$$\partial_z^2\,\phi(t,z) + k_0^2\,\phi(t,z) = 0$$

Amplitude of the modes of the nanofiber considering the tapers



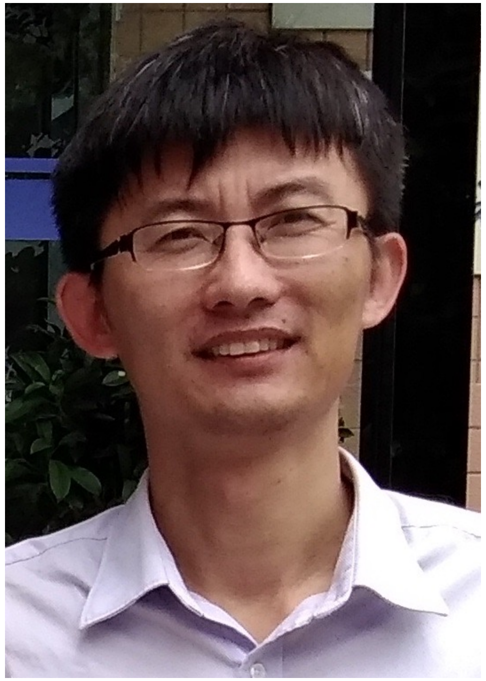
PhD Thesis Christian Wuttke

Questions?

Pablo

Jeff

Su Dianquiang



Zhao Yanting