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# An introduction to light-matter interaction, from cavity QED to waveguide QED

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The presentation will be available at:

http://www.physics.umd.edu/rgroups/amo/orozco/results/2019/Results19.htm

# 1. A review of Electricity and Magnetism and Polarization

# Maxell's Equations:

$$\nabla \cdot (\varepsilon_0 \mathbf{E} + \mathbf{P}) = 0,$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$

$$\nabla \times \mathbf{B} = \mu_0 \frac{\partial (\varepsilon_0 \mathbf{E} + \mathbf{P})}{\partial t},$$

$$\nabla \cdot \mathbf{B} = 0.$$

# Wave Equation:

$$\nabla \times (\nabla \times \mathbf{E}) = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E}$$

$$= -\frac{\partial}{\partial t}(\nabla \times \mathbf{B}) = -\mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} - \mu_0 \frac{\partial^2 \mathbf{P}}{\partial t^2}$$

$$-\nabla(\nabla \cdot \mathbf{E}) + \nabla^2 \mathbf{E} - \mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} = \mu_0 \frac{\partial^2 \mathbf{P}}{\partial t^2}.$$

In free space, $\nabla \cdot \mathbf{E} = 0$ and $\mathbf{P} = \mathbf{0}$,

$$\nabla^2 \mathbf{E} - \frac{1}{v_0^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0$$

# The slowly varying envelop equations (Towards the Maxwell Bloch equations)

$$\frac{\partial F}{\partial z} + \frac{1}{c}\frac{\partial F}{\partial t} = -\alpha P$$

Where F and P are the slowly varying envelops of the Electric field and the polarization and $\alpha$ is the absorption coefficient

# A note about polarization

Gauss's Law in free space:

$$\nabla \cdot \vec{E} = 0$$

$$\nabla_T \cdot \vec{E} + \frac{2\pi}{\lambda} i E_Z = 0$$

If there is a "transverse gradient" in the radiation field propagating in *z*, there is a longitudinal polarization also in *z*

# Polarization at the waist of an optical nanofiber

$$\nabla \cdot \vec{E} = 0$$

$$\nabla_T \cdot \vec{E} + \frac{2\pi}{\lambda} i E_Z = 0$$

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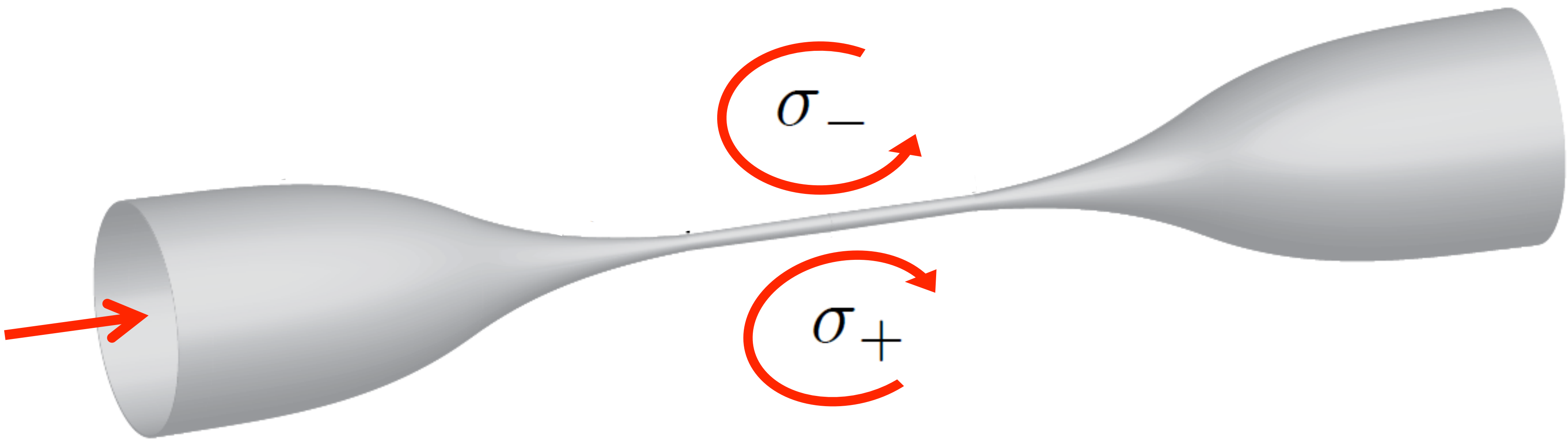
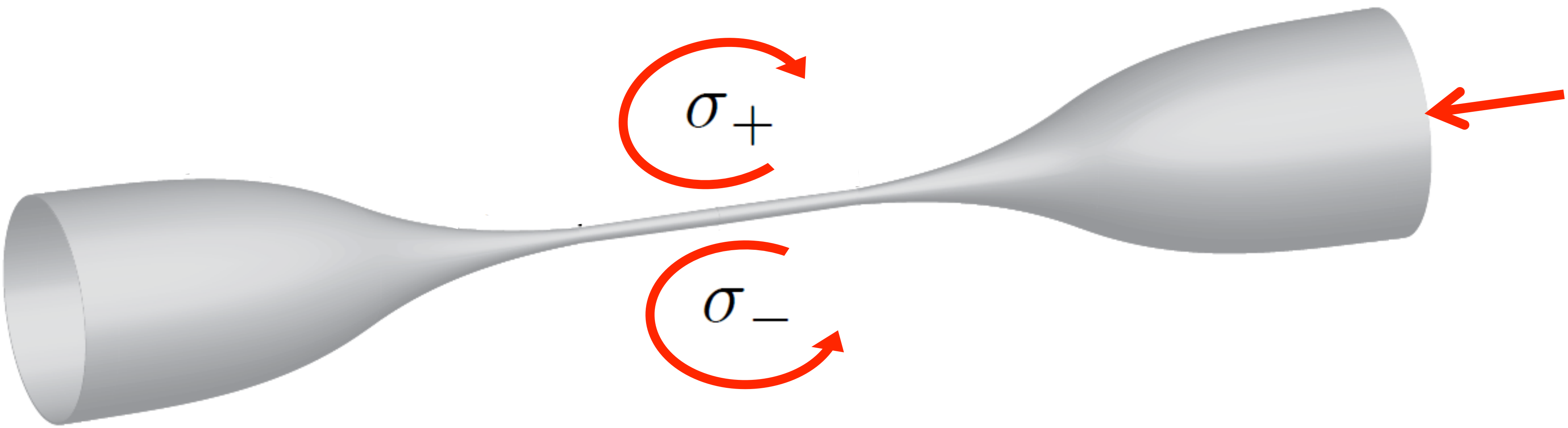



Bicycle polarization not propeller

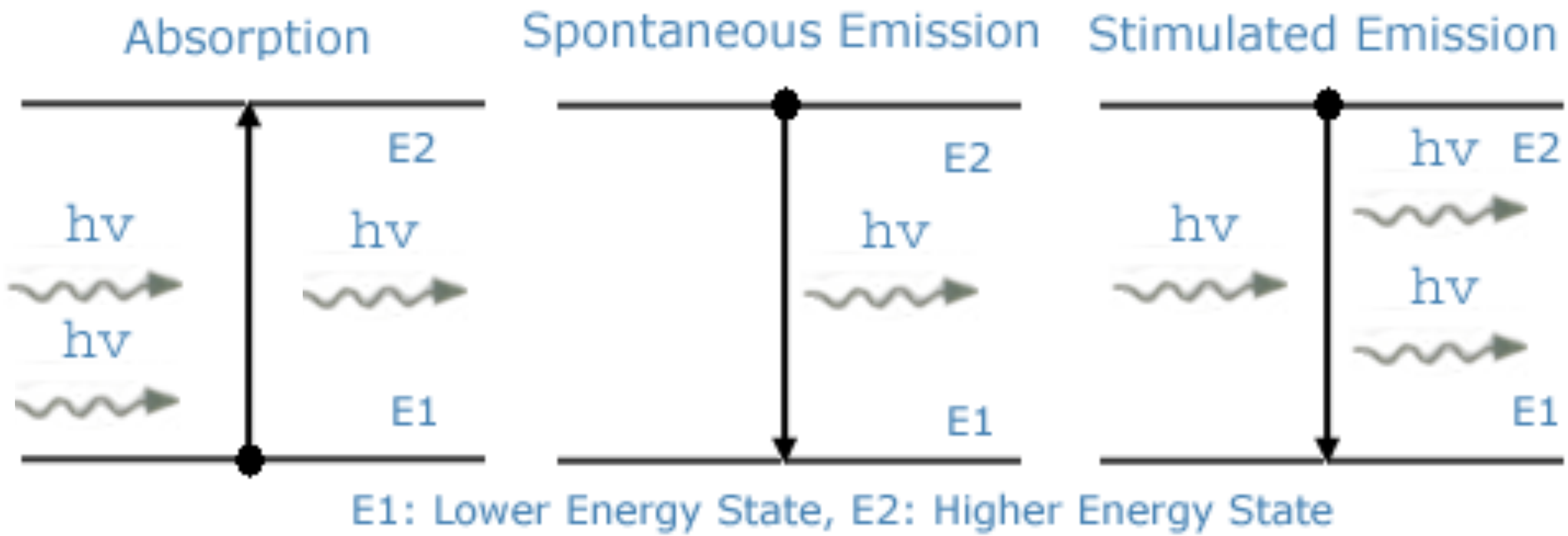
# Polarization at the waist of an optical nanofiber

$$\begin{aligned}\nabla \cdot \vec{E} &= 0 \\ \nabla_T \cdot \vec{E} + \frac{2\pi}{\lambda} i E_Z &= 0\end{aligned}$$




$$E_z = \pm \frac{i}{k} \nabla \cdot \mathbf{E}_\perp$$

# 2. One atom interacting with light in free space.




Absorption and Stimulated Emission are time reversals of each other, you can say this is the classical part. Spontaneous emission is the quantum, that is the jump.

Dipole cross section (same result for a classical dipole or from a two level atom):

$$\sigma_0 = \frac{3\lambda_0^2}{2\pi}$$

This is the “shadow” caused by a dipole on a beam of light.

Energy due to the interaction between a dipole and an electric field.

$$H = \vec{d} \cdot \vec{E}$$

The dipole matrix element between two states is fixed by the properties of the states (radial part) and the Clebsh-Gordan coefficients from the angular part of the integral. It is a few times $a_0$ (Bohr radius) times the electron charge e between the S ground and P first excited state in alkali atoms.

$$\vec{d} = e\left\langle 5S_{1/2}\middle|\vec{r}\middle|5P_{3/2}\right\rangle$$

# Beer-Lambert law for intensity attenuation

$\frac{dI}{dz} = -\alpha I$ if $\alpha$ is resonant and independent

of $I$, $\alpha_0$ (does not saturate)

$I = I_0 \exp(-\alpha_0 l)$

where $\alpha_0 = \sigma_0 \rho$

and $\rho = N / V$ the density of absorbers in a length $l$

# Rate of decay (Fermi's golden rule)

$$\gamma_{rad} \approx \frac{2\pi}{\hbar}\rho\left(k\right)\left\langle H_{int}\right\rangle^{2}$$

Phase space density Interaction

# Rate of decay free space (Fermi's golden rule)

$$\gamma_0 = \frac{\omega_0^3 d^2}{\pi \varepsilon_0 \hbar c^3}$$

Where *d* is the dipole moment

Saturation intensity:
One photon every two lifetimes over the cross section of the atom (resonant)

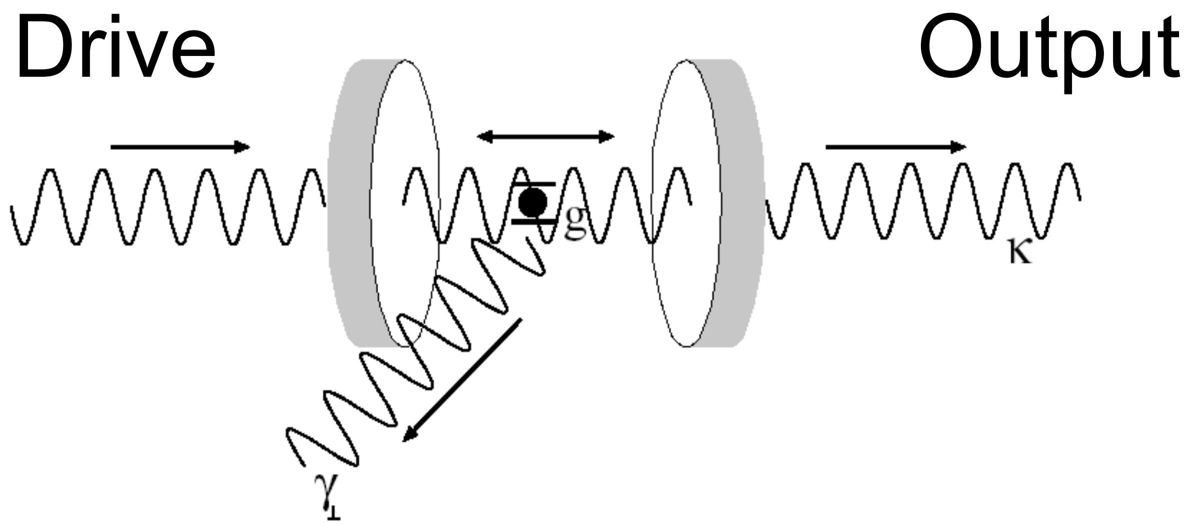
$$I_s = \frac{\hbar\omega_0}{2\tau_0\sigma_0} = \frac{\pi}{3}\frac{\gamma_0\hbar\omega_0}{\lambda_0^2}$$

If $I=I_0$ the rate of stimulated emission is equal to the rate of spontaneous emission (Rabi Frequency Ω) and the population on the excited state 1/4.

$$\Omega = \frac{\vec{d} \bullet \vec{E}}{\hbar} = \gamma \sqrt{I/I_s}$$

$$\text{Excited Population} = \frac{1}{2} \frac{I/I_s}{1 + I/I_s}$$

# Coupled atoms and cavities




Collection of N Two level atoms coupled to a single mode of the electromagnetic field (g). Driven with dissipation (atoms γ, cavity κ).

Microwaves

Micromaser

Visible light

Optical Bistability

Cavity QED

# Absorptive Element

A saturable absorber has an absorption coefficient which is a non-linear function of I:

$$\alpha = \frac{\alpha_o}{1 + I / I_s}$$

$$for \quad I / I_s < \quad 1$$

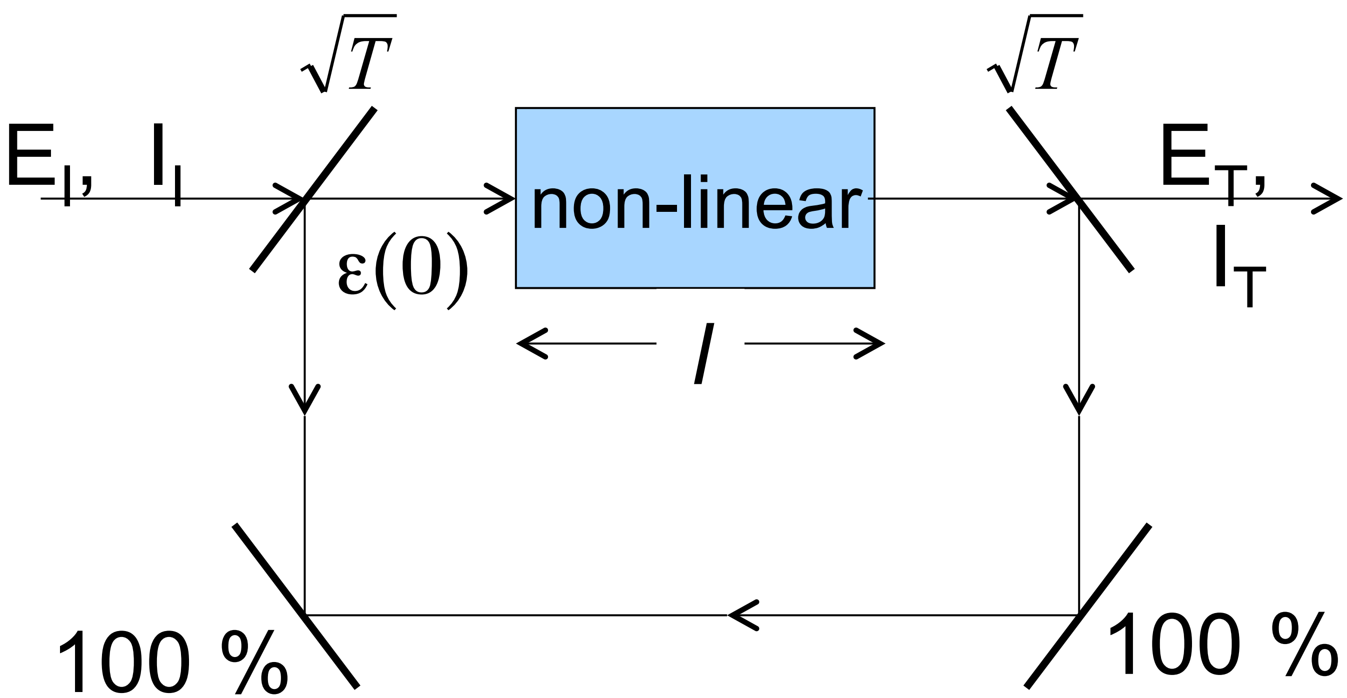
$$\alpha \cong \alpha_o \left(1 - I / I_s + \cdots \right)$$

- The cavity is resonant.
- At small intensities, the absorption due to the element is high and the output is low.
- As the intensity is increase beyond $I_s$, the absorption decreases and the output goes to high.

The field inside the cavity comes from the addition of the drive and what is already there

$$\text{Let } \varepsilon_{n+1}(0) = \sqrt{T}E_I + \mathrm{Re}^{-\alpha l} \; e^{iKL}\varepsilon_n(0)$$

Where e $_{n+1}$ is the electric field after the *n+1* path around the cavity, *L* is the round-trip length, a is the absorption coefficient and *R=1-T* the mirror reflectivity



- At steady state the electric field inside the cavity must be constant so that $\varepsilon_{n+1}(0) = \varepsilon_n(0) = \varepsilon_0$

$$\therefore \quad \varepsilon_0 = \sqrt{T}E_1 + \mathrm{Re}^{-\alpha l}\, e^{iKL}\varepsilon_0$$

rearanging this gives: $\varepsilon_0 = \dfrac{\sqrt{T}E_I}{(1-\mathrm{Re}^{-\alpha l + iKL})}$

- The output field is given by the mirror transmittance times the internal electric field at a distance $l$.

$$E_T = \sqrt{T}\varepsilon(l) = \sqrt{T}\varepsilon_o e^{(-\alpha + iK)l}$$

- the amplitude transmission function is:

$$\frac{E_T}{\mathrm{E_I}} = \frac{Te^{iK(l-L)}}{e^{\alpha l - iKL} - R}$$

# Absorptive Bistability

A saturable absorber, at resonance has an absorption coefficient which is a non-linear function of I:

$$\alpha = \frac{\alpha_o}{1 + I / I_s}$$

assuming that $\alpha l \ll 1$, gives on resonance:

$$\frac{E_T}{E_I} = \frac{1}{1 + \alpha l / T}$$

$$E_I = E_T\left[1+\frac{\alpha_o l/T}{1+I_T/I_s T}\right] \quad \text{with} \quad I=\frac{I_T}{T}$$

The ratio of losses: atomic losses per round trip ($\alpha l$) to cavity losses per round trip ($T$) is the Cooperativity

$$C=\frac{\alpha_o l}{T}=\frac{\sigma_0 \rho l}{T}=\frac{\sigma_0 N}{Area_{\text{mode}}}\frac{1}{T}$$

The steady state for normalized input y and output x fields:

On resonance :

field:

$$y = x\left(1 + \frac{2C}{1+x^2}\right)$$

intensity:

$$Y = X\left(1 + \frac{2C}{1+X}\right)^2$$

For low intensity, the input field and the output field are linearly related,

$y = x\ (1+2C)$ ; $x/y=1/(1+2C)$ goes as $1/N$

For the intensity $Y=y^2$ ; $X=x^2$

$Y=X(1+2C)^2$ ; $X/Y=1/(1+2C)^2$ goes as $1/N^2$

For very high field and intensity,

$y = x$ ; $Y=X +4C$

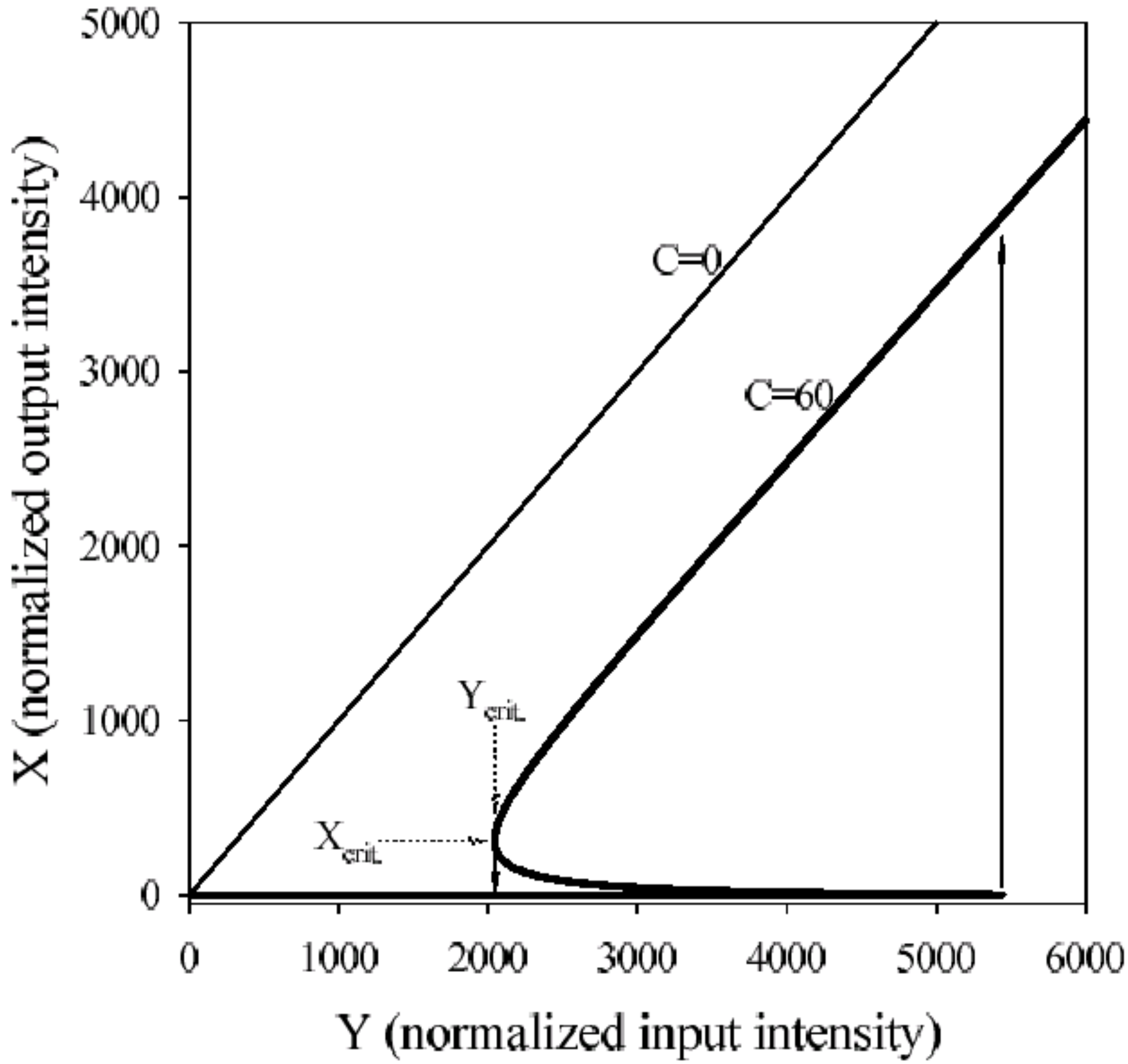
Almost an empty cavity

At intermediate intensity, there can be saturation (denominator of 1+X).

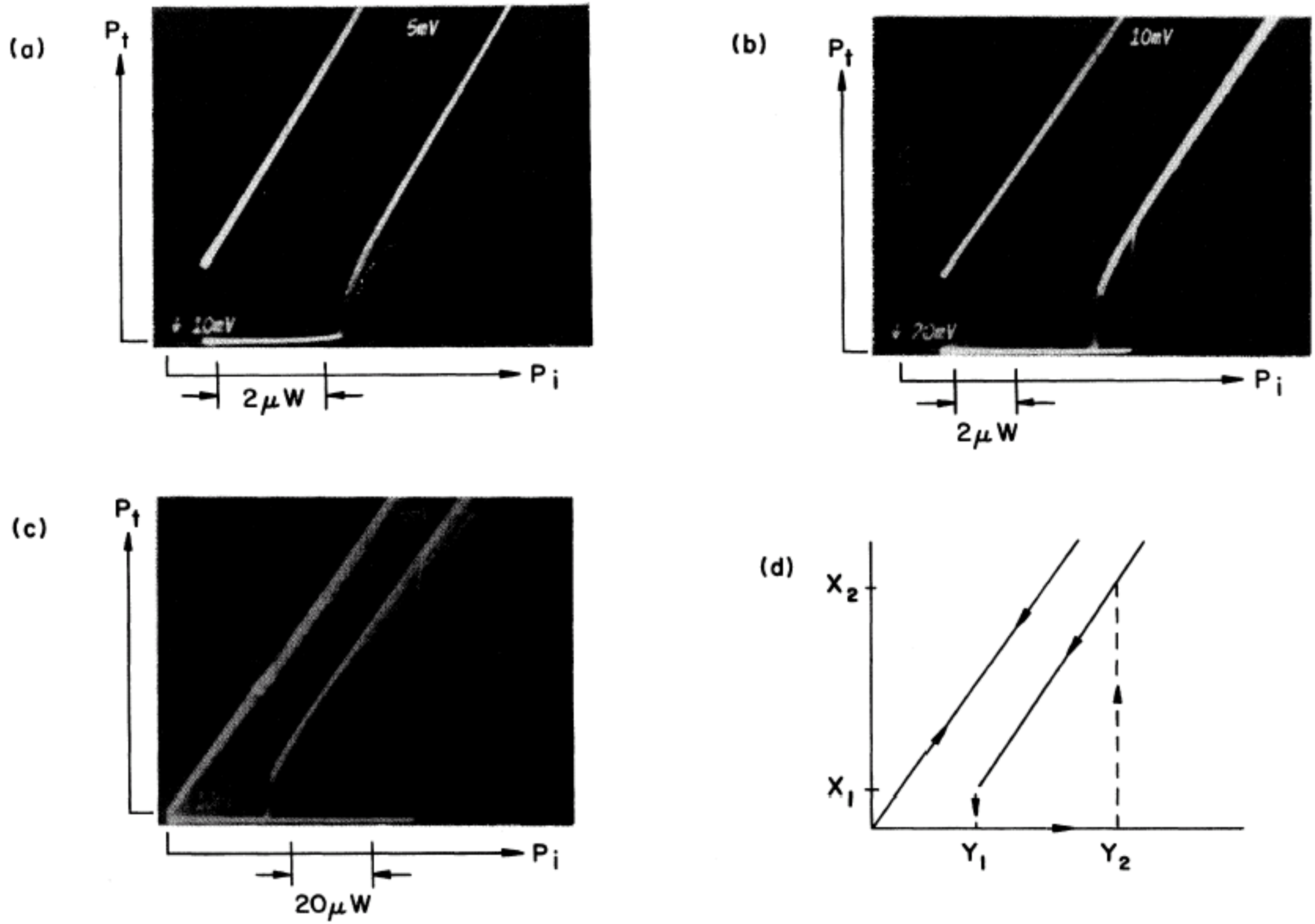
It happens in this simple model for the case of C>4. C (Cooperativity) is the negative of the laser pump paramenter.

C is a figure of merit.
It is the ratio of the atomic losses to the cavity losses or also can be read as the ratio between the good coupling (g) and the bad couplings ($\kappa$,$\gamma$), it is a ratio of areas.

Input-Output response of the atoms-cavity system for two different cooperativities C=0 is with no atoms, C=60 has plenty of atoms, with a drive that can saturate them and we recover the linear relationship with unit slope between Y and X.



# Increasing the number of atoms in the cavity:



# Cavity QED

# Optical Cavity QED

Quantum electrodynamics for pedestrians. No need for renormalization. One or a finite number of modes from the cavity.

ATOMS + CAVITY

Regimes:

Perturbative: Coupling<< Dissipation. Atomic decay suppressed or enhanced (cavity smaller than $\lambda/2$), changes in the energy levels.

Non Perturbative: Coupling>>Dissipation

Vacuum Rabi Splittings. Conditional dynamics.

Dipolar coupling between the atom and the mode of the cavity:

$$g = \frac{d \cdot E_v}{\hbar}$$

El electric field associated with one photon on average in the cavity with volume: $V_{eff}$ is:

$$E_v = \sqrt{\frac{\hbar\omega}{2\varepsilon_0 V_{eff}}}$$

# A note about the dipole approximation:

- The interaction is **d•E**
- The **E** wave has an exp(i**k•r**) term
- Since the extent of **d** (a few Bohr radius) is small compared to the wavelength expand the exponential such that we only keep the $1^{st}$ term.

There is another length scale in the problem:

The extent of the ground state ($z_0$) in the bottom of the well.

Lamb Dicke parameter: $kz_0$

You want to make sure that the change in the kinetic energy of the trapped particle when absorbing or emmiting a photon does not excite the mechanical (external) motion.

# 谢谢