From single atom absorption to waveguide QED; considerations for the light-matter coupling

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The presentation is available at:

http://www.physics.umd.edu/rgroups/amo/orozco/results/2019/Results19.htm

1. A review of Electricity and Magnetism and Polarization

Maxell’s Equations:

$$\nabla \cdot (\varepsilon_0 \mathbf{E} + \mathbf{P}) = 0,$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$

$$\nabla \times \mathbf{B} = \mu_0 \frac{\partial(\varepsilon_0 \mathbf{E} + \mathbf{P})}{\partial t},$$

$$\nabla \cdot \mathbf{B} = 0.$$

Wave Equation:

$$\nabla \times (\nabla \times \mathbf{E}) = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E}$$

$$= -\frac{\partial}{\partial t}(\nabla \times \mathbf{B}) = -\mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} - \mu_0 \frac{\partial^2 \mathbf{P}}{\partial t^2}$$

$$-\nabla(\nabla \cdot \mathbf{E}) + \nabla^2 \mathbf{E} - \mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} = \mu_0 \frac{\partial^2 \mathbf{P}}{\partial t^2}.$$

In free space, $\nabla \cdot \mathbf{E} = 0$ and $\mathbf{P} = \mathbf{0}$,

$$\nabla^2 \mathbf{E} - \frac{1}{v_0^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0$$

A note about polarization

Gauss's Law in free space:

$$\nabla \cdot \vec{E} = 0$$

$$\nabla_T \cdot \vec{E} + \frac{2\pi}{\lambda} i E_Z = 0$$

If there is a "transverse gradient" in the radiation field propagating in *z*, there is a longitudinal polarization also in *z*

Polarization at the waist of an optical nanofiber

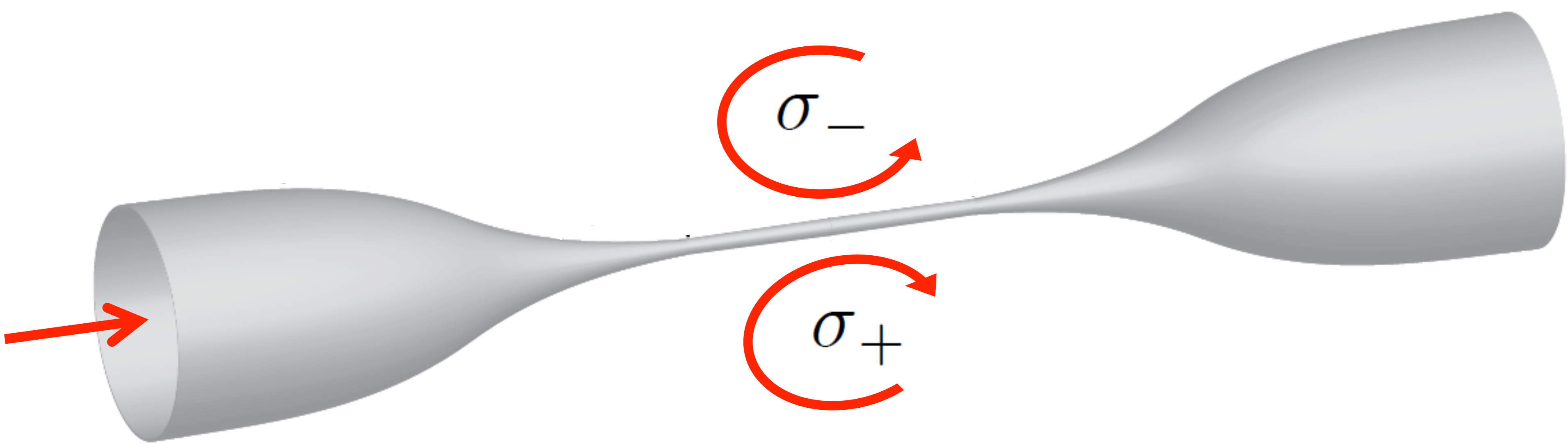
$$\nabla \cdot \vec{E} = 0$$

$$\nabla_T \cdot \vec{E} + \frac{2\pi}{\lambda} i E_Z = 0$$

Polarization at the waist of an optical nanofiber

$$\nabla \cdot \vec{E} = 0$$

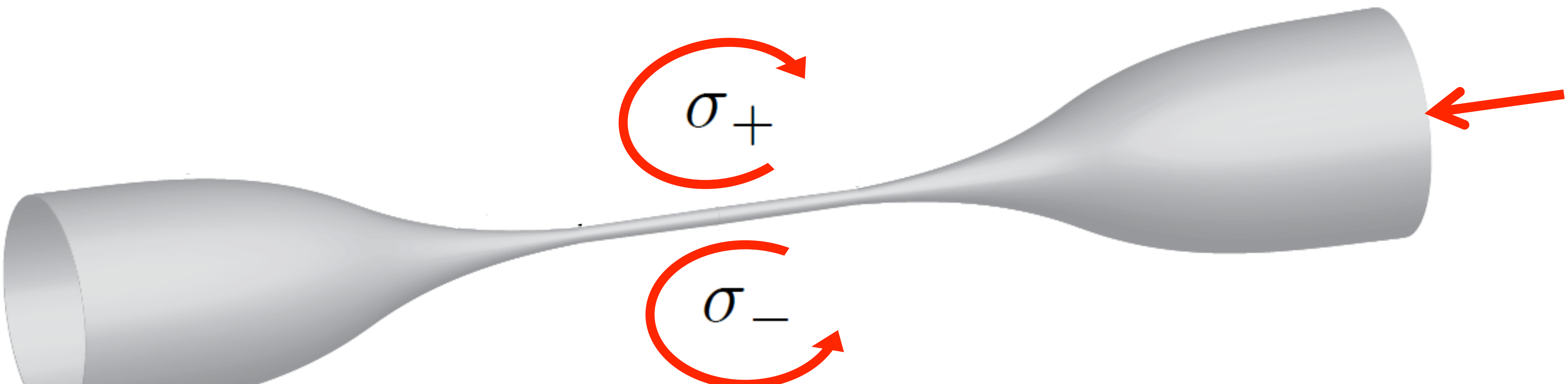
$$\nabla_T \cdot \vec{E} + \frac{2\pi}{\lambda} i E_Z = 0$$



Bicycle polarization not propeller

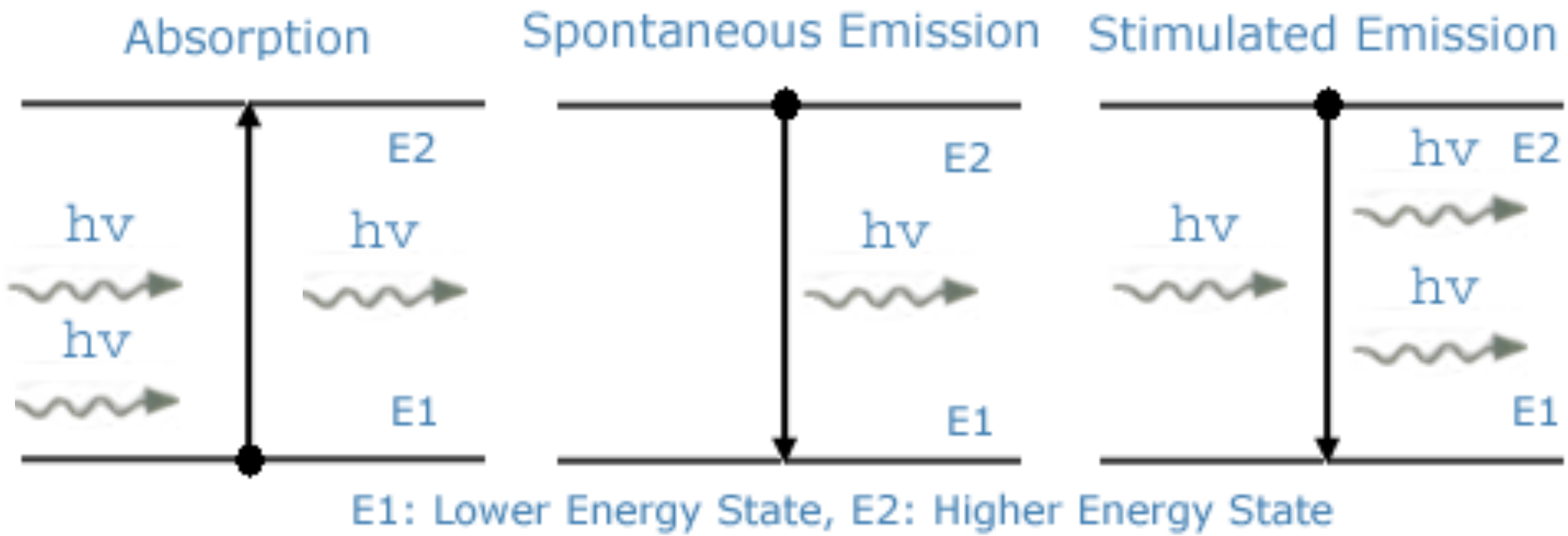
Polarizaciónen la cintura de la nanofibra

$$\begin{aligned}\nabla \cdot \vec{E} &= 0\\ \nabla_T \cdot \vec{E} + \frac{2\pi}{\lambda} i E_Z &= 0\end{aligned}$$



$$E_z = \pm \frac{i}{k} \nabla \cdot \mathbf{E}_\perp$$

2. One atom interacting with light in free space.



Absorption and Stimulated Emission are time reversals of each other, you can say this is the classical part. Spontaneous emission is the quantum, that is the jump.

Dipole cross section (same result for a classical dipole or from a two level atom):

$$\sigma_0 = \frac{3\lambda_0^2}{2\pi}$$

This is the “shadow” caused by a dipole on a beam of light.

Energy due to the interaction between a dipole and an electric field.

$$H = \vec{d} \cdot \vec{E}$$

The dipole matrix element between two states is fixed by the properties of the states (radial part) and the Clebsh-Gordan coefficients from the angular part of the integral. It is a few times a_0 (Bohr radius) times the electron charge e between the S ground and P first excited state in alkali atoms.

$$\vec{d} = e\left\langle 5S_{1/2} \middle| \vec{r} \middle| 5P_{3/2} \right\rangle$$

Beer-Lambert law for intensity attenuation

$$\frac{dI}{dz} = -\alpha I \quad \text{if } \alpha \text{ is resonant and independent}$$

of I, α_0 (does not saturate)

$$I = I_0 \exp(-\alpha_0 l)$$

where $\alpha_0 = \sigma_0 \rho$

and $\rho = N/V$ the density of absorbers in a length l

Rate of decay (Fermi's golden rule)

$$\gamma_{rad} \approx \frac{2\pi}{\hbar}\rho(k)\left\langle H_{int}\right\rangle^2$$

Phase space density

Interaction

Rate of decay free space (Fermi's golden rule)

$$\gamma_0 = \frac{\omega_0^3 d^2}{\pi \varepsilon_0 \hbar c^3}$$

Where *d* is the dipole moment

Saturation intensity:
One photon every two lifetimes over the cross section of the atom (resonant)

$$I_s = \frac{\hbar\omega_0}{2\tau_0\sigma_0} = \frac{\pi}{3}\frac{\gamma_0\hbar\omega_0}{\lambda_0^2}$$

If $I=I_0$ the rate of stimulated emission is equal to the rate of spontaneous emission (Rabi Frequency Ω) and the population on the excited state 1/4.

$$\Omega = \frac{\vec{d} \bullet \vec{E}}{\hbar} = \gamma \sqrt{I / I_s}$$

$$\text{Excited Population} = \frac{1}{2} \frac{I / I_s}{1 + I / I_s}$$

3a. Cavity QED

Optical Cavity QED

Quantum electrodynamics for pedestrians. No need for renormalization. One or a finite number of modes from the cavity.

ATOMS + CAVITY

Regimes:

Perturbative: Coupling<< Dissipation. Atomic decay suppressed or enhanced (cavity smaller than $\lambda/2$), changes in the energy levels.

Non Perturbative: Coupling>>Dissipation

Vacuum Rabi Splittings. Conditional dynamics.

Dipolar coupling between the atom and the mode of the cavity:

$$g = \frac{d \cdot E_v}{\hbar}$$

El electric field associated with one photon on average in the cavity with volume: V_{eff} is:

$$E_v = \sqrt{\frac{\hbar\omega}{2\varepsilon_0 V_{eff}}}$$

Radiation field:

$$\frac{\partial}{\partial t}\langle\hat{a}\rangle = -\kappa(1+i\theta)\langle\hat{a}\rangle + \sum_{j=1}^{N} g_j\langle\hat{\sigma}_j^-\rangle + \mathcal{E},$$

Atomic polarization:

$$\frac{\partial}{\partial t}\langle\hat{\sigma}_j^-\rangle = -\gamma_\perp(1+i\Delta)\langle\hat{\sigma}_j^-\rangle + g_j\langle\hat{a}\rangle\langle\hat{\sigma}_j^z\rangle,$$

Atomic inversion:

$$\frac{\partial}{\partial t}\langle\hat{\sigma}_j^z\rangle = -\gamma_\parallel\left(\langle\hat{\sigma}_j^z\rangle + 1\right) - 2g_j\left(\langle\hat{a}\rangle\langle\hat{\sigma}_j^+\rangle + \langle\hat{a}^\dagger\rangle\langle\hat{\sigma}_j^-\rangle\right).$$

The cavity and atomic detunings θ and Δ are defined as

$$\theta = \frac{\omega_c - \omega_l}{\kappa} \;\text{ and }\; \Delta = \frac{\omega_a - \omega_l}{\gamma_\perp}\;.$$

$$\dot{x} = \kappa\left(+2Cp + y - (i\Theta + 1)x\right)$$
$$\dot{p} = \gamma\left(-(1 + i\Delta)p + xD\right)/2$$
$$\dot{D} = \gamma\left(2(1 - D) - (x^*p + xp^*)\right)$$

$$\Theta = \frac{\omega_c - \omega_l}{\kappa}; \quad \Delta = \frac{\omega_a - \omega_l}{\gamma / 2}$$

γ is the rate of spontaneous emission (energy decay)

κ is the rate of escape of the field

$\omega_{a,c,l}$ refer to atom, cavity, laser

Low intensity $x<<1$: with $D=0$, resonant $\Delta=0$ and $\Theta=0$ weakly driven.

Two coupled oscillators

$$\dot{x} = \kappa(-x + 2Cp + y)$$

Two coupled oscillators

$$\dot{p} = \gamma(-p - x)$$

Steady state

$$y = x - 2Cp$$

$$p = -x$$

$$y = x(1 + 2C)$$

$$\kappa >> \gamma \quad \dot{p} = -\gamma(1 + 2C)p - \gamma y$$

$$\gamma >> \kappa \quad \dot{x} = -\kappa(1 + 2C)x + \kappa y$$

Enhanced emission

Steady state with detuning and at all intensities:

$$y = x\left(1 + \frac{2C}{1+\Delta^2+|x|^2}\right) + ix\left(\theta - \frac{2C\Delta}{1+\Delta^2+|x|^2}\right)$$

Dispersive limit when $\Theta=0$ and $\Delta >> 1$:

$$y = -ix\frac{2C\Delta}{1+\Delta^2+|x|^2}$$

Steady state with detuning and at all intensities:

$$y = x\left(1 + \frac{2C}{1+\Delta^2+|x|^2}\right) + ix\left(\theta - \frac{2C\Delta}{1+\Delta^2+|x|^2}\right)$$

The transmission spectrum:

for $\omega_c = \omega_a$ $\quad \Omega = (\gamma/2)\Delta = \kappa\Theta \quad \Omega_{V.R.} = g\sqrt{N} = \sqrt{C\kappa\gamma}$

$$x = y\frac{\kappa(\gamma_\perp + i\Omega)}{(\kappa+i\Omega)(\gamma_\perp+i\Omega) + \Omega_{V.R.}^2/(1+\gamma_\perp^2|x|^2/(\gamma_\perp^2+\Omega^2))}$$

Two coupled oscillators

$$\frac{x}{y} = \frac{A}{i\Omega - \Omega_1} + \frac{B}{i\Omega - \Omega_2} \, ,$$

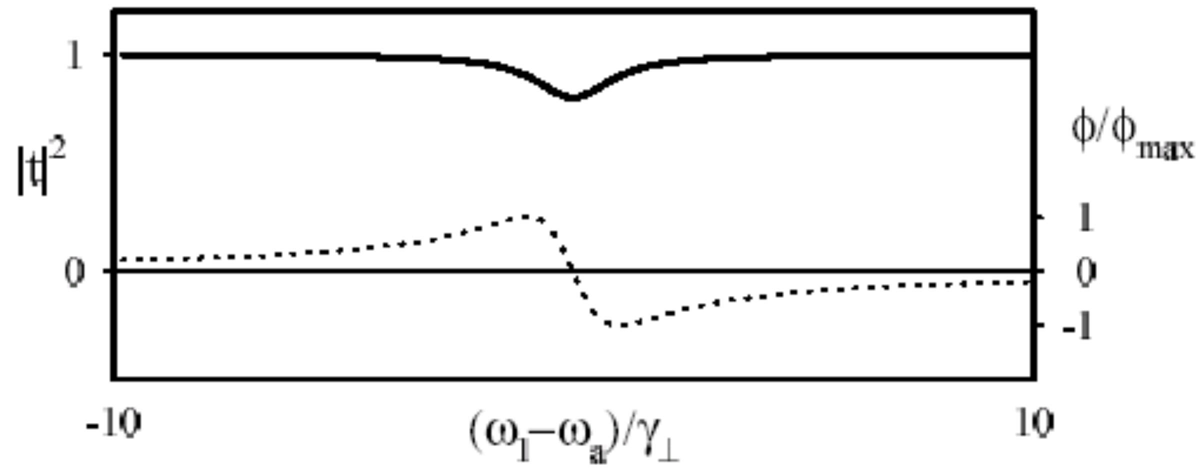
$$A = \kappa \frac{\gamma_\perp + \Omega_1}{\Omega_1 - \Omega_2} \, ,$$

$$B = \kappa \frac{\gamma_\perp + \Omega_2}{\Omega_2 - \Omega_1} \, ,$$

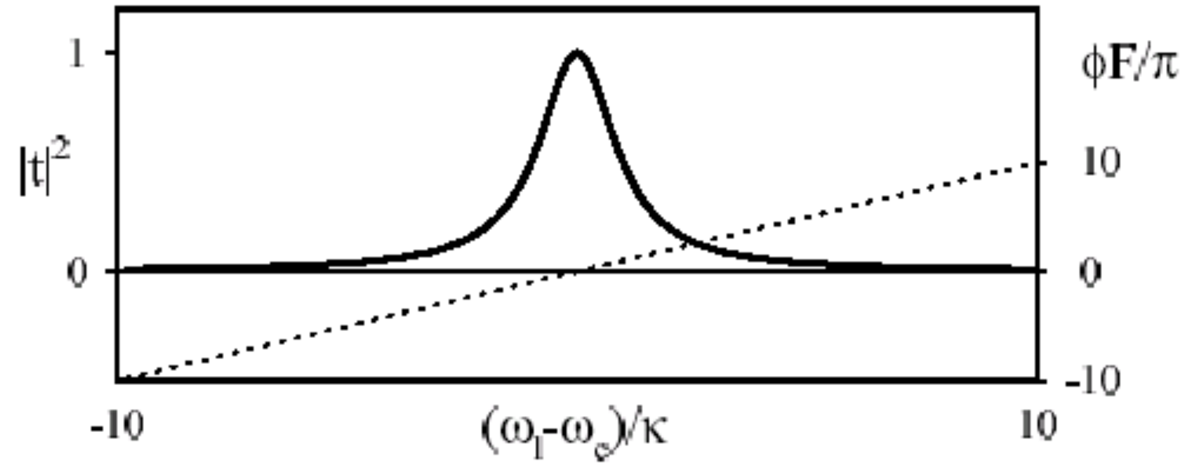
$$\Omega_{1,2} = -\frac{\kappa + \gamma_\perp}{2} \pm i \sqrt{-\left(\frac{\kappa - \gamma_\perp}{2}\right)^2 + \frac{\Omega_{V.R.}^2}{1 + \gamma_\perp^2 |x|^2 / (\gamma_\perp^2 + \Omega^2)}} \, .$$

Cavity mode and atomic polarization

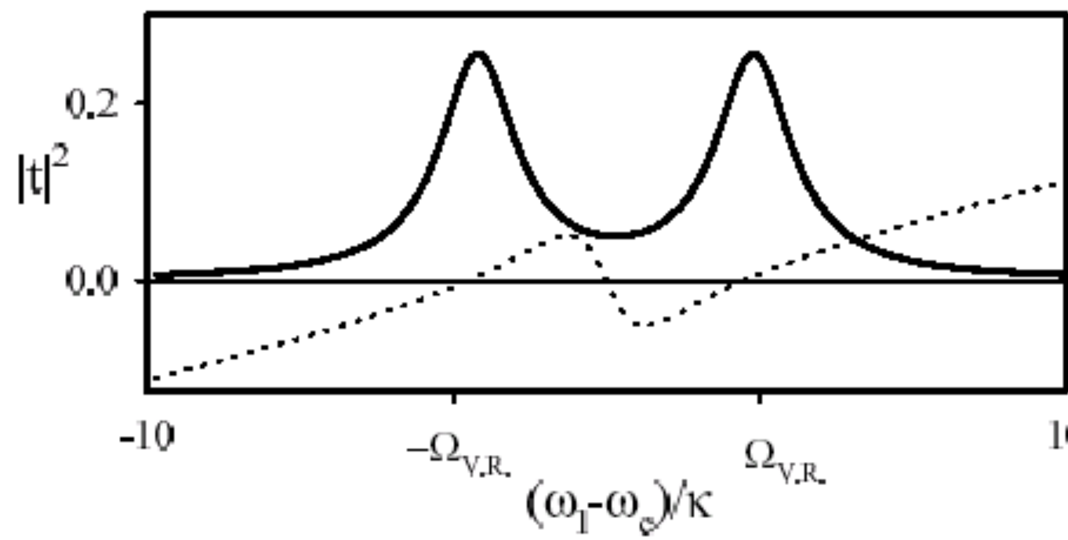
Atomic absorption



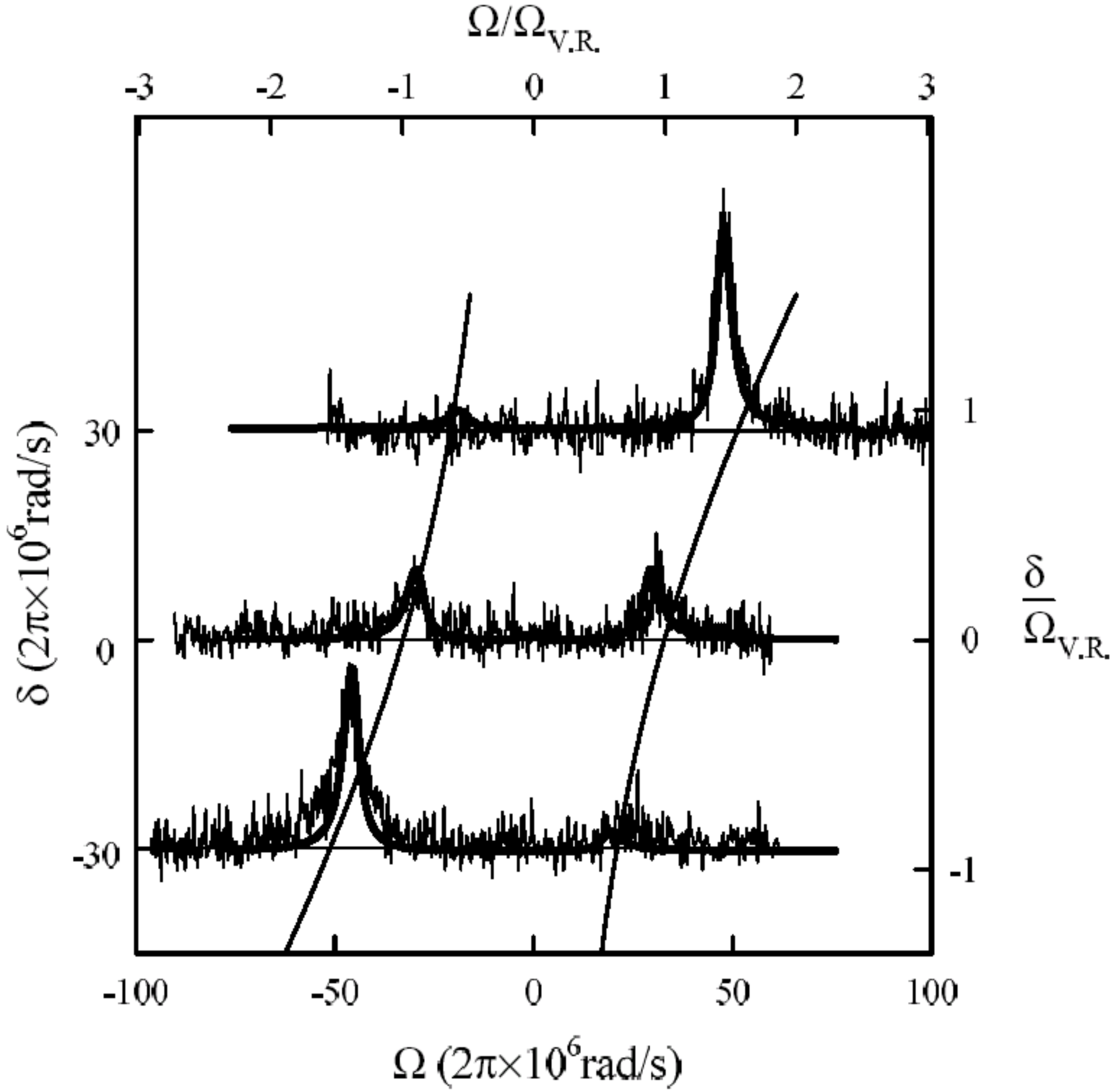
Fabry Perot



Coupled modes



Transmission spectrum at low intensity for varying atomic detunings.



Quantum Hamiltonian for *N* atoms

$$\hat{H} = \hat{H}_1 + \hat{H}_1 + \hat{H}_2 + \hat{H}_3 + \hat{H}_4 + \hat{H}_5 \ ,$$

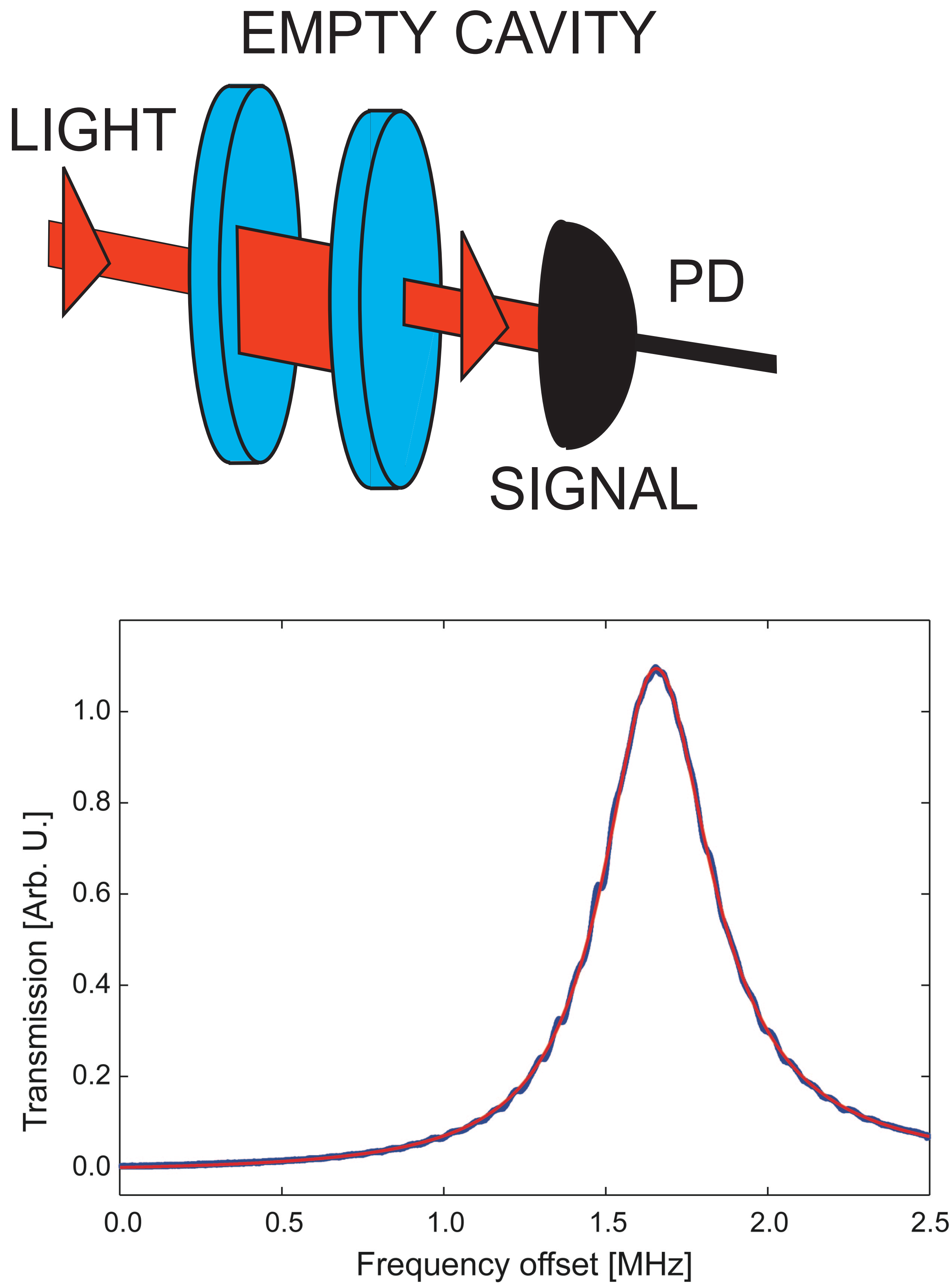
$$\hat{H}_1 = \hbar\omega_c \hat{a}^\dagger \hat{a} + \frac{1}{2}\hbar\omega_a \sum_{j=1}^{N} \hat{\sigma}_j^z \ ,$$ Free atoms free field

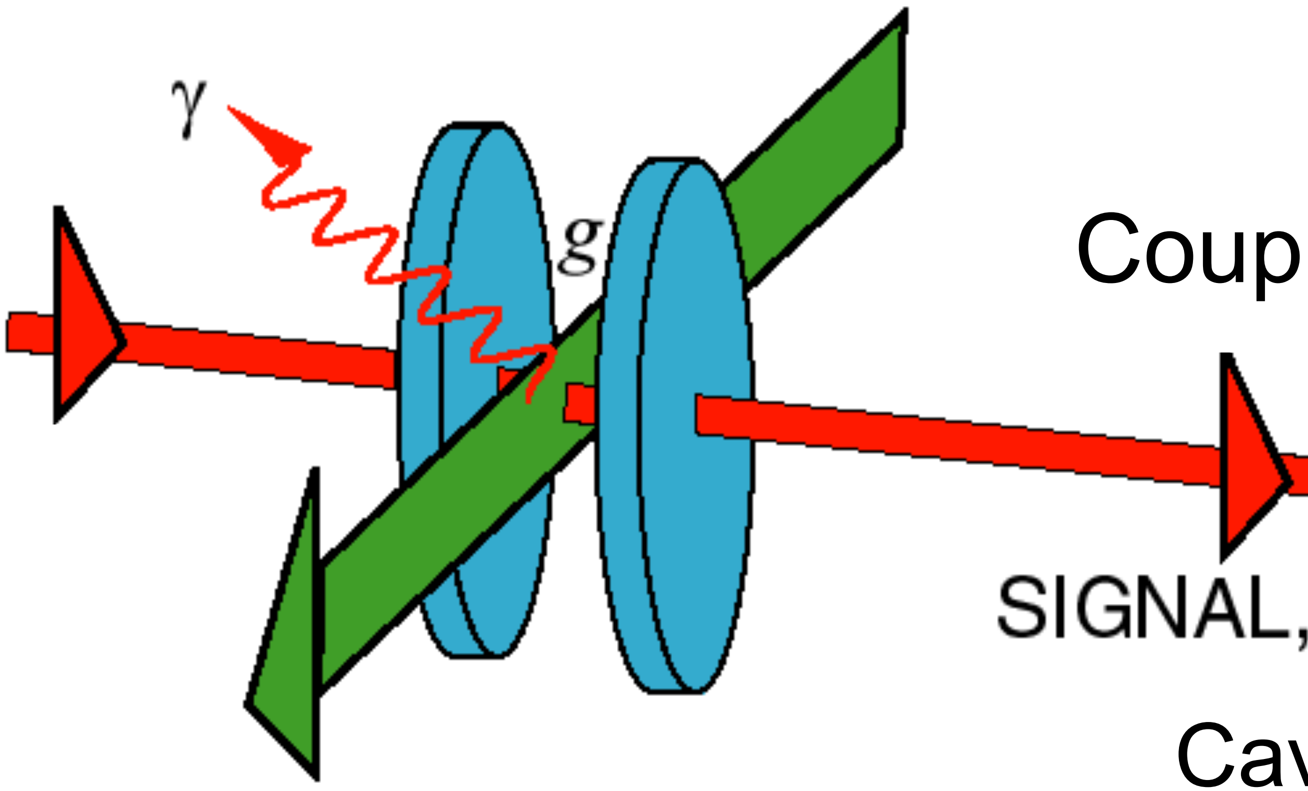
J.C $$\hat{H}_2 = i\hbar \sum_{j=1}^{N} g_j \left(\hat{a}^\dagger \hat{\sigma}_j^- e^{-i\vec{k}\cdot\vec{r}_j} - \hat{a}\hat{\sigma}_j^+ e^{i\vec{k}\cdot\vec{r}_j} \right)$$ Interaction

$$\hat{H}_3 = \sum_{j=1}^{N} \left(\hat{\Gamma}_A \hat{\sigma}_j^+ + \hat{\Gamma}_A^\dagger \hat{\sigma}_j^- \right) \ ,$$ Atomic decay

$$\hat{H}_4 = \hat{\Gamma}_F \hat{a}^\dagger + \hat{\Gamma}_F^\dagger \hat{a} \ ,$$ Cavity decay

$$\hat{H}_5 = i\hbar \left(\hat{a}^\dagger \mathcal{E} e^{-i\omega_l t} - \hat{a}\mathcal{E}^* e^{i\omega_l t} \right) \ .$$ Drive


EMPTY CAVITY
LIGHT
PD
SIGNAL
Transmission [Arb. U.]
0.0
0.2
0.4
0.6
0.8
1.0
0.0
0.5
1.0
1.5
2.0
2.5
Frequency offset [MHz]



Cooperativity for one atom: C_1

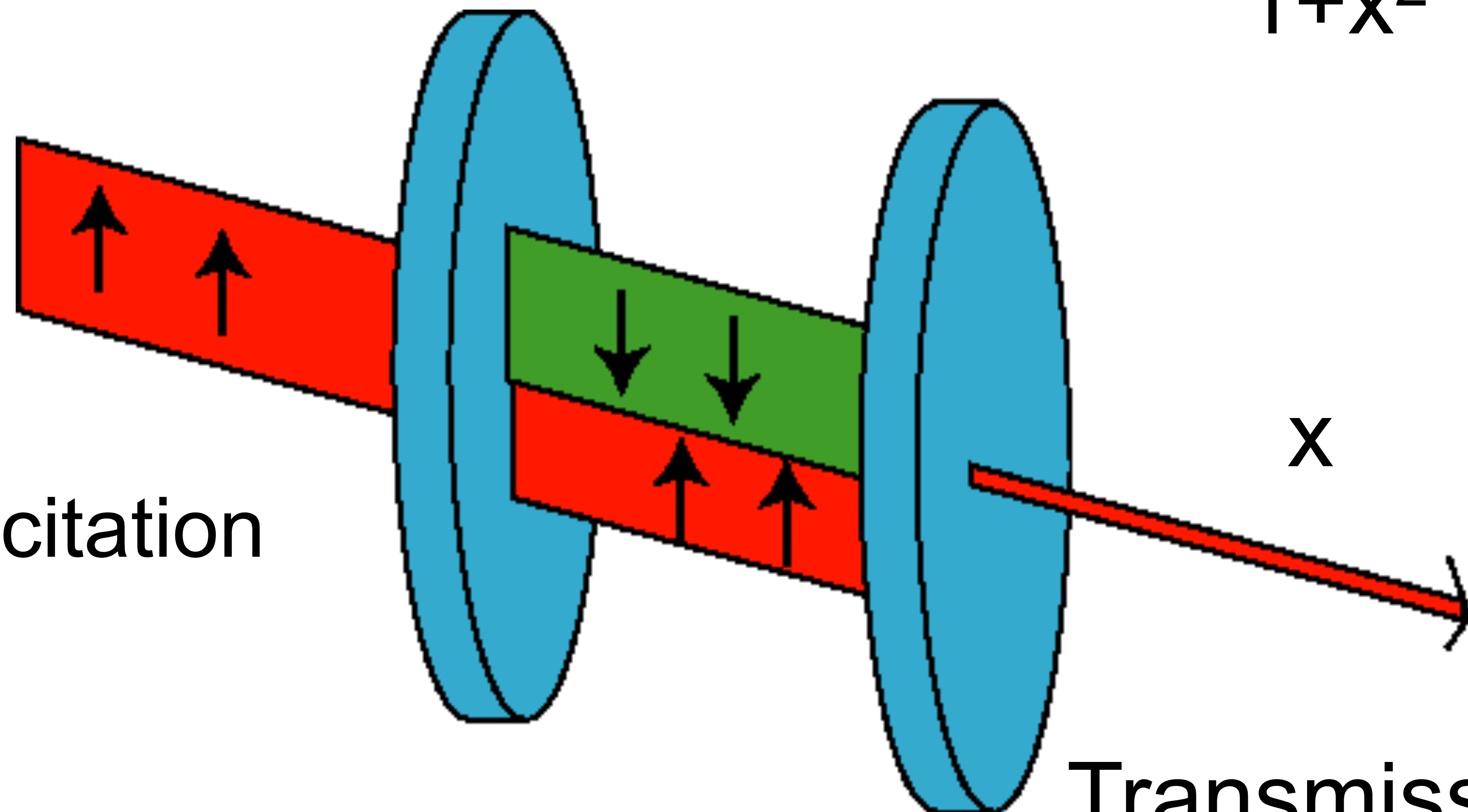
$$C_1 = \frac{g^2}{\kappa\gamma} \qquad C = C_1 N$$

Cooperativity for N atoms: C

$$g \approx \kappa \approx \gamma$$

Steady State

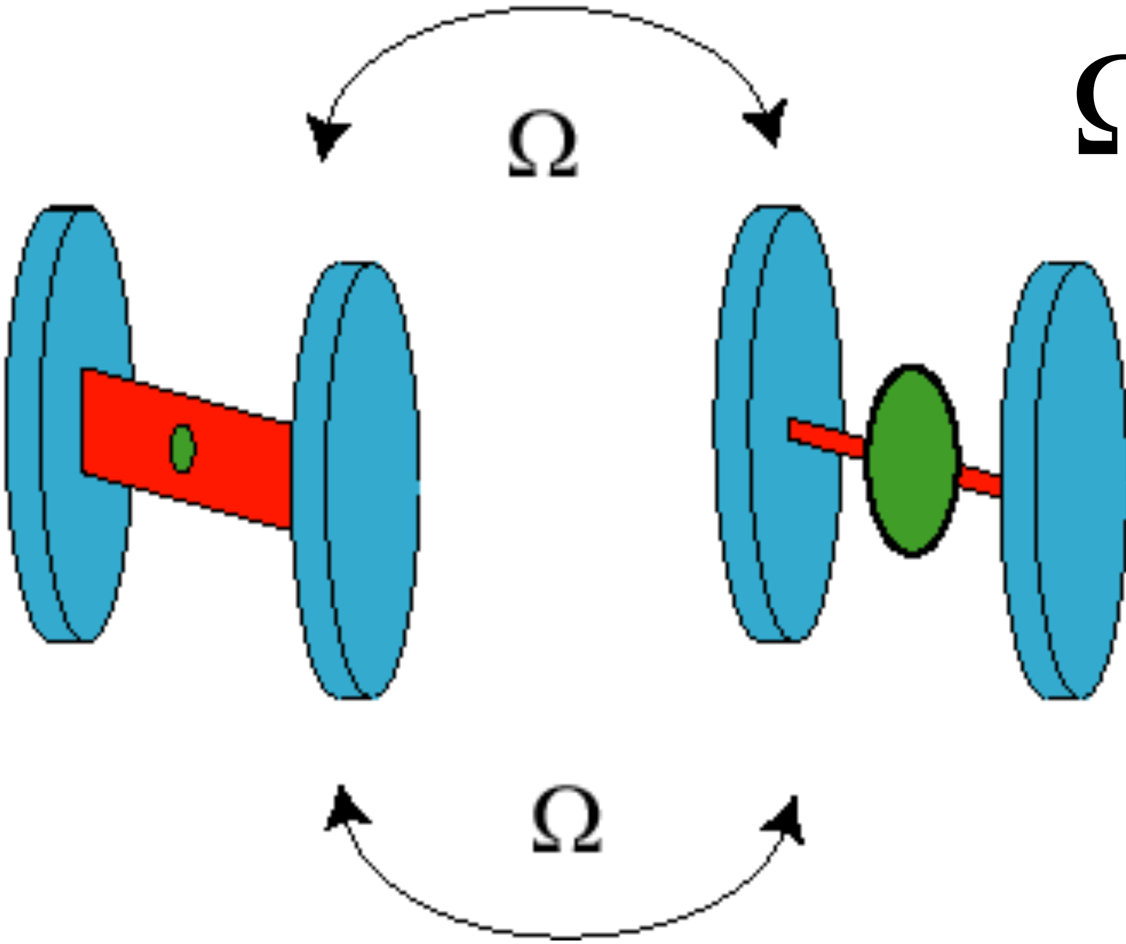
Atomic polarization: $\frac{-2Cx}{1+x^2}$



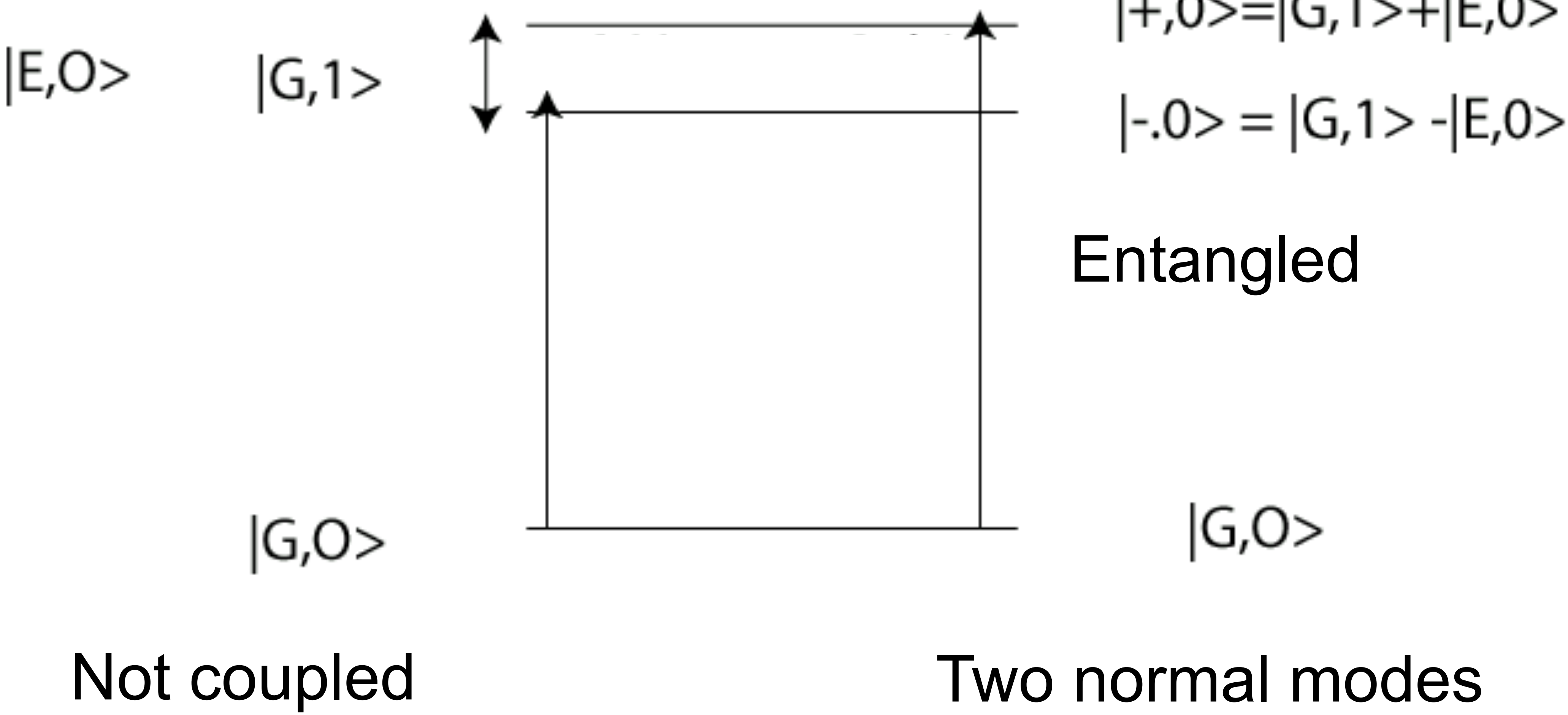
Jaynes Cummings Dynamics
Rabi Oscillations

Exchange of excitation for *N* atoms:

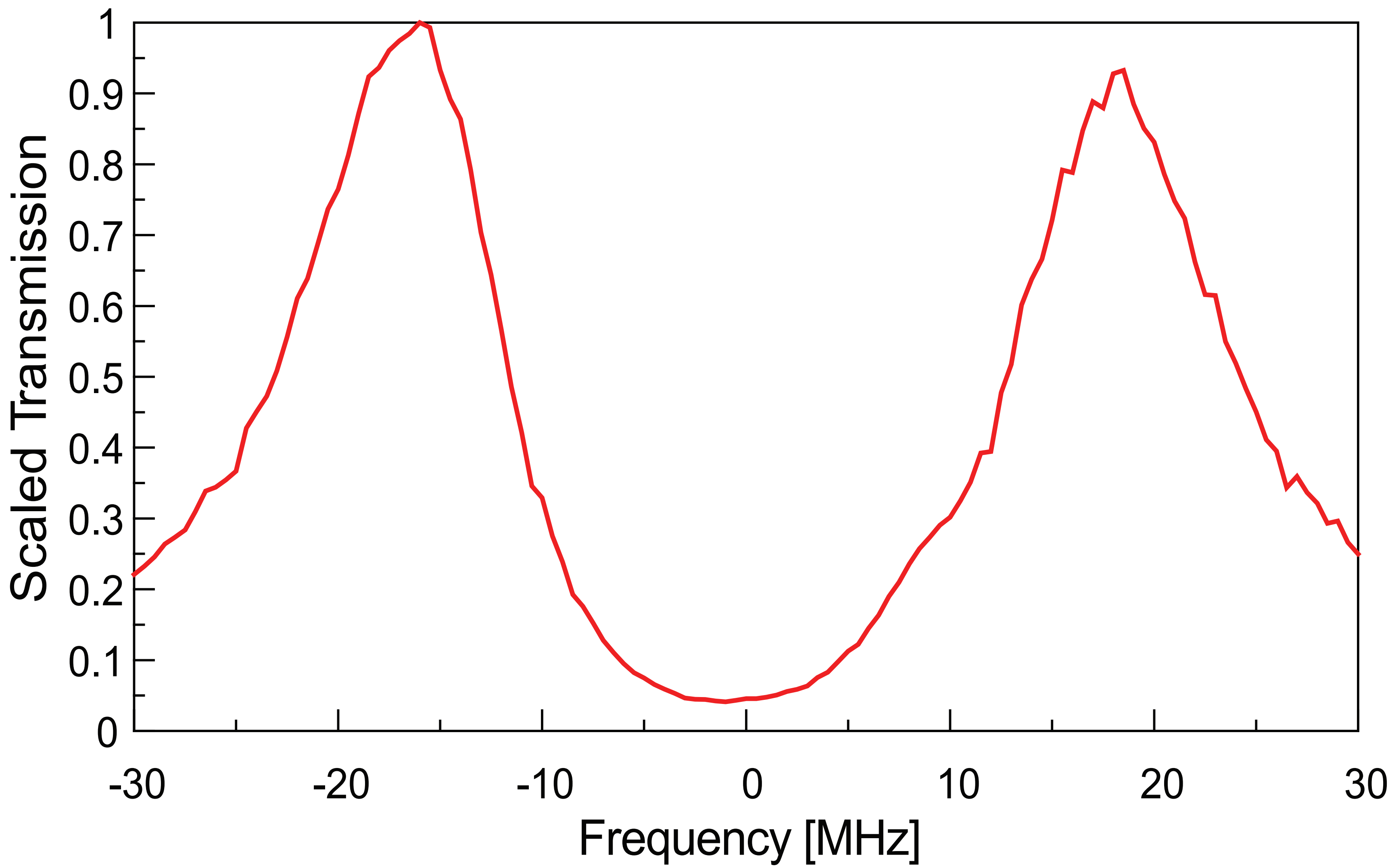
$$\Omega \approx g\sqrt{N}$$



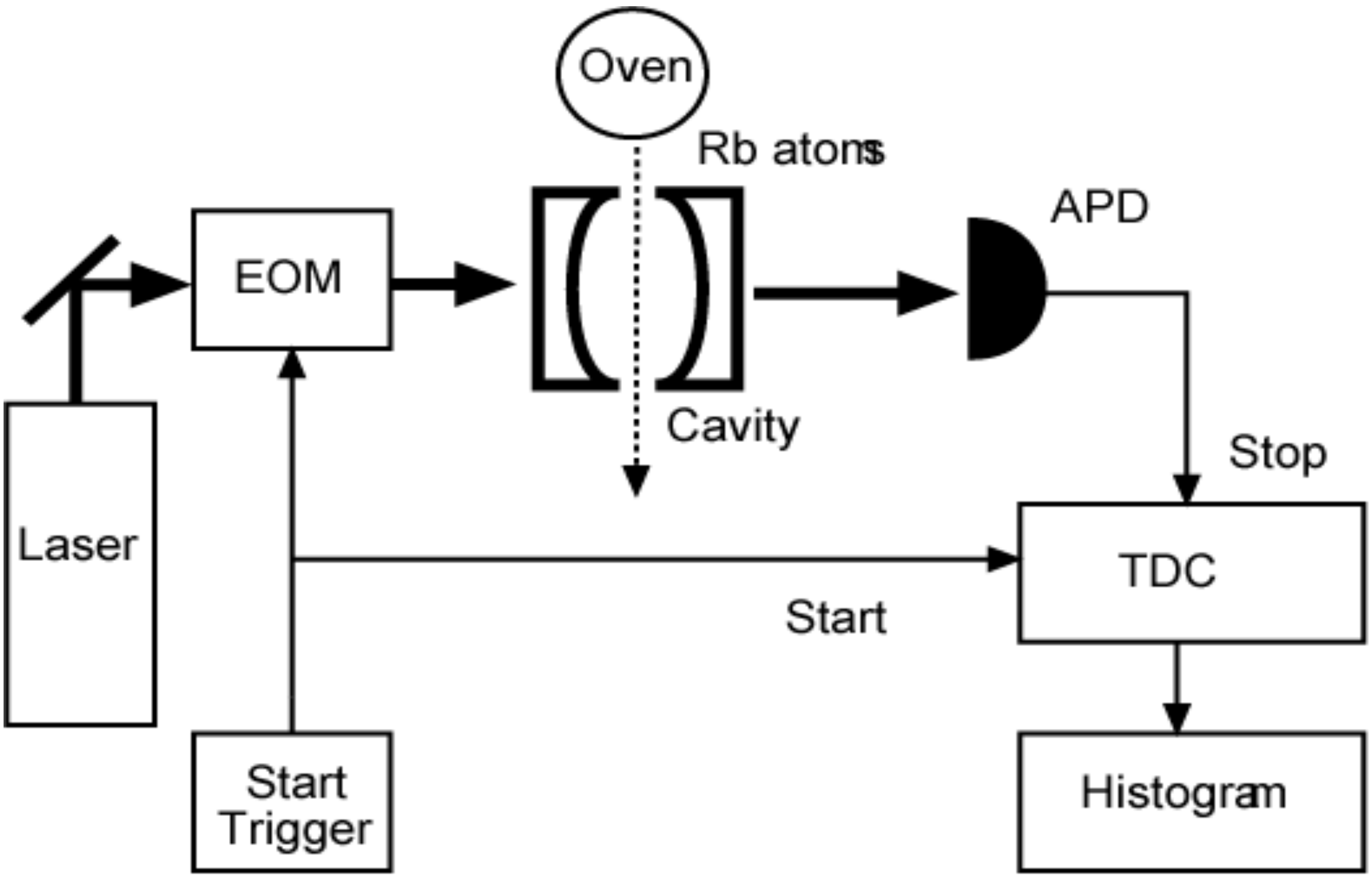
2g Vacuum Rabi Splitting



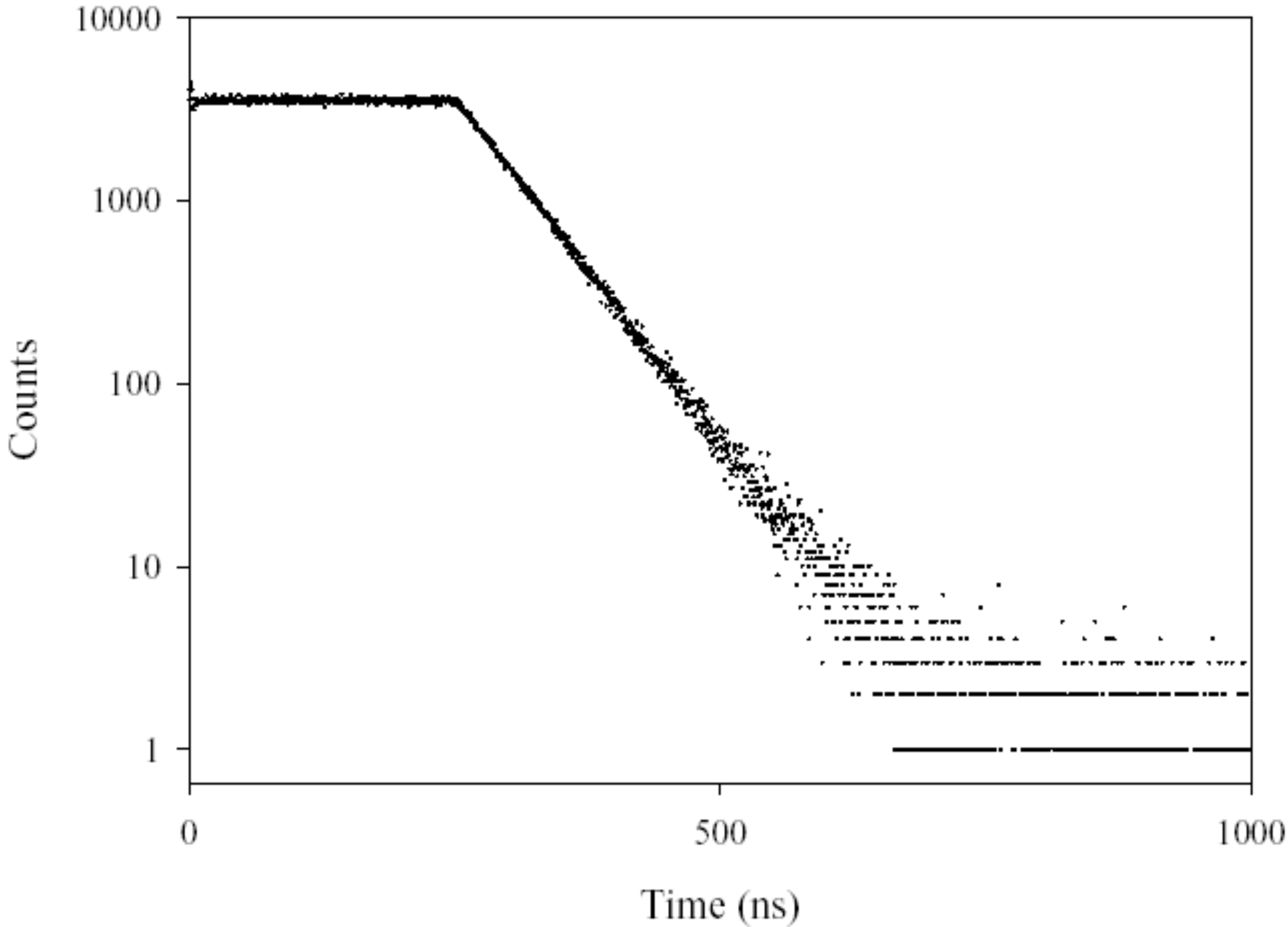
Transmission doublet different from the Fabry Perot resonance



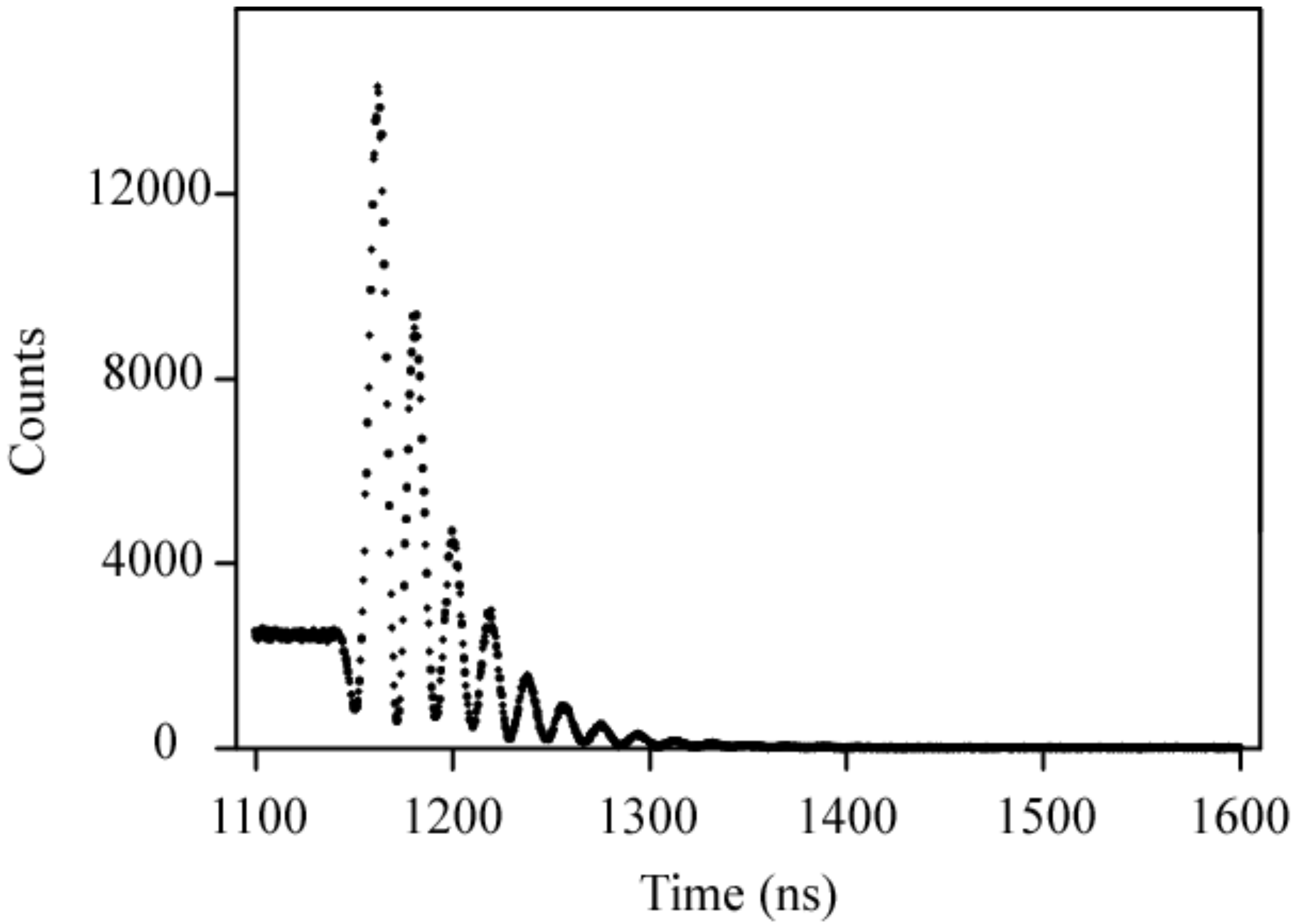
Study the system dynamics classically by providing a step function.

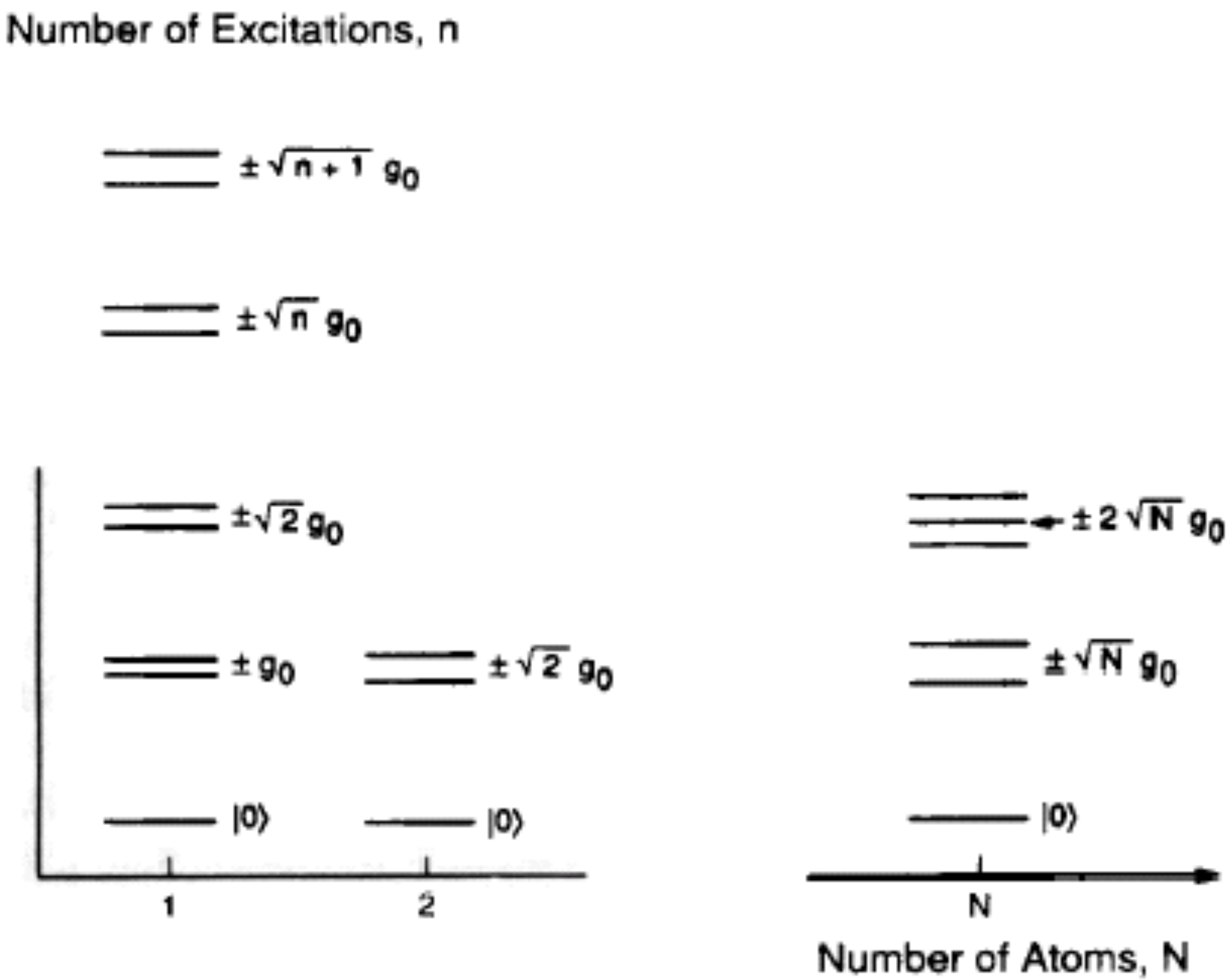


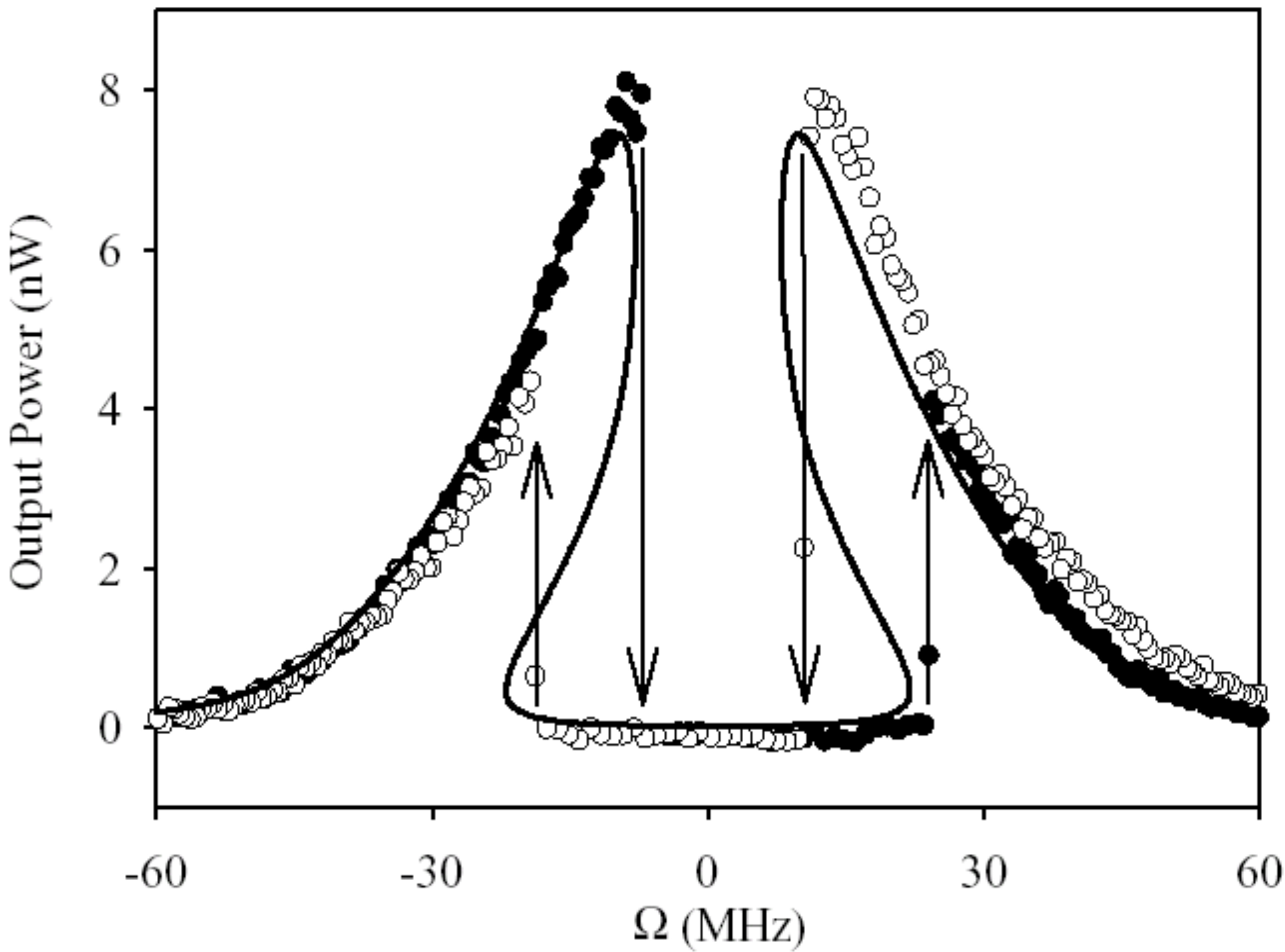
Decay of the empty cavity



Response to step down excitation



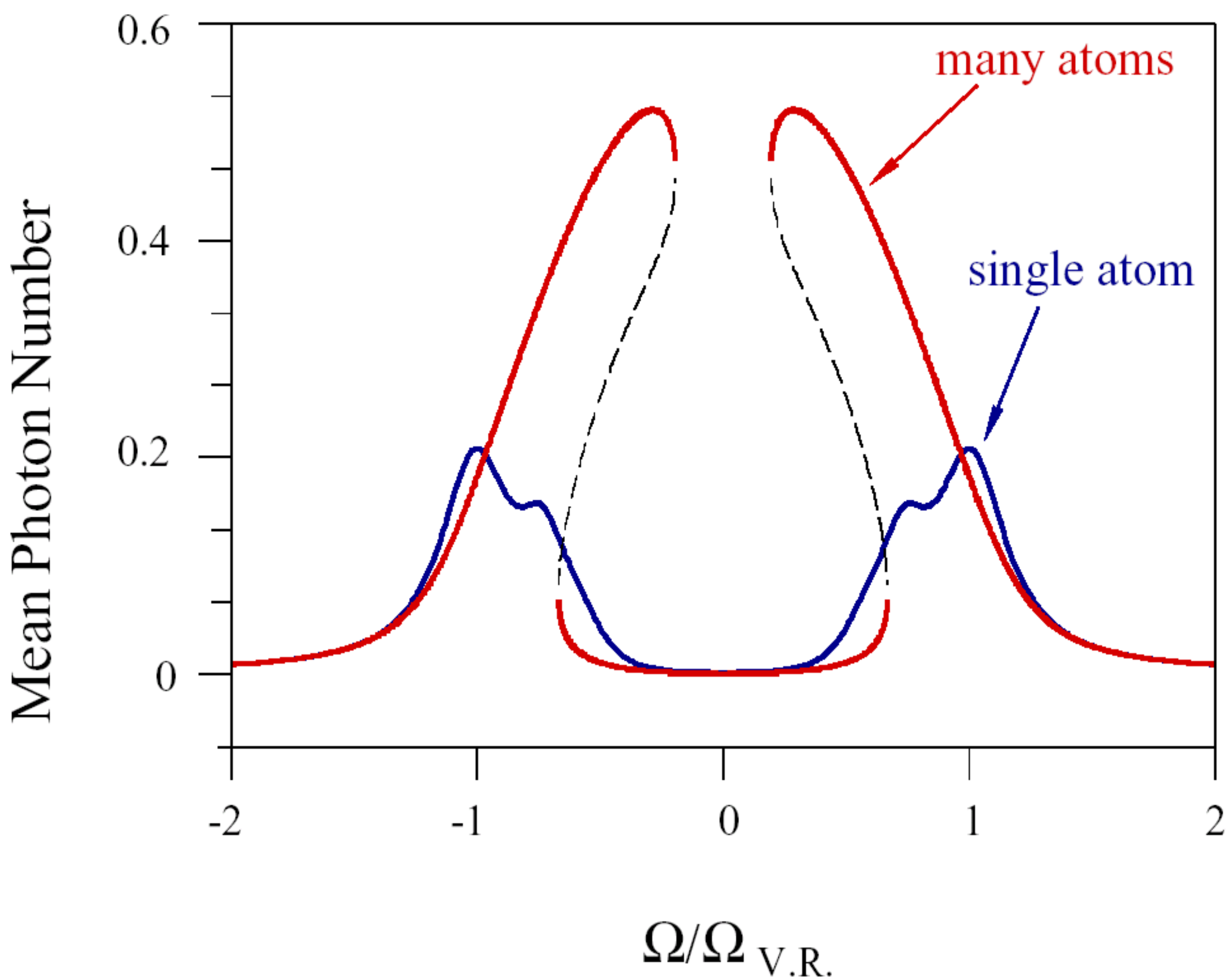

Number of Excitations, n
$\pm\sqrt{n+1}\,g_0$
$\pm\sqrt{n}\,g_0$
$\pm\sqrt{2}\,g_0$
$\pm g_0$
$\pm\sqrt{2}\,g_0$
$|0\rangle$
$|0\rangle$
1
2
$\pm 2\sqrt{N}\,g_0$
$\pm\sqrt{N}\,g_0$
$|0\rangle$
N
Number of Atoms, N



Hysteresis for a frequency scan of the light from the coupled atoms-cavity system.

Many atoms solved with Maxwell Bloch equations.

Single atom solved with the full Hamiltonian, no decorrelation. The system does not show hysteresis.



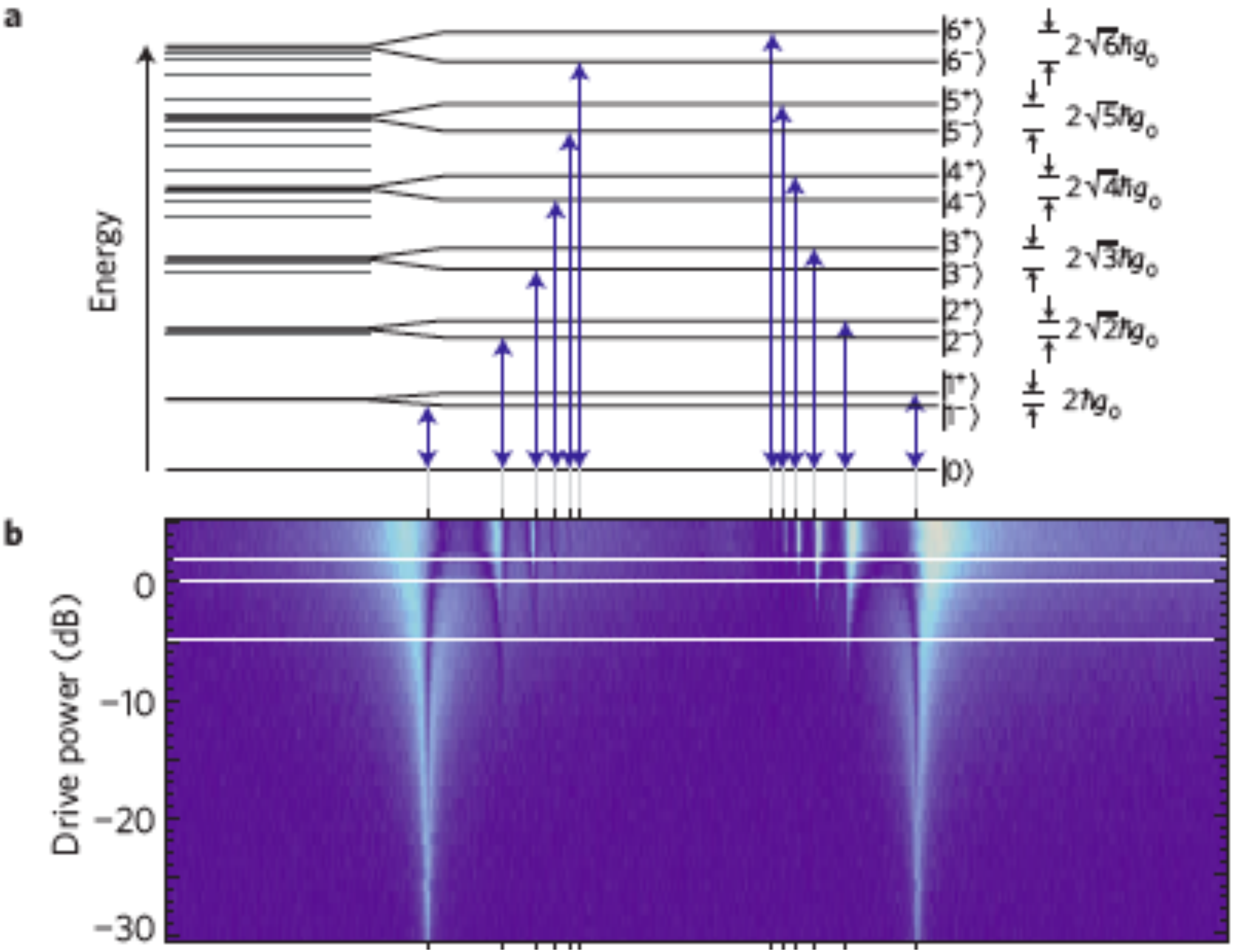
nature physics

LETTERS

PUBLISHED ONLINE: 14 DECEMBER 2008; CORRECTED ONLINE: 15 DECEMBER 2008 | DOI: 10.1038/NPHYS1154

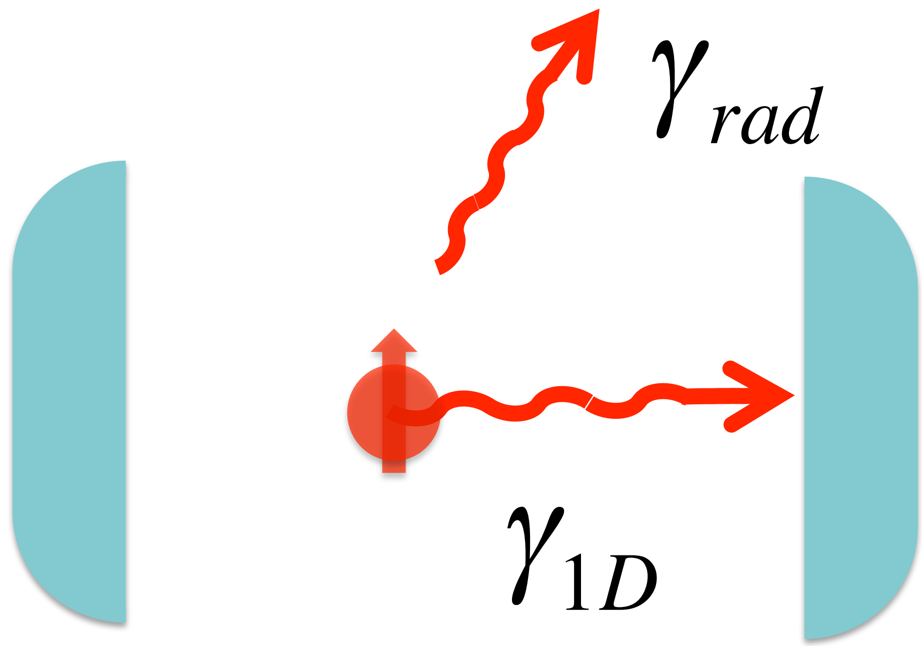
Nonlinear response of the vacuum Rabi resonance

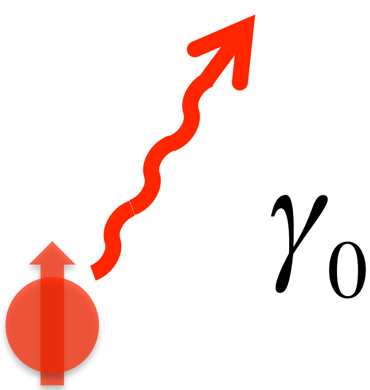
Lev S. Bishop[1], J. M. Chow[1], Jens Koch[1], A. A. Houck[1], M. H. Devoret[1], E. Thuneberg[2], S. M. Girvin[1] and R. J. Schoelkopf[1]*



a
Energy
|6+⟩
|6−⟩
2√6ℏg0
|5+⟩
|5−⟩
2√5ℏg0
|4+⟩
|4−⟩
2√4ℏg0
|3+⟩
|3−⟩
2√3ℏg0
|2+⟩
|2−⟩
2√2ℏg0
|1+⟩
|1−⟩
2ℏg0
|0⟩
b
Drive power (dB)
0
−10
−20
−30

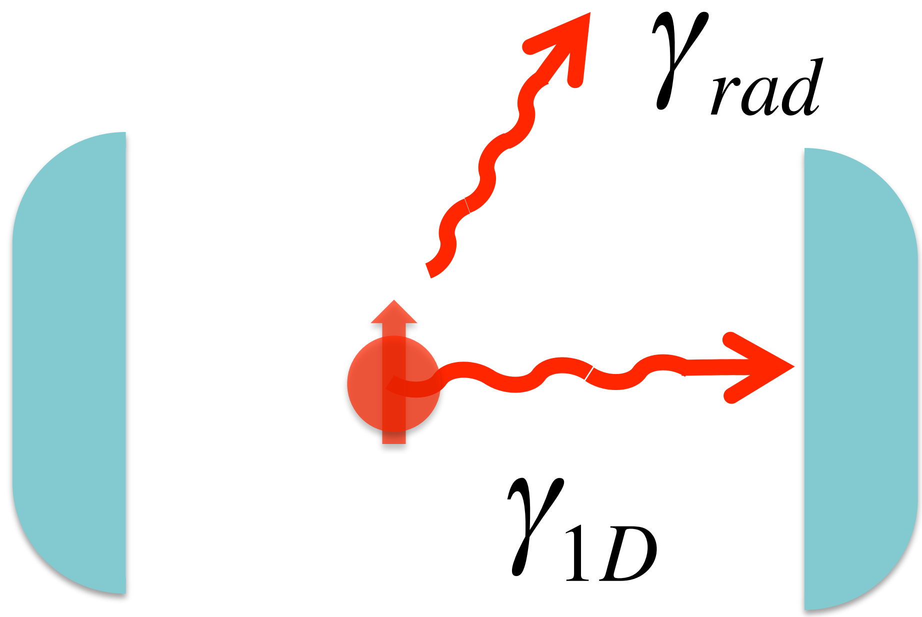
Coupling Enhancement

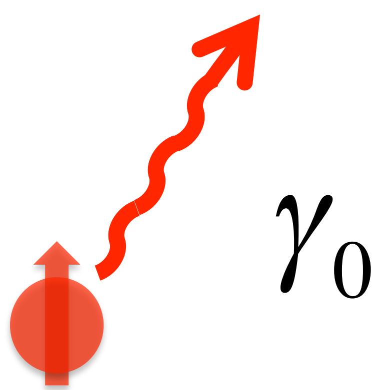




$$\alpha = \frac{\gamma_{1D}}{\gamma_0}$$

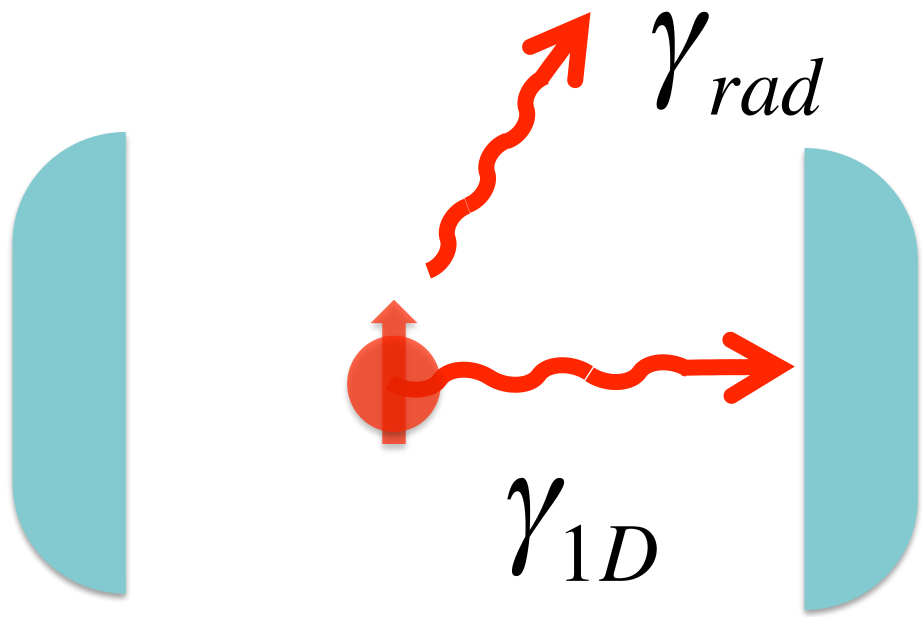
Coupling Efficiency

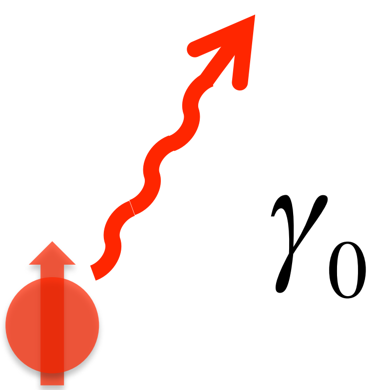




$$\beta = \frac{\gamma_{1D}}{\gamma_{Tot}} \quad ; \quad \gamma_{Tot} = \gamma_{1D} + \gamma_{rad}$$

Purcell Factor

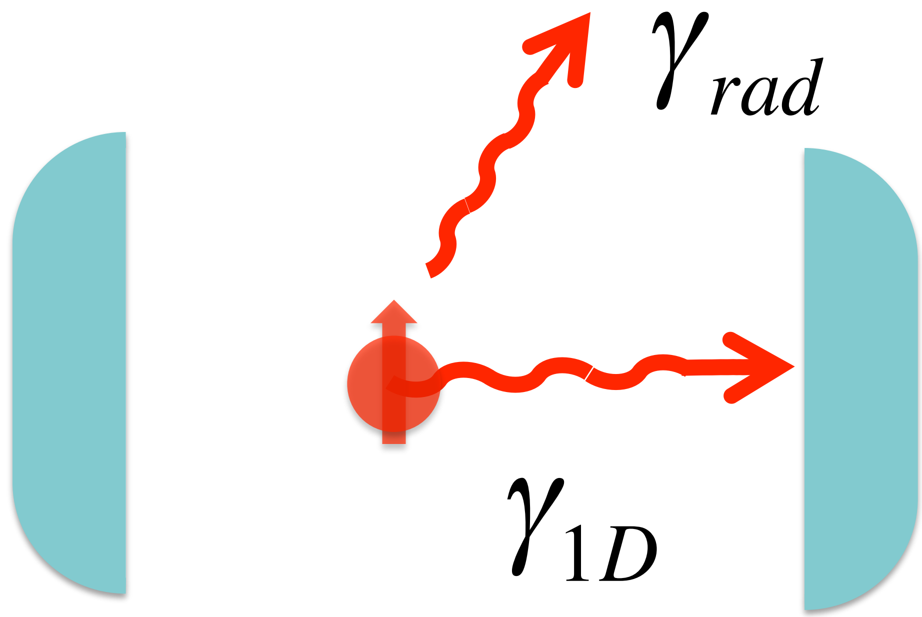


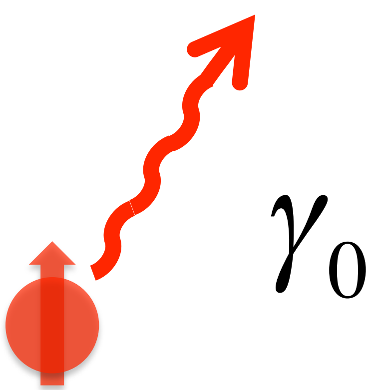


$$F_P = \frac{\gamma_{tot}}{\gamma_0} = \frac{\alpha}{\beta}$$

$$\gamma_{Tot} = \gamma_{1D} + \gamma_{rad}$$

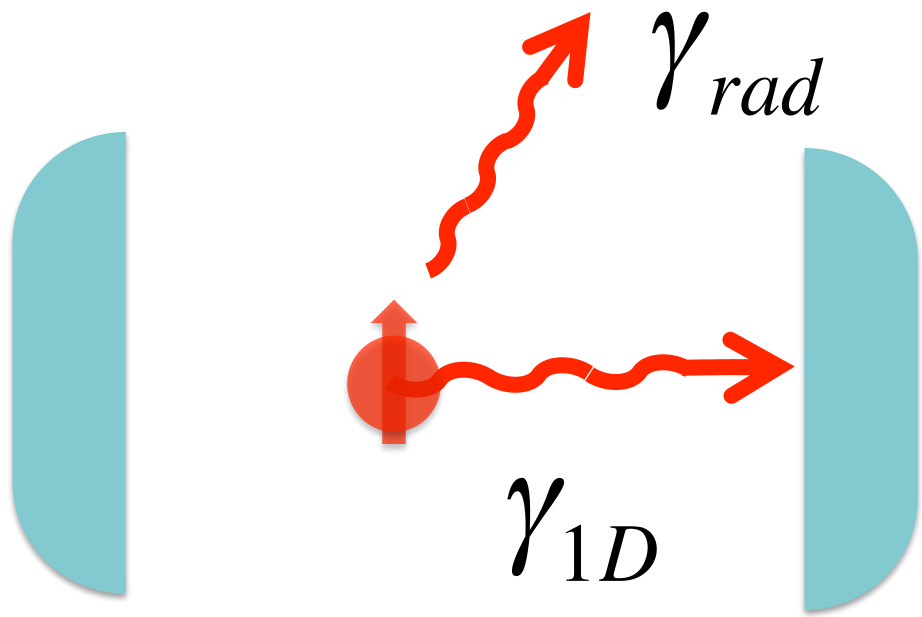
Cooperativity

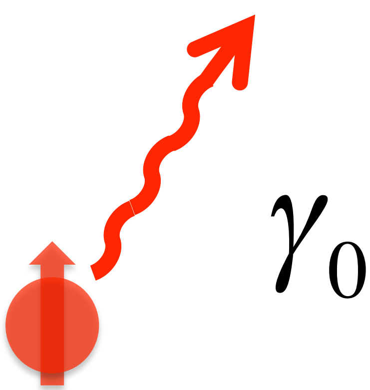




$$C_1 = \frac{\beta}{(1-\beta)} = \frac{\gamma_{1D}}{\gamma_{rad}}$$

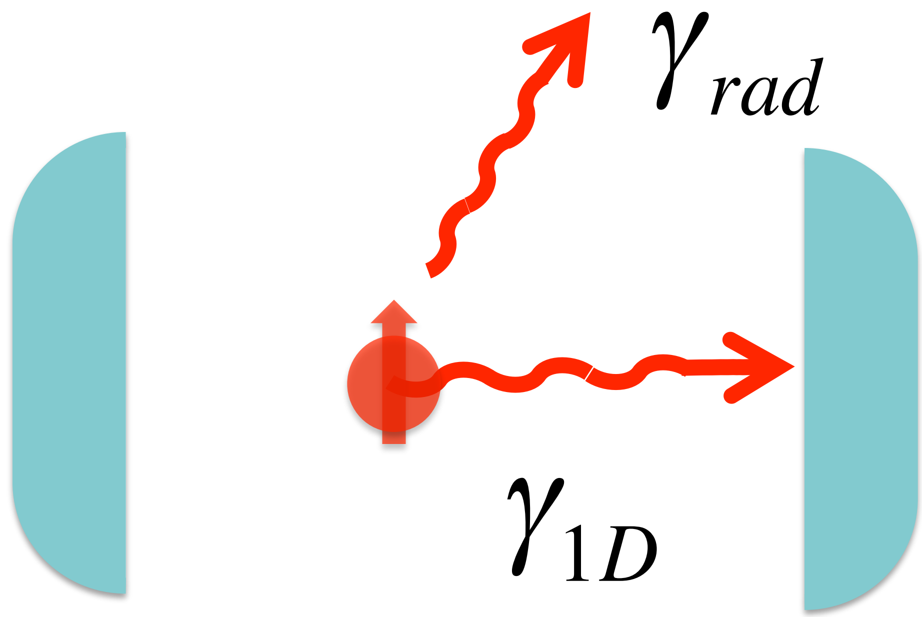
Cooperativity

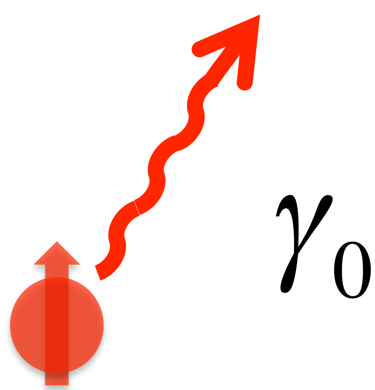




$$C_1 = \frac{g^2}{\kappa\gamma_0}$$

Cooperativity





$$C_1 = \frac{\sigma_0}{Area_{\mathrm{mode}}} \frac{1}{T}$$

The coupling enhancement α is proportional to the total number of photons emitted into the cavity mode,

The coupling efficiency β is the percentage of photons emitted into the mode relative to the total number of emitted photons.

The cooperativity is the ratio between the photons going into the mode and those emitted out to other modes. It is the inverse of the number of atoms that are necessary to observe non-linear effects in the cavity.

Some Implementations

- Rydbergs on Superconducting cavities (Microwaves)
- Alkali atoms on Optical Cavities (Optical)
- Quantum dots on microcavities (Optical)
- Trapped ions and vibrational mode (phonons)
- Circuit QED Superconducting qbits on microwave resonators (Microwaves)
- Polaritons on optical microcavities (photons)

The cooperativity has become the figure of merit for many quantum optics experiments, it is not limited to cavity QED.

How to choose a platform?

$$C = \frac{\sigma_0}{Area_{\mathrm{mode}}} \frac{1}{T} N \qquad C = \frac{g^2}{\kappa \gamma_0} N$$

Take the area of the mode to be $\pi(\lambda/2)^2$, and σ as $3\lambda^2/2\pi$ then C does not depend on the "atom"

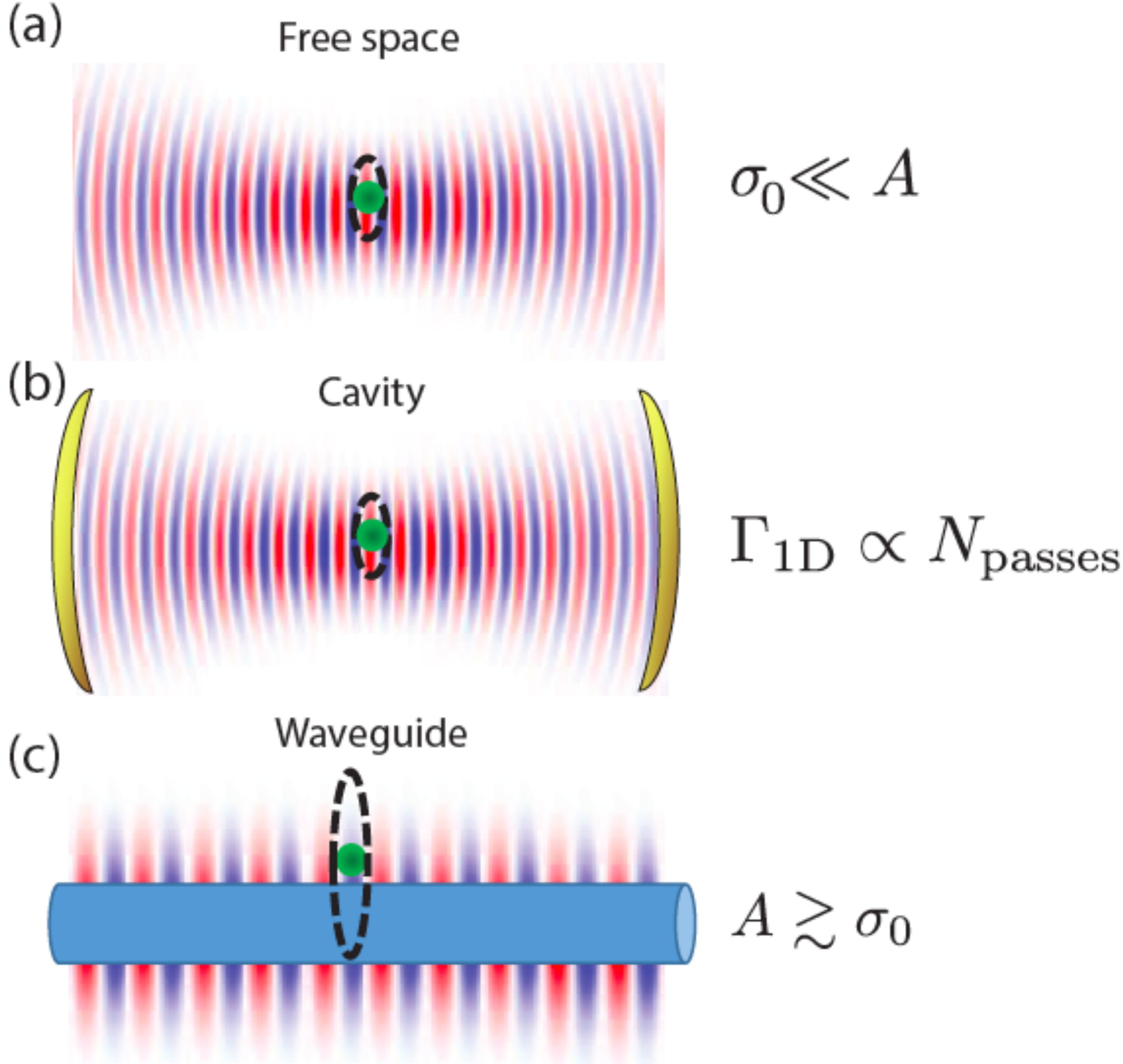
Another approach is to maximize g through a large E_0, then minimize the cavity volume V

The solutions are guided by your resources and where you can approach the ideals

Microwaves can be confined to cavities with mode areas close to the atomic cross section of the Rydberg Atoms. (Experiments led by S. Haroche)

This is more difficult in the visible for free space with atoms, but recent developments at ENS on making micrometric mirrors are helping.

3b. From Cavity QED to Waveguide QED



PhD Thesis Jonathan Hood

Decay into the nanofiber mode

Density of modes in 1D

$$\gamma_{1D} \approx \frac{2\pi}{\hbar} \rho(k) \langle H_{int} \rangle^2$$

Decay into the nanofiber mode

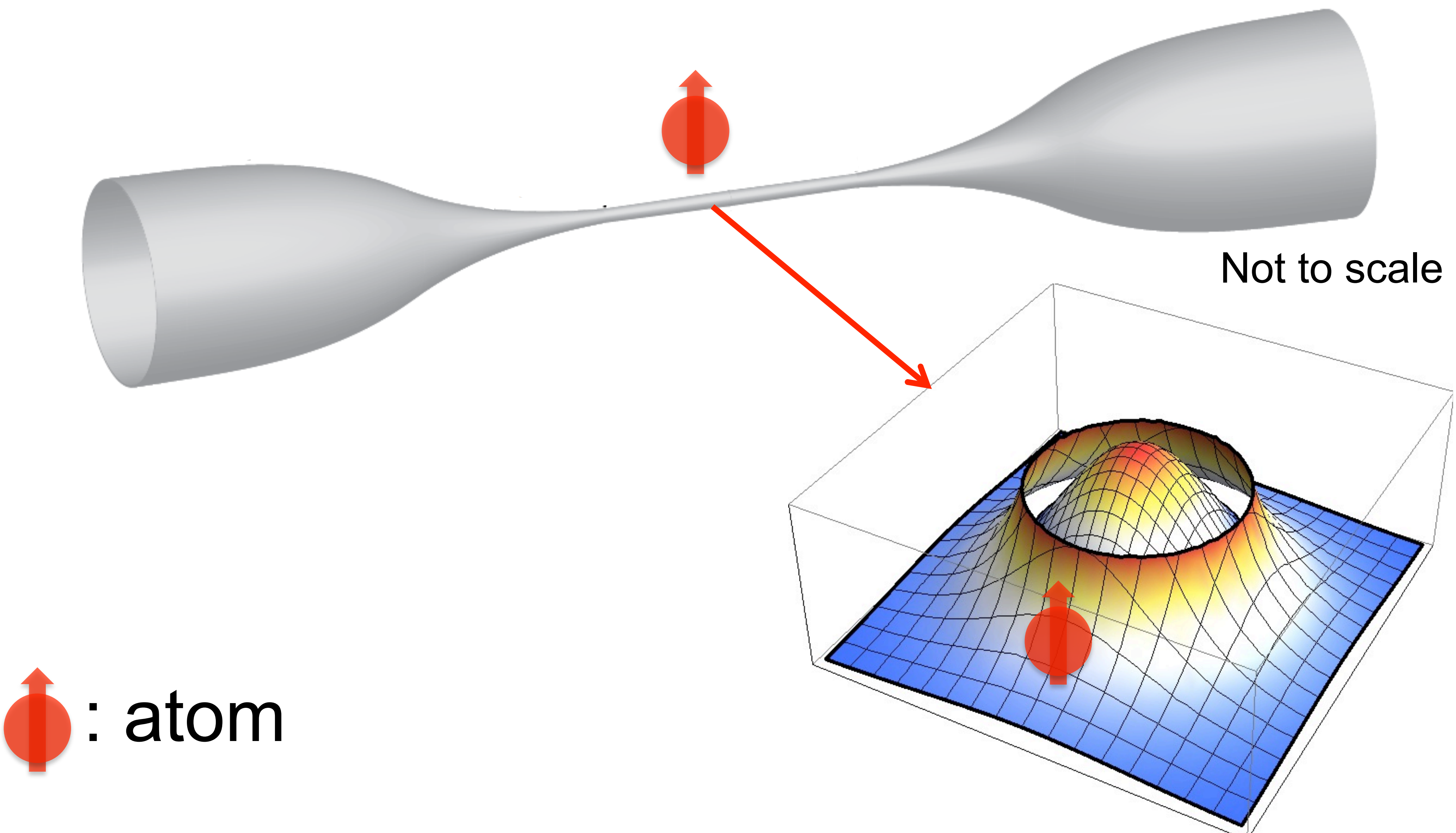
Density of modes

$$\gamma_{1D} \approx \frac{2\pi}{\hbar}\rho\left(k\right)\left\langle H_{int}\right\rangle^{2}$$

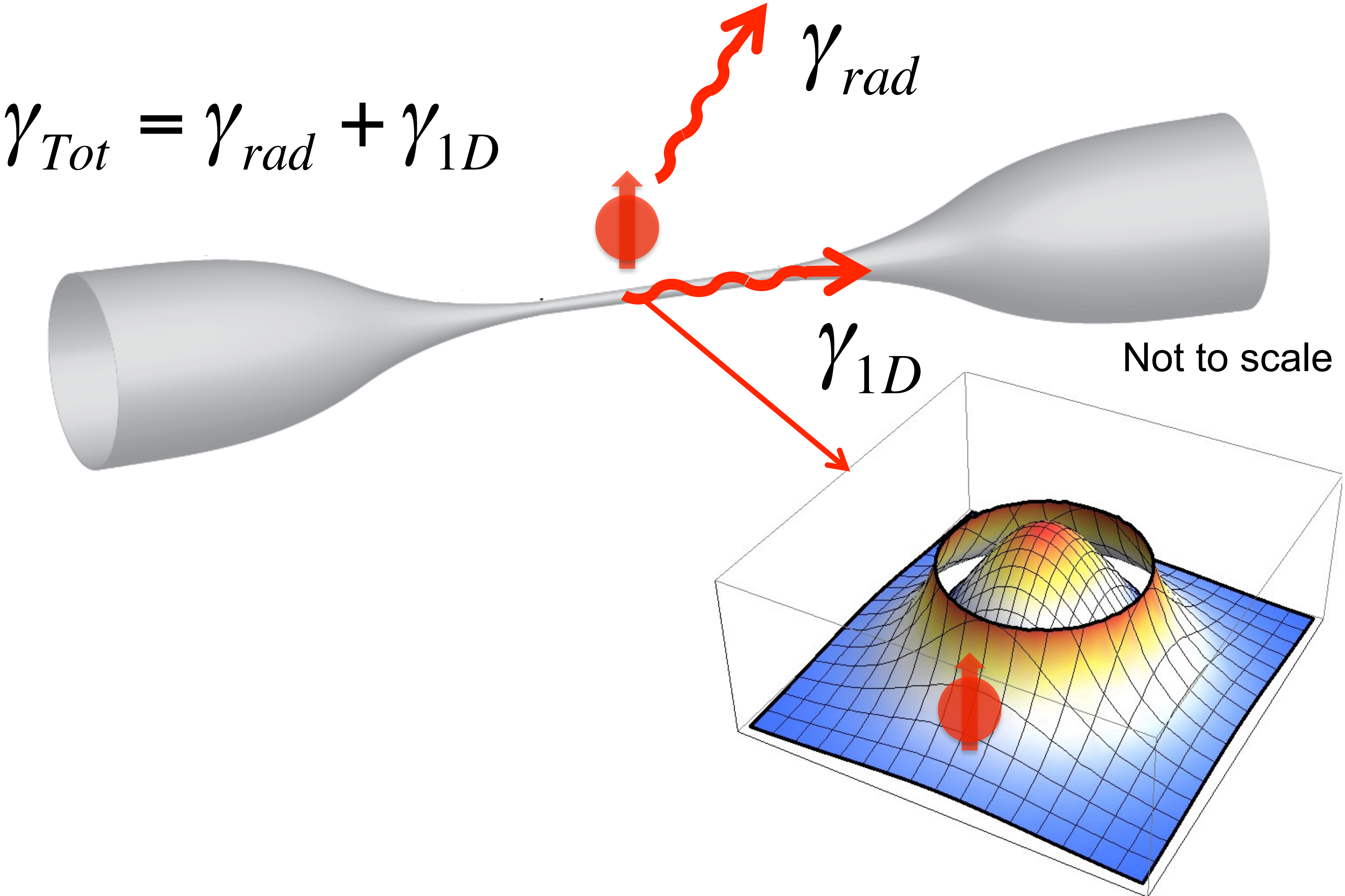
Proportional to the electric field of the guided mode

$$|E|^2 = \mathcal{E}^2\left[K_0^2(qr) + wK_1^2(qr) + fK_2^2(qr)\right]$$

Evanescent Coupling

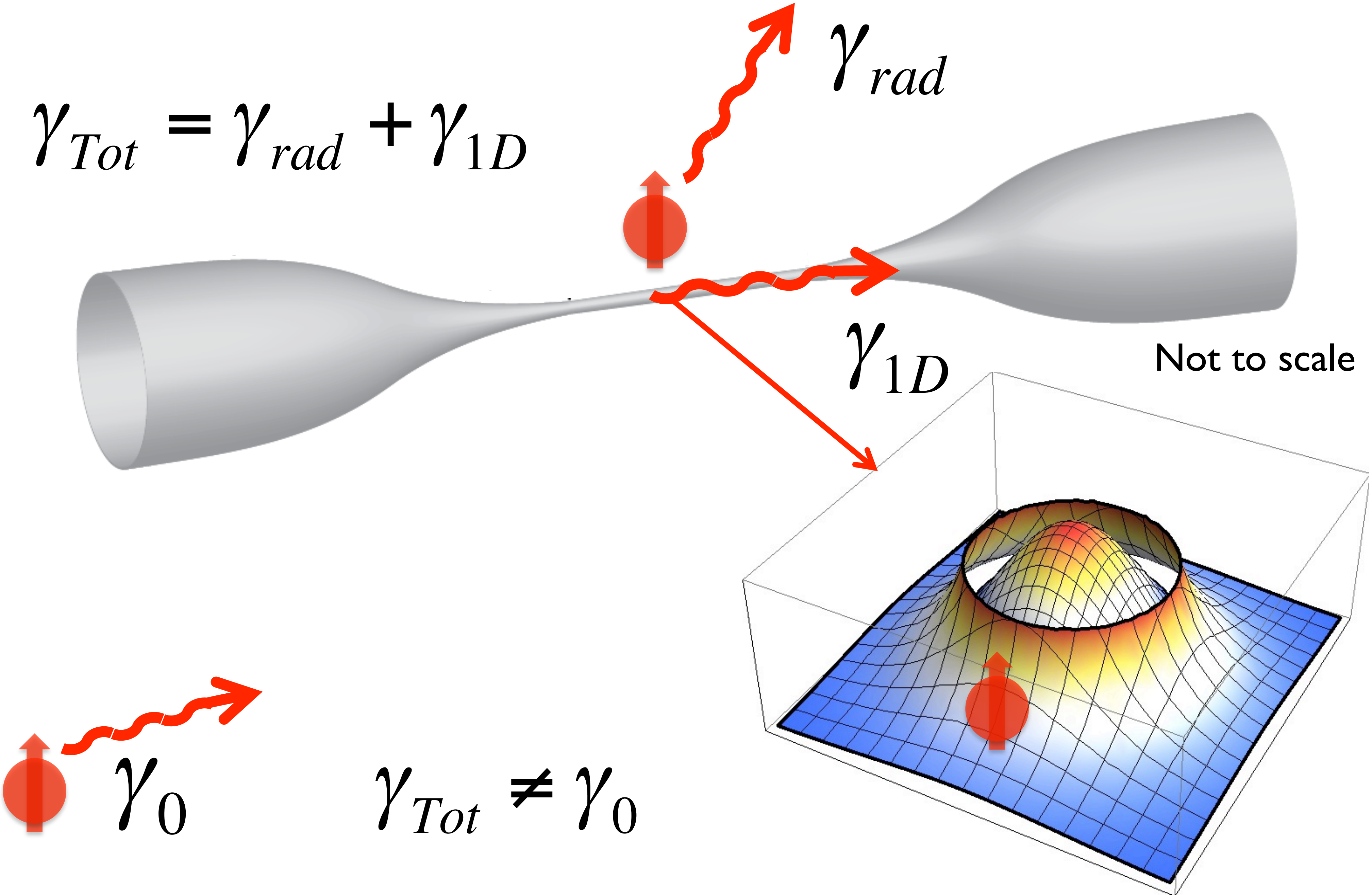


Evanescent Coupling



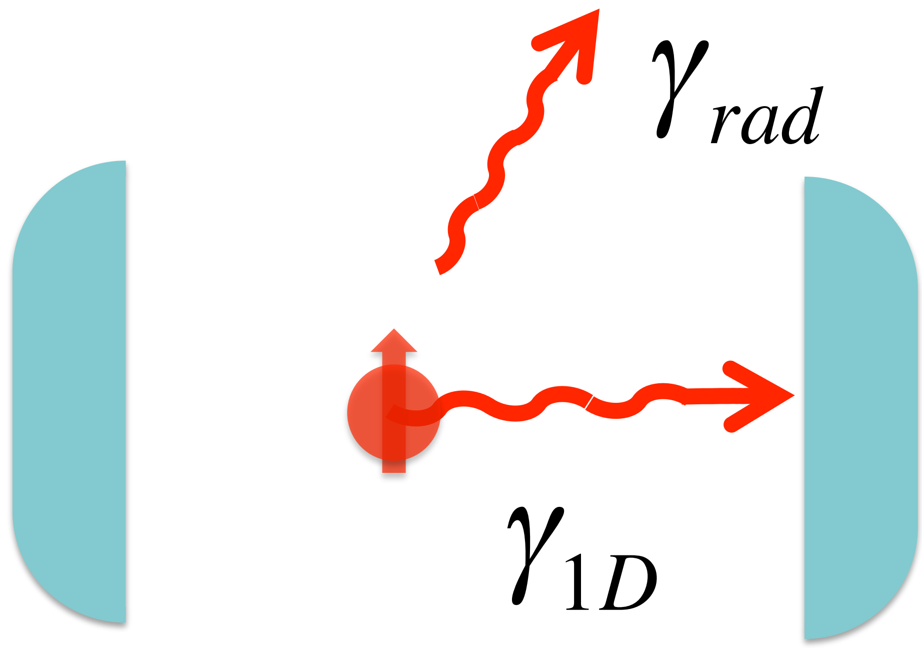
Evanescent Coupling

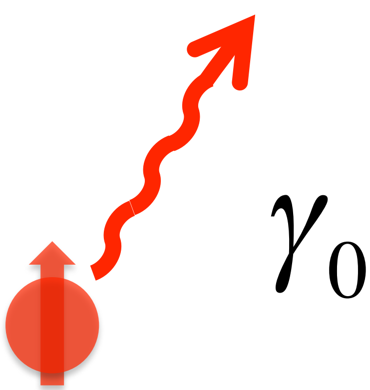
$$\gamma_{Tot} = \gamma_{rad} + \gamma_{1D}$$



$$\gamma_{Tot} \neq \gamma_0$$

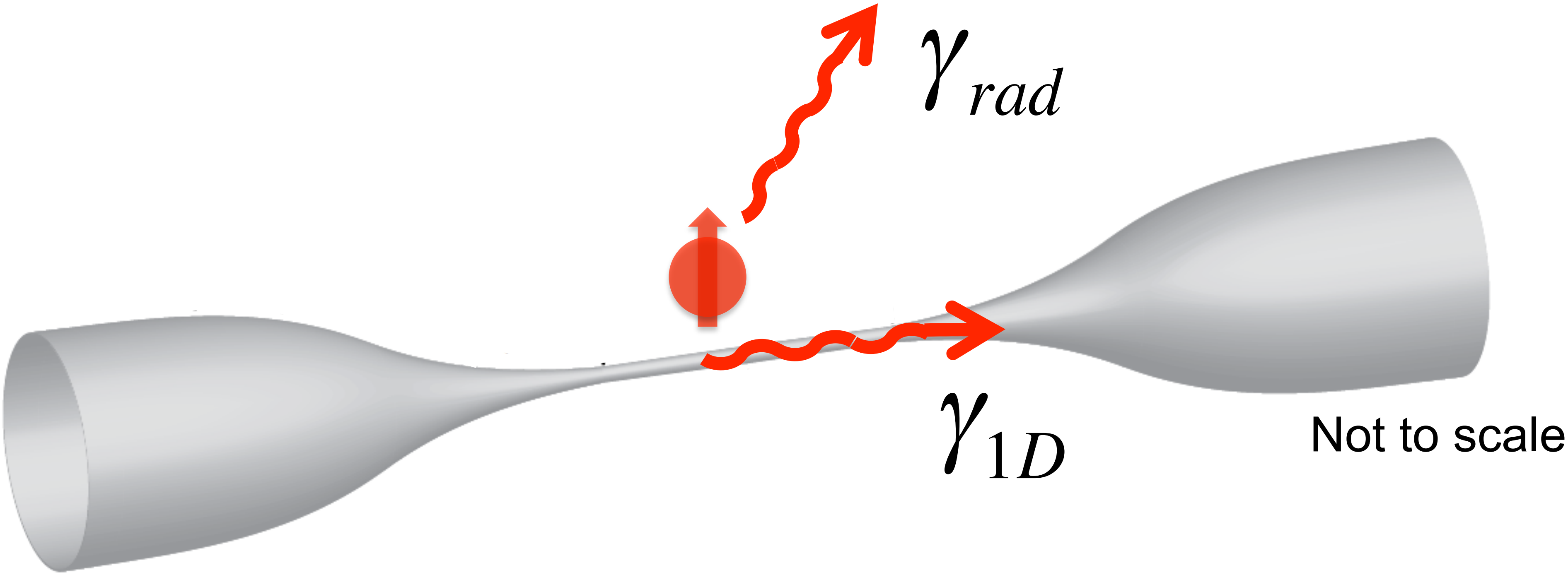
Coupling Enhancement



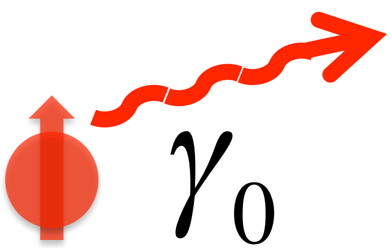


$$\alpha = \frac{\gamma_{1D}}{\gamma_0}$$

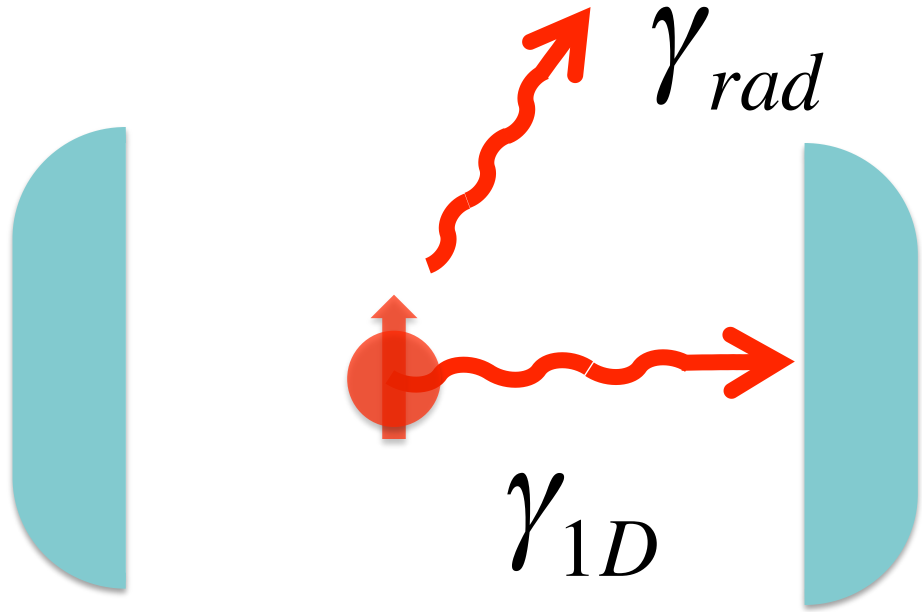
Coupling Enhancement

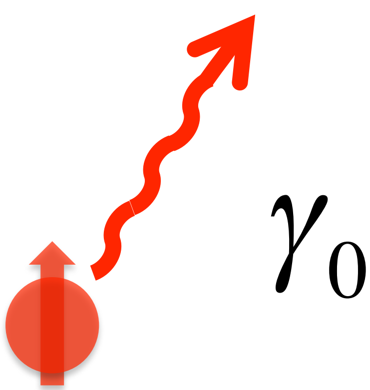


$$\alpha = \frac{\gamma_{1D}}{\gamma_0}$$



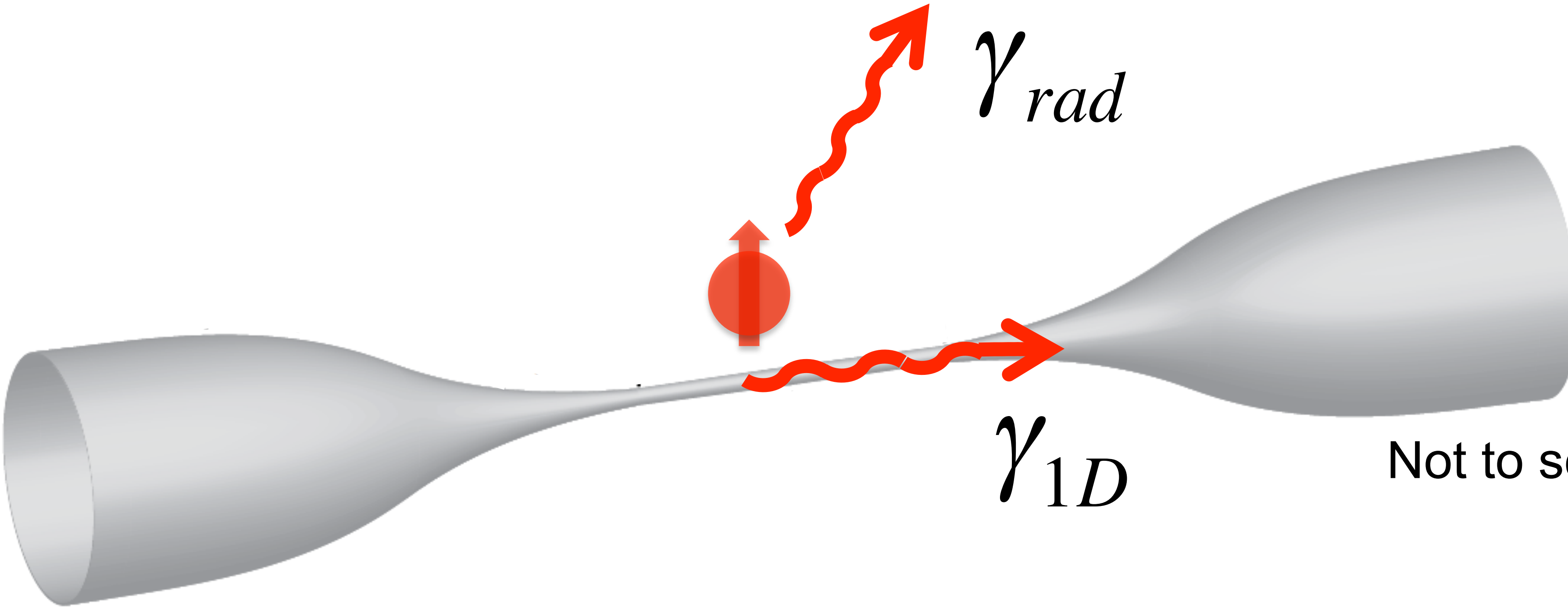
Coupling Efficiency





$$\beta = \frac{\gamma_{1D}}{\gamma_{Tot}} \quad ; \quad \gamma_{Tot} = \gamma_{1D} + \gamma_{rad}$$

Coupling Efficiency

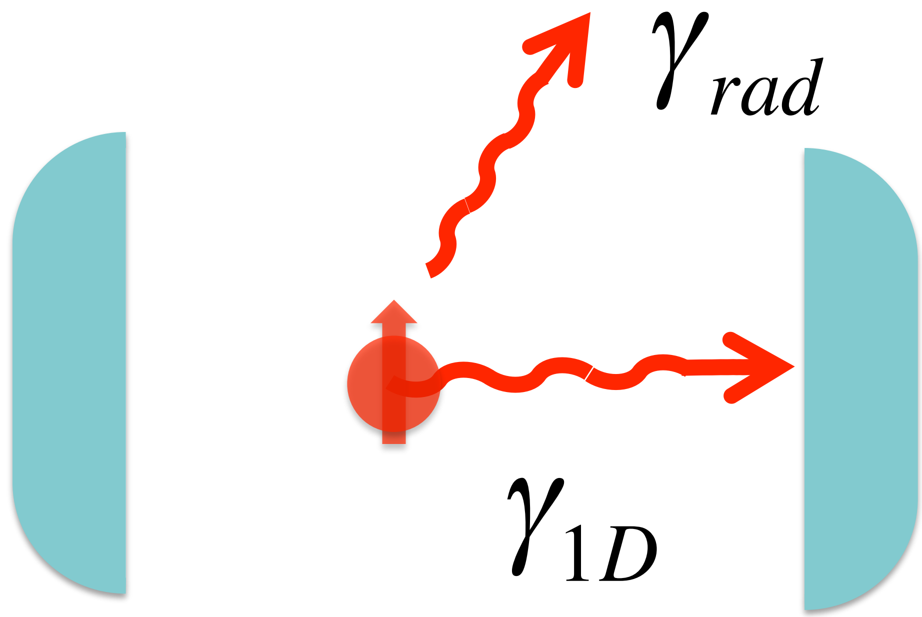


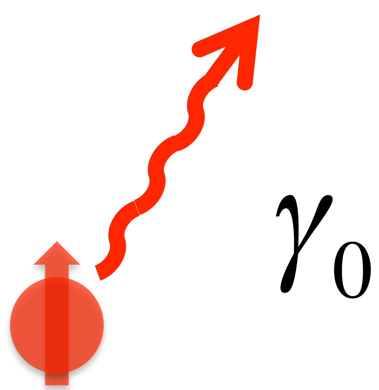
Not to scale

$$\beta = \frac{\gamma_{1D}}{\gamma_{Tot}}$$

$$\gamma_{Tot} = \gamma_{rad} + \gamma_{1D}$$

Purcell Factor

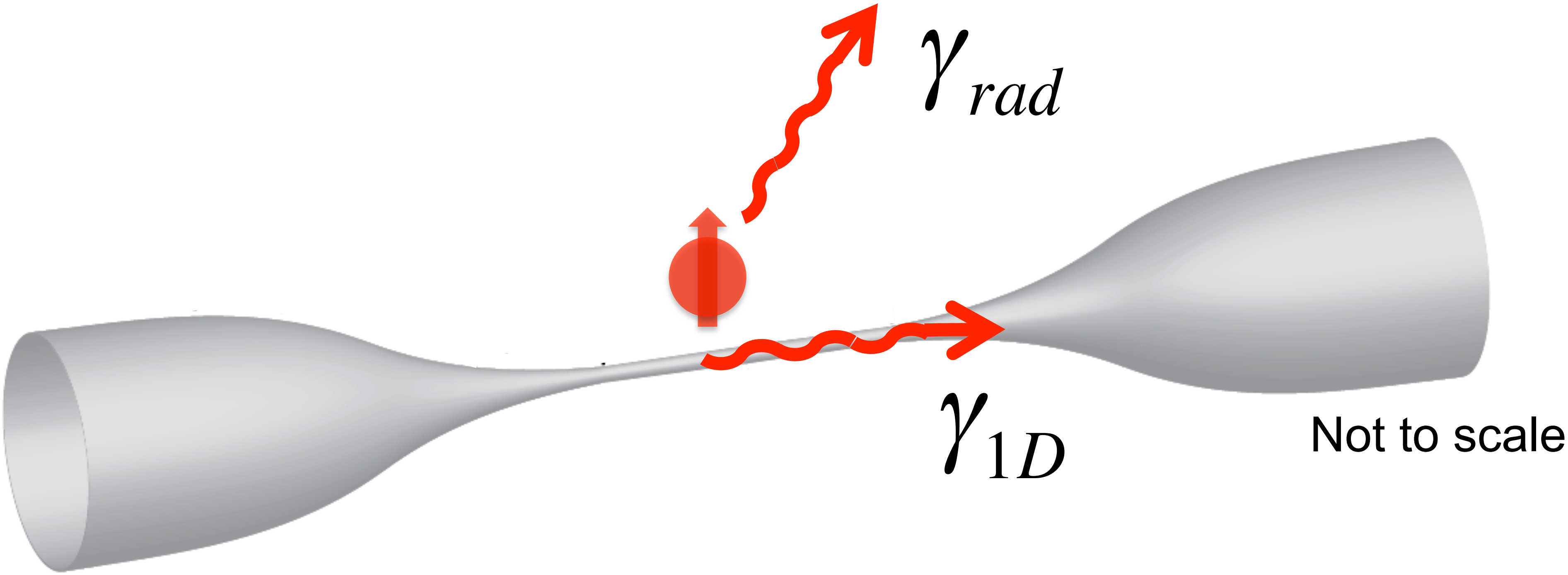




$$F_P = \frac{\gamma_{tot}}{\gamma_0} = \frac{\alpha}{\beta}$$

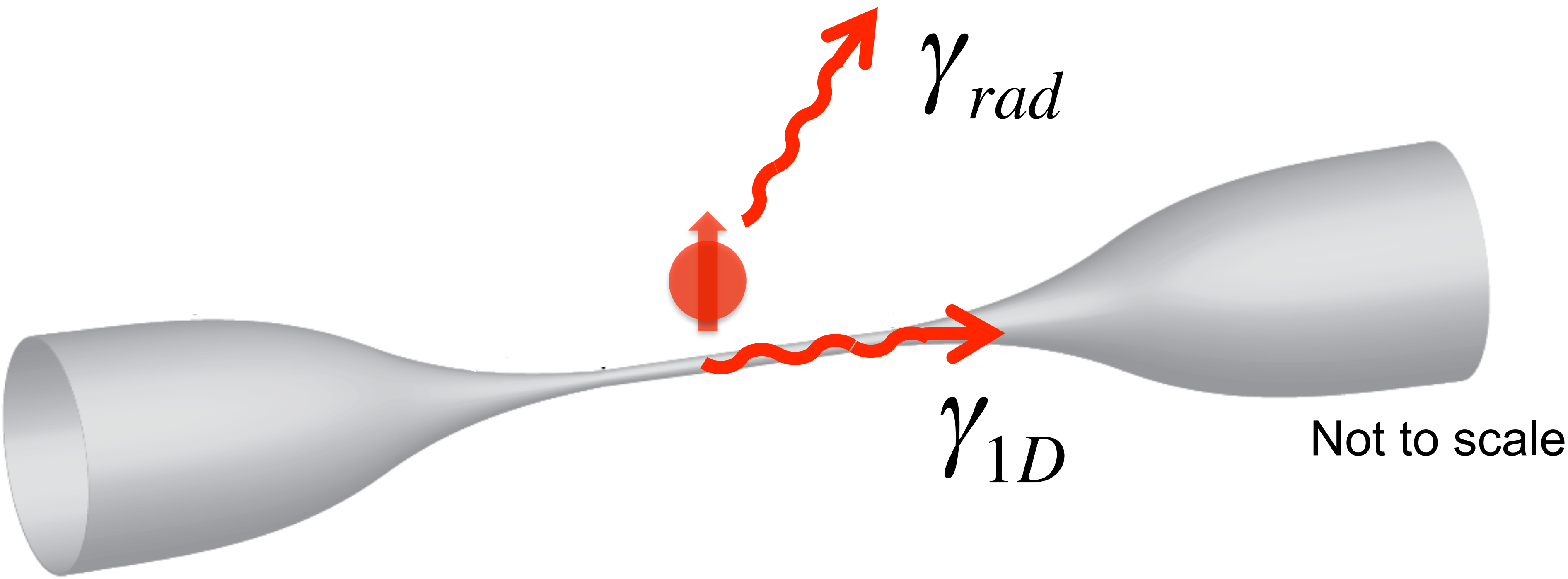
$$\gamma_{Tot} = \gamma_{1D} + \gamma_{rad}$$

Purcell Factor



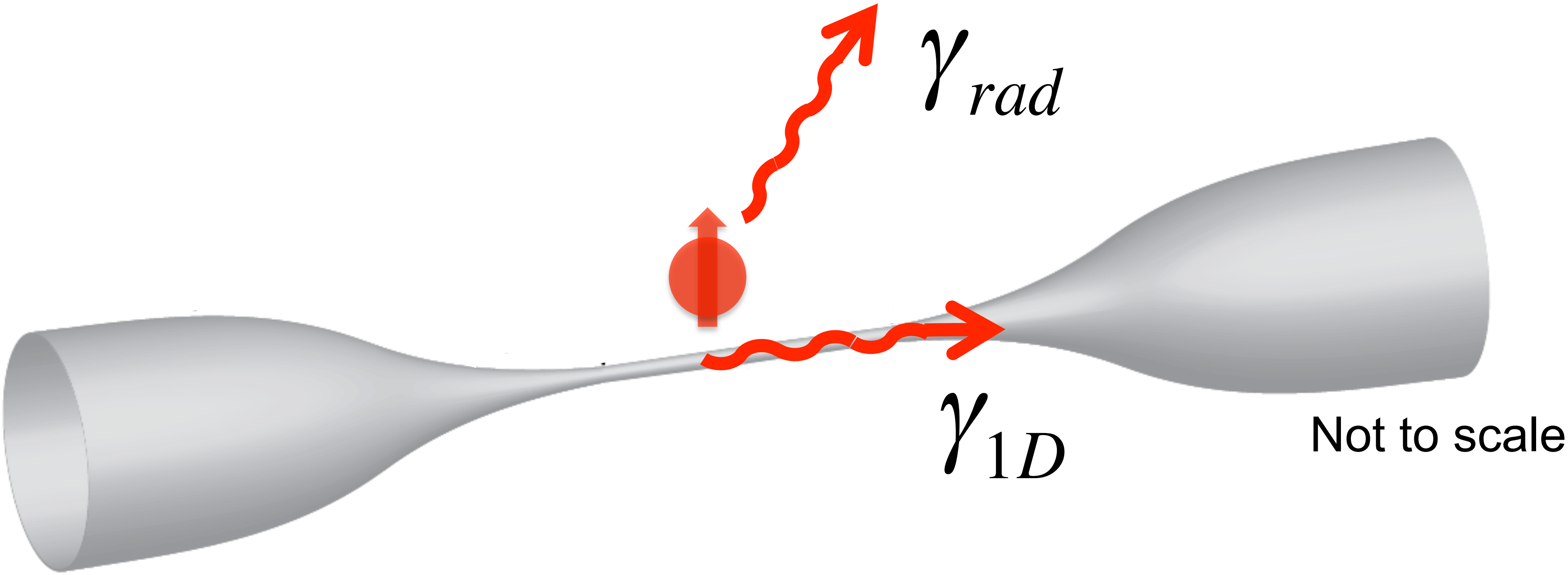
$$F_P = \frac{\gamma_{tot}}{\gamma_0} = \frac{\alpha}{\beta}$$

Cooperativity



$$C_1 = \frac{\beta}{(1-\beta)} = \frac{\gamma_{1D}}{\gamma_{rad}}$$

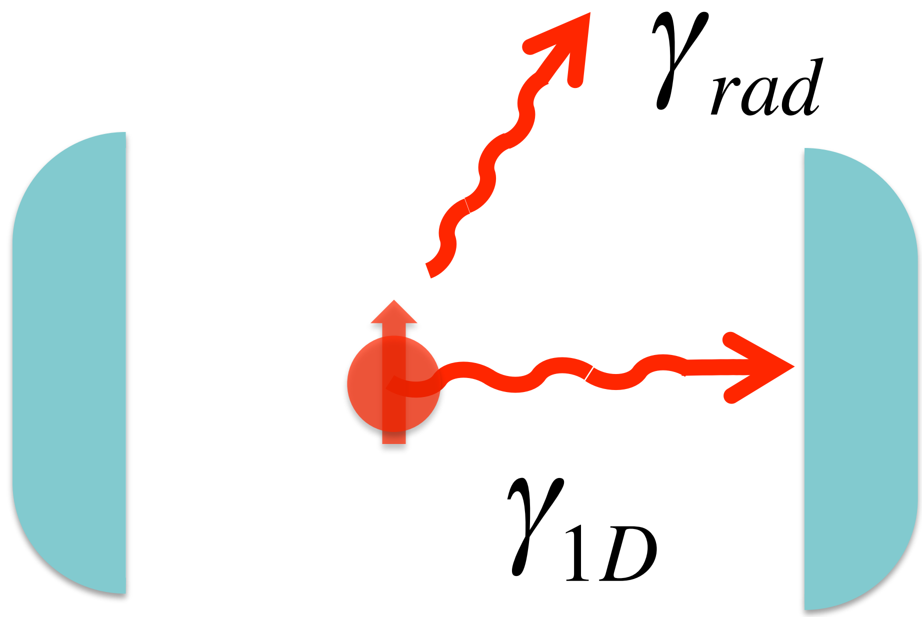
Cooperativity

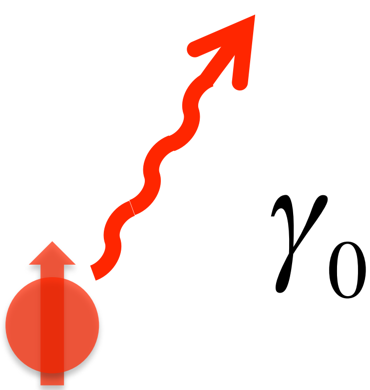


$$C_1 = \frac{\beta}{(1-\beta)} = \frac{\gamma_{1D}}{\gamma_{rad}}$$

C_1 is the ratio of what goes into the selected mode to what goes into all the rest

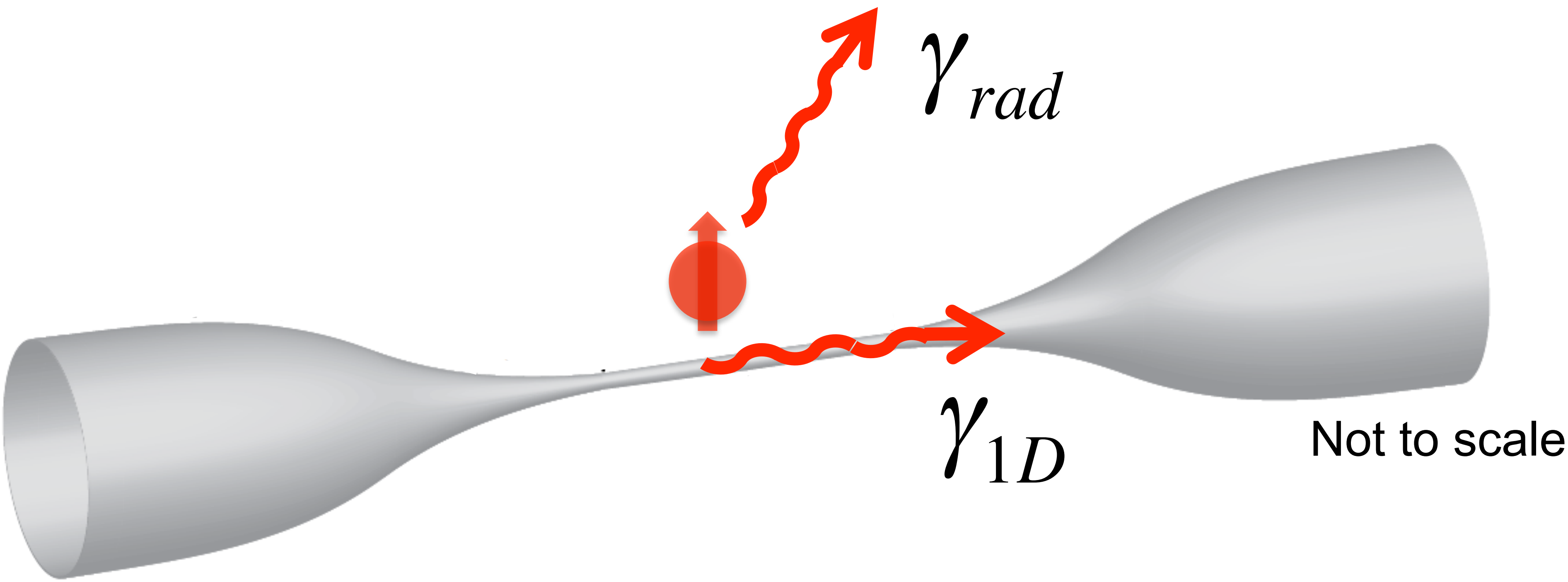
Cooperativity





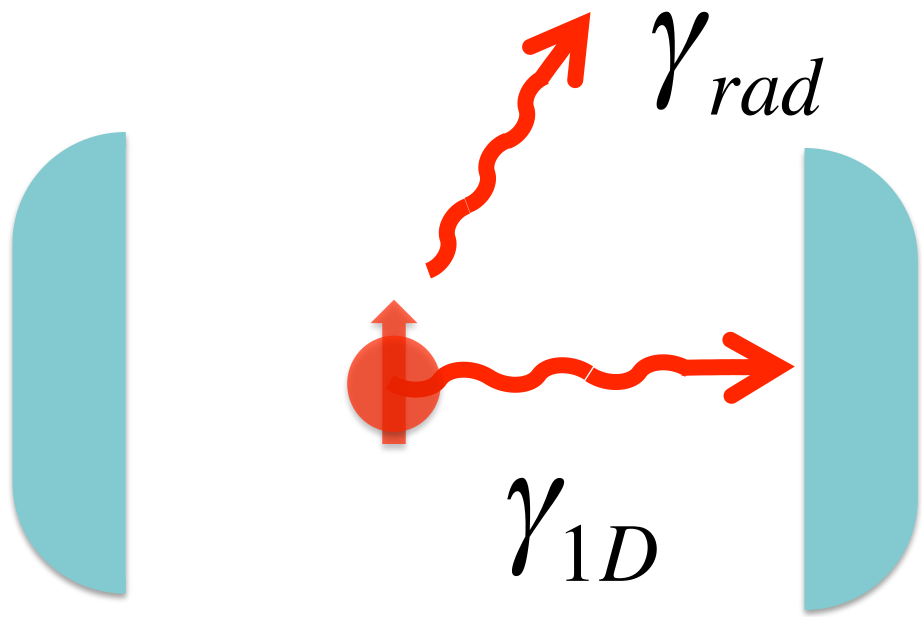
$$C_1 = \frac{\beta}{(1-\beta)} = \frac{\gamma_{1D}}{\gamma_{rad}}$$

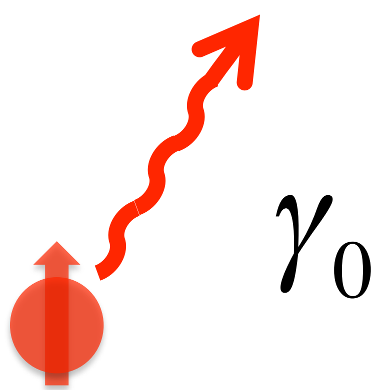
Cooperativity



$$C_1 = \frac{\beta}{(1-\beta)} = \frac{\gamma_{1D}}{\gamma_{rad}}$$

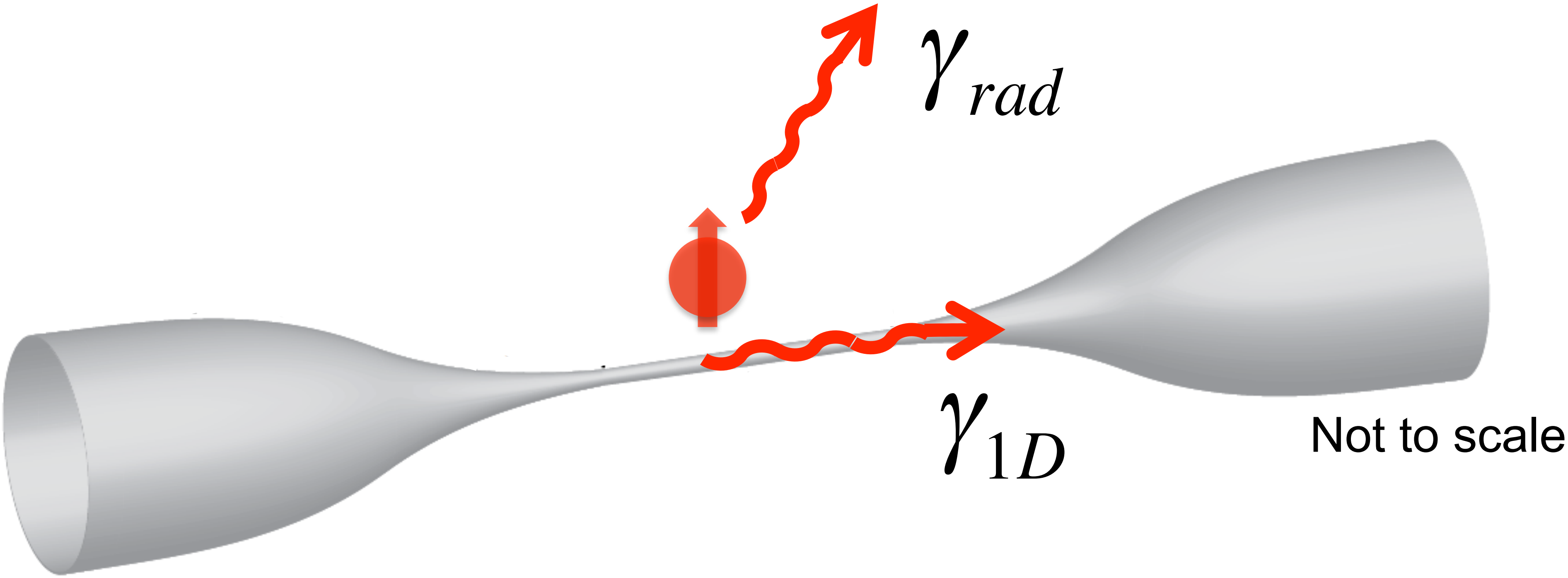
Cooperativity





$$C_1 = \frac{\sigma_0}{Area_{\mathrm{mode}}} \frac{1}{T}$$

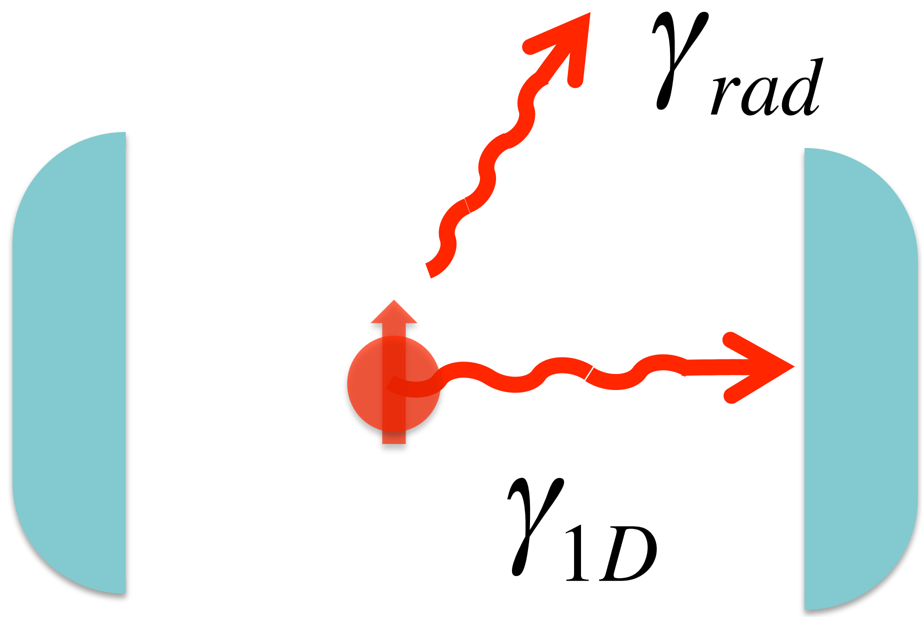
Cooperativity



no mirrors
T=1

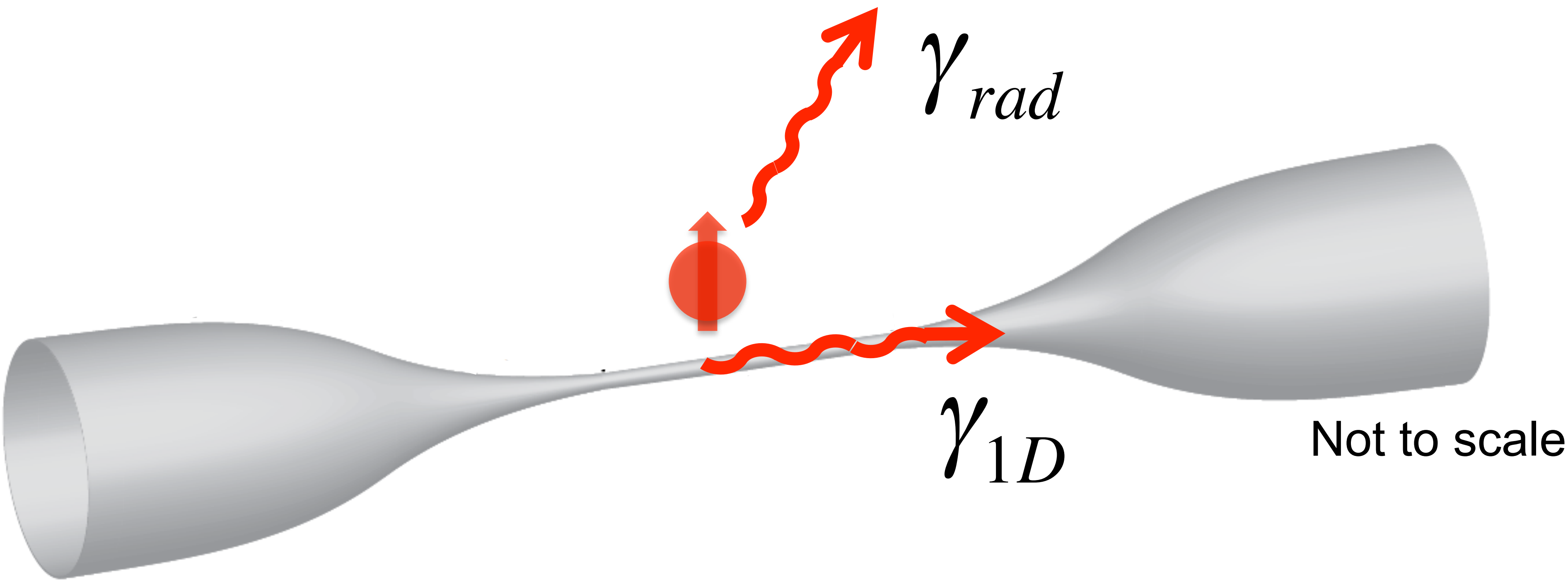
$$C_1 = \frac{\sigma_0}{Area_{\text{mode}}}$$

Cooperativity



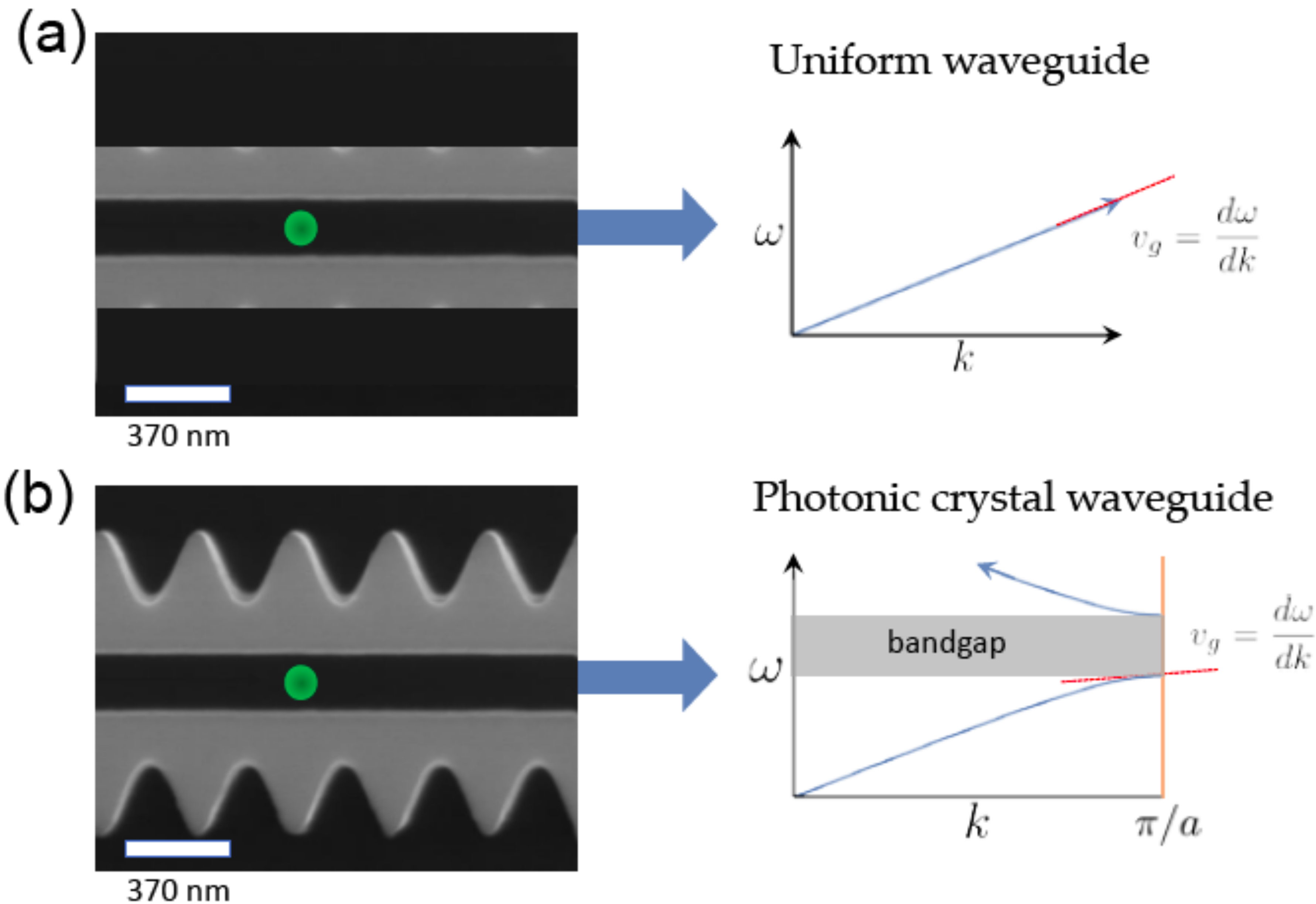
$$C_1 = \frac{g^2}{\kappa \gamma_{rad}} = \left(\frac{\sigma_0}{A_{\text{mode}}}\right)\left(\frac{c}{v_g}\right) = \frac{\gamma_{1D}}{\gamma_{rad}}$$

Cooperativity



$$C_1 = \frac{\sigma_0}{Area_{\text{mode}}} n_{eff} = \frac{\gamma_{1D}}{\gamma_{rad}}$$

4.What happens on a photonic structure?



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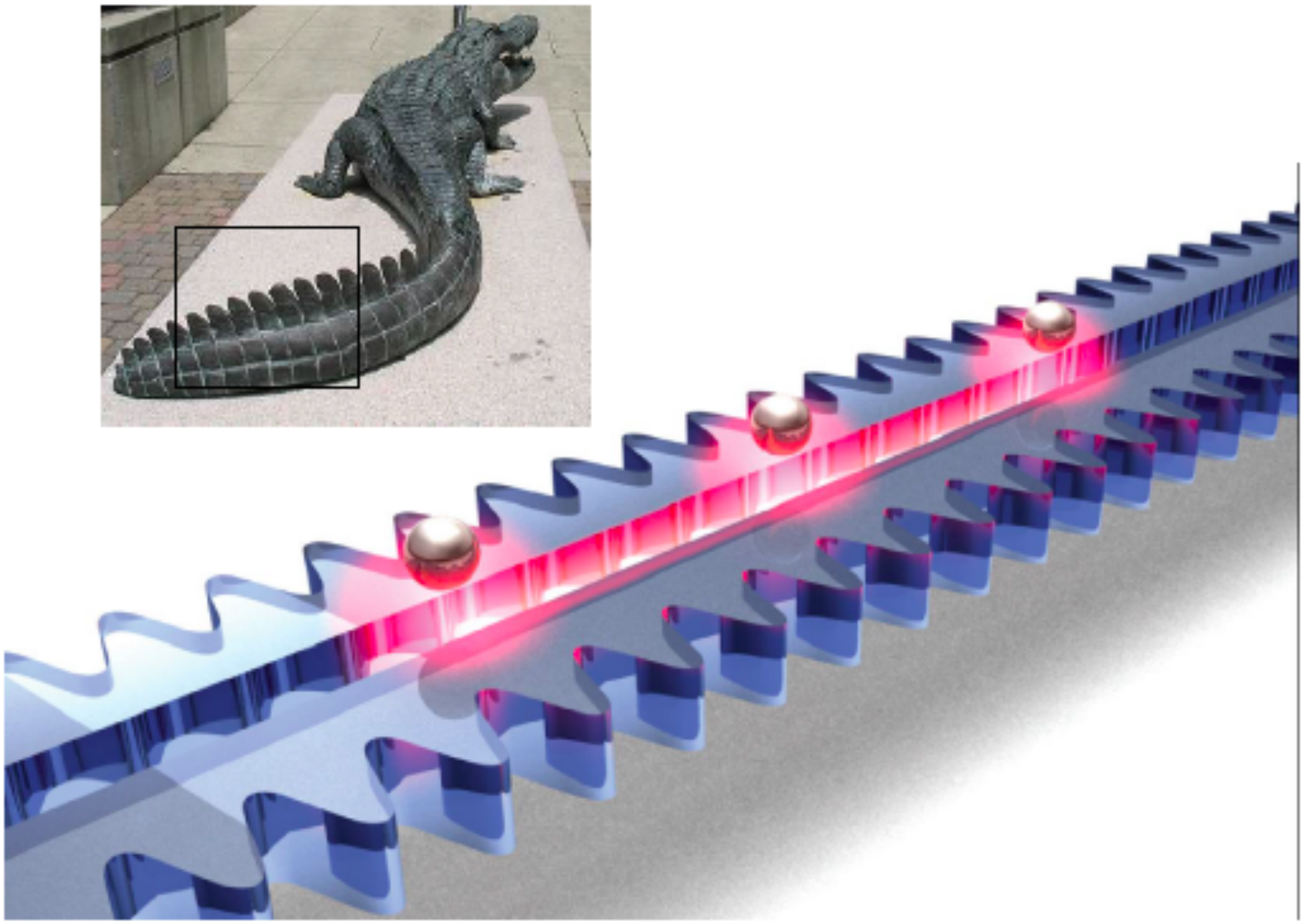
The alligator photonic crystal waveguide (Cal Tech)



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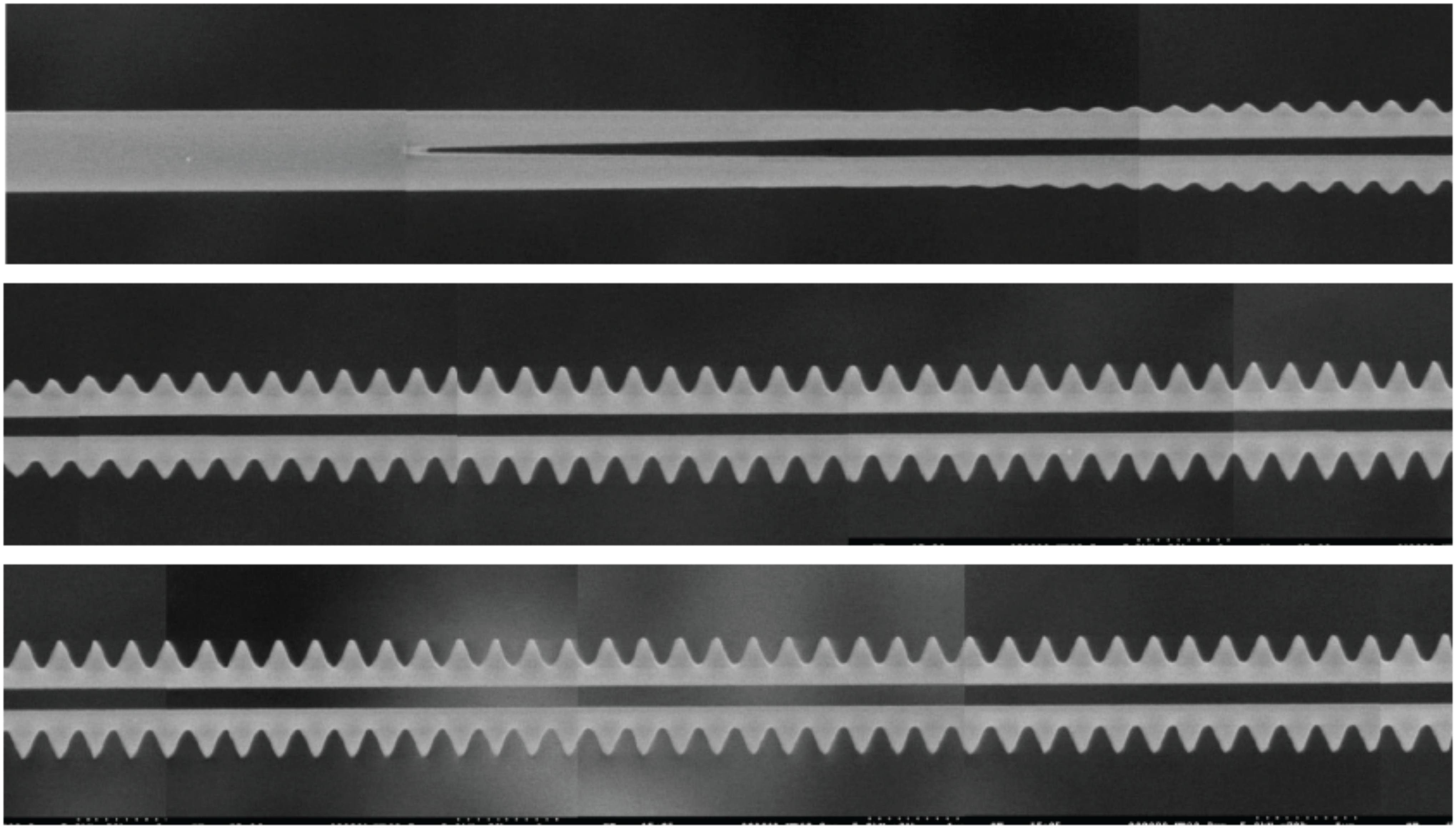
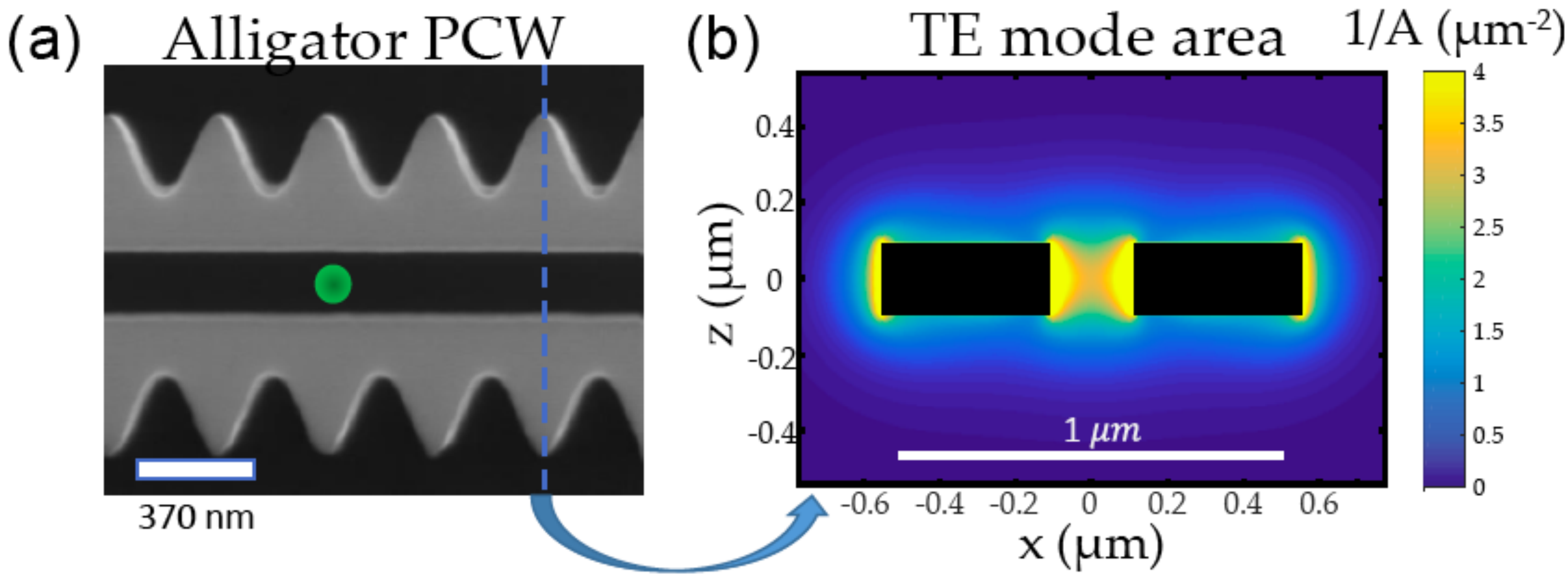


Mode area: $$A_k = \frac{\int_{\text{area}} \mathrm{d}^2\mathbf{r}\ \epsilon(\mathbf{r})|\mathbf{E}_k(\mathbf{r})|^2}{\max\left[\ \epsilon(\mathbf{r})|\mathbf{E}_k(\mathbf{r})|^2\ \right]}.$$



Scanning electron microscope

Cross section of the intensity

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Because there is a bandgap, the cooperativity grows with it. It can also create a “cavity mode” that does not move attached to the atom

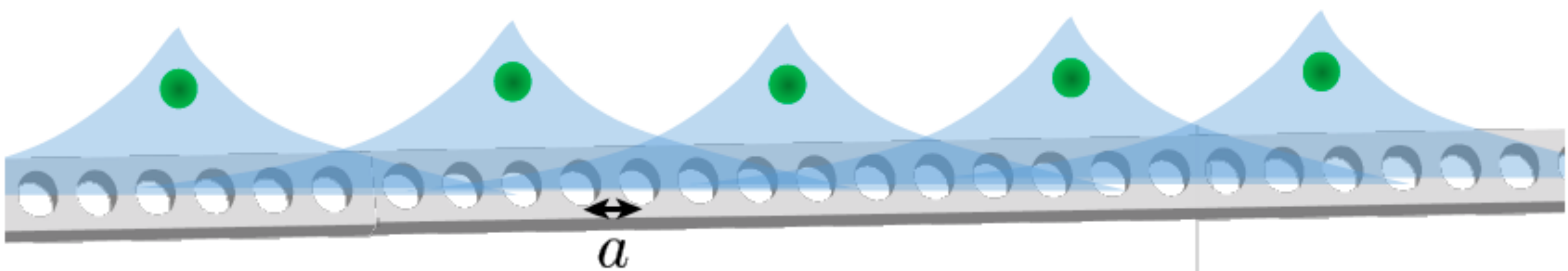

Figure 1.12: Atoms coupled to the bandgap of a photonic crystal waveguide. The atoms and photon cloud form atom-photon bound states.

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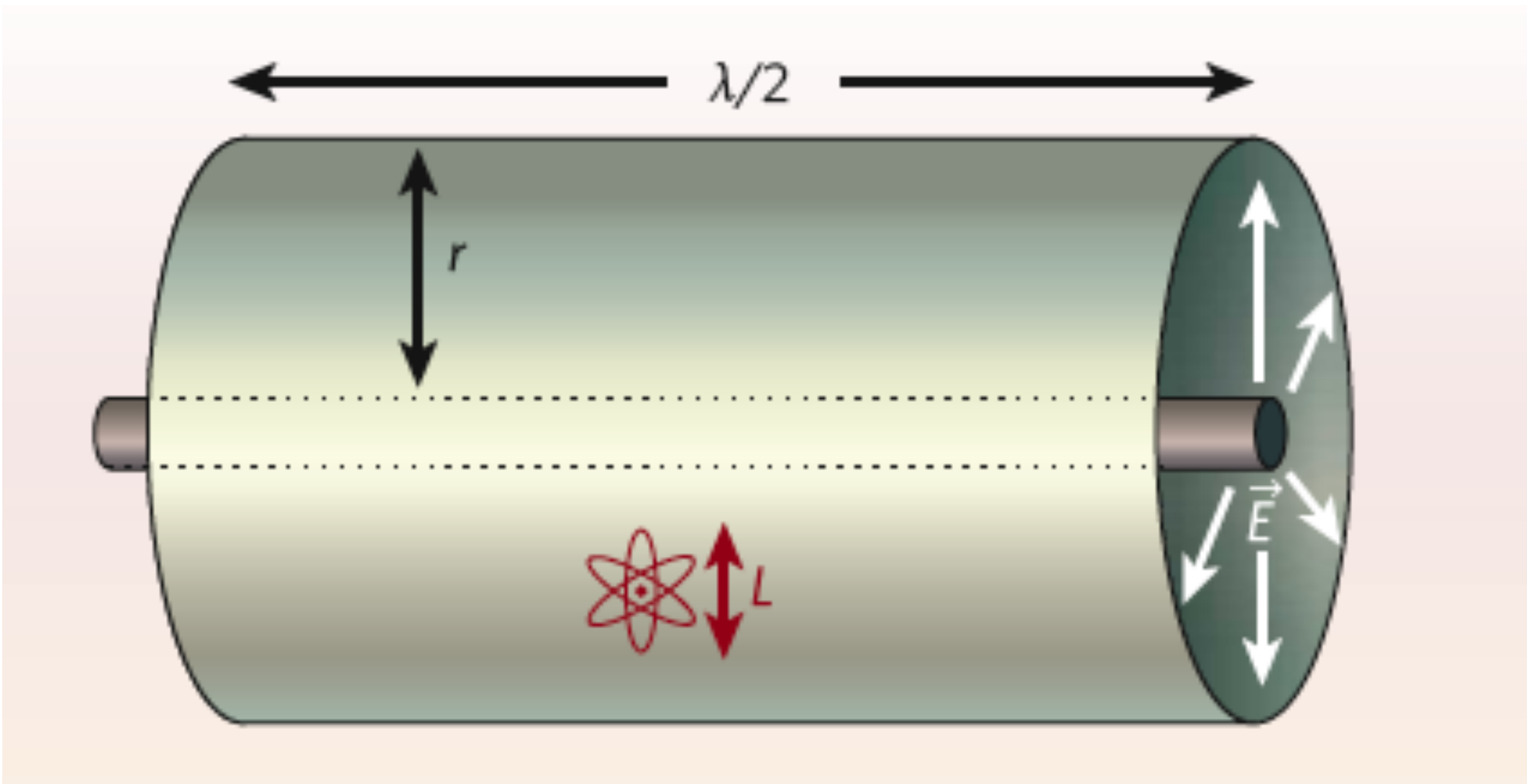
5. Limit of coupling atom and electromagnetic field, the case of circuit QED



The dipole *d* with characteristic length *L* is in a coaxial cavity of lengh λ/2 and radius *r*

The coaxial mode volume is much more confined than λ^3

$$g = \frac{dE_\upsilon}{\hbar}; \quad d = eL$$

$$V_{eff} = \pi r^2 \lambda / 2;$$

$$E_\upsilon = \frac{1}{r}\sqrt{\frac{\hbar\omega^2}{2\pi^2\varepsilon_0 c}}$$

$$\frac{g}{\omega} = \left(\frac{L}{r}\right)\sqrt{\frac{e^2}{2\pi^2\varepsilon_0\hbar c}} = \left(\frac{L}{r}\right)\sqrt{\frac{2\alpha}{\pi}}$$

Now the coupling constant can be a percentage of the frequency!

$$\frac{g}{\omega} = \left(\frac{L}{r}\right)\sqrt{\frac{2\alpha}{\pi}} = 0.068\left(\frac{L}{r}\right)$$

Be careful as the Jaynes Cummings model may no longer be adequate

Thanks