Top

Left side

TEST

Right side

Bottom

Datos 3

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https://www.physics.umd.edu/rgroups/amo/orozco/results/2024/Results24.htm

Cálculo de la media utilizando χ^2

Cálculo de la media μ con incertidumbre utilizando χ^2

x_i son n mediciones con sus respectivas incertidumbres σ_i

Cantidad a minimizar:

$$\chi^2 = \sum_{i=1}^{n} \left(\frac{(\mu - x_i)}{\sigma_i} \right)^2$$

Observaciones: si los errores son solamente estadísticos, en la diferencia del numerador será del orden del error, por lo tanto la suma dará n.

Vamos a minimizar χ^2 con respecto a μ

$$\frac{\partial \chi^2}{\partial \mu} = \sum_{i=1}^{n} \frac{\partial}{\partial \mu}\left(\frac{(\mu - x_i)}{\sigma_i}\right)^2 = \sum_{i}^{n} \frac{2(\mu - x_i)}{\sigma_i^2} = 0$$

$$\frac{\partial \chi^2}{\partial \mu} = \sum_{i}^{n} \frac{(\mu - x_i)}{\sigma_i^2} = -\sum_{i}^{n} \frac{(x_i)}{\sigma_i^2} + \sum_{i}^{n} \frac{(\mu)}{\sigma_i^2} = 0$$

$$\sum_{i}^{n} \frac{(x_i)}{\sigma_i^2} = \sum_{i}^{n} \frac{(\mu)}{\sigma_i^2} = \mu \sum_{i}^{n} \frac{1}{\sigma_i^2}$$

$$\sum_i^n \frac{(x_i)}{\sigma_i^2} \Bigg/ \sum_i^n \frac{1}{\sigma_i^2} = \mu$$

La fórmula para las medias de una muestra con pesos diferentes

$$\left. \sum_i^n \frac{(x_i)}{\sigma_i^2} \middle/ \sum_i^n \frac{1}{\sigma_i^2} \right. = \mu$$

Fórmula para las medias de una muestra con pesos diferentes

$$\left. \frac{1}{\sigma^2} \sum_i^n x_i \middle/ \frac{1}{\sigma^2} n \right. = \mu$$

Fórmula para las medias de una muestra con pesos iguales que se reduce a la definición tradicional de la media.

¿Cuál va a ser la incertidumbre de la media?

Sabemos cómo calcular la incertidumbre de un parámero con incertidumbres no correlacionados entonces hagamos la propagación de errores para calcular el valor.

$$\sigma_\mu^2 = \sum\left[\sigma_i^2\left(\frac{\partial\mu'}{\partial x_i}\right)^2\right] \qquad \frac{\partial\mu'}{\partial x_i} = \frac{\partial}{\partial x_i}\frac{\Sigma(x_i/\sigma_i^2)}{\Sigma(1/\sigma_i^2)} = \frac{1/\sigma_i^2}{\Sigma(1/\sigma_i^2)}$$

$$\sigma_\mu^2 = \sum\frac{1/\sigma_i^2}{[\Sigma(1/\sigma_i^2)]^2} = \frac{1}{\Sigma(1/\sigma_i^2)}$$

si todas las incertidumbres son iguales a s se reduce a

$$\sigma_\mu = \frac{s}{\sqrt{n}}$$

Errores con Chi cuadrada χ^2

es el valor que temenos que incrementar a la variable para que Chi cuadrada se incremente en 1

$$P(a_1, a_2, \ldots, a_m) = \prod \left[\frac{1}{\sigma_i \sqrt{2\pi}}\right] \exp\left\{-\frac{1}{2}\sum\left[\frac{y_i - y(x_i)}{\sigma_i}\right]^2\right\}$$

$$\chi^2 \equiv \sum \left\{\frac{1}{\sigma_i^2}[y_i - y(x_i)]^2\right\}$$

$$\frac{\partial \chi^2}{\partial a_j} = \frac{\partial}{\partial a_j} \sum \left\{\frac{1}{\sigma_i^2}[y_i - y(x_i)]^2\right\} = 0$$

$$= -2\sum\left\{\frac{1}{\sigma_i^2}[y_i - y(x_i)]\frac{\partial y(x_i)}{\partial a_j}\right\}$$

si la muestra es lo suficientemente grande:

$$P(a_j) = Ae^{-(a_j - a_j')^2/2\sigma_i^2}$$

$$\chi^2 = -2\ \ln[P(a_1, a_2, \ldots, a_m)] + 2\sum \ln(\sigma_i\sqrt{2\pi})$$

Muy cerca del mínimo la variación es cuadrática.

$$\chi^2 = \frac{(a_j - a_j')^2}{\sigma_j^2} + C$$

Si se incrementa el estimado a’ en una variación estándar produce en χ^2 una cambio de uno. Podemos saber la incertidumbre en ese parámetro.

Expansion de χ^2 en una serie de Taylor alrededor del mínimo

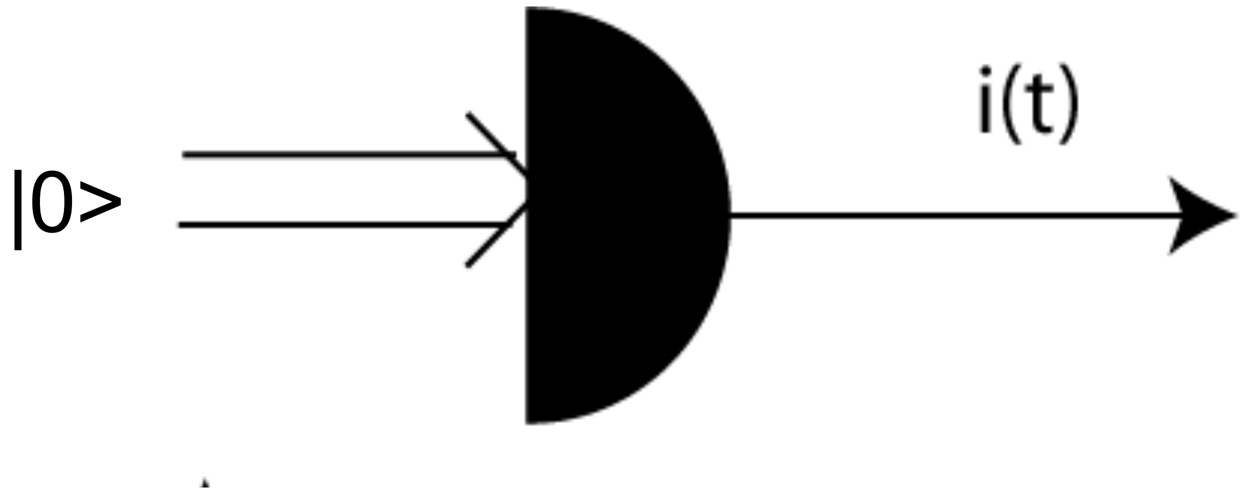
$$\chi^2 \simeq \chi_0^2 + \sum_{j=1}^{m} \left\{ \frac{\partial \chi_0^2}{\partial a_j} (a_j - a_j') \right\} + \frac{1}{2} \sum_{k=1}^{m} \sum_{j=1}^{m} \left\{ \frac{\partial^2 \chi_0^2}{\partial a_k \partial a_j} (a_k - a_k')(a_j - a_j') \right\}$$

El caso de la media

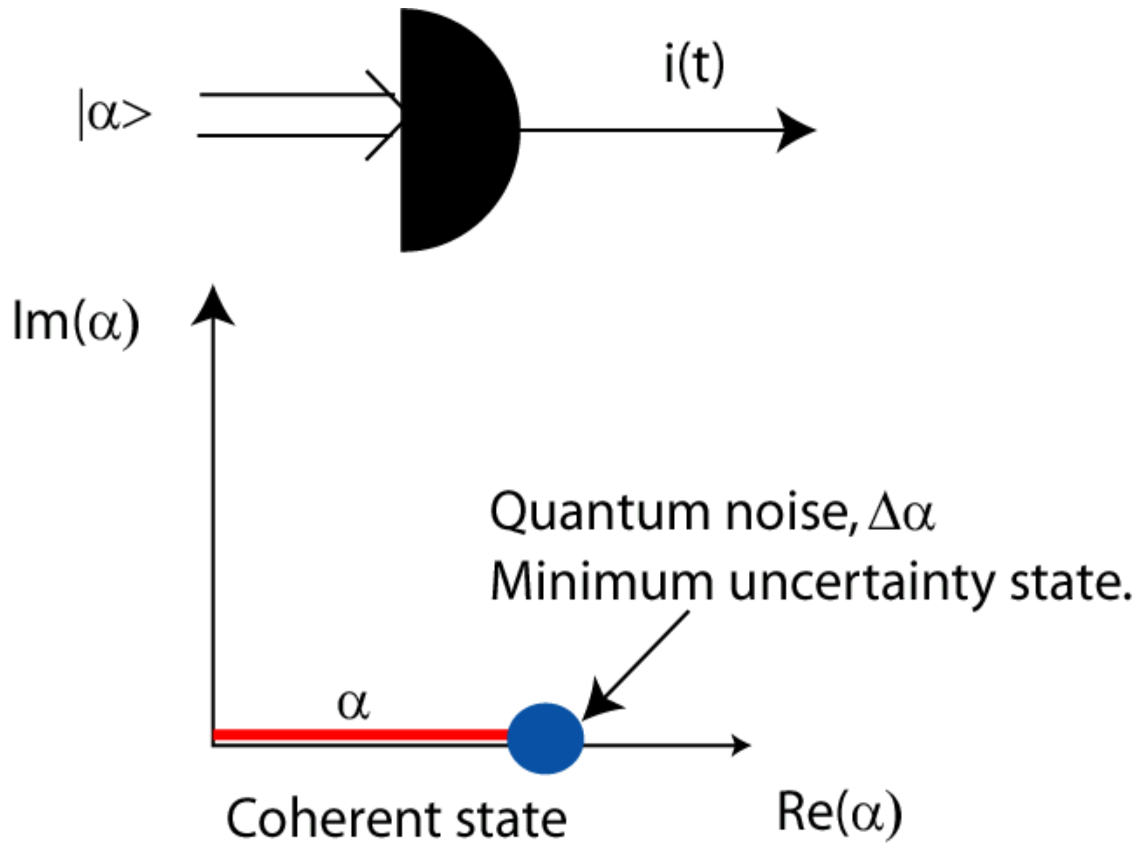
$$\frac{\partial^2 \chi^2}{\partial \mu^2} = \frac{2}{\sigma_\mu^2} \qquad \chi^2 = \sum_{i=1}^{n} \left(\frac{(\mu - x_i)}{\sigma_i} \right)^2$$

$$\frac{\partial^2 \chi^2}{\partial \mu^2} = \frac{\partial}{\partial \mu} \sum_{i=1}^{n} \left(\frac{2(\mu - x_i)}{\sigma_i^2} \right) = 2 \sum_{i=1}^{n} \frac{1}{\sigma_i^2} \qquad \sigma_\mu^2 = \frac{1}{\sum_{i=1}^{n} \frac{1}{\sigma_i^2}}$$

¿Cuál debe ser la distribución de los residuos normalizados? Si la muestra es suficientemente grande Gaussiana con una varianza de uno. Si la distribución tiene dos picos, probablemente tienen un prooblema sistemático. Si la varianza no es uno, modificar los errores estadísticos (agrandandolos) hasta que de uno. El factor de escala va a ser $\sqrt{\chi^2_{red}}$. Esa decision puede depender de otros factores, ver las guías de PDG



¿Cuánto es la corriente de salida?

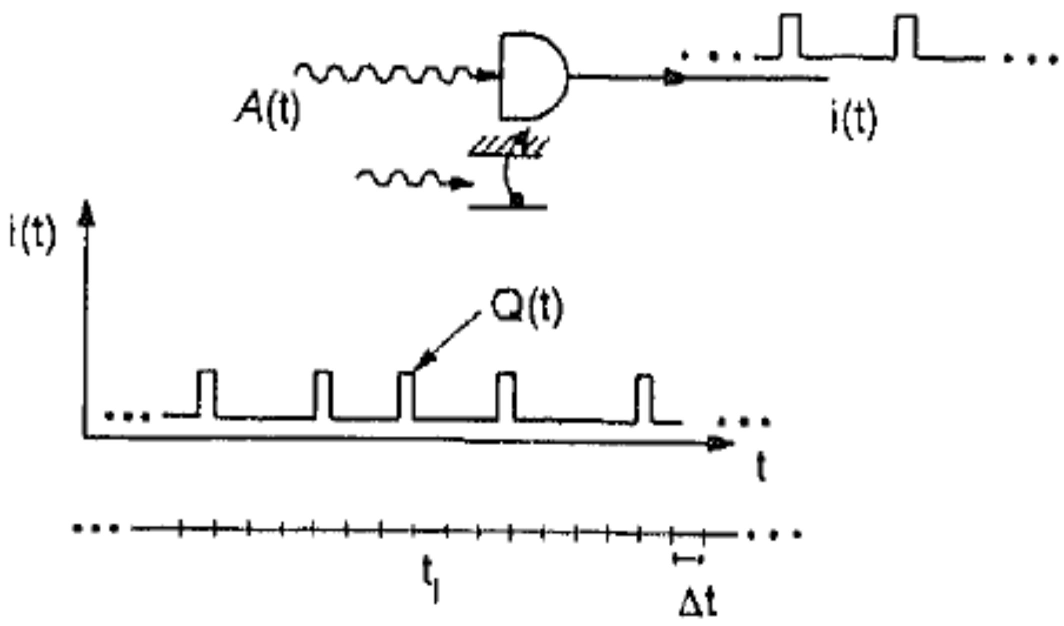


Detector perfecto $i(t) = |\alpha + \Delta\alpha|^2$

$i(t) = |\alpha|^2 + 2\ \alpha\ \Delta\alpha + |\Delta\alpha|^2$; $<\alpha^*\alpha>=n$

DC~n Ruido de disparo~$n^{1/2}$ despreciar.

Detección de Luz (ruido de disparo)



El campo A(t) produce una fotocorriente con carga Q. La ionización sucede en un periodo Δt tal que es lo suficientemente pequeño para solo tener un electrón en cada Δt. p_k es una variable aleatoria.

$$i(t) = \sum_k Q(t - t_k) p_k.$$

H. J. Kimble

Tenemos que calcular la densidad espectral de potencia

$$\Phi(\Omega) \equiv \int \langle \Delta i(t) \Delta i(t+\tau) \rangle \, e^{-i\Omega\tau} \, d\tau,$$

Función de correlación

$$\langle \Delta i(t) \Delta i(t+\tau) \rangle = \langle i(t) i(t+\tau) \rangle - \langle i \rangle^2$$

La correlación es:

$$\langle i(t) i(t+\tau) \rangle = \left\langle \sum_{k=-\infty}^{\infty} Q(t-t_k) p_k \sum_{j=-\infty}^{\infty} Q(t+\tau-t_j) p_j \right\rangle$$

H. J. Kimble

$$= \sum_k Q(t - t_k)Q(t + \tau - t_k)\langle p_k\rangle$$

$$+ \sum_{k \neq j} Q(t - t_k)Q(t + \tau - t_j)\langle p_k p_j\rangle,$$

$$\langle p_1 p_2 \dots p_k\rangle = W_k(t_1, t_2, \dots, t_k)\Delta t_1 \Delta t_2 \dots \Delta t_k,$$

$$W_k(t_1, t_2, \dots, t_k) = \alpha^k \langle : I(t_1)I(t_2) \dots I(t_k) : \rangle$$

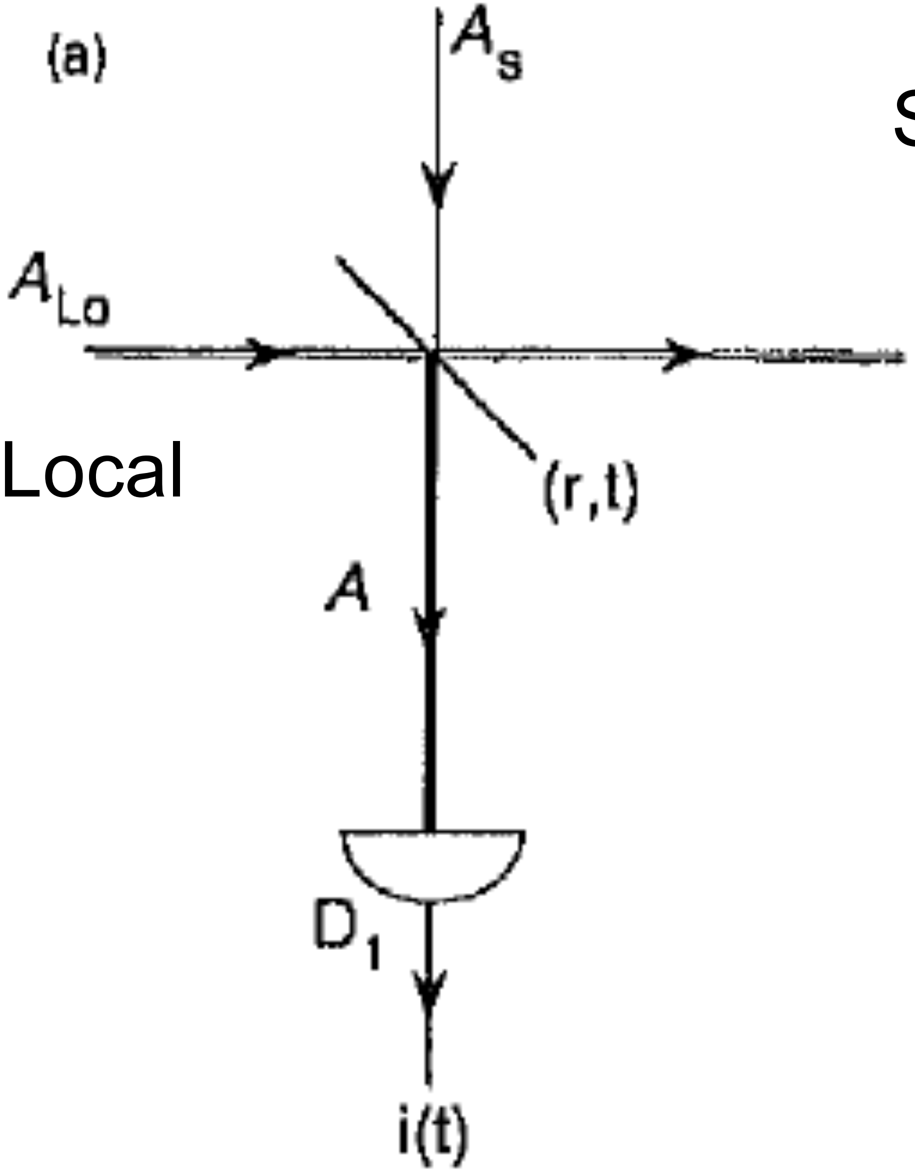
$$Q(t - t') = Q_0 \delta(t - t'),$$

$$\begin{aligned}\langle i(t)i(t+\tau)\rangle = & \int \mathrm{d}t'\ W_1(t')Q(t-t')Q(t+\tau-t') \\ & + \int \mathrm{d}t' \int \mathrm{d}t''\ W_2(t',t'')Q(t-t')Q(t+\tau-t'') \\ = & \ Q_0^2\alpha\ \langle : I(\tau) : \rangle \delta(\tau) \\ & + Q_0^2\alpha^2\ \langle : I(t)I(t+\tau) : \rangle .\end{aligned}$$

$$\langle \Delta i(t)\Delta i(t+\tau)\rangle = Q_0^2\alpha\ \langle : I(t) : \rangle\ \delta(\tau) + Q_0^2\alpha^2 C(\tau)$$

$$C(\tau) \equiv \langle : I(t)I(t+\tau) : \rangle - \langle : I : \rangle^2 .$$

Pero la fotocorriente se debe a un batido, pues queremos ver las fluctuaciones



Señal

Oscilador Local

r>>t

$R=r^2$

$T=t^2$

Lo siguiente es calcular la correlación con el campo A

$$A = rA_{\mathrm{LO}} + tA_{\mathrm{s}}.$$

A primer orden en A_s

$$C(\tau) = RTA_0^2[e^{-2i\theta}\langle A_{\mathrm{s}}(t+\tau), A_{\mathrm{s}}(t)\rangle + e^{2i\theta}\langle A_{\mathrm{s}}^\dagger(t), A_{\mathrm{s}}^\dagger(t+\tau)\rangle$$
$$+ \langle A_{\mathrm{s}}^\dagger(t), A_{\mathrm{s}}(t+\tau)\rangle + \langle A_{\mathrm{s}}^\dagger(t+\tau), A_{\mathrm{s}}(t)\rangle],$$

Pero las cuadraturas de las fluctuaciones del campo electromagnético:

$$z_\theta(t) = e^{-i\theta} A_{\mathrm{s}}(t) + A_{\mathrm{s}}^\dagger(t)\, e^{i\theta},$$

$$C(\tau) = RTA_0^2 \langle : z_\theta(t), z_\theta(t+\tau) : \rangle$$

$$\langle \Delta i(t) \Delta i(t+\tau) \rangle = Q_0 i_0 [\delta(\tau) + \alpha T \langle : z_\theta(t), z_\theta(t+\tau) : \rangle],$$

$$\Phi(\Omega, \theta) = Q_0 i_0 [1 + \alpha T S_{\mathrm{s}}(\Omega, \theta)],$$

$$S_{\mathrm{s}}(\Omega, \theta) = \int d\tau \; \mathrm{e}^{-\mathrm{i}\Omega\tau} \langle : z_\theta(t), z_\theta(t+\tau) : \rangle.$$

La densidad espectral de potencia del ruido de disparo cuando S=0:

$$\langle(\Delta i(\Omega,\theta))^2\rangle = \frac{1}{2\pi}\left[\int_{-\Omega-\Delta\Omega/2}^{-\Omega+\Delta\Omega/2} \Phi(\Omega,\theta)\, \mathrm{d}\Omega + \int_{\Omega-\Delta\Omega/2}^{\Omega+\Delta\Omega/2} \Phi(\Omega,\theta)\, \mathrm{d}\Omega\right]$$

$$\langle(\Delta i(\Omega))^2\rangle = 2Q_0 i_0 B.$$

Pero en el caso cuando S≠0

$$\langle(\Delta i(\Omega))^2\rangle = 2Q_0 i_0 B[1 + \xi S(\Omega,\theta)]$$

Tomando en cuenta la propagacion, la detección alpha, la eficiencia de batido eta, y la eficiencia de escape de la cavidad rho, la transmisión del espejo T_0

$$\xi \equiv \alpha\eta^2 T_0 \rho$$

¿Como afecta el fondo (background) la incertidumbre?

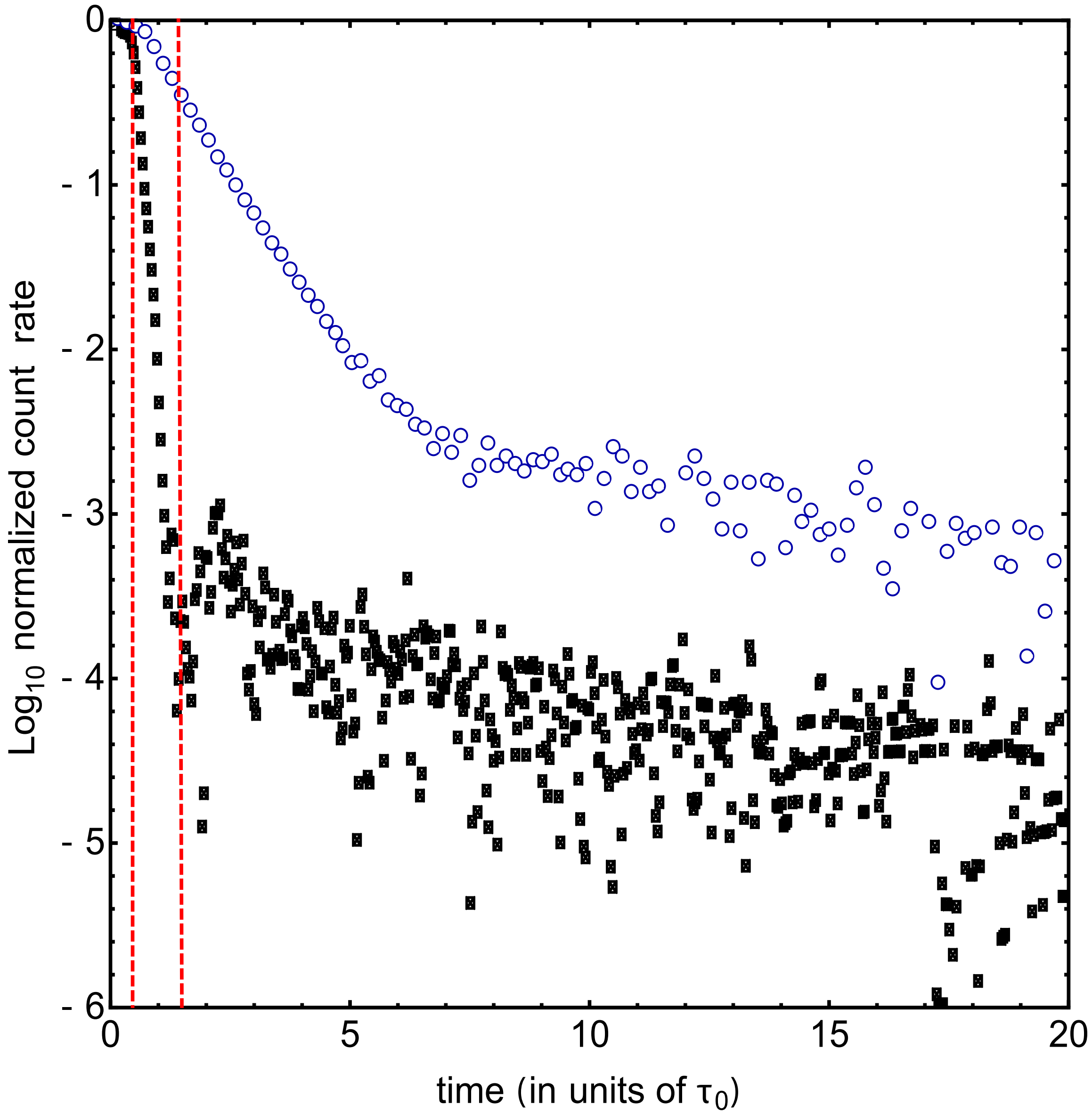
Si la señal S está sobre un fondo F y el ruido es R (Poissoniano)

$$\frac{S}{N} = \frac{S}{\sqrt{S + F}}$$

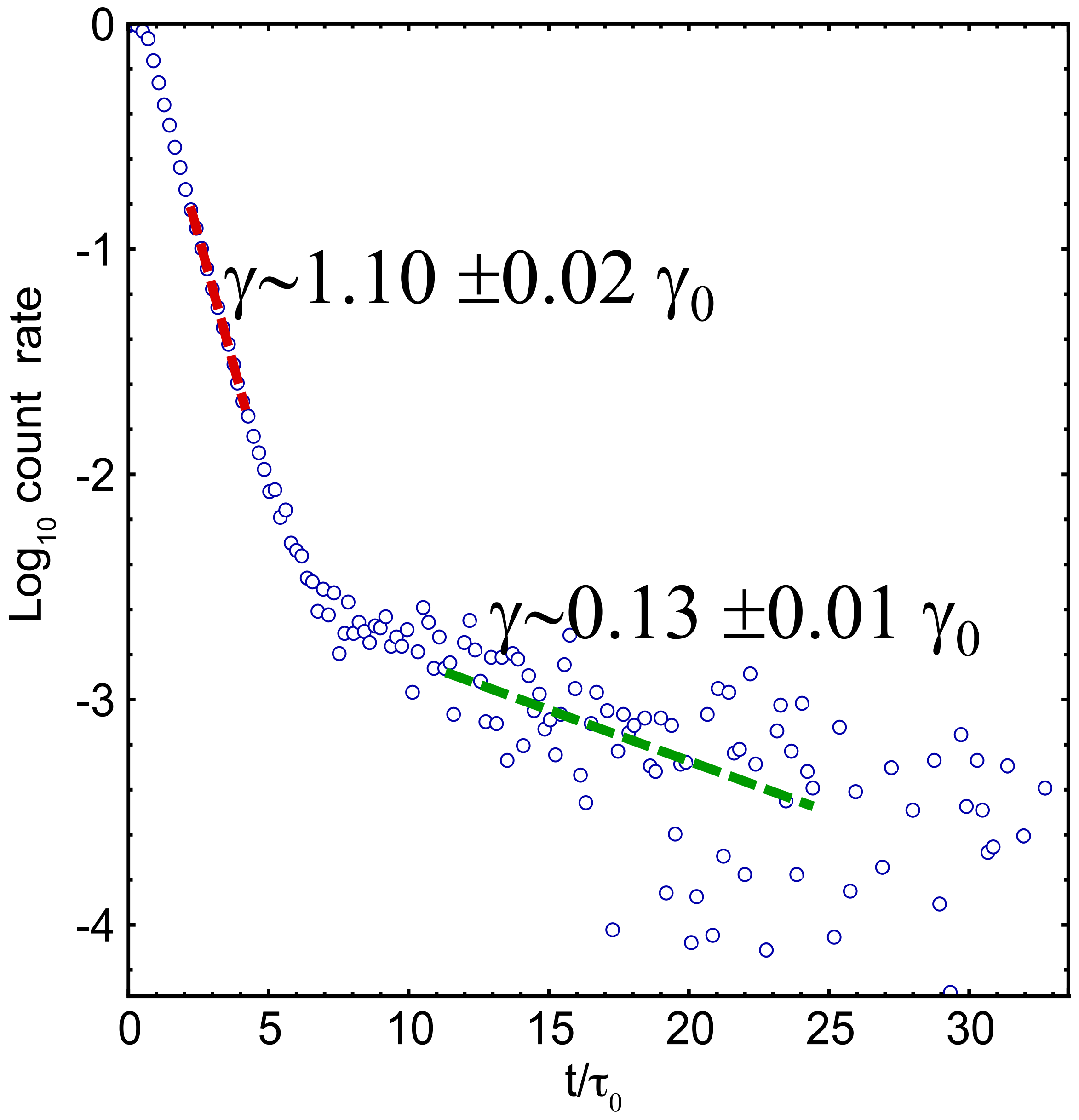
Mucho cuidado pues el fondo puede dominar completamente
la razón señal a ruido.

Además el fondo puede ser afectado por otros sistemáticos.

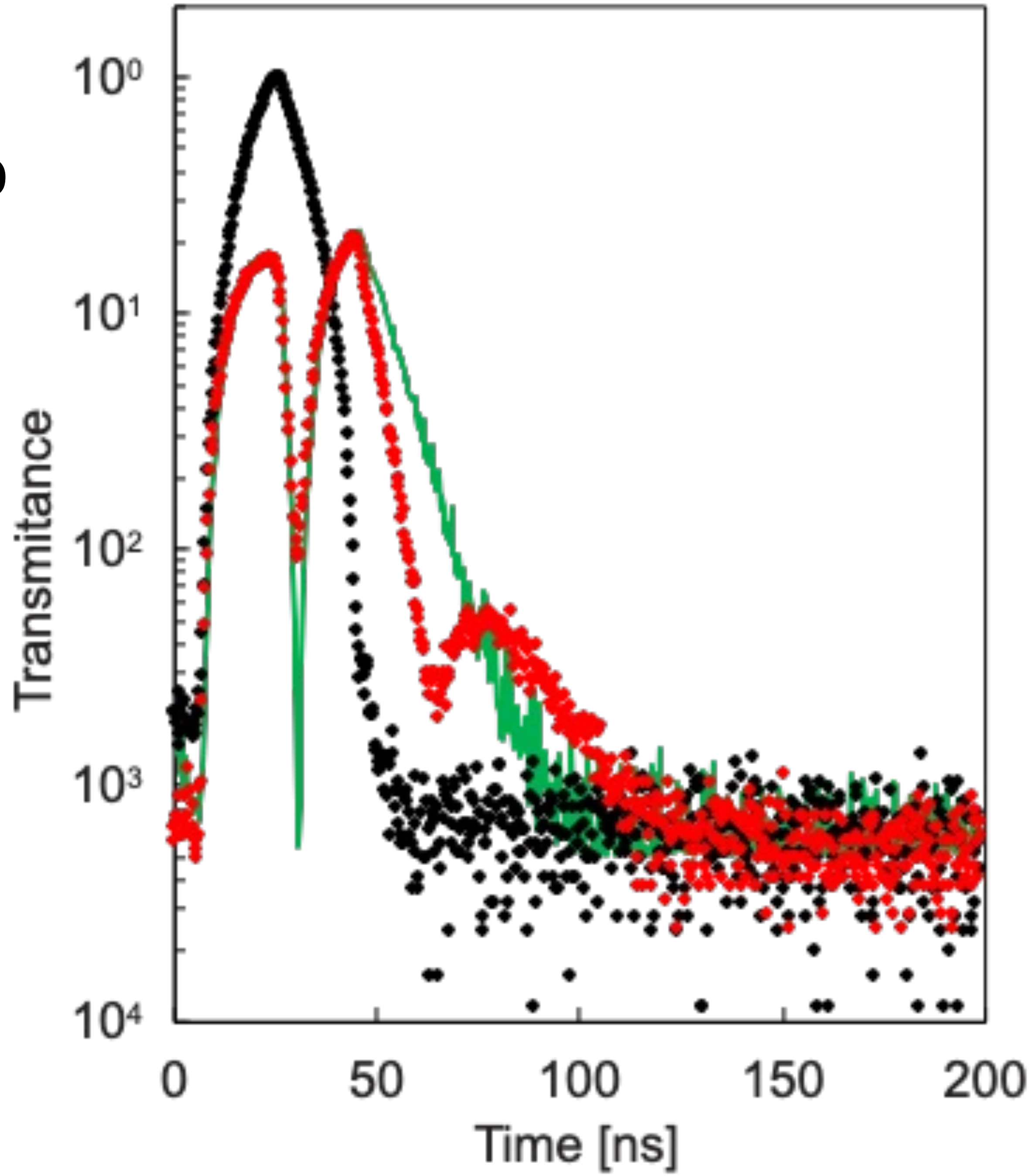
Pulso y señal



Dos tasas de decaimiento distintas



Verde modelo con un solo modo
pulso ~13 ns



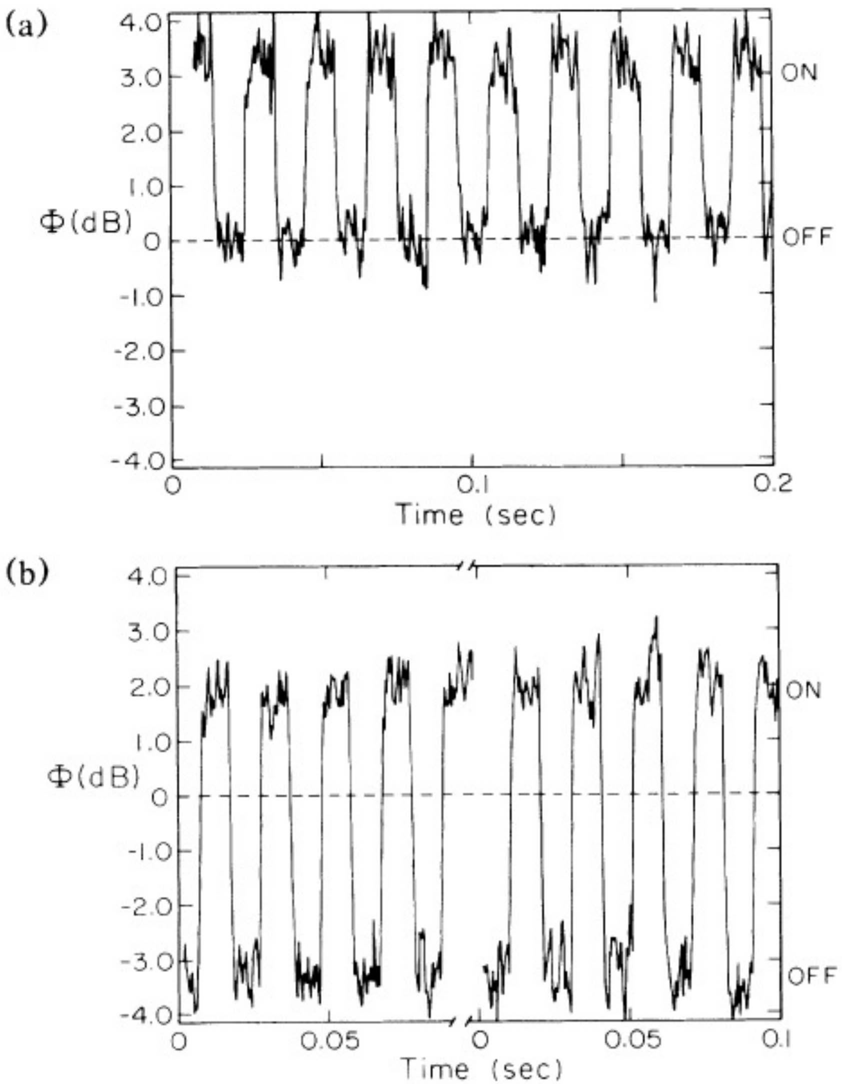


FIG. 2. Level of fluctuations Φ of the difference photocurrent i vs time for fixed analysis frequency $\nu/2\pi = 1.6$ MHz, analysis bandwidth $B = 100$ kHz, and two video filters of time

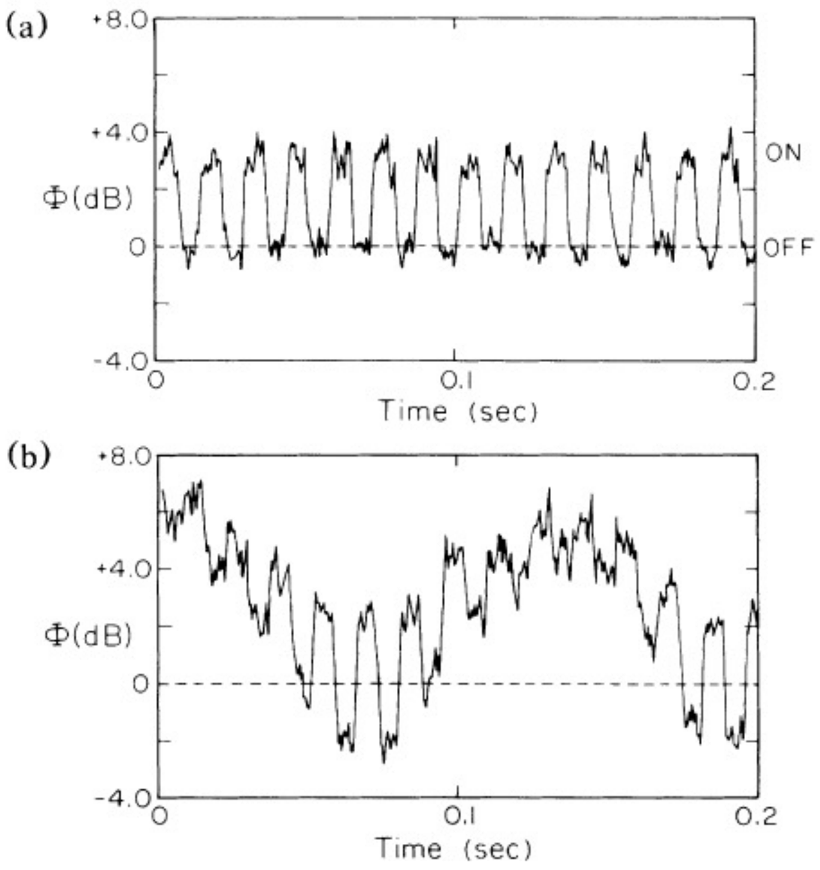


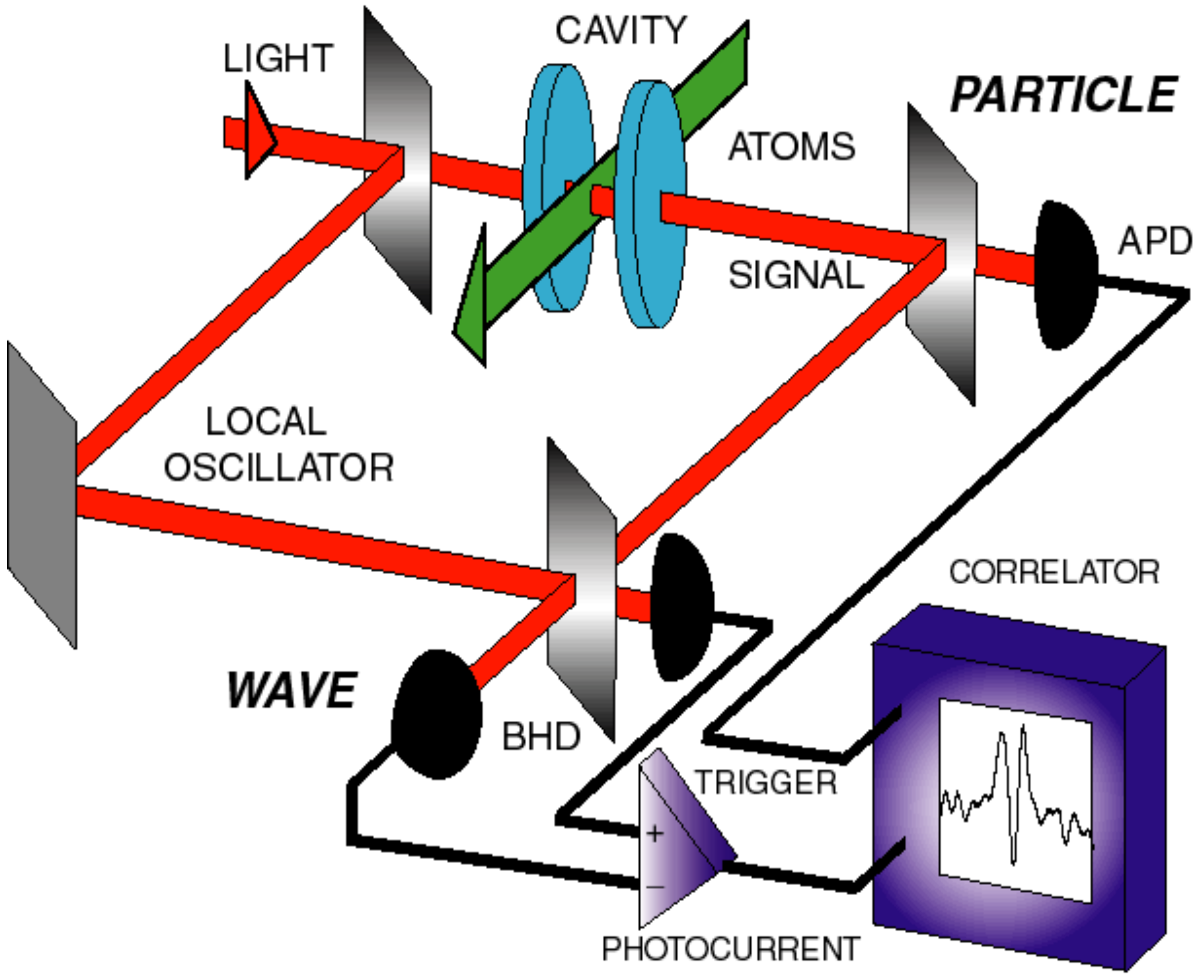
FIG. 3. Level of fluctuations Φ of the photocurrent i vs time. Parameters are similar to those in Fig. 2 with the exception that the phase angle θ is slowly swept with a linear ramp. (a) Vacuum-state input for the field $\hat{E}_s$; (b) Squeezed-state input $\hat{E}_s$. The variation of θ produces alternately a degradation and an improvement in signal to noise as first the increased $(S > 0)$ and then the decreased $(S < 0)$ fluctuations of the squeezed state are combined with the coherent field $\hat{E}_1$.

Mejora de la señal a ruido promediando

El campo de un fotón:

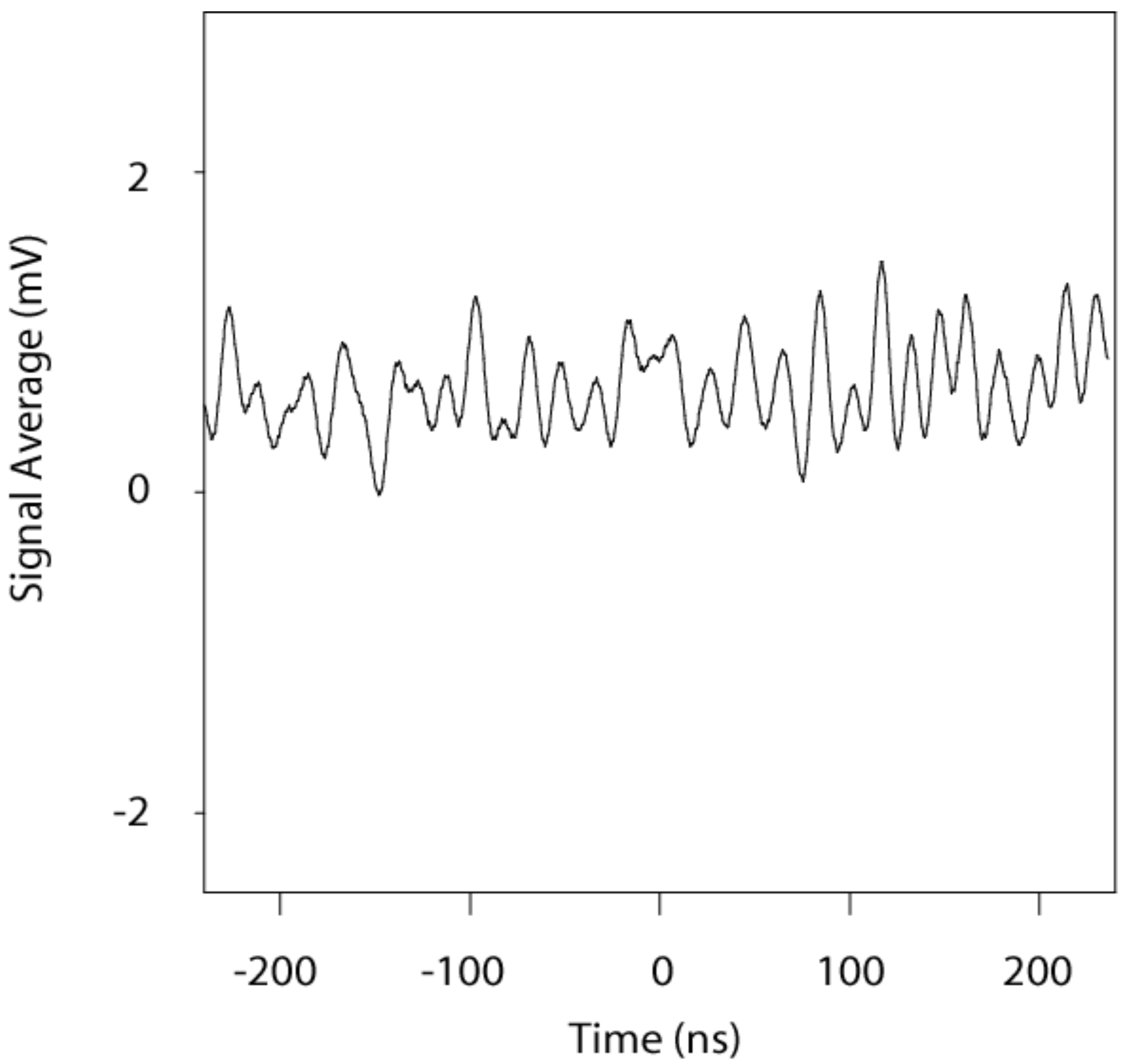
G. T. Foster, L. A. Orozco, H. J. Carmichael, and H. M. Castro-Beltran " Quantum State Reduction and Conditional Time Evolution of Wave-Particle Correlations in Cavity QED," Phys. Rev. Lett. **85**, 3149 (2000).

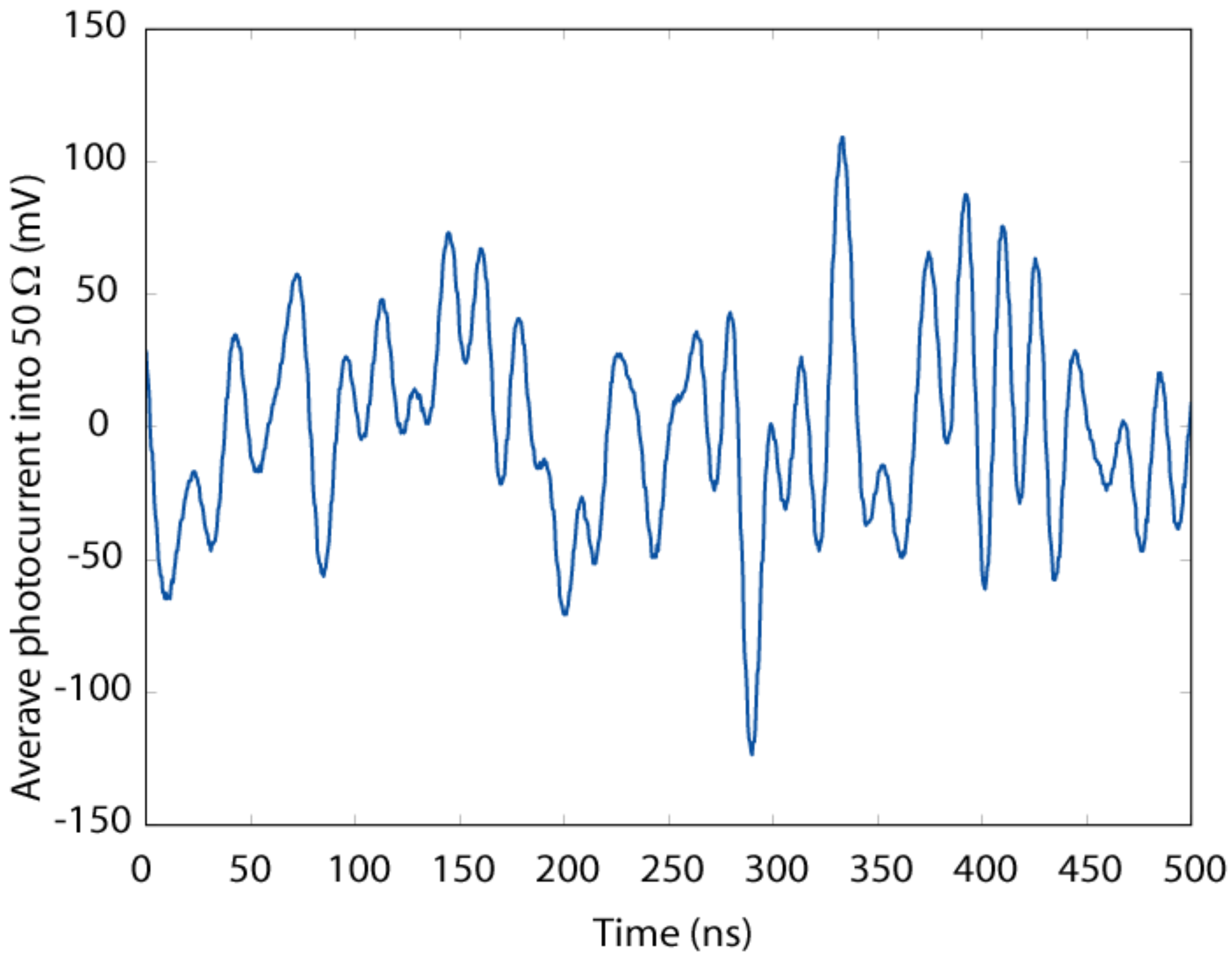
G. T. Foster, W. P. Smith, J. E. Reiner, and L. A. Orozco," Time-dependent electric field fluctuations at the subphoton level," Phys. Rev. A. **66**, 033807, (2002).



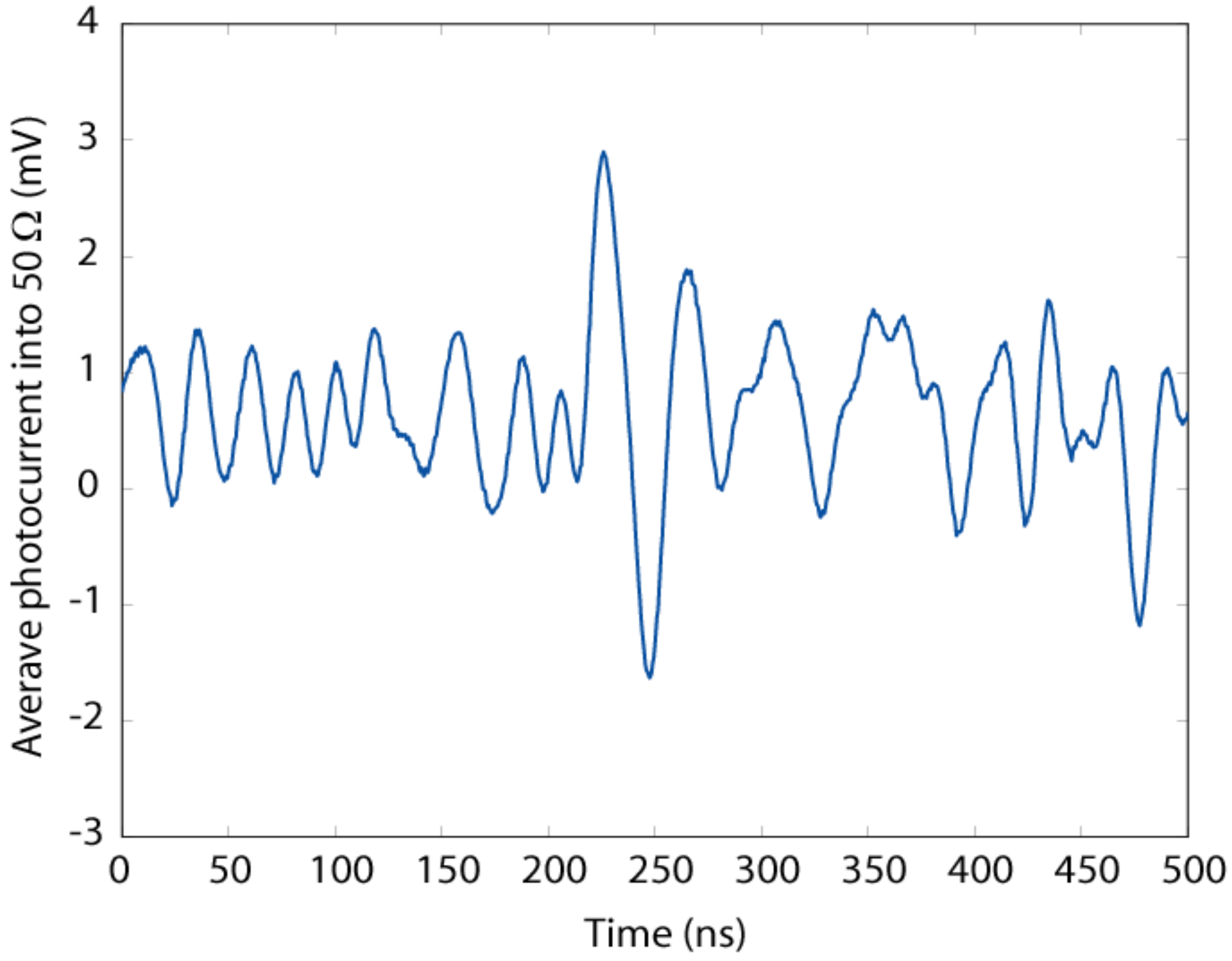
Solo se promedia la fotocorriente si se detecta una fluctuación (fotón)

Fotocorriente condicional sin átomos en la cavidad.

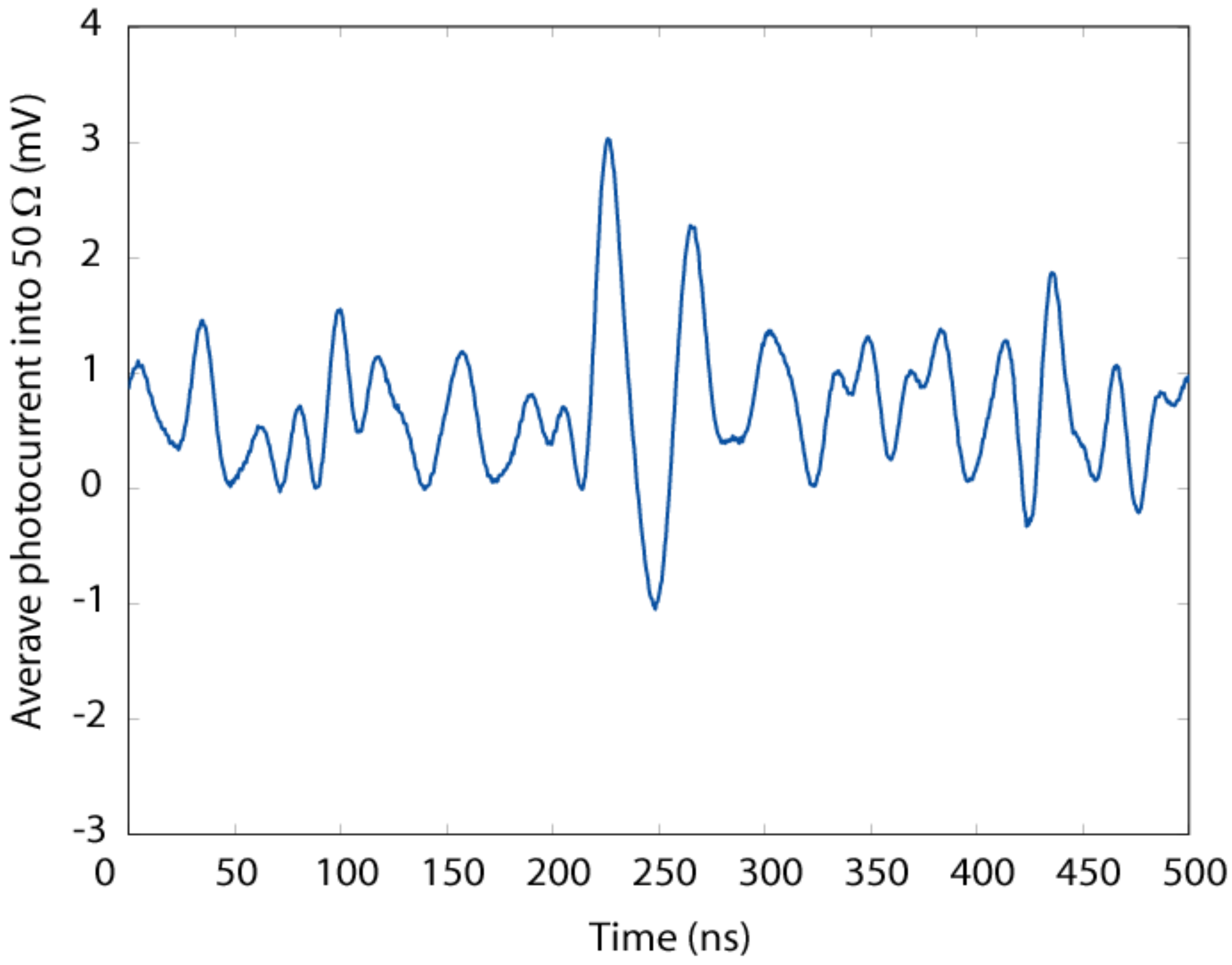




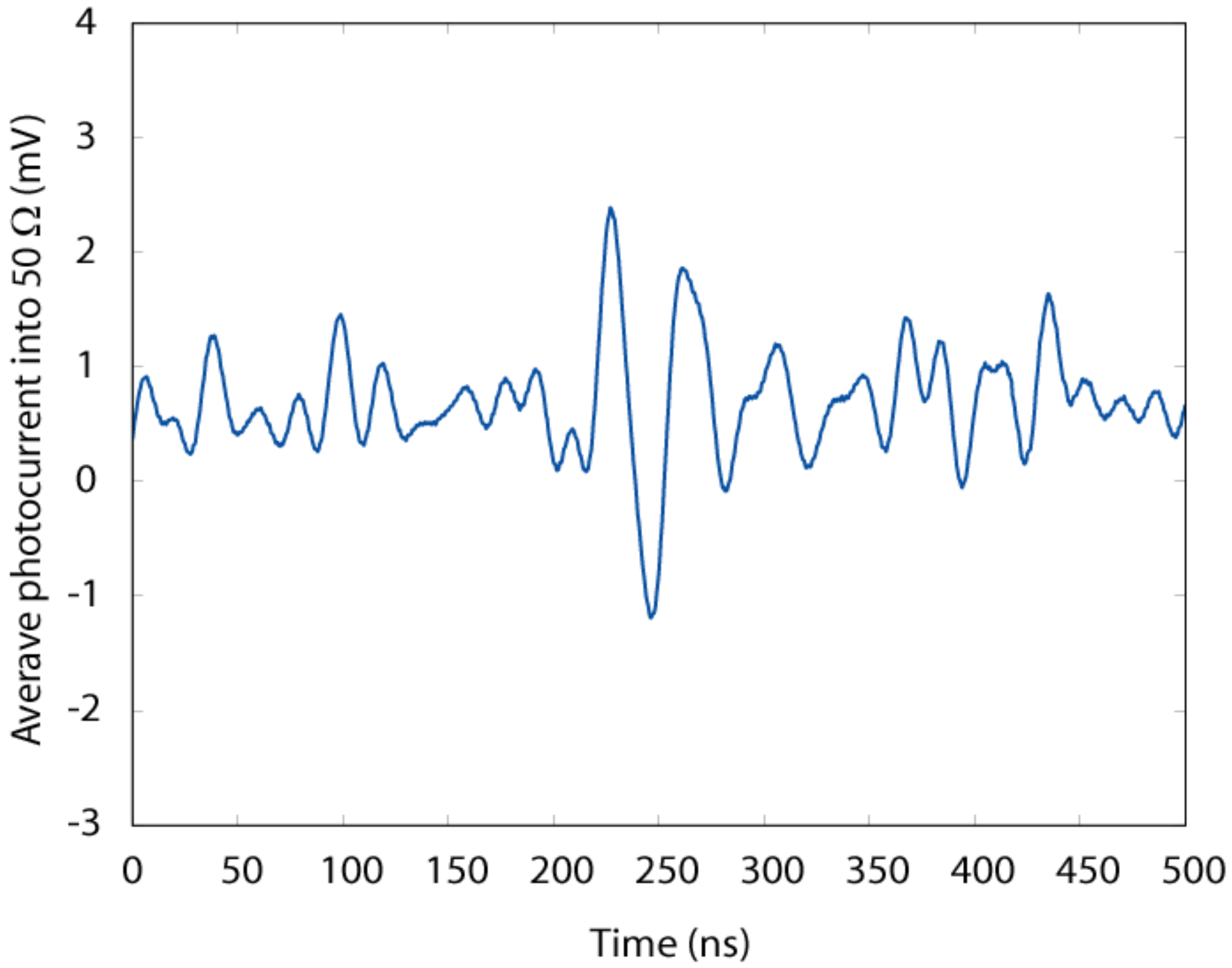
Después de 1 promedio, pp~200 mV



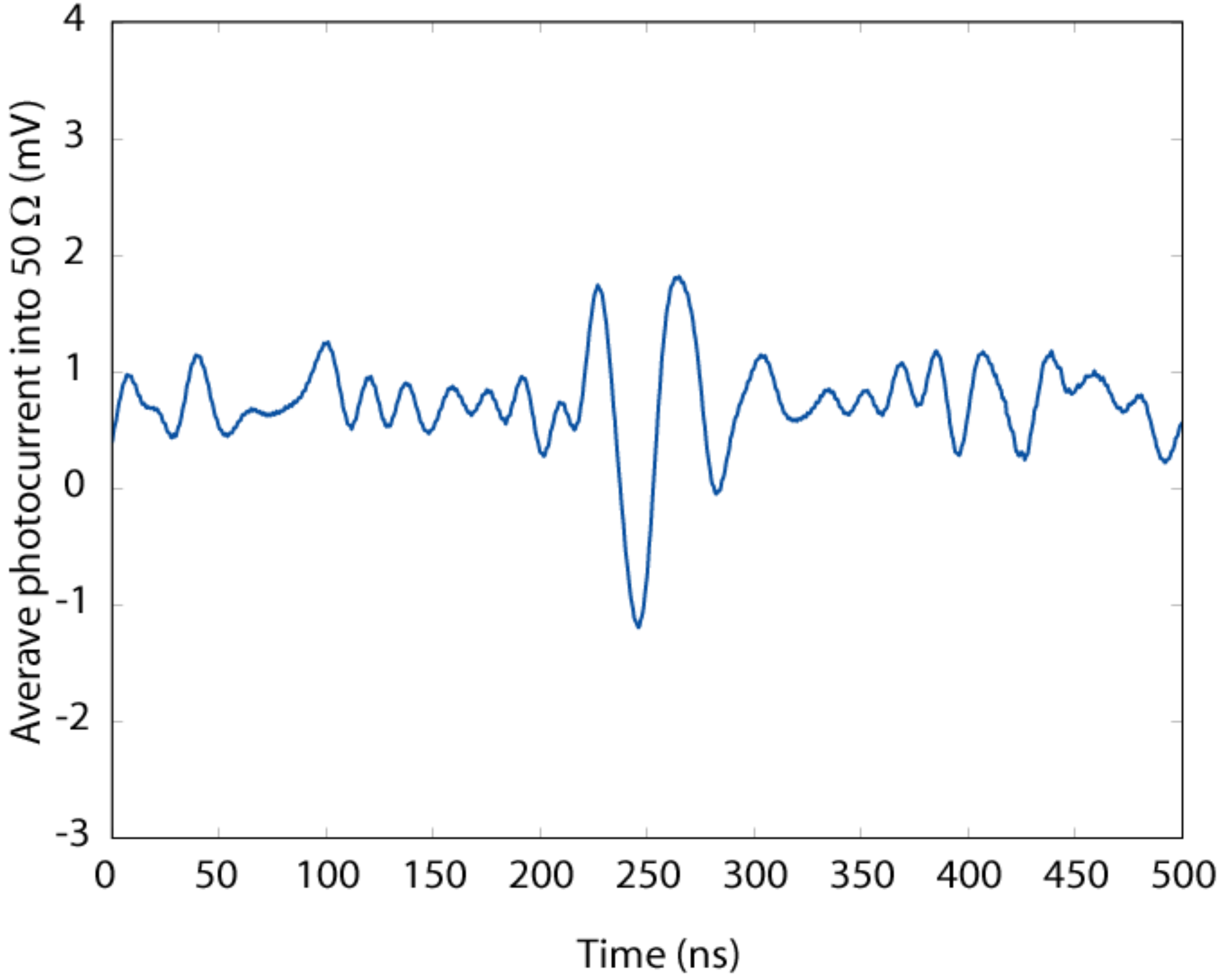
Después de 6.000 promedios



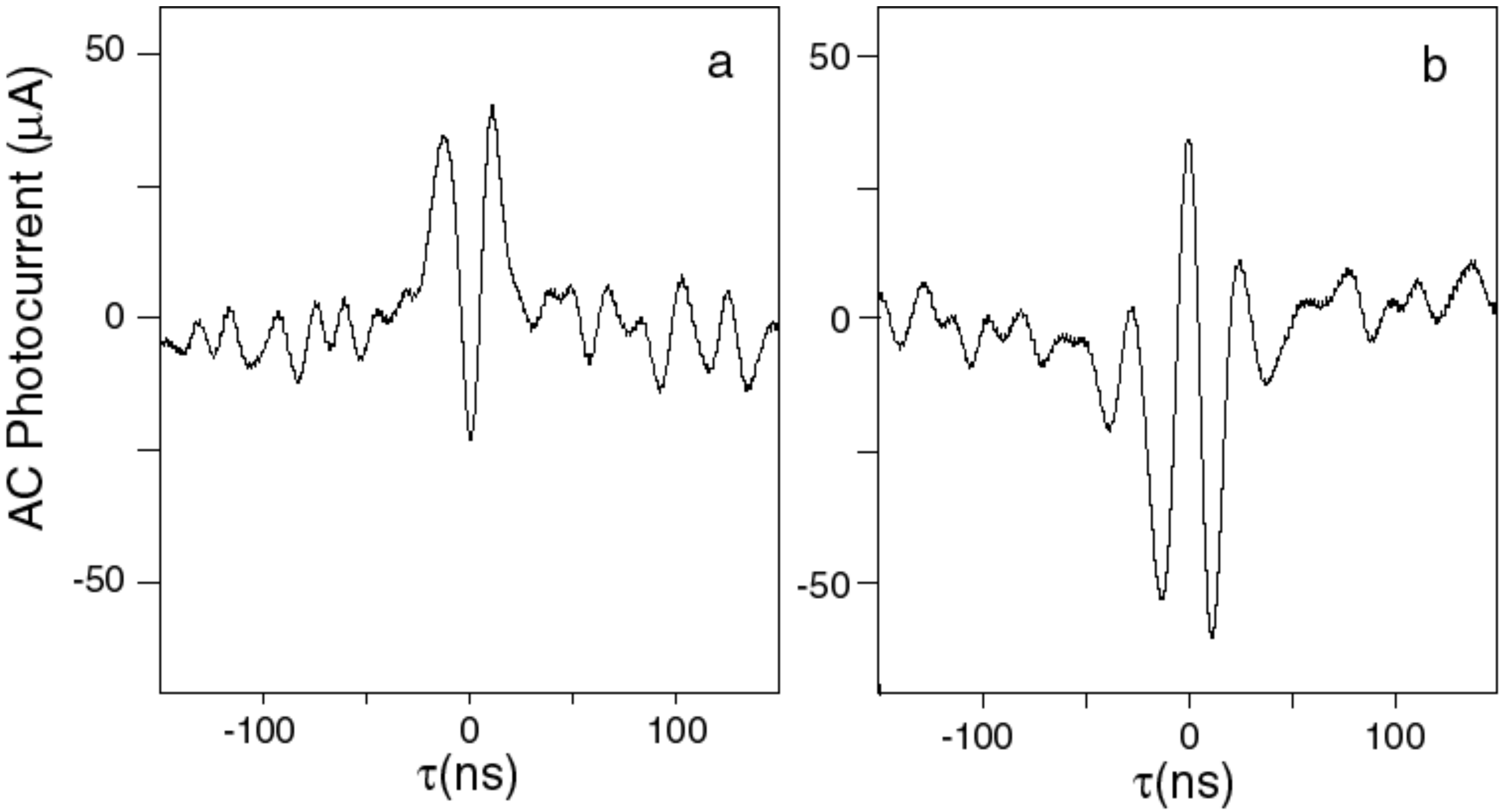
Después de 10.000 promedios



Después de 30.000 promedios

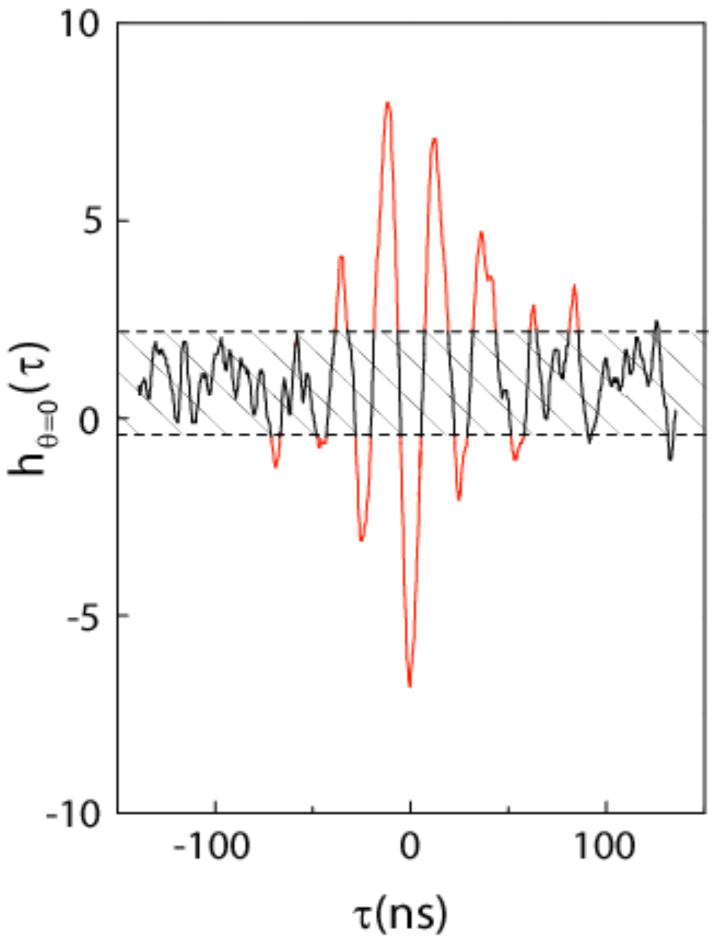


Después de 65,000 promedios, ruido pp ~ 1mV



Cambio la fase del Mach-Zehnder por 146º

Simulaciones Monte Carlo para excitación débil:



en negro la región clásicamente permitida

Haz atómico N=11

Esta es la evolución condicional del campo de una fracción de un fotón [B(t)] a partir de la función de correlación.

$g^{(3/2)}(\tau) = h_\theta(\tau) = \langle E(\tau)\rangle_c / \langle E \rangle$

El campo condicional preparado por el clic es:

$A(t)|0\rangle + B(t)|1\rangle$ con $A(t) \approx 1$ y $B(t) << 1$

¡Medimos el campo de una fracción de fotón!

Las fluctuaciones son muy importantes.

Notas:

El DC que se ve en el promedio del ruido lo entendemos.

Si uno promedia lo suficente la señal persiste y el ruido se reduce

Si tienen dudas al respecto contáctenme.

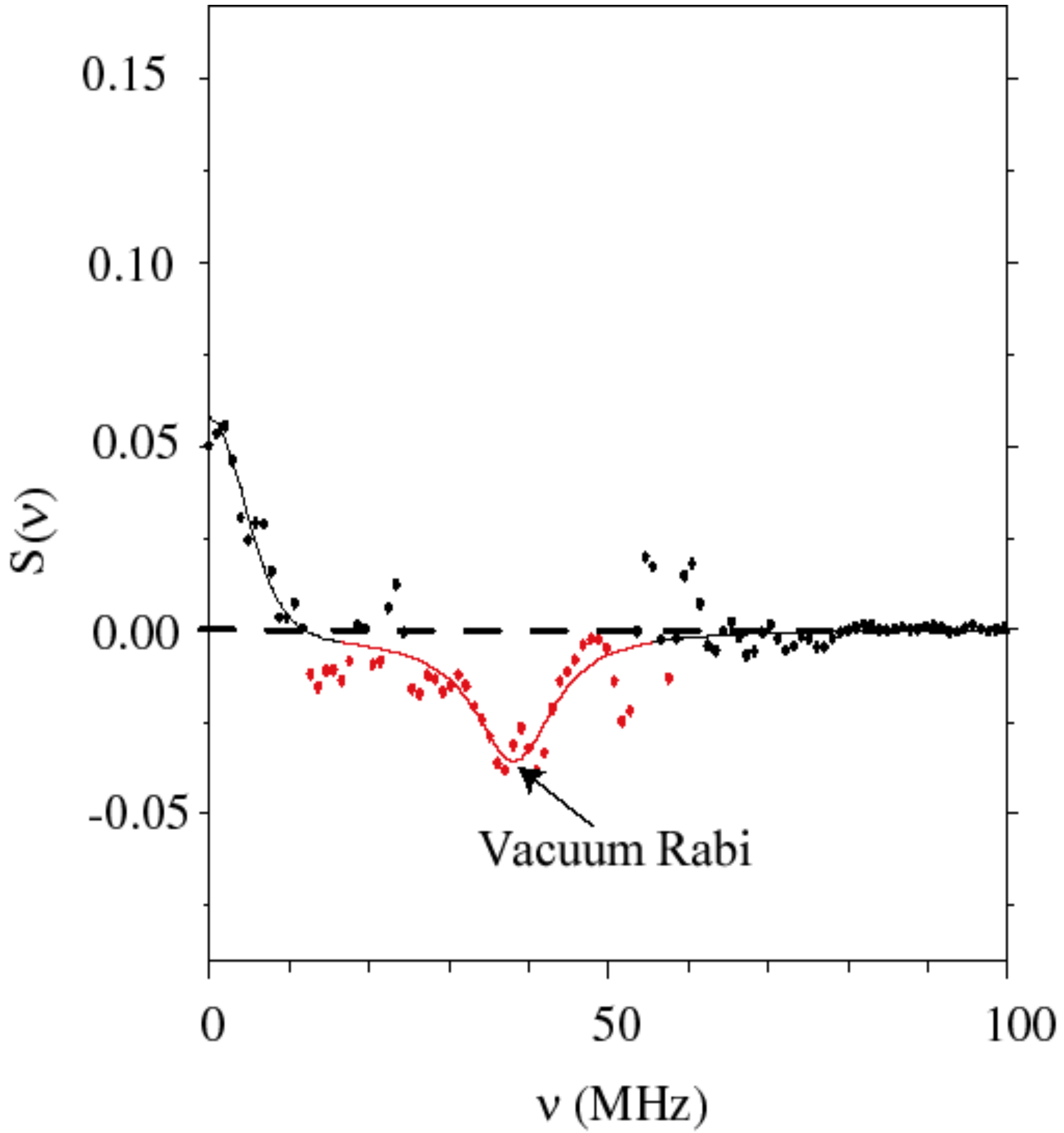
Por cierto la transformada de Fourier de la correlación produce el espectro de Squeezing

Las fluctuaciones del campo electromagnético se miden por el espectro de compresión. Mira el espectro de ruido de la fotocorriente.

$$S(\nu,0^\circ) = 4F\int_0^\infty \cos(2\pi\nu\tau)\,[g^{(3/2)}(\tau)\text{-}1]\; d\tau$$

F es el flujo de fotones en el correlacionador.

Espectro de squeezing procedente de la T. F. de $g^{(3/2)}(\tau)=h_0(\tau)$



Datos experimentales

J. E. Simsarian, L. A. Orozco, G. D. Sprouse, W. Z. Zhao, "Lifetime Measurement of the 7p Levels of Francium," Phys. Rev. A **57,** 2448 (1998).

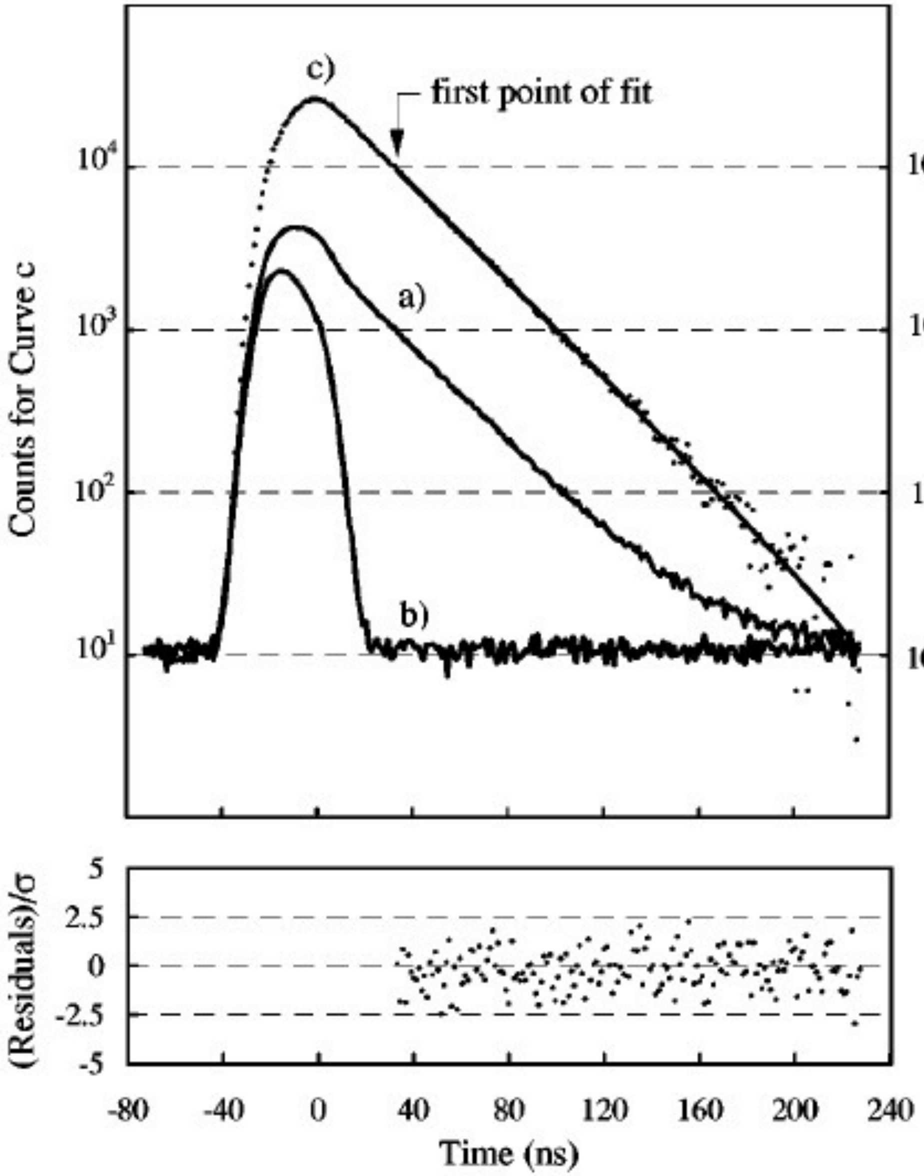


FIG. 6. Decay curves of Fr $7P_{1/2}$ level. Curve (a) is the raw data with Fr in the trap and curve (b) is the background. Curve (c) is the subtraction of a minus (b) and the straight line is a pure exponential fit to (c). The bottom shows the residuals of the fit divided by the statistical uncertainty of each point. The reduced $\chi^2 = 0.98$ for this measurement.

DOI=179

$$\sigma_{\chi^2_{red}} = \sqrt{\frac{2}{(n-1)}}$$

$$\chi^2_{red} = 0.98 \pm 0.11$$

TABLE I. Error budget for the lifetimes of the D_2 and D_1 lines of Fr in percentage.

Error	Fr $P_{3/2}$(%)	Fr $P_{1/2}$(%)
Systematic		
TAC-MCA nonlinearity	±0.03	±0.03
Time calibration	±0.04	±0.04
Truncation error	±0.39	±0.19
Zeeman quantum beat	±0.04	±0.00
Other	±0.23	±0.25
Total systematic	±0.46	±0.32
Statistical	±0.24	±0.18
Sum in quadrature	±0.52	±0.37

PHYSICAL REVIEW A **70**, 042504 (2004)

Lifetimes of the 9*s* and 8*p* levels of atomic francium

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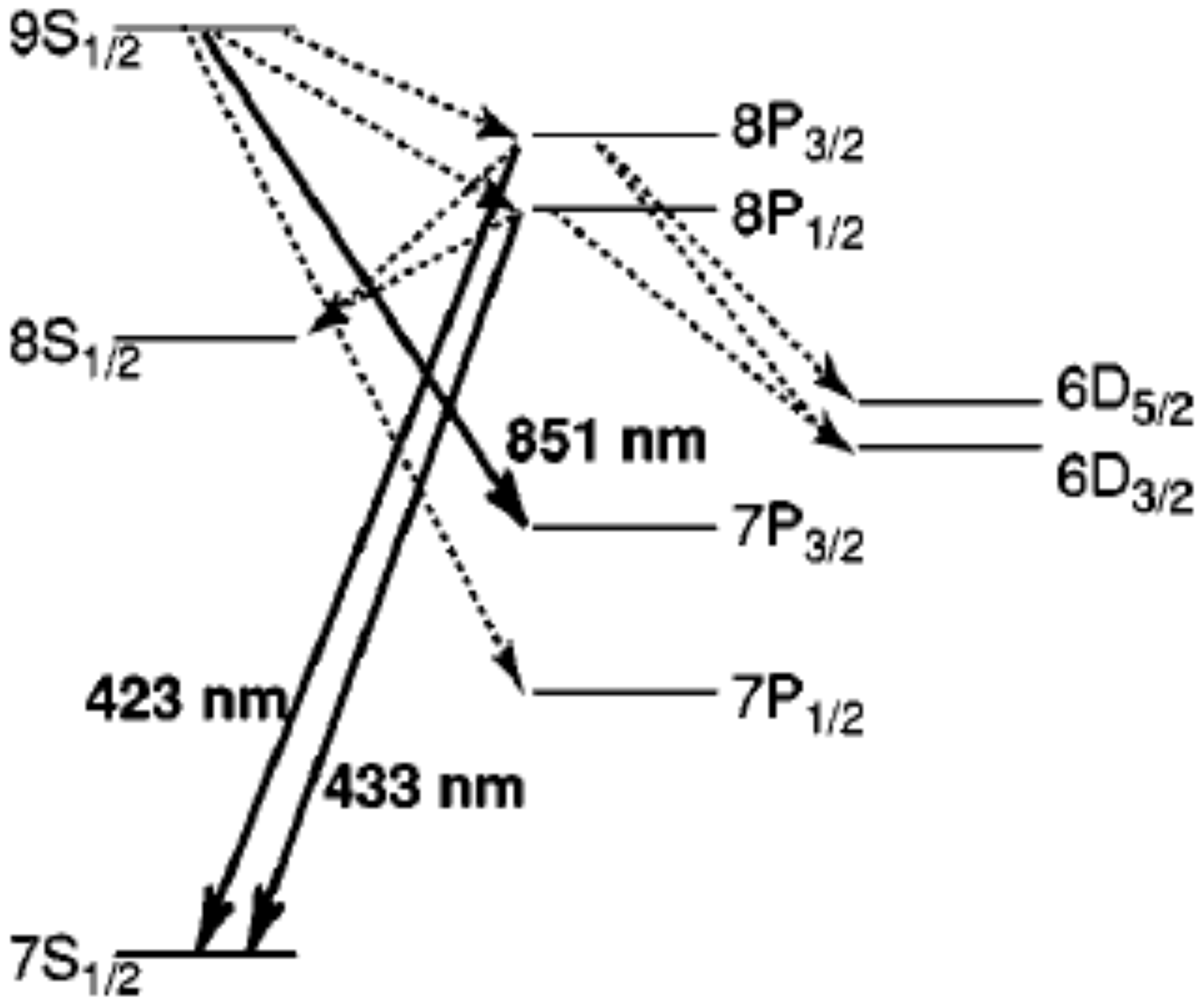
(Received 10 May 2004; published 20 October 2004)

We use time-correlated single-photon counting techniques on a sample of ^{210}Fr atoms confined and cooled in a magneto-optical trap to measure the lifetimes of the $9S_{1/2}$, $8P_{3/2}$, and $8P_{1/2}$ excited levels. We populate the $9S_{1/2}$ level by two-photon resonant excitation through the $7P_{1/2}$ level. The direct measurement of the $9S_{1/2}$ decay through the $7P_{3/2}$ level at 851 nm gives a lifetime of 107.53 ± 0.90 ns. We observe the decay of the $9S_{1/2}$ level through the $8P_{3/2}$ level at 423 nm and the $8P_{1/2}$ level at 433 nm down to the $7S_{1/2}$ ground level, and indirectly determine the lifetimes of these to be 83.5 ± 1.5 ns and 149.3 ± 3.5 ns, respectively.



9S1/2
8P3/2
8P1/2
8S1/2
6D5/2
6D3/2
851 nm
7P3/2
7P1/2
423 nm
433 nm
7S1/2

(a) *TAC and MCA nonlinearity*. We calibrate the timing of the $8P_{3/2}$ pulse detection apparatus in the same manner as the for $9S_{1/2}$ measurement. While the technique is the same, the apparatus is physically distinct, and consequently shows some difference in performance. We measure the linear time calibration of the TAC and MCA with an uncertainty of $\pm 0.02\%$. The time scale has a nonlinearity, which adds an uncertainty of $\pm 0.13\%$ to the lifetime of the fitted data. We find that the height scale of the TAC and MCA taken together varies by roughly 1% over 1000 channels. This nonuniformity in the height scale influences the fitted lifetime by less than 0.13%.

(b) *Pulse pileup correction*. We keep this correction small by maintaining count rates of about one event every 50 cycles. The correction to the data provided by Eq. (2) affects the fitted lifetime of our data by less than 0.07%.

(c) *Truncation error*. We find that varying the start and end points of the data set does not lead to statistically significant changes in the fitted lifetime.

(d) *Quantum beats*. We populate the energy eigenstates of the $8p$ levels incoherently from the decay of the $9s$ level through the spontaneous emission of a photon. The loss of coherence between energy eigenstates during spontaneous emission eliminates the possibility of quantum beats in the decay of the $8p$ levels. The error due to variations in the filling rate of the $8P_{3/2}$ level from quantum beats in the decay of the $9s$ level are automatically included in the Bayesian error (see below) due to uncertainty in the $9s$ lifetime.

(e) *Contamination shift*. We find a contamination shift of $\delta\tau_2 = -0.10 \pm 0.04$ ns. The shift is much smaller than the statistical error on τ_2.

(f) *Bayesian error*. Since we do not extract the $9s$ lifetime, τ_1, from fits of the $8p$ data, the uncertainty in τ_1 affects the precision with which τ_2 can be extracted in a fit of the data. This source of uncertainty is Bayesian, since it is conditioned on our knowledge of τ_1. τ_1 and τ_2 are not independent variables. The probability that the $8P_{3/2}$ lifetime is given by τ_2' is

$$P(\tau_2') = \int P(\tau_1', \tau_2') d\tau_1' = \int P(\tau_2'|\tau_1') P(\tau_1') d\tau_1'$$

$$= \frac{1}{\sqrt{2\pi}\sigma_1} \int P(\tau_2'|\tau_1') \exp\left(-\frac{1}{2} \frac{(\tau_1' - \tau_1)^2}{\sigma_1^2} \right) d\tau_1', \tag{13}$$

$$P(\tau_2'|\tau_1') = \frac{1}{\sqrt{2\pi}\sigma_2} \exp\left(-\frac{1}{2}\frac{[\tau_2' - \tau_2(\tau_1')]^2}{\sigma_2^2}\right),$$

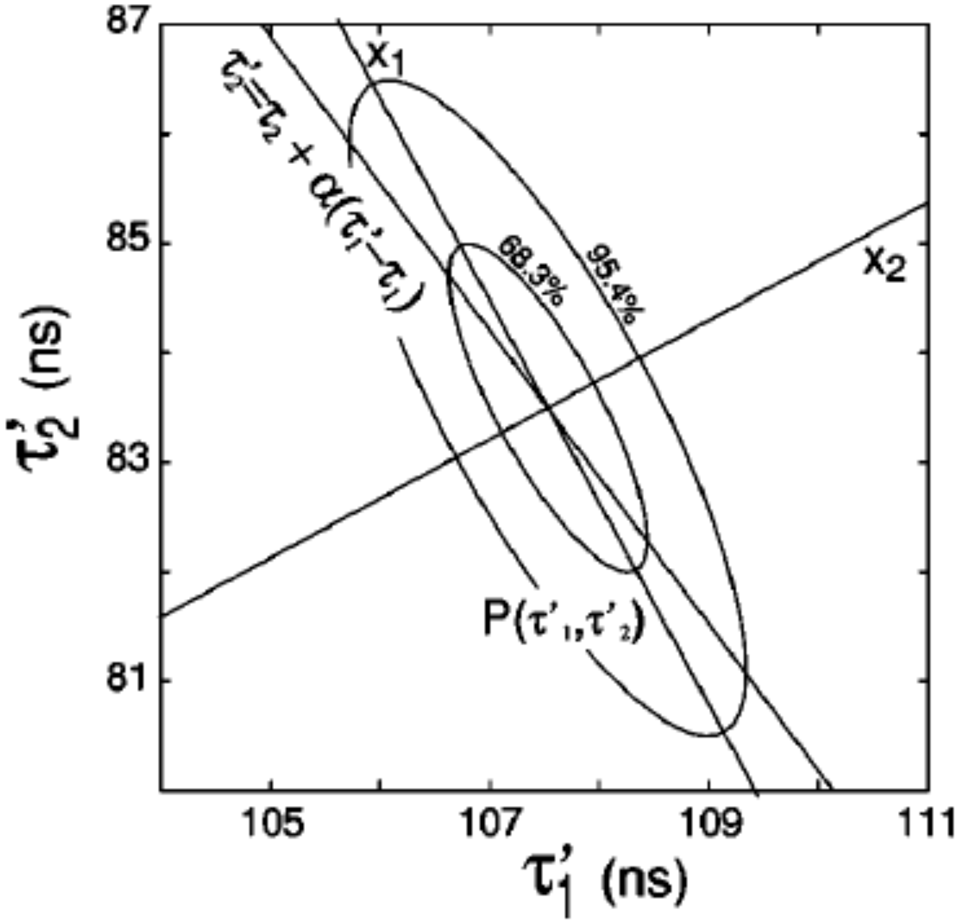


FIG. 11. Probability distribution plot of the 9s and 8$P_{3/2}$ lifetimes, τ_1 and τ_2. The percentages indicate boundaries of the 68.3% and 95.4% confidence level regions for $P(\tau_1', \tau_2')$. Note that the principal axes of the Gaussian distribution do not coincide with the linear relation for $\tau_2'(\tau_1')$ of Eq. (14).

Un ejemplo de cómo dar incertidumbres basado en diferentes mediciones

Survey of Hyperfine Structure Measurements in Alkali Atoms

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For each atomic species, we mention in the text the states where a very high precision is obtained or where a disagreement between the measured values exists. When several (n) measurements are associated with a single state, the tables include a weighted average, w.a., representing a reference for further work. We follow the procedure of the Particle Data Group in Zyla *et al.* (2020) in the Introduction, Sec. 5.2.2, *Unconstrained averaging*, to find the weighted error (w.e.). We calculate it first based on the n individual errors (e_l), w.e. = $(1/\sum 1/e_l{}^2)^{1/2}$. We also calculate the reduced χ-squared (χ^2_{red}) with $n-1$ degrees of freedom to test the size of the w.e. If (χ^2_{red}) is greater than unity by more than one standard deviation $(2/(n-1))^{1/2}$, then we increase the w.e. of the w.a. by the factor $(\chi^2_{\text{red}})^{1/2}$ so that the weighted enhanced error (w.e.e.) is w.e.e. = $(\chi^2_{\text{red}})^{1/2}\times$ w.e. We report in the table either the w.a. with its w.e. or the w.e.e., which we explicitly state. Such averaging is not performed when the precision of one measurement is greater than all the remaining ones, which is then denoted by "Recommended" in the table's last column. The last column contains a "See text" statement, if one or more values are not included into the w.a.

This supplementary information (SI) material to the Allegrini, Arimondo, and Orozco (2022) “Survey of hyperfine structure measurements in alkali atoms” presents a comparison of the weighted error (w.e.) and the weighted enhanced error (w.e.e.) reported in our main text with the values we get by using the cluster maximum likelihood estimator (CMLE) introduced by Rukhin (2009, 2019) .

TABLE V. ^{23}Na $3^2P_{3/2}$ state: A constant

χ^2 method		
$\chi^2_{\rm red}$	$\sigma_{\rm rcs}$	$(\chi^2_{\rm red})^{1/2}$
4.375	0.5	2.092
w.a.	w.e.	w.e.e.
18.532	0.003	0.006
CMLE method		
w.a.CMLE		w.e.CMLE
18.531		0.003
Δ		
0.167		

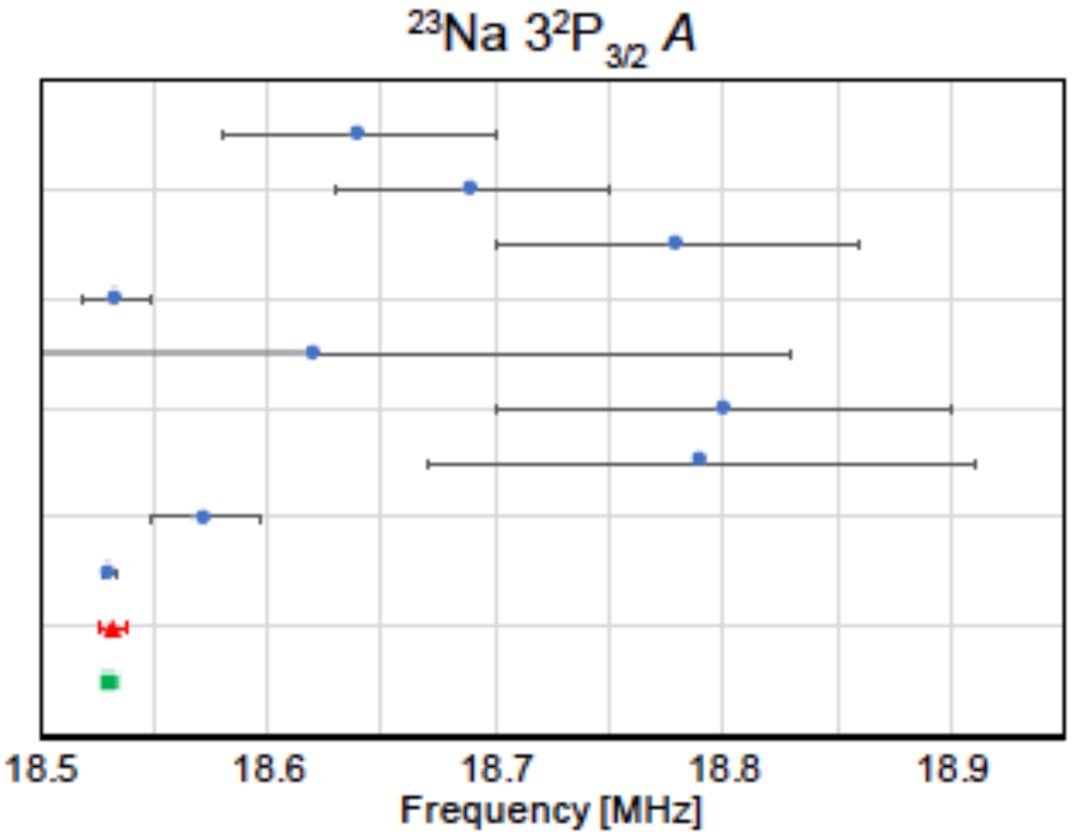

FIG. 5. ^{23}Na $3^2P_{3/2}$ state: A constant.

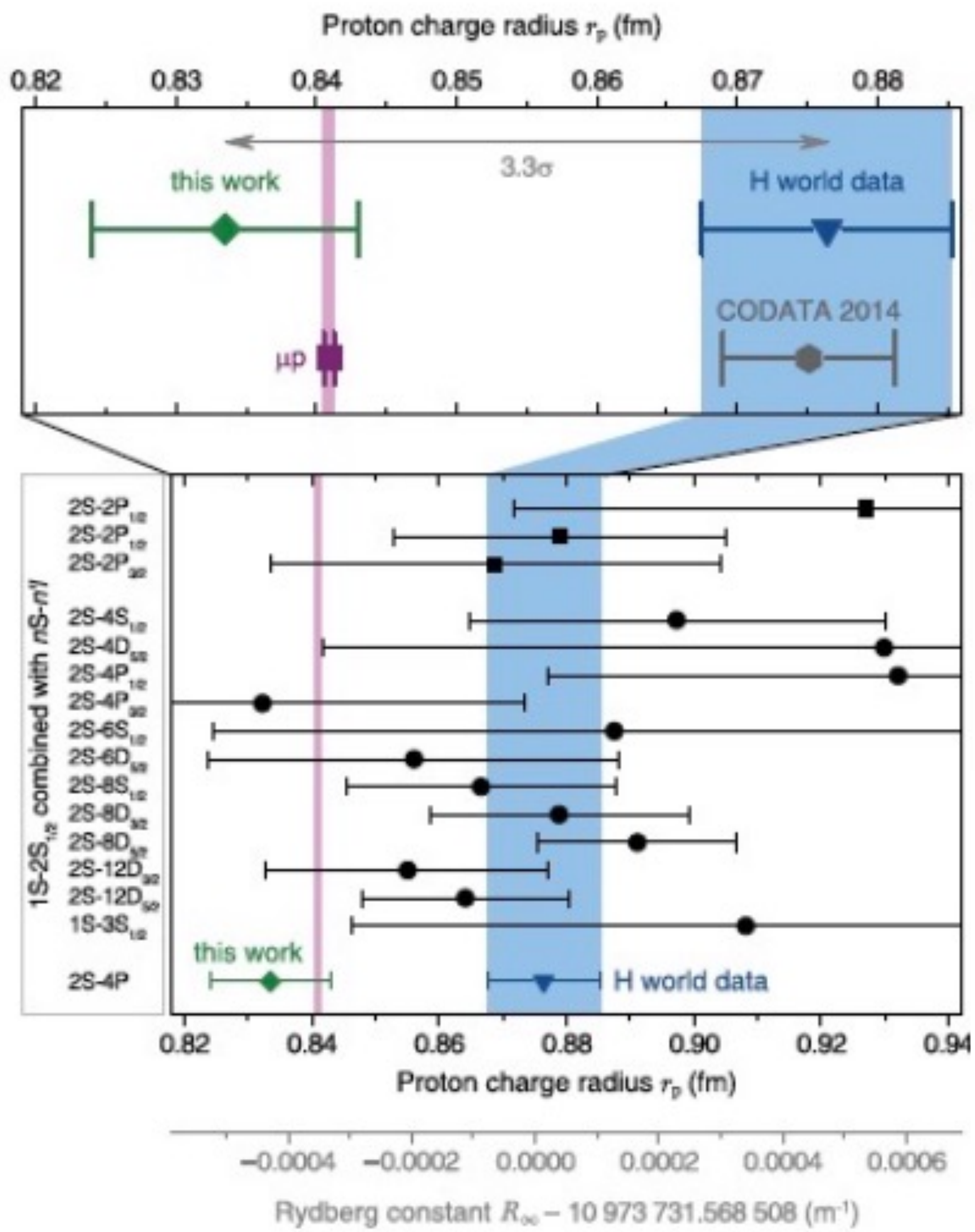

Fig. 1 Rydberg constant R_∞ and proton RMS charge radius r_p. Values of r_p derived from this work (green diamond) and spectroscopy of μp (μp; pink bar and violet square) agree. We find a discrepancy of 3.3 and 3.7 combined standard deviations with respect to the H spectroscopy world data (*12*) (blue bar and blue triangle) and the CODATA 2014 global adjustment of fundamental constants (*3*) (gray hexagon), respectively. The H world data consist of 15 individual measurements (black circles, optical measurements; black squares, microwave measurements). In addition to H data, the CODATA adjustment includes deuterium data (nine measurements) and elastic electron scattering data. An almost identical plot arises when showing R_∞ instead of r_p because of the strong correlation of these two parameters. This is indicated by the R_∞ axis shown at the bottom.

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