Top

Left side

TEST

Right side

bottom

An introduction to light-matter interaction, from cavity QED to waveguide QED 2

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Luis A. Orozco

www.jqi.umd.edu

UNIVERSITY OF MARYLAND 1856

PHYSICS FRONTIER CENTER

JOINT QUANTUM INSTITUTE

NIST

The presentation will be available at:

http://www.physics.umd.edu/rgroups/amo/orozco/results/2019/Results19.htm

Another definition of the Cooperativity

- Atomic decay rate γ
- Cavity decay rate κ
- Atom-cavity coupling rate g

$$C_1 = \frac{g^2}{\kappa\gamma} \quad C = NC_1$$

$$\dot{x} = \kappa\left(+2Cp + y - (i\Theta + 1)x\right)$$
$$\dot{p} = \gamma\left(-(1 + i\Delta)p + xD\right)/2$$
$$\dot{D} = \gamma\left(2(1 - D) - (x^*p + xp^*)\right)$$

$$\Theta = \frac{\omega_c - \omega_l}{\kappa}; \quad \Delta = \frac{\omega_a - \omega_l}{\gamma/2}$$

γ is the rate of spontaneous emission (energy decay)

κ is the rate of escape of the field

$\omega_{a,c,l}$ refer to atom, cavity, laser

Low intensity $x<<1$: with $D=0$, resonant $\Delta=0$ and $\Theta=0$ weakly driven.

Two coupled oscillators

$$\dot{x} = \kappa(-x + 2Cp + y)$$

$$\dot{p} = \gamma(-p - x)$$

Two coupled oscillators

Steady state

$$y = x - 2Cp$$

$$p = -x$$

$$y = x(1 + 2C)$$

$$\kappa >> \gamma \quad \dot{p} = -\gamma(1 + 2C)p - \gamma y$$

$$\gamma >> \kappa \quad \dot{x} = -\kappa(1 + 2C)x + \kappa y$$

Enhanced emission

Steady state with detuning and at all intensities:

$$y = x\left(1 + \frac{2C}{1+\Delta^2+|x|^2}\right) + ix\left(\theta - \frac{2C\Delta}{1+\Delta^2+|x|^2}\right)$$

Dispersive limit when Θ=0 and Δ >> 1 :

$$y = -ix\frac{2C\Delta}{1+\Delta^2+|x|^2}$$

Atoms in cavities

$$y = x\left(1 + \frac{2C}{1+\left|x^2\right|}\right)$$

The laser steady state:

- Do not drive the system (y=0)
- Invert the absorption to gain C goes to -G
- Then the steady state for low field

$$0 = x\left(1 - \frac{2G}{1+\left|x\right|^2}\right) = x - 2Gx(1+\left|x\right|^2) = x(1-2G+\left|x\right|^2)$$

$$x = 0, \quad \left|x\right|^2 = 2G-1$$

Transmission spectra at low intensity

Steady state with detuning and at all intensities:

$$y = x\left(1 + \frac{2C}{1+\Delta^2+|x|^2}\right) + ix\left(\theta - \frac{2C\Delta}{1+\Delta^2+|x|^2}\right)$$

The transmission spectrum:

$$\text{for } \omega_c = \omega_a \quad \Omega = (\gamma/2)\Delta = \kappa\Theta \quad \Omega_{V.R.} = g\sqrt{N} = \sqrt{C\kappa\gamma}$$

$$x = y\frac{\kappa(\gamma_\perp + i\Omega)}{(\kappa + i\Omega)(\gamma_\perp + i\Omega) + \Omega_{V.R.}^2/(1+\gamma_\perp^2|x|^2/(\gamma_\perp^2+\Omega^2))}$$

Two coupled oscillators

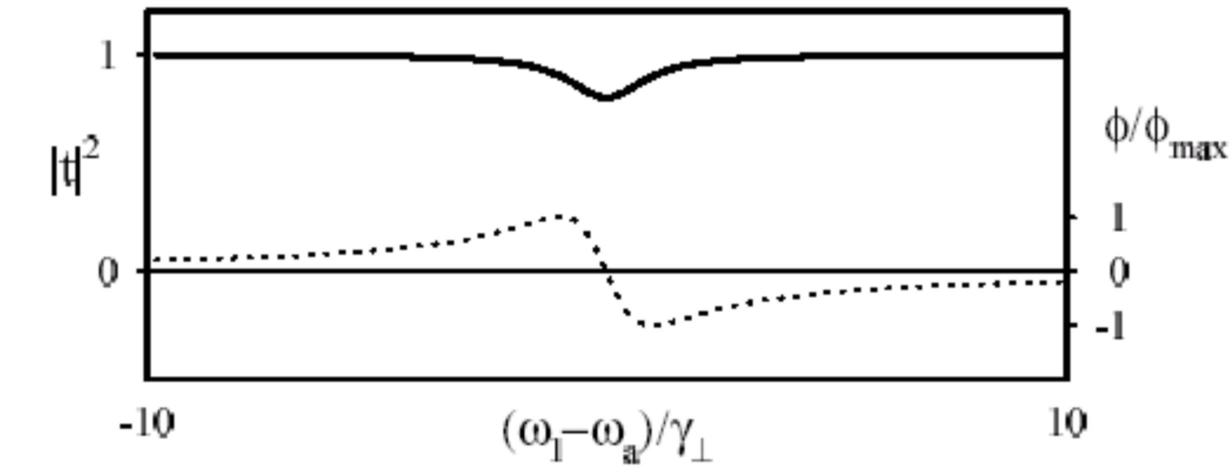
$$\frac{x}{y} = \frac{A}{i\Omega - \Omega_1} + \frac{B}{i\Omega - \Omega_2} \,, \qquad A = \kappa\frac{\gamma_\perp + \Omega_1}{\Omega_1 - \Omega_2} \,, \qquad B = \kappa\frac{\gamma_\perp + \Omega_2}{\Omega_2 - \Omega_1} \,,$$

Cavity mode and atomic polarization

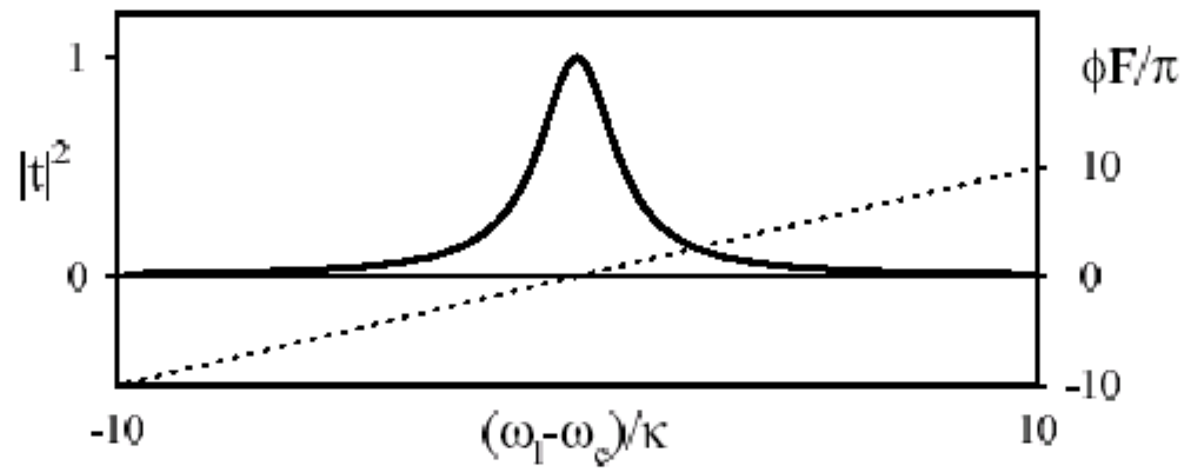
At low intensity and equal decay rates

$$\Omega_{1,2} = \pm\, \Omega_{VR}$$

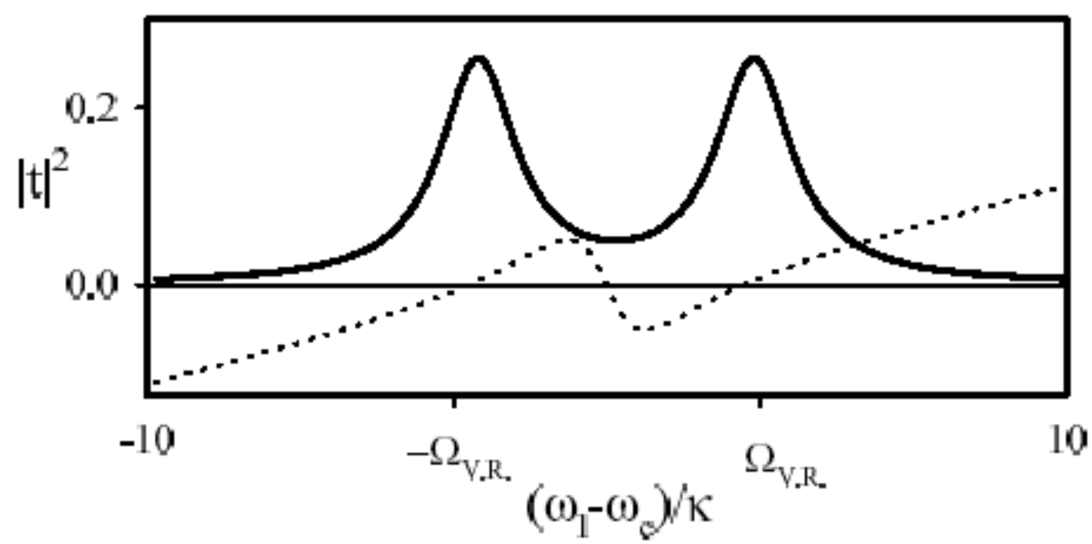
Atomic absorption

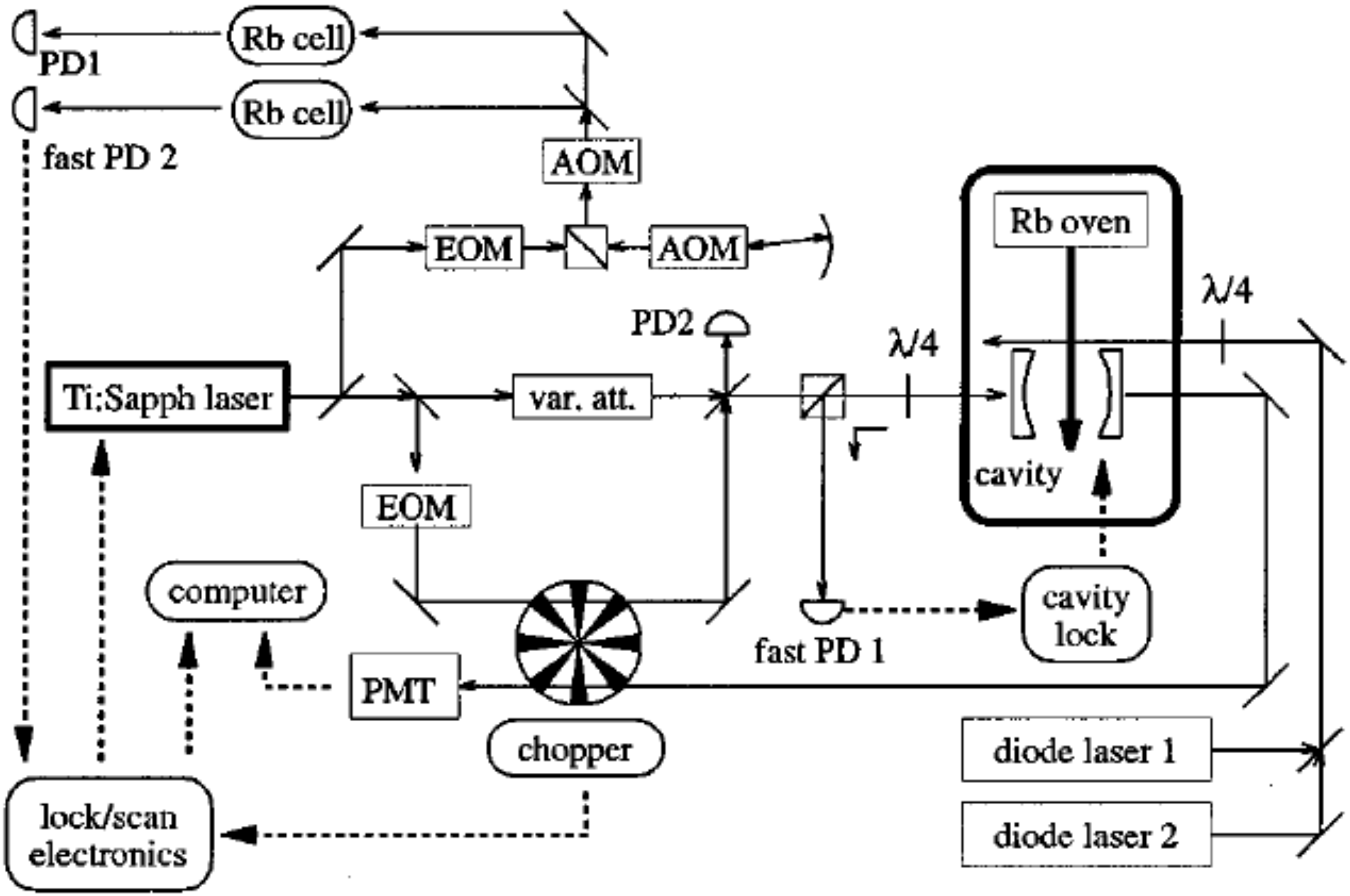


Fabry Perot

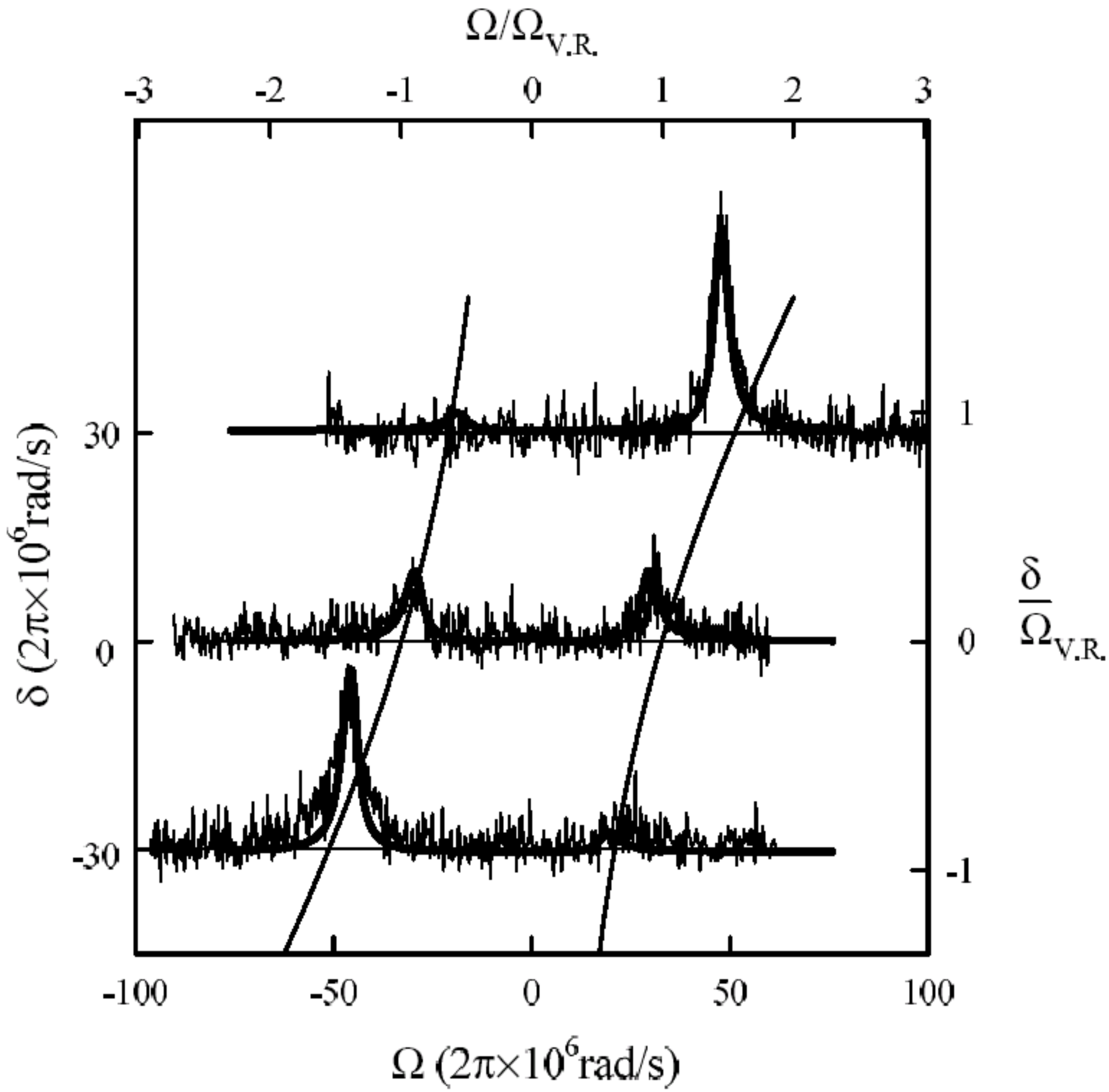


Coupled modes



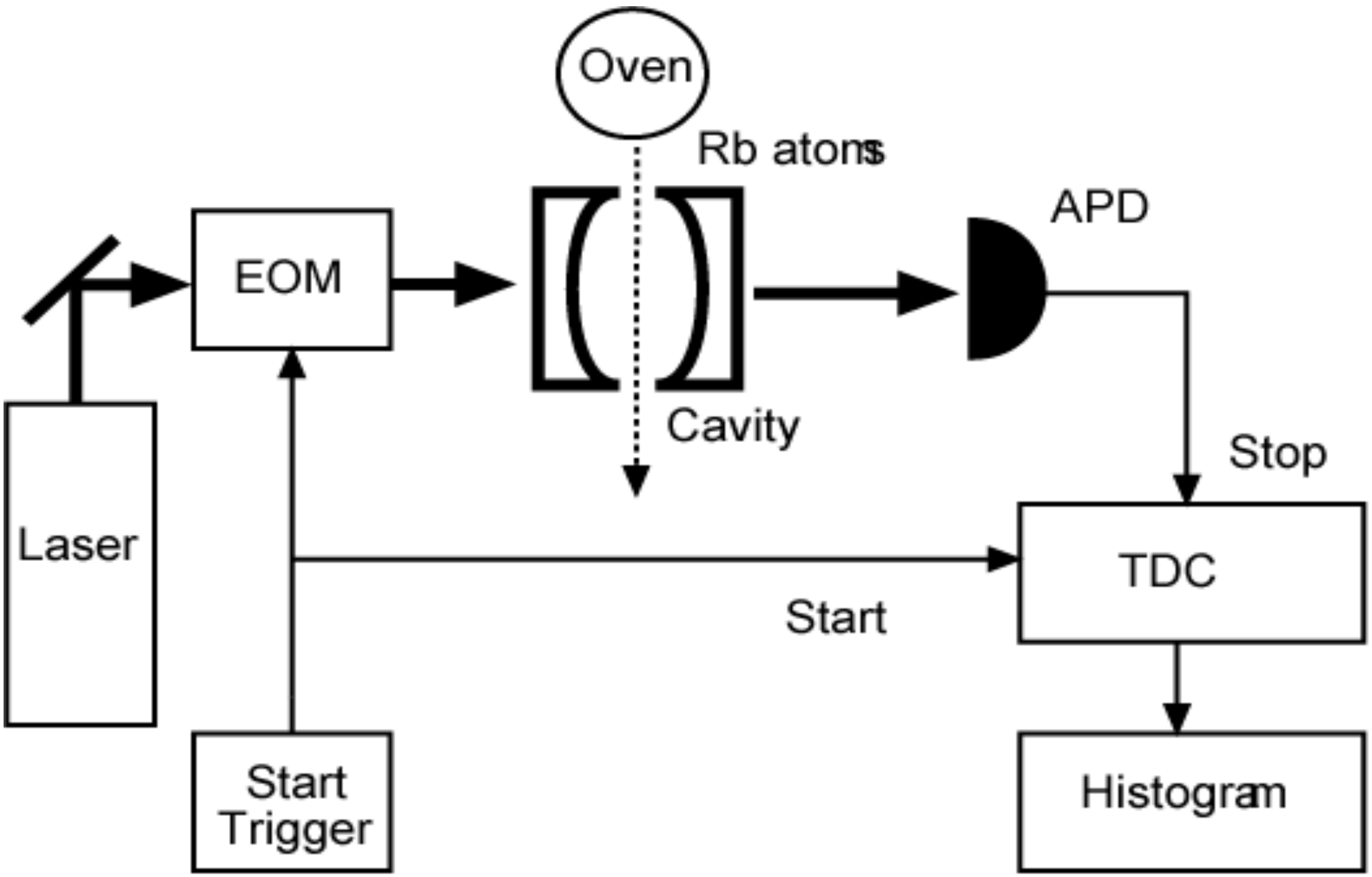

PD1
Rb cell
Rb cell
fast PD 2
AOM
EOM
AOM
Rb oven
λ/4
PD2
λ/4
Ti:Sapph laser
var. att.
cavity
EOM
computer
cavity lock
fast PD 1
PMT
chopper
diode laser 1
diode laser 2
lock/scan electronics

Transmission spectrum at low intensity for varying atomic detunings.

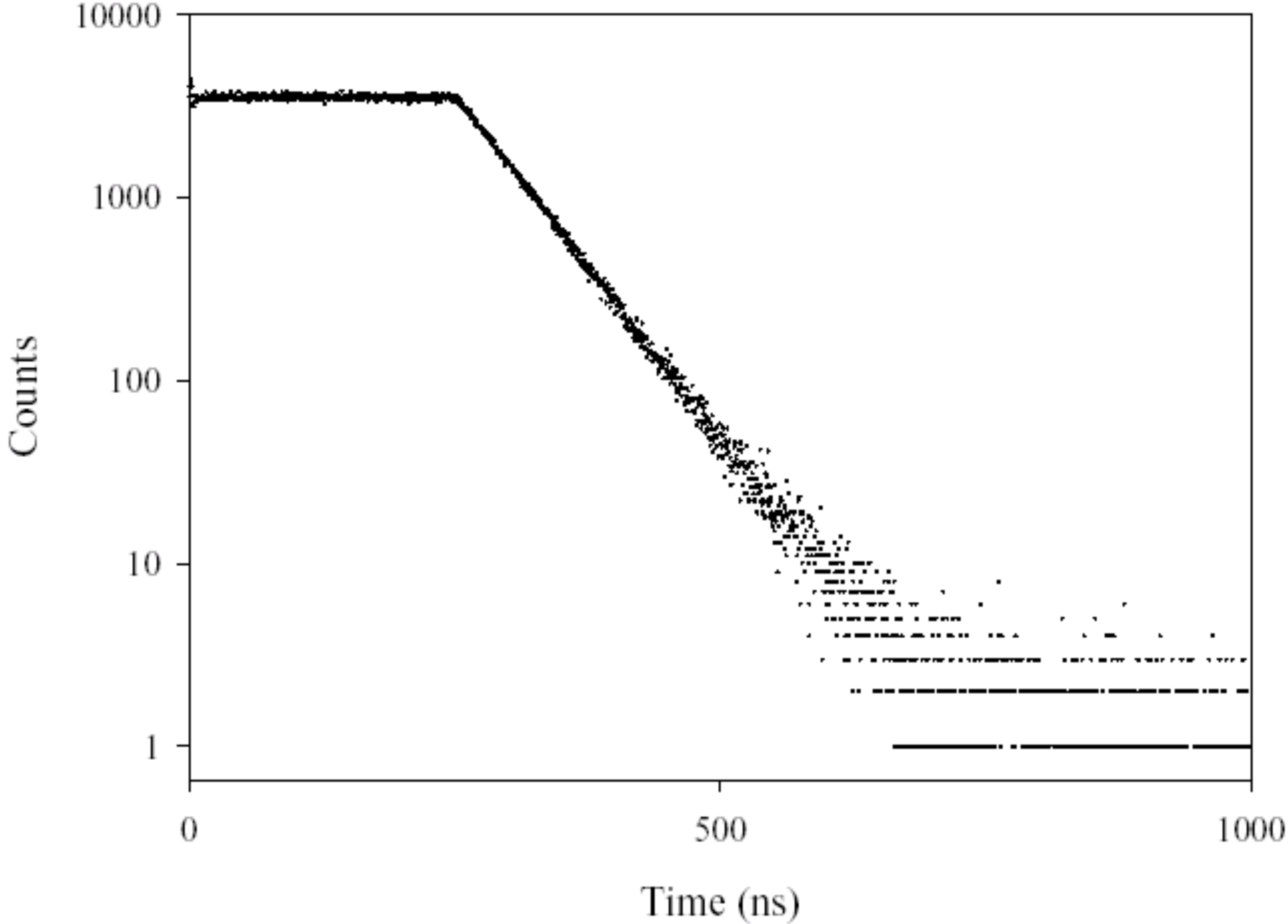


Time response to excitation

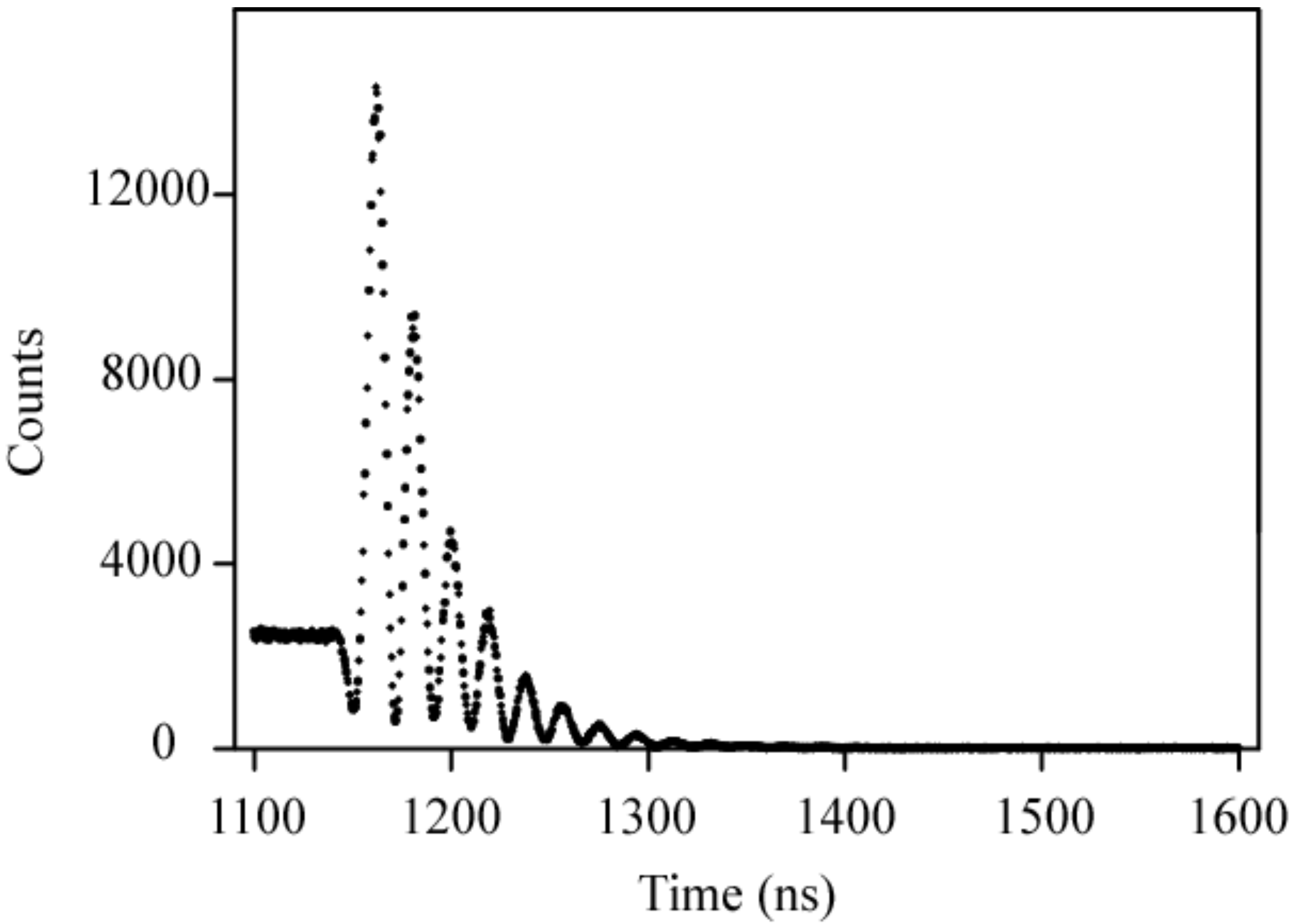
Study the system dynamics classically by providing a step function.



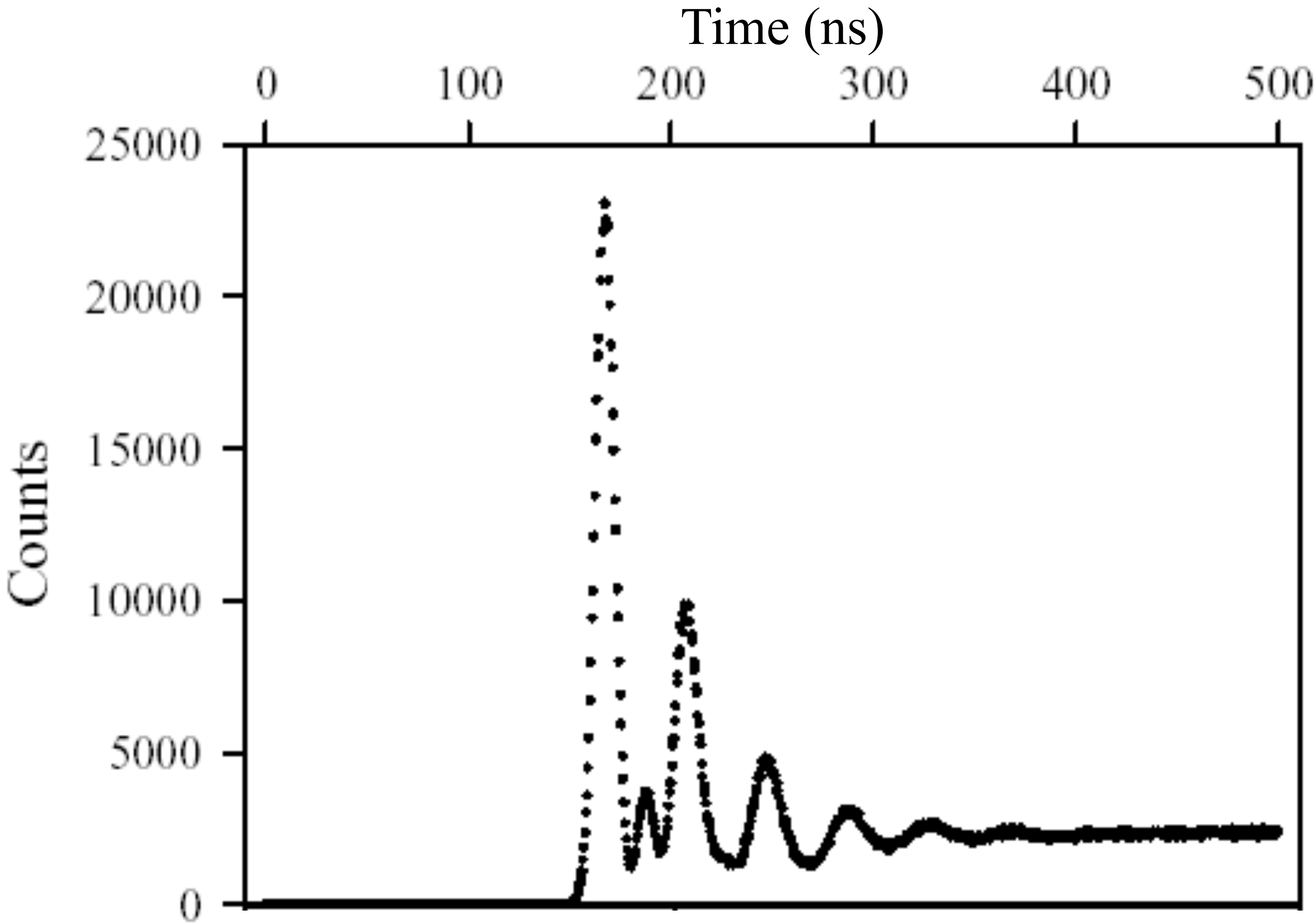
Decay of the empty cavity



Response to step down excitation



Response to step up excitation



Transmission spectra at arbitrary intensity

Two coupled oscillators

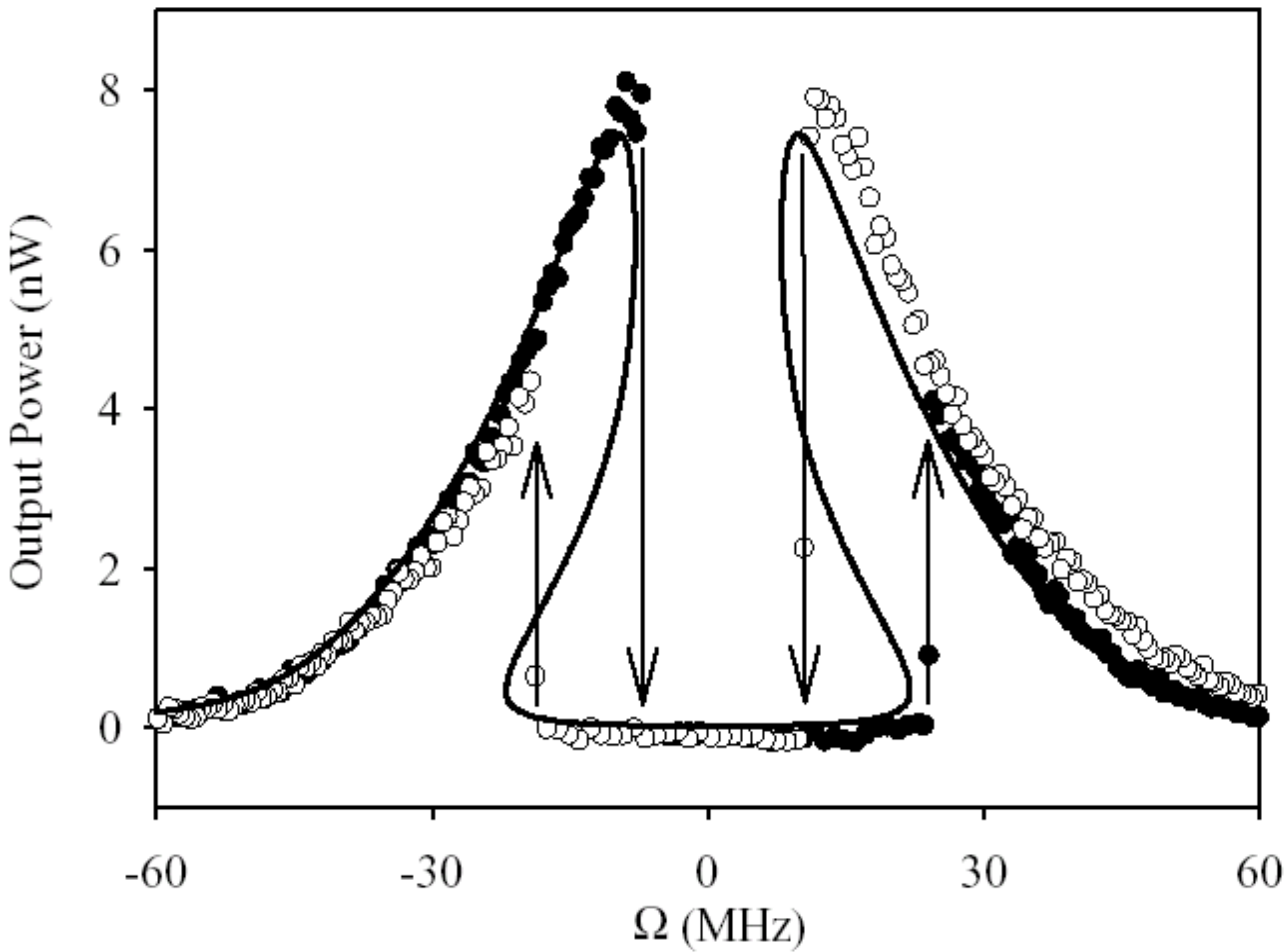
$$\frac{x}{y} = \frac{A}{i\Omega - \Omega_1} + \frac{B}{i\Omega - \Omega_2} \,,$$

$$A = \kappa \frac{\gamma_\perp + \Omega_1}{\Omega_1 - \Omega_2} \,,$$

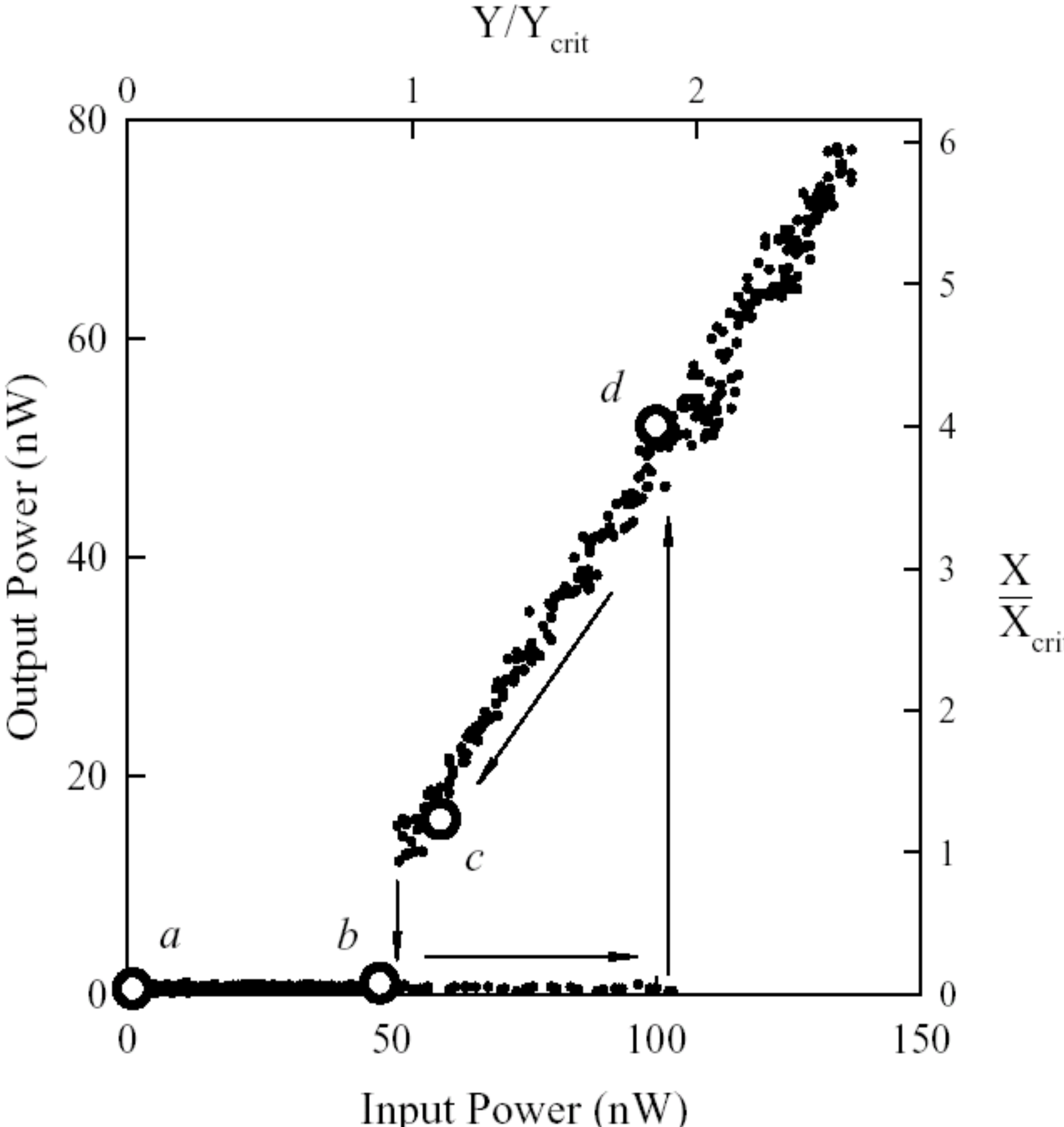
$$B = \kappa \frac{\gamma_\perp + \Omega_2}{\Omega_2 - \Omega_1} \,,$$

Cavity mode and atomic polarization

$$\Omega_{1,2} = -\frac{\kappa + \gamma_\perp}{2} \pm i \sqrt{-\left(\frac{\kappa - \gamma_\perp}{2}\right)^2 + \frac{\Omega^2_{V.R.}}{1 + \gamma_\perp^2 |x|^2 / (\gamma_\perp^2 + \Omega^2)}} \,.$$

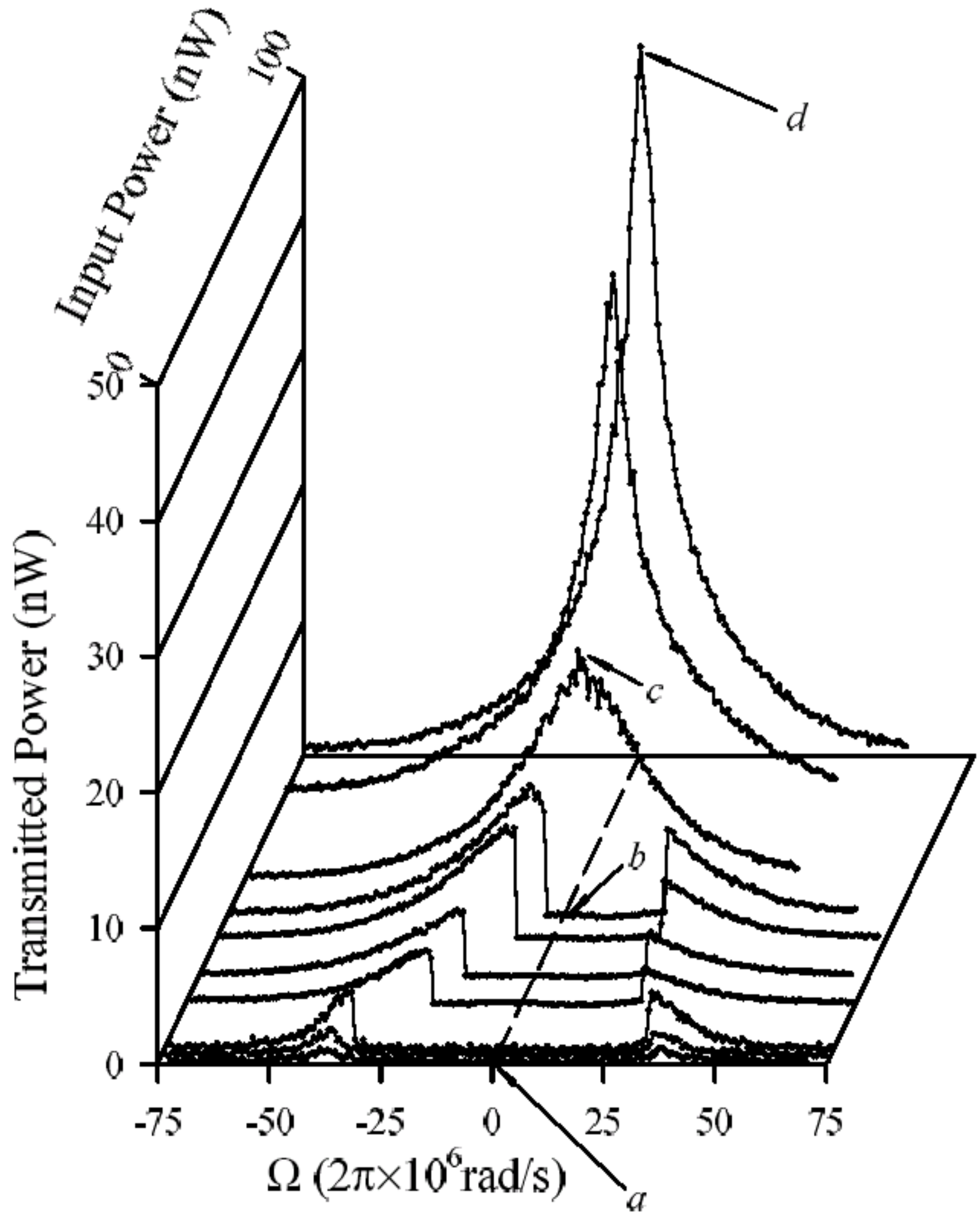


Hysteresis for a frequency scan of the light from the coupled atoms-cavity system.

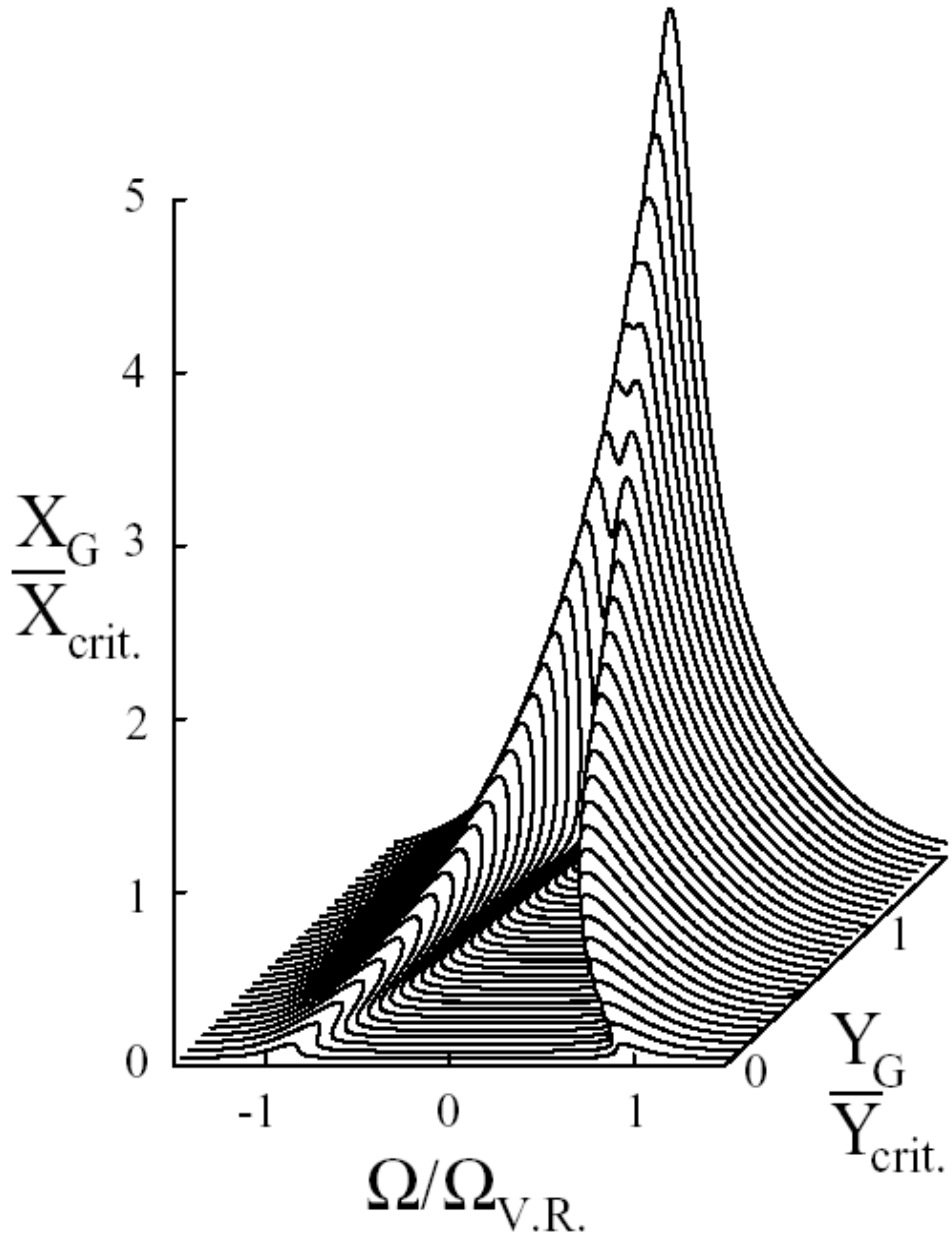


Y/Y$_{crit}$
0
1
2
80
60
40
20
0
Output Power (nW)
6
5
4
3
2
1
0
$\frac{X}{X_{crit}}$
a
b
c
d
0
50
100
150
Input Power (nW)

Transmission spectra for different intensities.



Theory



The quantum model

Quantum Hamiltonian for *N* atoms

$$\hat{H} = \hat{H}_1 + \hat{H}_1 + \hat{H}_2 + \hat{H}_3 + \hat{H}_4 + \hat{H}_5 \ ,$$

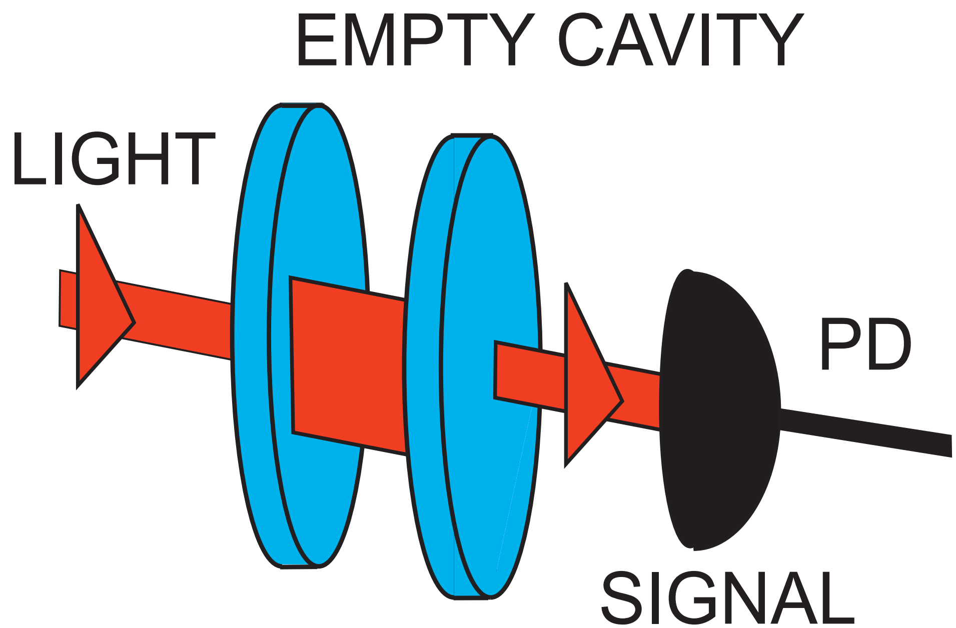
$$\hat{H}_1 = \hbar\omega_c \hat{a}^\dagger \hat{a} + \frac{1}{2}\hbar\omega_a \sum_{j=1}^{N} \hat{\sigma}_j^z \ ,$$ Free atoms free field

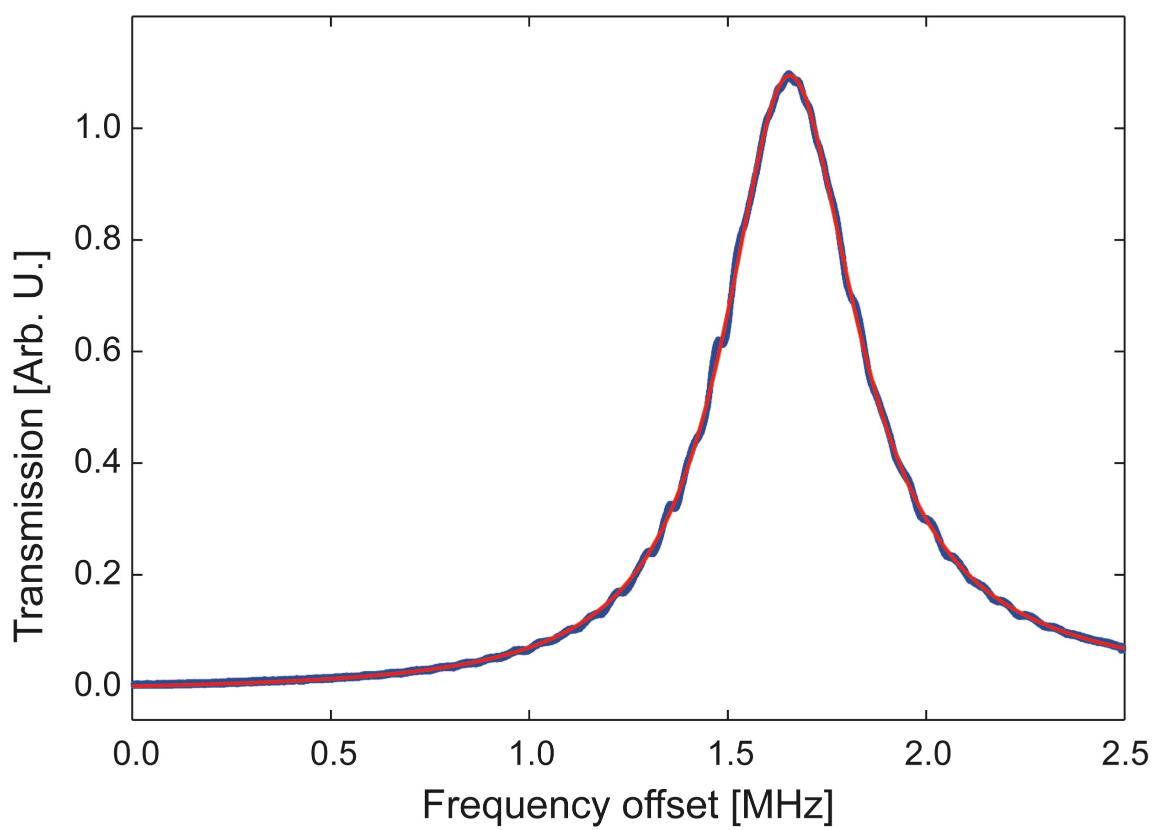
J.C $$\hat{H}_2 = i\hbar \sum_{j=1}^{N} g_j \left(\hat{a}^\dagger \hat{\sigma}_j^- e^{-i\vec{k}\cdot\vec{r}_j} - \hat{a}\hat{\sigma}_j^+ e^{i\vec{k}\cdot\vec{r}_j} \right)$$ Interaction

$$\hat{H}_3 = \sum_{j=1}^{N} \left(\hat{\Gamma}_A \hat{\sigma}_j^+ + \hat{\Gamma}_A^\dagger \hat{\sigma}_j^- \right) \ ,$$ Atomic decay

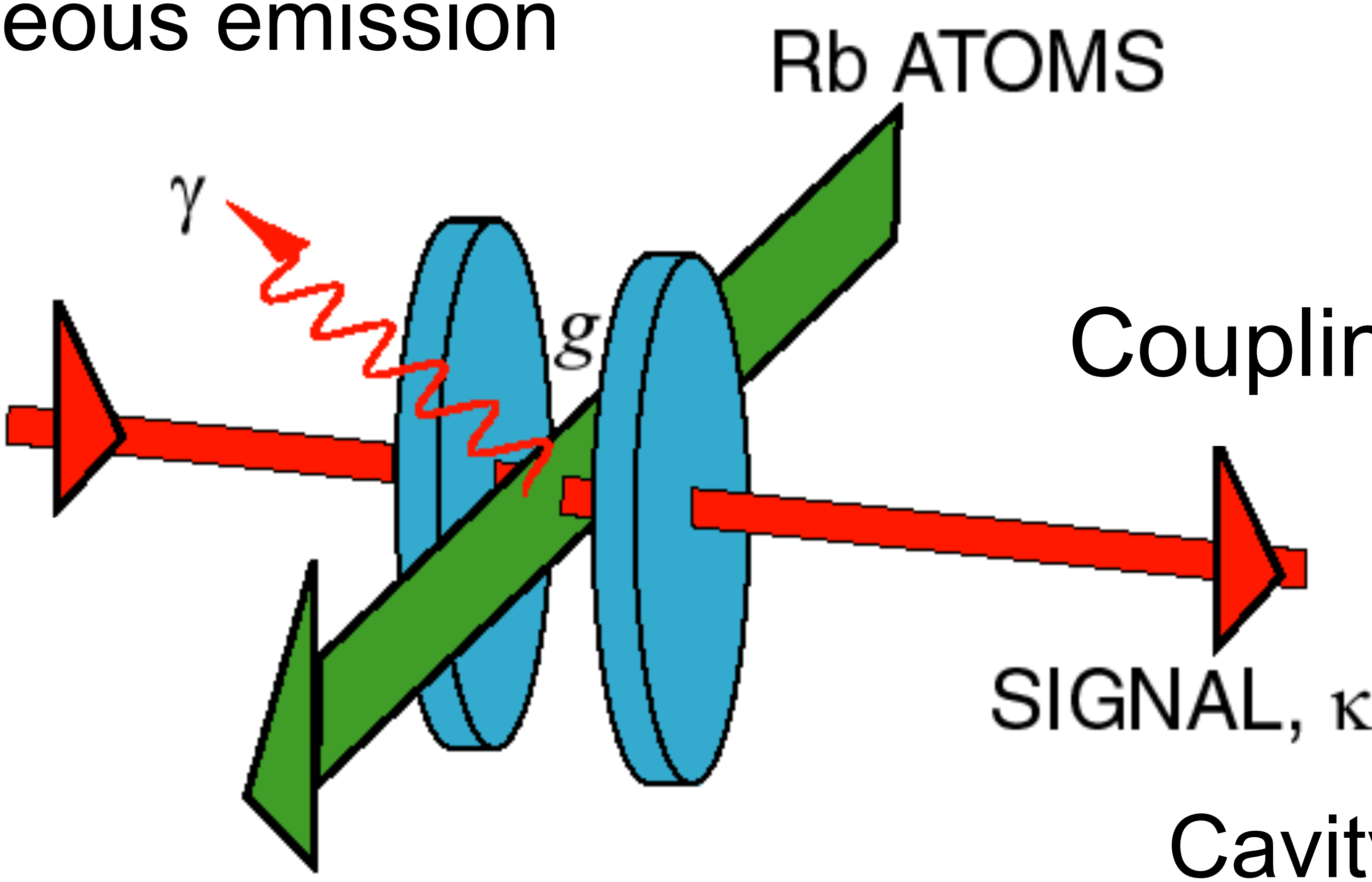
$$\hat{H}_4 = \hat{\Gamma}_F \hat{a}^\dagger + \hat{\Gamma}_F^\dagger \hat{a} \ ,$$ Cavity decay

$$\hat{H}_5 = i\hbar \left(\hat{a}^\dagger \mathcal{E} e^{-i\omega_l t} - \hat{a}\mathcal{E}^* e^{i\omega_l t} \right) \ .$$ Drive


EMPTY CAVITY
LIGHT
PD
SIGNAL


Transmission [Arb. U.]
0.0
0.2
0.4
0.6
0.8
1.0
0.0
0.5
1.0
1.5
2.0
2.5
Frequency offset [MHz]

Spontaneous emission



Coupling

Cavity decay

Cooperativity for one atom: C_1

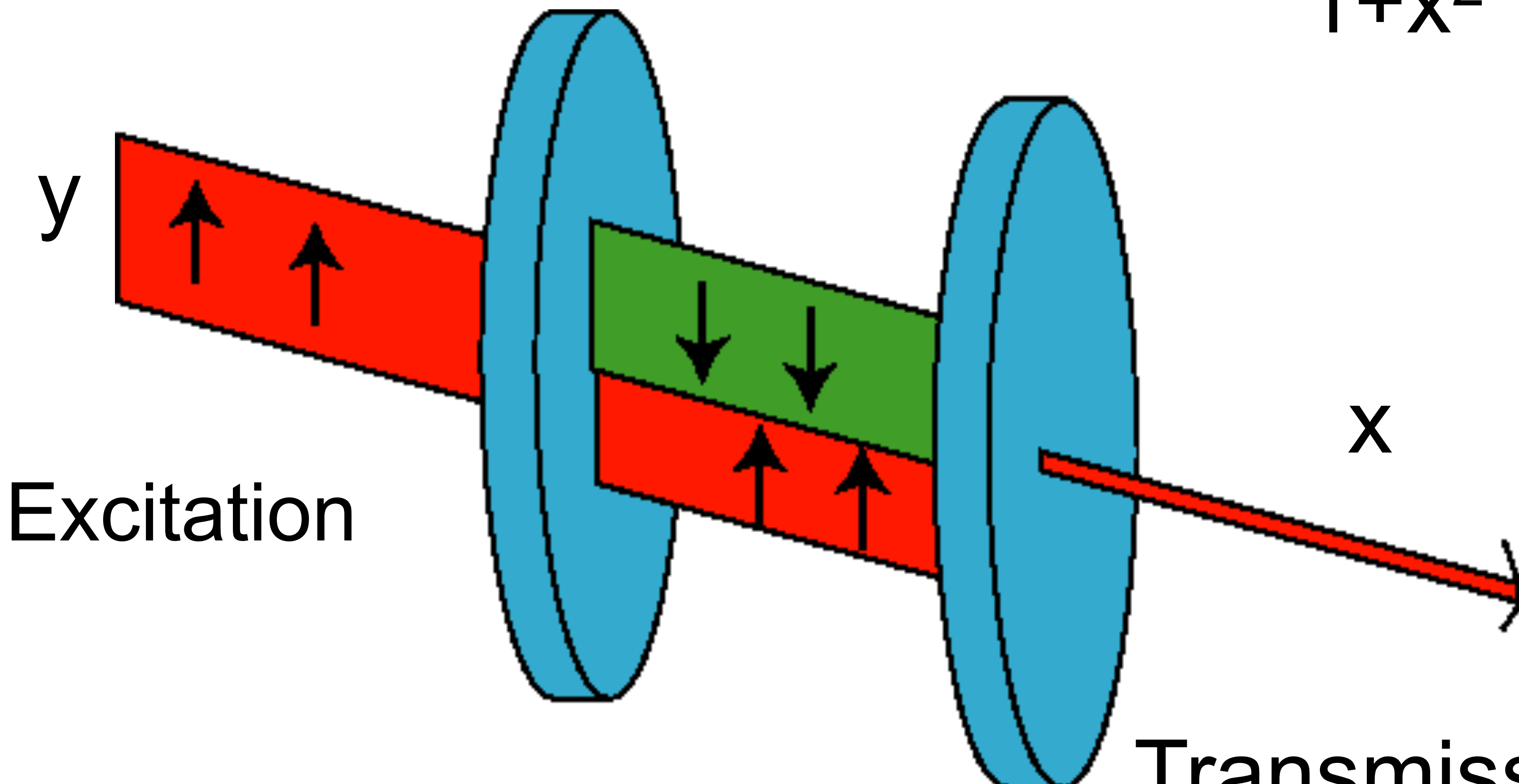
$$C_1 = \frac{g^2}{\kappa\gamma} \qquad C = C_1 N$$

Cooperativity for N atoms: C

$$g \approx \kappa \approx \gamma$$

Steady State

Atomic polarization: $\dfrac{-2Cx}{1+x^2}$



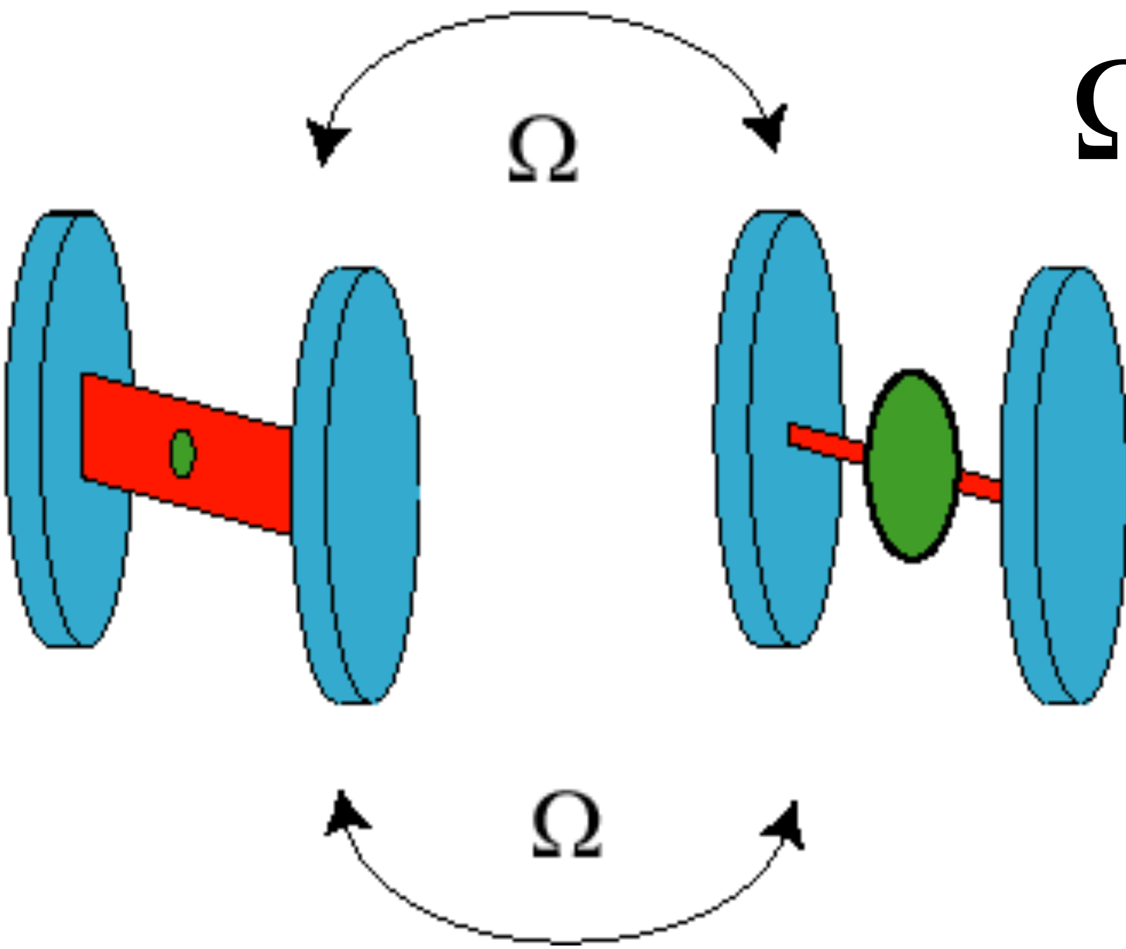
Transmission

$x/y = 1/(1+2C)$

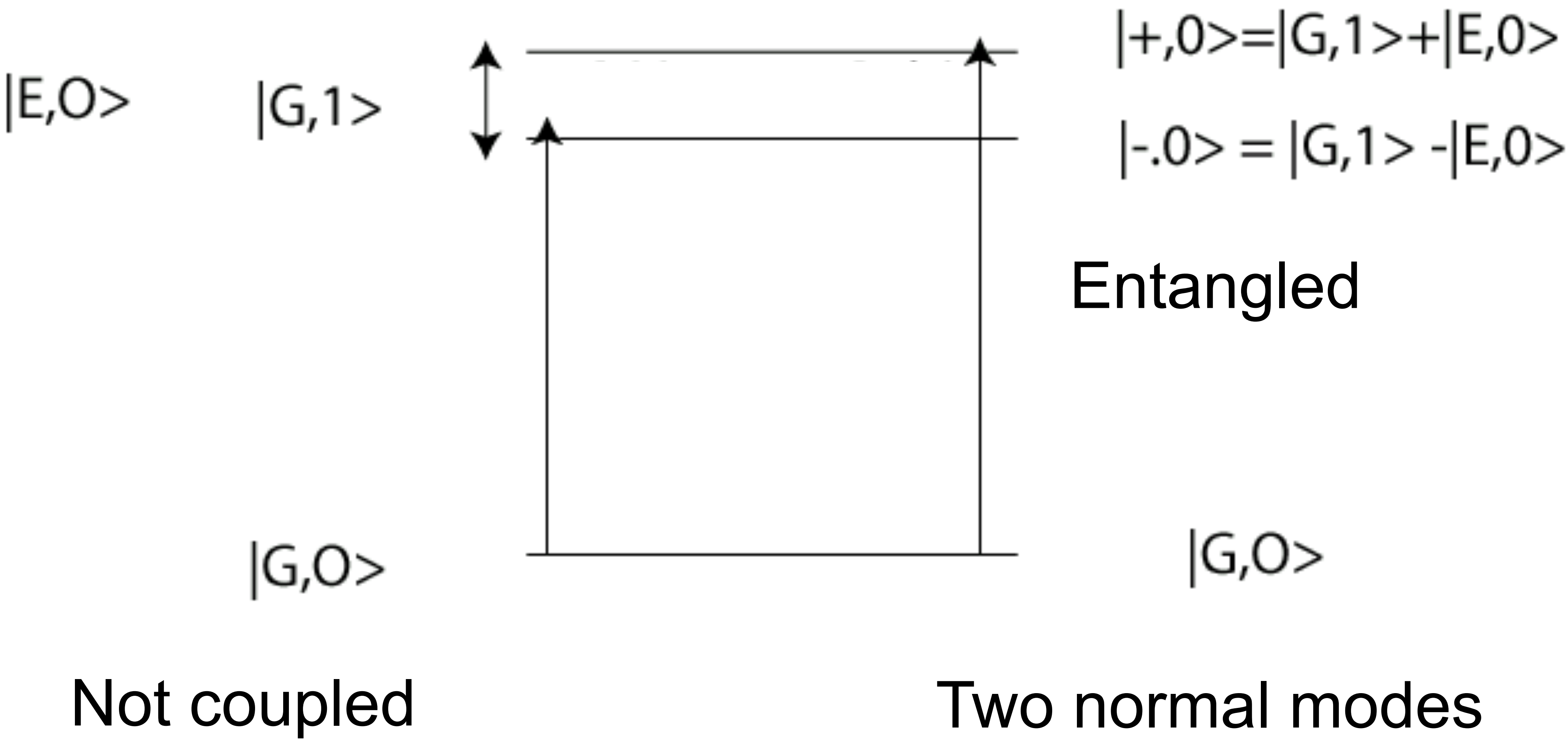
Jaynes Cummings Dynamics
Rabi Oscillations

Exchange of excitation for *N* atoms:

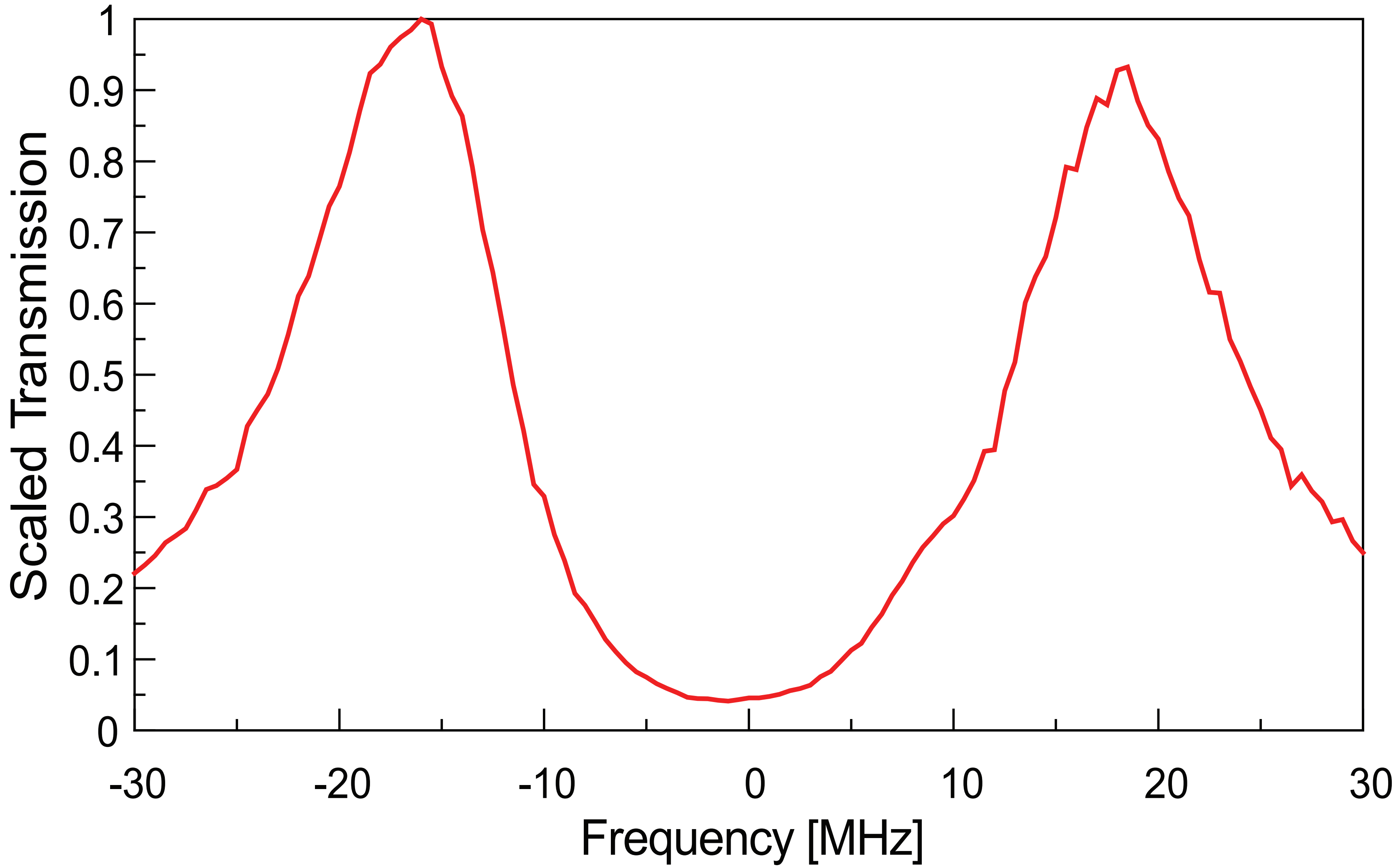
$$\Omega \approx g\sqrt{N}$$

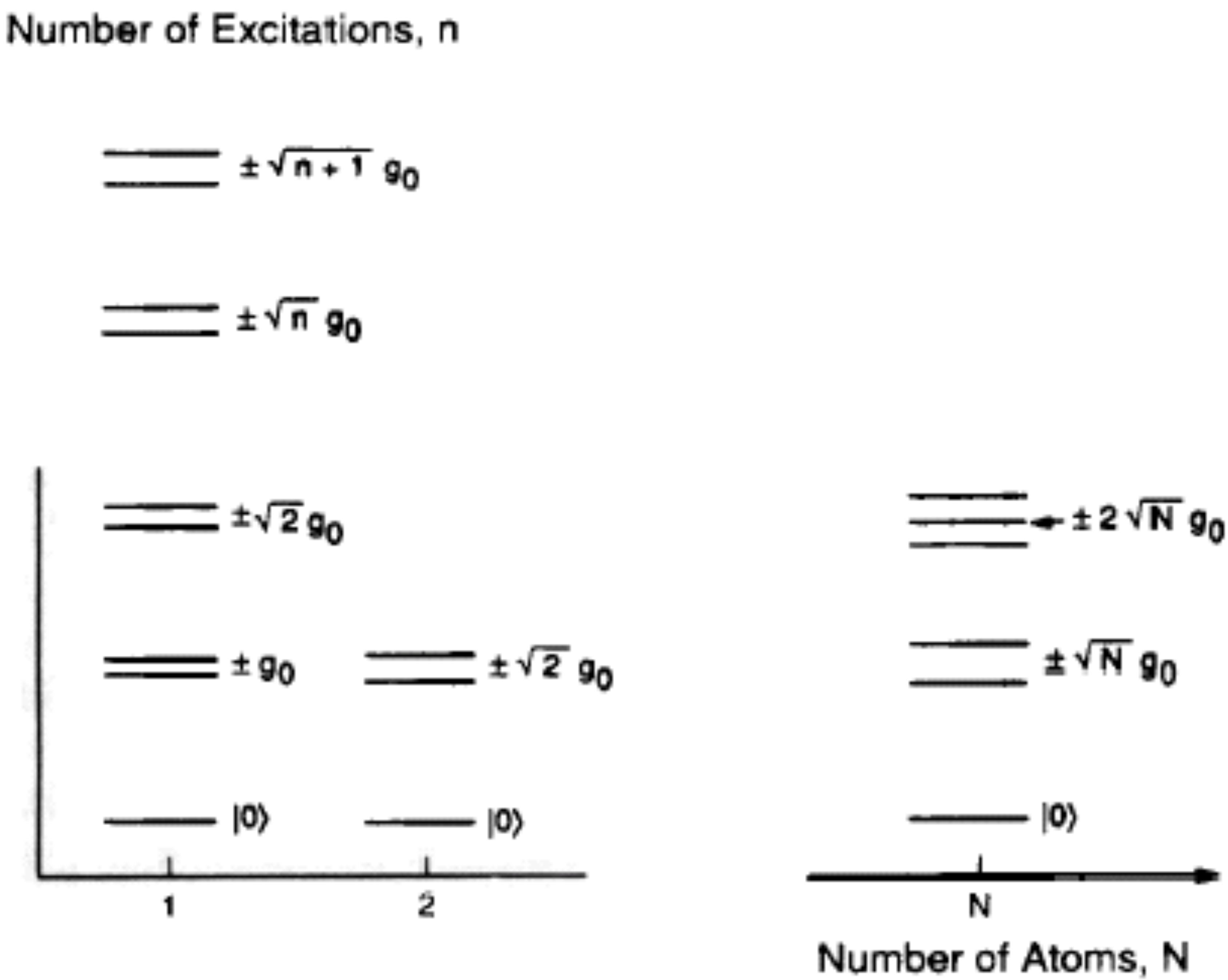


2g Vacuum Rabi Splitting

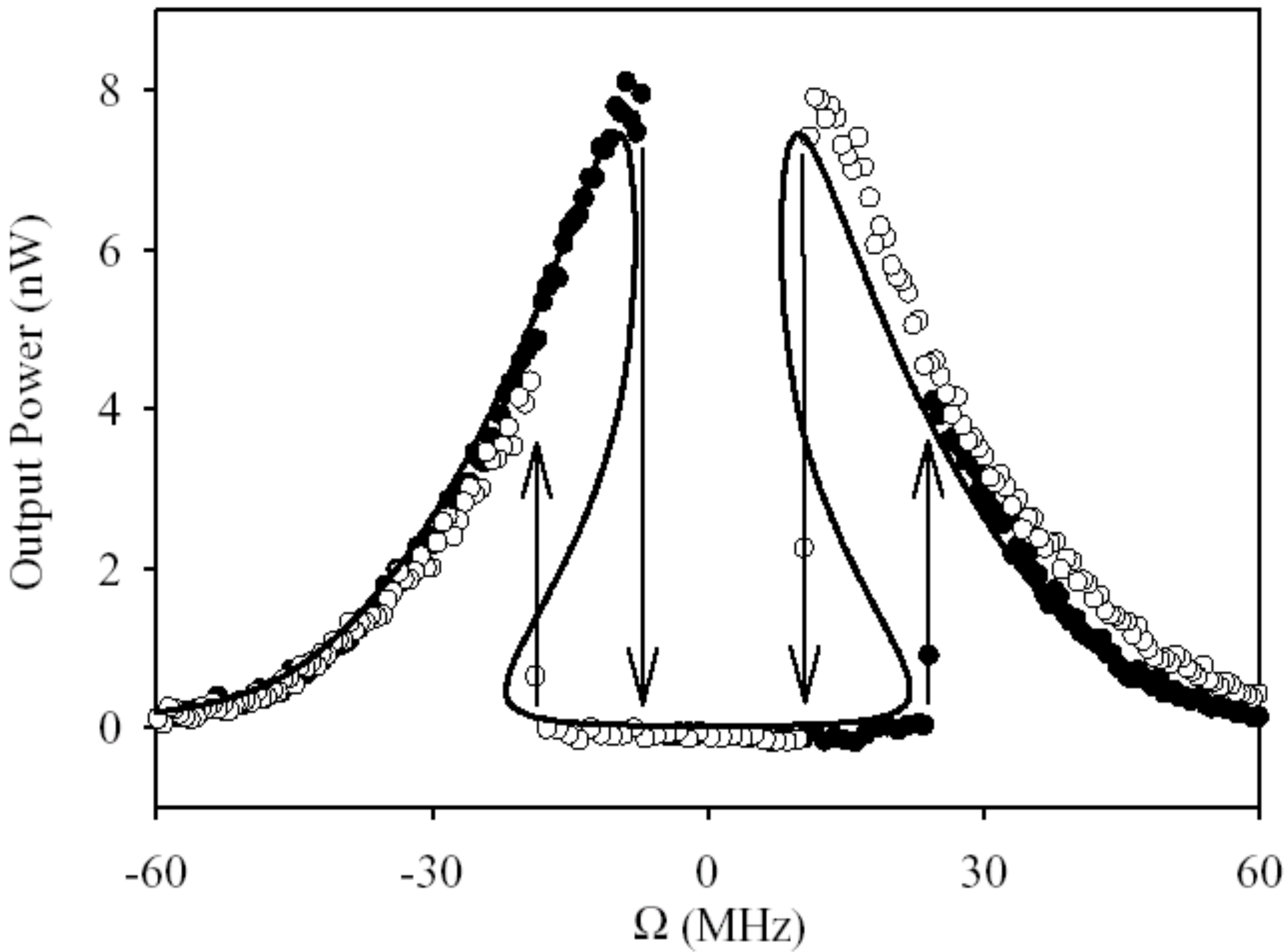


Transmission doublet different from the Fabry Perot resonance





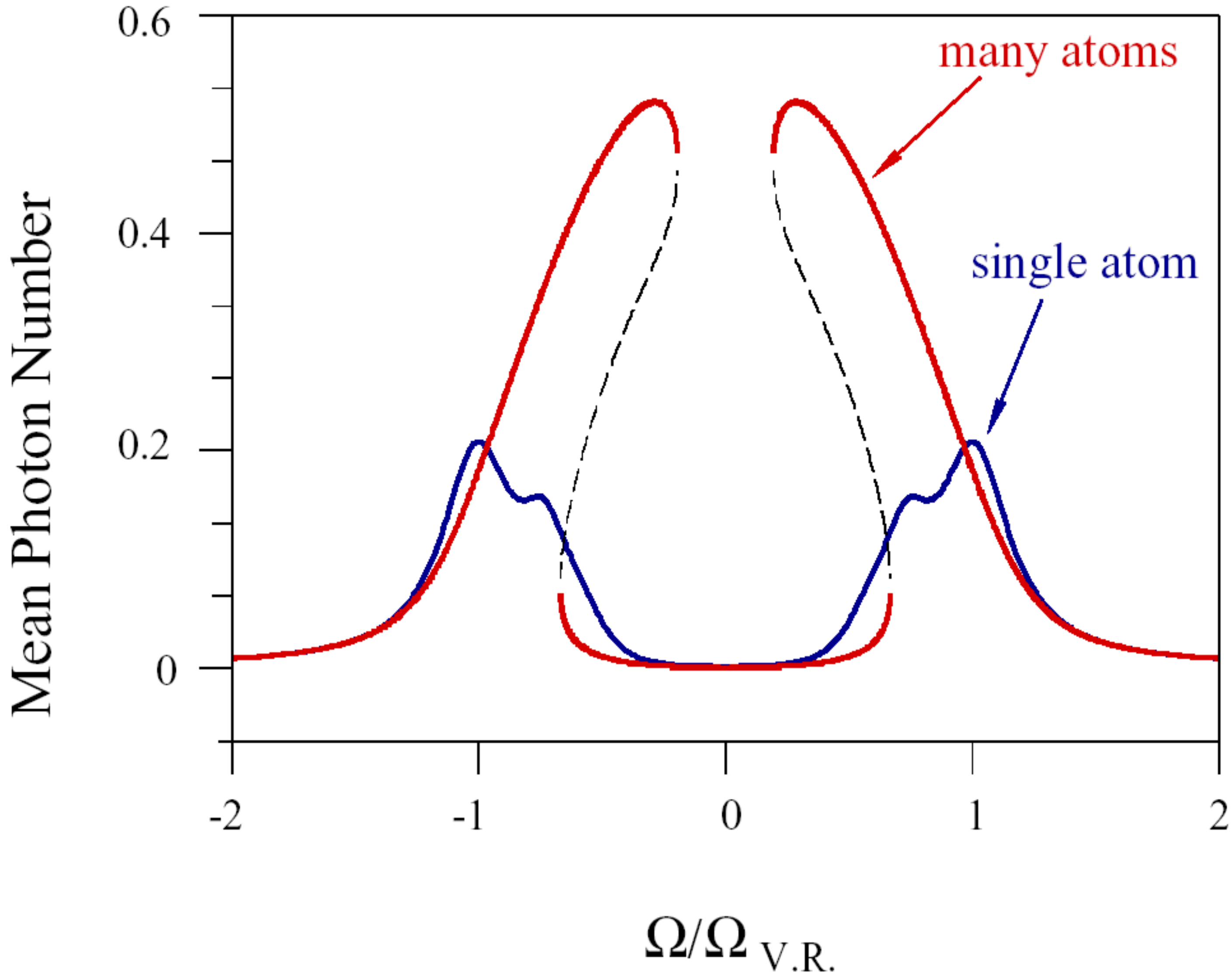
Number of Excitations, n
$\pm\sqrt{n+1}\,g_0$
$\pm\sqrt{n}\,g_0$
$\pm\sqrt{2}\,g_0$
$\pm g_0$
$\pm\sqrt{2}\,g_0$
$|0\rangle$
$|0\rangle$
1
2
$\pm 2\sqrt{N}\,g_0$
$\pm\sqrt{N}\,g_0$
$|0\rangle$
N
Number of Atoms, N



Hysteresis for a frequency scan of the light from the coupled atoms-cavity system.

Many atoms solved with Maxwell Bloch equations.

Single atom solved with the full Hamiltonian, no decorrelation. The system does not show hysteresis.



nature physics

LETTERS

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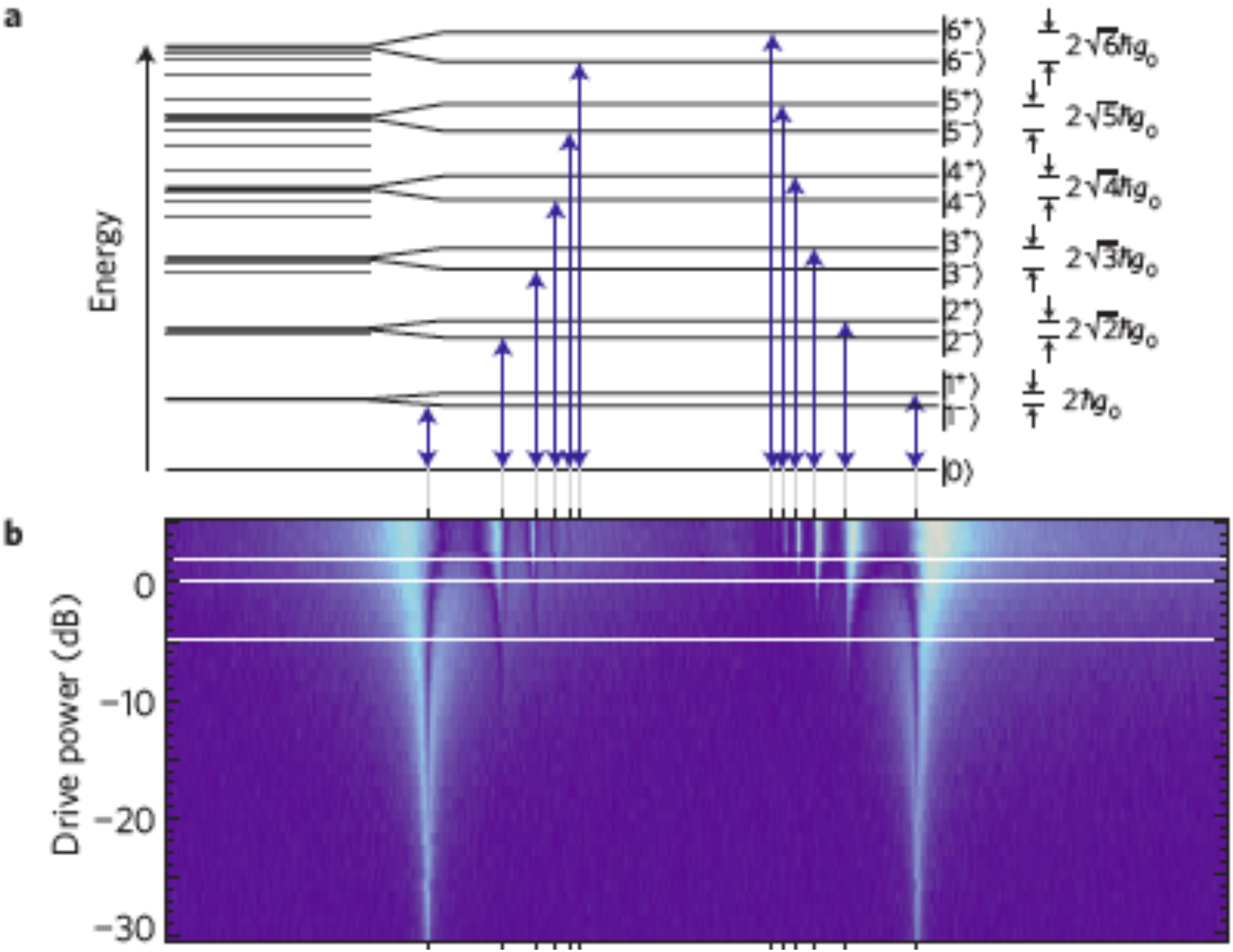
Nonlinear response of the vacuum Rabi resonance

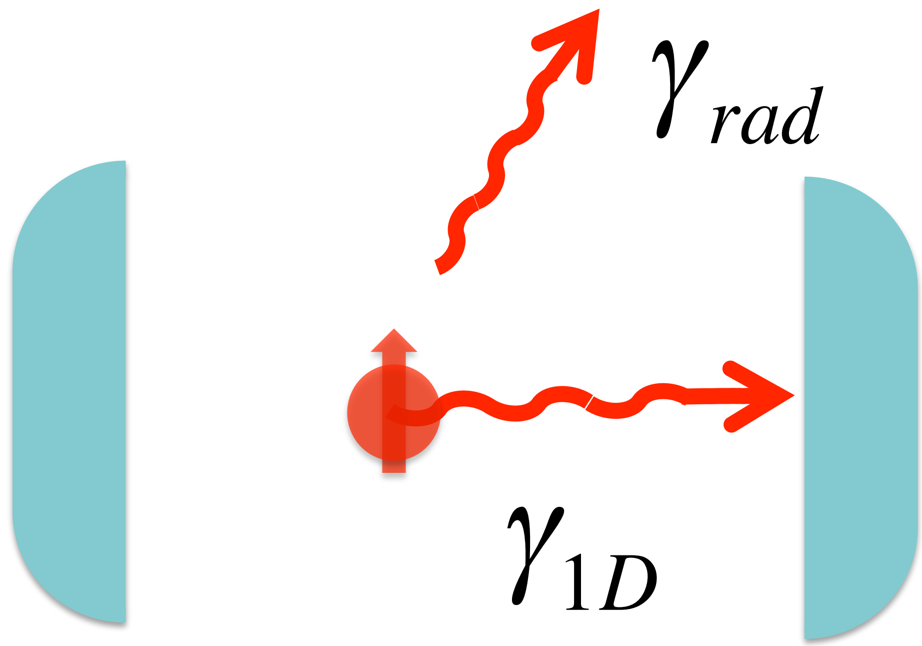
Lev S. Bishop[1], J. M. Chow[1], Jens Koch[1], A. A. Houck[1], M. H. Devoret[1], E. Thuneberg[2], S. M. Girvin[1] and R. J. Schoelkopf[1]*

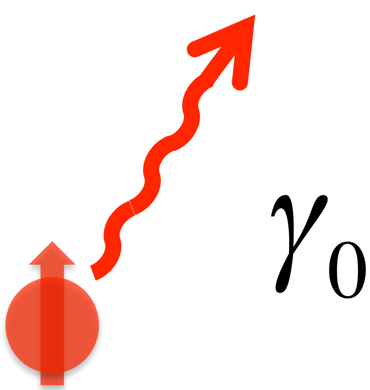


a
Energy
|6⁺⟩
|6⁻⟩
2√6ħg₀
|5⁺⟩
|5⁻⟩
2√5ħg₀
|4⁺⟩
|4⁻⟩
2√4ħg₀
|3⁺⟩
|3⁻⟩
2√3ħg₀
|2⁺⟩
|2⁻⟩
2√2ħg₀
|1⁺⟩
|1⁻⟩
2ħg₀
|0⟩
b
Drive power (dB)
0
−10
−20
−30

How to quantify the coupling?

Coupling Enhancement

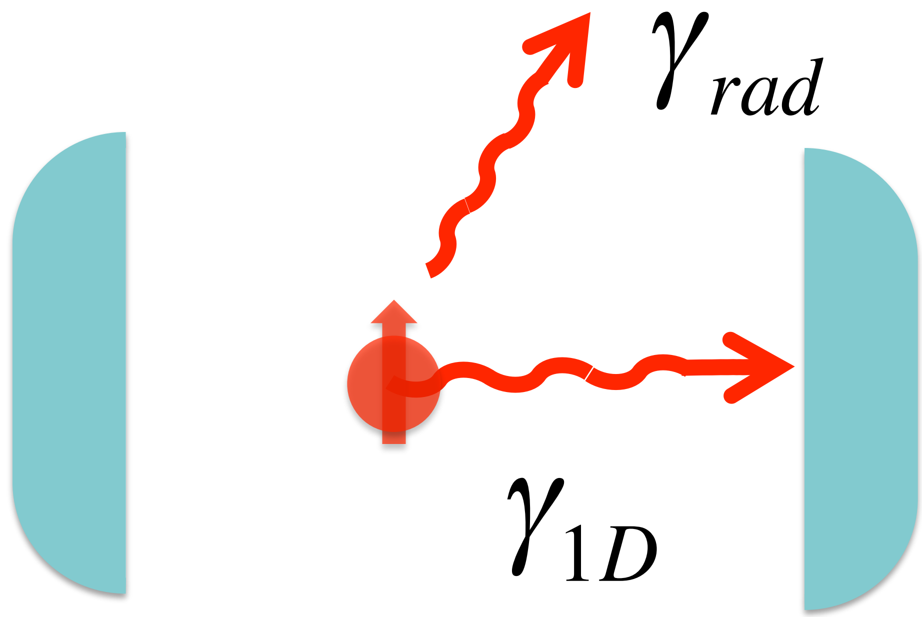


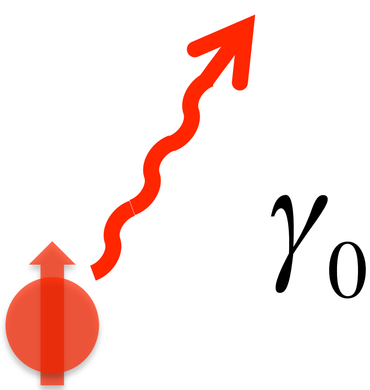


$$\alpha = \frac{\gamma_{1D}}{\gamma_0}$$

The coupling enhancement α is proportional to the total number of photons emitted into the cavity mode.

Coupling Efficiency





$$\beta = \frac{\gamma_{1D}}{\gamma_{Tot}} \quad ; \quad \gamma_{Tot} = \gamma_{1D} + \gamma_{rad}$$

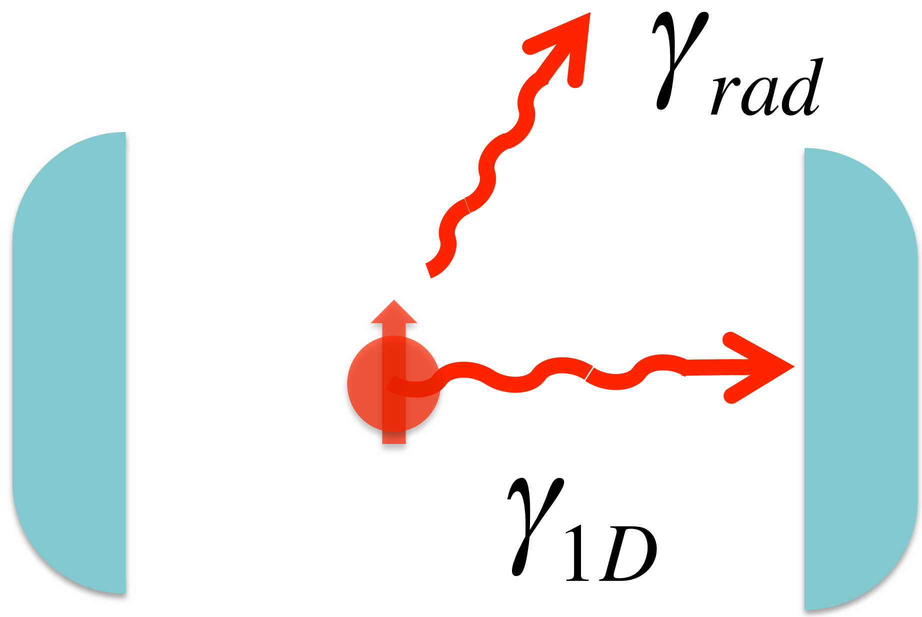
The coupling efficiency β is the percentage of photons emitted into the mode relative to the total number of emitted photons.

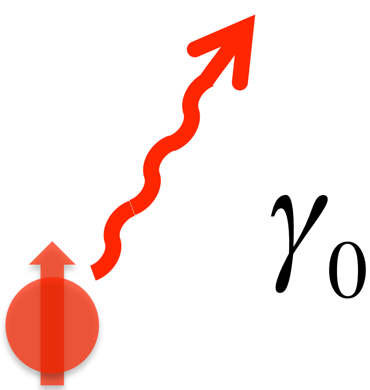
The difference between α and β :

When the coupling efficiency β is very large, close to one, most of the emitted photons couple to the cavity (waveguide). However, the total number of photons emitted into the waveguide can still be close to zero if the total spontaneous emission were to be greatly inhibited.

The amplitude of the signal measured through the output mode is represented by the emission enhancement parameter α .

Purcell Factor



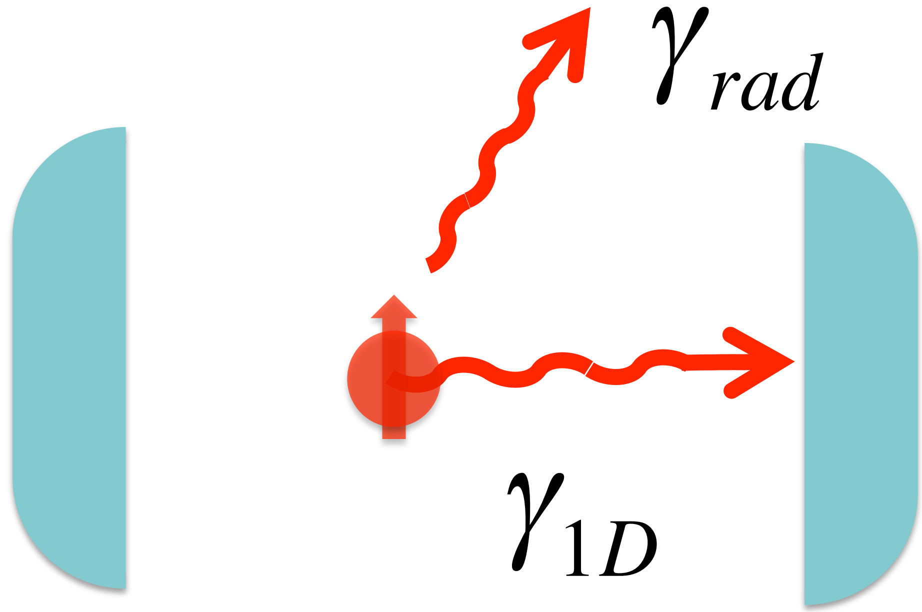


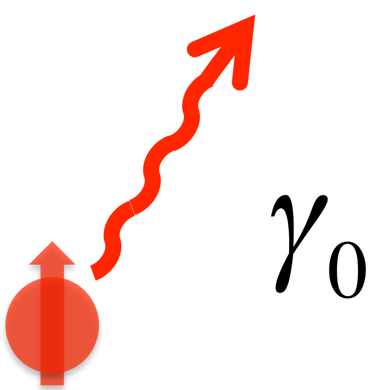
$$F_P = \frac{\gamma_{tot}}{\gamma_0} = \frac{\alpha}{\beta}$$

$$\gamma_{Tot} = \gamma_{1D} + \gamma_{rad}$$

The Purcell factor is the ratio of the total decay rate compared to what would be in free space.

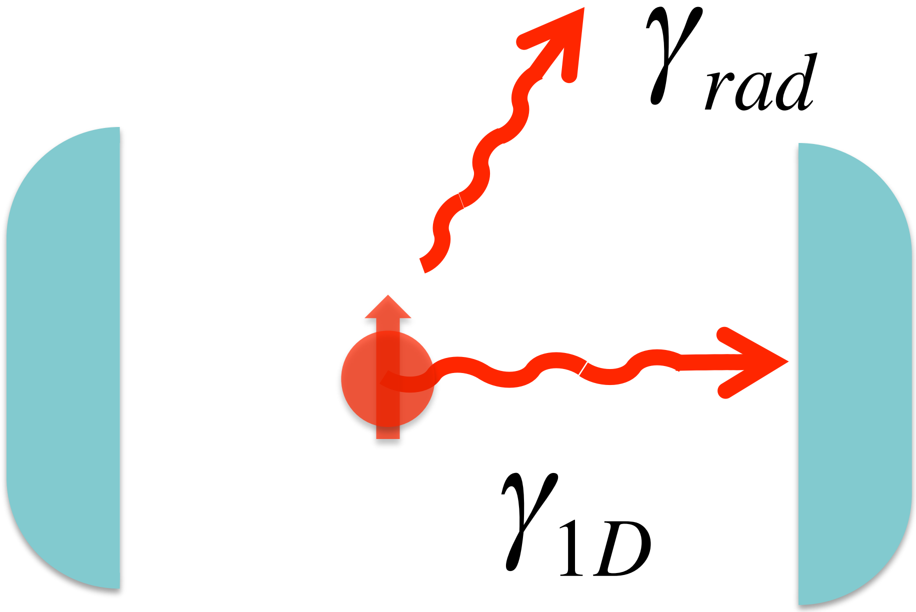
Cooperativity

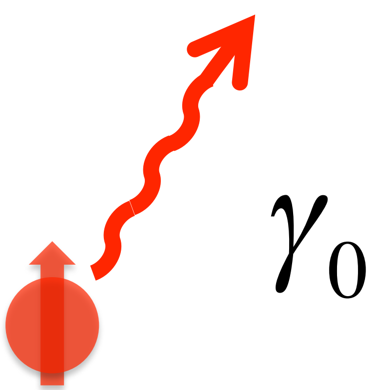




$$C_1 = \frac{\beta}{(1-\beta)} = \frac{\gamma_{1D}}{\gamma_{rad}}$$

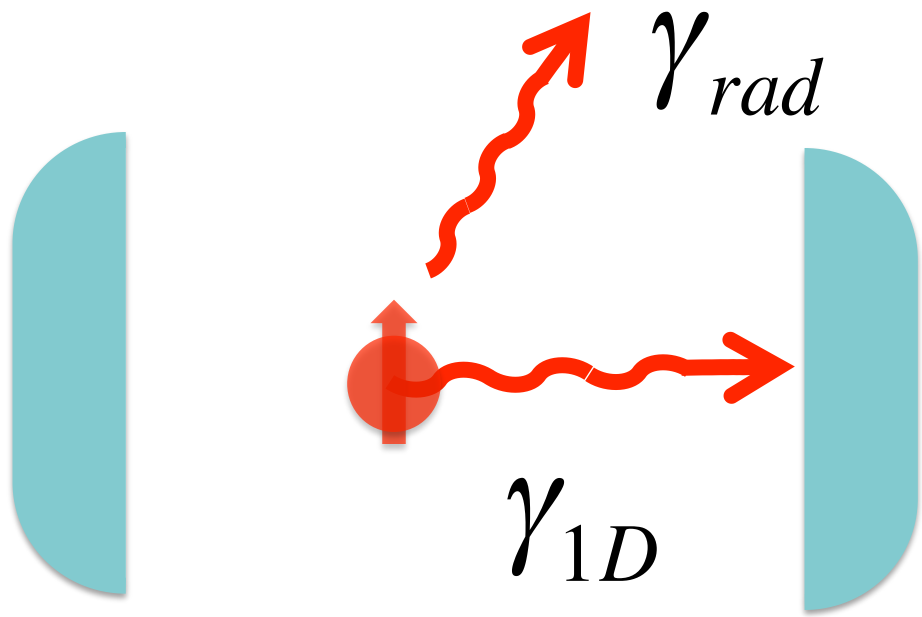
Cooperativity

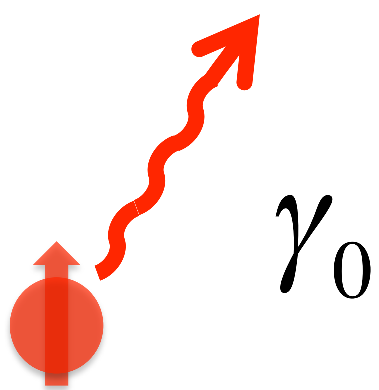




$$C_1 = \frac{g^2}{\kappa\gamma_0}$$

Cooperativity





$$C_1 = \frac{\sigma_0}{Area_{\mathrm{mode}}} \frac{1}{T}$$

The cooperativity is the ratio between the photons going into the mode and those emitted out to other modes. It is the inverse of the number of atoms that are necessary to observe non-linear effects in the cavity.

Some Implementations

- Rydbergs on Superconducting cavities (Microwaves)
- Alkali atoms on Optical Cavities (Optical)
- Quantum dots on microcavities (Optical)
- Trapped ions and vibrational mode (phonons)
- Circuit QED Superconducting qbits on microwave resonators (Microwaves)
- Polaritons on optical microcavities (photons)

The cooperativity has become the figure of merit for many quantum optics experiments, it is not limited to cavity QED.

How to choose a platform?

$$C = \frac{\sigma_0}{Area_{\mathrm{mode}}} \frac{1}{T} N \qquad\qquad C = \frac{g^2}{\kappa \gamma_0} N$$

Take the area of the mode to be $\pi(\lambda/2)^2$, and σ as $3\lambda^2/2\pi$ then C does not depend on the choise of atom

Another approach is to maximize g through a large E_0, then minimize the cavity volume V

The solutions are guided by your resources and where you can approach the ideals

Microwaves can be confined to cavities with mode areas close to the atomic cross section of the Rydberg Atoms. (Experiments led by S. Haroche)

This is more difficult in the visible for free space with atoms, but recent developments at ENS on making micrometric mirrors are helping.

谢谢