Quantum Optics

Luis A. Orozco Physics Colloquium Marshall University, April 2019 Supported by DLS APS www.jqi.umd.edu







- M. Born and E. Wolf, *Principles of Optics* Cambridge University Press, Cambridge, 1999, 7th expanded.
- L. Mandel and E. Wolf, *Optical Coherence* and Quantum Optics Cambridge University Press, Cambridge, 1995.
- E. Wolf, Introduction to the Theory of Coherence and Polarization of Light Cambridge University Press, Cambridge, 2007.

Michelson measures the diameter of a star

Michelson – Stellar Interferometer (1920)



Handbury Brown and Twiss

Can we use intensity fluctuations, noise, to measure the size of a star? Yes. They were radio astronomers and had done it around 1952,

R. Hanbury Brown and R.Q. Twiss, "A New Type of Interferometer for Use in Radio Astronomy," Phil. Mag. **46**, 663 (1954).

The Hanbury Brown Twiss stellar interferometer, Mk 1

R. Hanbury Brown & R. Q. Twiss, A Test of a New Type of Stellar Interferometer on Sirius, *Nature* **178**, 1046-1053 (1956).



Fig. 1. Simplified diagram of the apparatus

Flux collectors at Narrabri

R.Hanbury Brown: The Stellar Interferometer at Narrabri Observatory Sky and Telescope 28, No.2, 64, August 1964

Narrabri intensity interferometer with its circular railway track R.Hanbury Brown: BOFFIN. A Personal Story of the Early Da of Radar, Radio Astronomy and Quantum Optics (1991)



The HBT controversy



The physics of Hanbury Brown–Twiss intensity interferometry: from stars to nuclear collisions.*

Gordon Baym

Presented at the XXXVII Cracow School of Theoretical Physics, Zakopane, Poland. May 30 - June 10, 1997. Source a and b are within a Star. Can we measure the angular distance R/L~θ so that we could know the diameter?

Source a:

$$\alpha e^{ik|\vec{r}-\vec{r}_a|+i\phi_a}/|\vec{r}-\vec{r}_a|$$

Source b:

$$eta e^{ik|\vec{r}-\vec{r}_b|+i\phi_b}/|\vec{r}-\vec{r}_b|$$

The amplitude at detector 1 from sources a and b is: $A_1 = \frac{1}{L} \left(\alpha e^{ikr_{1a} + i\phi_a} + \beta e^{ikr_{1b} + i\phi_b} \right)$

And the intensity I_1

 $=\frac{1}{L^2}\left(|\alpha|^2+|\beta|^2+\alpha^*\beta e^{i(k(r_{1b}-r_{1a})+\phi_b-\phi_a)}+\alpha\beta^*e^{-i(k(r_{1b}-r_{1a})+\phi_b-\phi_a)}\right)$

The average over the random phases ϕ_a and ϕ_b gives zero $\langle I_1 \rangle = \langle I_2 \rangle = \frac{1}{L^2} \left(\langle |\alpha|^2 \rangle + \langle |\beta|^2 \rangle \right)$

And the product of the intensity of each of the detectors $<I_1><I_2>$ is independent of the separation of the detectors.

Multiply the two intensities and then average.

$$\langle I_1 I_2 \rangle = \langle I_1 \rangle \langle I_2 \rangle + \frac{2}{L^4} |\alpha|^2 |\beta|^2 \cos\left(k(r_{1a} - r_{2a} - r_{1b} + r_{2b})\right)$$
$$= \frac{1}{L^4} \left[(|\alpha|^4 + |\beta|^4) + 2|\alpha|^2 |\beta|^2 (1 + \cos\left(k(r_{1a} - r_{2a} - r_{1b} + r_{2b})\right) \right].$$

$$g^{(2)} = \frac{\langle I_1 I_2 \rangle}{\langle I_1 \rangle \langle I_2 \rangle}$$

= $1 + 2 \frac{\langle |\alpha|^2 \rangle \langle |\beta|^2 \rangle}{(\langle |\alpha|^2 \rangle + \langle |\beta|^2 \rangle)^2} \cos \left(k (r_{1a} - r_{2a} - r_{1b} + r_{2b}) \right).$

 $(L \gg R), k(r_{1a} - r_{2a} - r_{1b} + r_{2b}) \to k(\vec{r}_a - \vec{r}_b) \cdot (\hat{r}_2 - \hat{r}_1) = \vec{R} \cdot (\vec{k}_2 - \vec{k}_1)$

This function changes as a function of the separation between the detectors.

 $d = \lambda/\theta_1$ with $\theta = R/L$

Relation to the Michelson Interferometer

$$|A_1 + A_2|^2 = |A_1|^2 + |A_1|^2 + (A_1^*A_2 + A_1A_2^*)$$

The term in parenthesis is the associated to the fringe Visibility (first order coherence) if we now take the square of the fringe visibility and average it:

$$\langle V^2 \rangle = 2 \langle |A_1|^2 |A_2|^2 \rangle + \langle A_1^{*2} A_2^2 \rangle + \langle A_1^2 A_2^{*2} \rangle$$

$$\langle V^2 \rangle \to 2 \langle I_1 I_2 \rangle$$

The solution of E. M. Purcell, *Nature* **178**, 1449 (1956).

Points to the work of Forrester as the frist real optical intensity correlation. A. T. Forrester, R. A. Gudmundsen and P. O. Johnson, "*Photoelectric Mixing of Incoherent Light,*" Phys. Rev. **99**, 1691 (1955).

Mentions that bosons tend to appear together

Does the calculation and relates it to the first order coherence.



Interferometry of the Intensity Fluctuations in Light II. An Experimental Test of the Theory for Partially Coherent Light

R. Hanbury Brown; R. Q. Twiss

Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences, Vol. 243, No. 1234 (Jan. 14, 1958), 291-319.

Stable URL:

http://links.jstor.org/sici?sici=0080-4630%2819580114%29243%3A1234%3C291%3AIOTIFI%3E2.0.CO%3B2-H

Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences is currently published by The Royal Society.



FIGURE 1. A simplified outline of an intensity interferometer.



FIGURE 4. A simplified outline of the optical system.

TABLE 2. THE EXPERIMENTAL AND THEORETICAL VALUES FOR THE NORMALIZED CORRELATION FACTOR FOR DIFFERENT CATHODE SPACINGS

			theoretical corre-		
			lation assuming		theoretical
		observed	cathodes	experimental value	value of the
	cathode	correlation	superimposed	of normalized	normalized
	separation	(r.m.s. signal	(r.m.s. signal	correlation factor	correlation
	(mm)	to noise ratio)	to noise ratio)	$\Gamma^2(n,d) = \frac{(S/N)}{N}$	factor
run no.	d	(S/N)	(S/N)'	$\Gamma^{-}(\nu_0,a) = \frac{1}{(S/N)'}$	$\Gamma^2(\nu_0,d)$
1	0	+17.55	+17.10	1.03 ± 0.04 (p.e.)	1.00
2	1.25	+ 8.25	+ 9.27	0.89 ± 0.07	0.928
3	2.50	+ 5.75	+ 8.85	0.65 ± 0.08	0.713
4	3.75	+ 3.59	+ 8.99	0.40 ± 0.07	0.461
5	5.00	+ 2.97	+ 9.00	0.33 ± 0.07	0.244
6	10.00	+ 0.90	+ 8.17	0.11 ± 0.08	0.012



separation of cathodes, d (mm)

Correlation measurements

The study of optical noisy signals uses correlation functions.

400

450

500

Photocurrent with noise: < $F(t) F(t+\tau) >$ $< F(t) G(t+\tau) >$ 150 100 For optical signals 50 the variables (JmV) usually are: Field 0 and Intensity, but -50 they can be cross -100 correlations as -150└ 0 50 100 150 200 250 300 350 well (ns)

How do we measure these functions?

 $G^{(1)}(\tau) = \langle E(t)^* E(t+\tau) \rangle$ field-field $G^{(2)}(\tau) = \langle I(t) | (t+\tau) \rangle$ intensity-intensity $H(\tau) = \langle I(t) | E(t+\tau) \rangle$ intensity-field

- Correlation functions tell us something about fluctuations.
- The correlation functions have classical limits.
- They are related to conditional measurements. They give the probability of an event given that something has happened.

Mach Zehnder or Michelson Interferometer Field – Field Correlation



This is the basis of Fourier Spectroscopy



Correlations of the intensity at $\tau=0$

$$g^{(2)}(0) = \frac{\langle I(t)^2 \rangle}{\langle I(t) \rangle^2}$$
$$= \frac{\langle (I_0 + \delta(t))^2 \rangle}{\langle I_0 + \delta(t) \rangle^2}$$
$$= 1 + \frac{\langle \delta(t)^2 \rangle}{I_0^2}$$

It is proportional to the variance

Intensity correlations (bounds) $g^{(2)}(0) = 1 + \frac{\langle \delta(t)^2 \rangle}{I_0^2}$ $q^{(2)}(0) - 1 \ge 0$ Cauchy-Schwarz $2I(t)I(t+\tau) \le I^2(t) + I^2(t+\tau)$ $|g^{(2)}(\tau) - 1| \le |g^{(2)}(0) - 1|$ The correlation is maximal at equal times $(\tau=0)$ and it can not increase.

How do we measure them?

Build a "Periodogram". The photocurrent is proportional to the intensity I(t)

$$I(t) \rightarrow I_i$$

$$I(t+\tau) \rightarrow I_j$$

$$\left\langle I(t)I(t+\tau) \right\rangle \rightarrow \sum_{i=0}^M \sum_{n=0}^N I_i I_{i+n}$$

- Discretize the time series.
- Apply the algorithm on the vector.
- Careful with the normalization.

Discretize:

I;

Multiply with displacement :

$$I_{i+n}$$

Add and average:



Another form to measure the correlation with with the waiting time distribution of the photons. The minimum size of the variance of the electromagnetic field.

- Store the time separation between two consecutive pulses (start and stop).
- Histogram the separations
- If the fluctuations are few you get after normalization $g^{(2)}(\tau)$.
- Work at low intensities (low counting rates).

Intensity (photons)



Use a time stamp card. Later process the data, then you can calculate all sorts of correlations.

Digitize the full signal (important to identify the nature of the event *e.g.* particle physics).

Quantum optics

- The photon is the smallest fluctuation of the intensity of the electromagnetic field, its variance.
- The photon is the quantum of energy of the electromagnetic field. With energy ħω at frequency ω.

An important point about the quantum calculation $g^{(2)}(\tau)$

Quantum Correlations (Glauber):

$$g^{(2)}(\tau) = \frac{\langle T : \hat{I}(t)\hat{I}(t+\tau) : \rangle}{\langle \hat{I}(t) \rangle^2}$$

The intensity operator I is proportional to the number of photons, but the operators have to be normal (:) and time (T) ordered. All the creation operators do the left and the annihilation operators to the right (just as a photodetector works). The operators act in temporal order.

R. Glauber, "The Quantum Theory of Optical Coherence," Phys. Rev. **130**, 2529 (1963).

At equal times (normal order) :

$$g^{(2)}(0) = \frac{\langle \hat{a}^{\dagger} \hat{a}^{\dagger} \hat{a} \hat{a} \rangle}{\langle \hat{a}^{\dagger} \hat{a} \rangle^2}.$$

Conmutator: $\hat{a}^{\dagger}\hat{a} = \hat{a}\hat{a}^{\dagger} - 1$

$$\left\langle \hat{a}^{\dagger} \hat{a}^{\dagger} \hat{a} \hat{a} \right\rangle = \left\langle \hat{a}^{\dagger} (\hat{a} \hat{a}^{\dagger} - 1) \hat{a} \right\rangle = \left\langle \hat{a}^{\dagger} \hat{a} \hat{a}^{\dagger} \hat{a} \right\rangle - \left\langle \hat{a}^{\dagger} \hat{a} \right\rangle$$
$$\left\langle \hat{a}^{\dagger} \hat{a}^{\dagger} \hat{a} \hat{a} \right\rangle = \left\langle \hat{n}^{2} \right\rangle - \left\langle \hat{n} \right\rangle \quad \text{where} \quad \hat{n} = \hat{a}^{\dagger} \hat{a}$$

The correlation requires detecting two photons, so if we detect one, we have to take that into consideration in the accounting.

In terms of the variance of the photon number:

$$\sigma^2 = \langle \hat{n}^2 \rangle - \langle \hat{n} \rangle^2$$
$$g^{(2)}(0) = 1 + \frac{\sigma^2 - \langle \hat{n} \rangle}{\langle \hat{n} \rangle^2}.$$

The classical result says:

$$= 1 + \frac{\langle \delta(t)^2 \rangle}{I_0^2}$$
The quantum correlation function can be zero, as the detection changes the number of photons in the field. This is related to the variance properties: is the variance larger or smaller than the mean (Poissonian, Super-Poissonian or Sub-Poissonian).

$$g^{(2)}(0) = 1 + \frac{\sigma^2 - \langle \hat{n} \rangle}{\langle \hat{n} \rangle^2}.$$

At equal times the value gives: $g^{(2)}(0)=1$ Poissonian $g^{(2)}(0)>1$ Super-Poissonian $g^{(2)}(0)<1$ Sub-Poissonian

The slope at equal times:

 $g^{(2)}(0) > g^{(2)}(0^+)$ Bunched $g^{(2)}(0) < g^{(2)}(0^+)$ Antibunched

Classically we can not have Sub-Poissonian nor Antibunched.

Quantum Correlations (Glauber): $g^{(2)}(\tau) = \frac{\left\langle : \hat{I}(t) \, \hat{I}(t+\tau) : \right\rangle}{\left\langle \hat{I}(t) \right\rangle^2}$

$$g^{(2)}(\tau) = \frac{\left\langle : \hat{I}(\tau) : \right\rangle_{c}}{\left\langle : \hat{I} : \right\rangle}$$

If we detect a photon at time t, $g^{(2)}(\tau)$ gives the probability of detecting a second photon after a time τ .

Correlation functions as conditional measurements in quantum optics.

- The detection of the first photon gives the initial state that is going to evolve in time.
- Bayesian probabities.
- $g^{(2)}(\tau)$ Hanbury-Brown and Twiss.

Correlation functions in Optics (Wolf 1954)

 The optical correlations propagate using the wave equation for the electromagnetic field (Wolf 1954, 1955).

Quantum regression theorem

 The correlation functions can be calculated using the master equation with the appropriate initial and boundary conditions (Lax 1968).

Antibunching in Resonance Fluorescence

Photon Antibunching in Resonance Fluorescence

H. J. Kimble,^(a) M. Dagenais, and L. Mandel

Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627 (Received 22 July 1977)

The phenomenon of antibunching of photoelectric counts has been observed in resonance fluorescence experiments in which sodium atoms are continuously excited by a dye-laser beam. It is pointed out that, unlike photoelectric bunching, which can be given a semiclassical interpretation, antibunching is understandable only in terms of a quantized electromagnetic field. The measurement also provides rather direct evidence for an atom undergoing a quantum jump.



An example of Intensity Intensity Correlations in cavity QED

Optical Cavity QED

Quantum electrodynamics for pedestrians. No need for renormalization. One or a finite number of modes from the cavity.

ATOMS + CAVITY MODE

Dipolar coupling between the atom and the mode of the cavity:

$$g = \frac{d \cdot E_v}{\hbar}$$

El electric field associated with one photon on average in the cavity with volume: V_{eff} is:

$$E_{v} = \sqrt{\frac{\hbar\omega}{2\varepsilon_{0}V_{eff}}}$$









Jaynes Cummings Dynamics Rabi Oscillations

Exchange of excitation for *N* atoms:





Transmission doublet different from the Fabry Perot resonance









mean = 25


































mean = 548









How to correlate fields and intensities?

H. J. Carmichael, G. T. Foster, L. A. Orozco, J. E. Reiner, and P. R. Rice "Intensity-Field Correlations of Non-Classical Light ". Progress in Optics, Vol. 46, 355-403, Edited by E. Wolf Elsevier, Amsterdam 2004.

Detection of the field: Homodyne.

Conditional Measurement: Only measure when we know there is a photon.

The Intensity-Field correlator.



Condition on a Click Measure the correlation function of the Intensity and the Field: $<I(t) E(t+\tau)>$ Normalized form: $h_{\theta}(\tau) = <E(\tau)>_{c} /<E>$

From Cauchy Schwartz inequalities:

$$0 \le \overline{h_0}(0) - 1 \le 2$$
$$\left| \overline{h_0}(\tau) - 1 \right| \le \left| \overline{h_0}(0) - 1 \right|$$



Photocurrent average with random conditioning



Conditional photocurrent with no atoms in the cavity.





After 1 average



After 6,000 averages



After 10,000 averages



After 30,000 averages





Flip the phase of the Mach-Zehnder by 146°

Monte Carlo simulations for weak excitation:



Atomic beam N=11

The fluctuations of the electromagnetic field are measured by the spectrum of squeezing. Look at the noise spectrum of the photocurrent.

$$S(\nu,0^{\circ}) = 4F \int_{0}^{\infty} \cos(2\pi\nu\tau) [\overline{h}_{0}(\tau) - 1] d\tau,$$

F is the photon flux into the correlator.

Monte Carlo simulations of the wave-particle correlation and the spectrum of squeezing in the low intensity limit for an atomic beam.



It has upper and a lower classical bounds



N=13; 1.2n₀

With quantum optics we can measure and then manipulate the fluctuations of the field (Squeezing) and the intensity.

Improve the S/N ratio in LIGO, in telecomunications, etc.

Thanks