Top

Left side

TEST

Right side

bottom

An introduction to light-matter interaction, from cavity QED to waveguide QED 2

Les Houches,
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NIST

The presentation will be available at:

http://www.physics.umd.edu/rgroups/amo/orozco/results/2019/Results19.htm

A note about the dipole approximation:

- The interaction is **d•E**
- The **E** wave has an exp(i**k•r**) term
- Since the extent of **d** (a few Bohr radius) is small compared to the wavelength expand the exponential such that we only keep the 1st term.

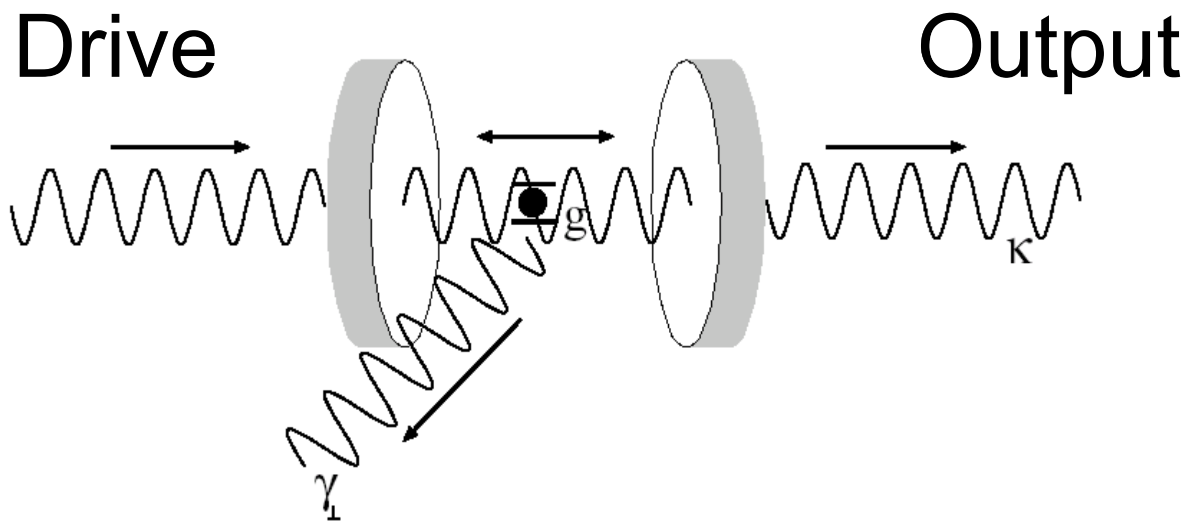
There is another length scale in the problem:

The extent of the ground state (z_0) in the bottom of the well.

Lamb Dicke parameter: kz_0

You want to make sure that the change in the kinetic energy of the trapped particle when absorbing or emmiting a photon does not excite the mechanical (external) motion.

Coupled atoms and cavities



Collection of N Two level atoms coupled to a single mode of the electromagnetic field (g). Driven with dissipation (atoms γ, cavity κ).

Microwaves

Micromaser

Visible light

Optical Bistability

Cavity QED

Absorptive Element

A saturable absorber has an absorption coefficient which is a non-linear function of I:

$$\alpha = \frac{\alpha_o}{1 + I / I_s}$$

$$for \quad I / I_s < \quad 1$$

$$\alpha \cong \alpha_o \left(1 - I / I_s + \cdots \right)$$

- The cavity is resonant.

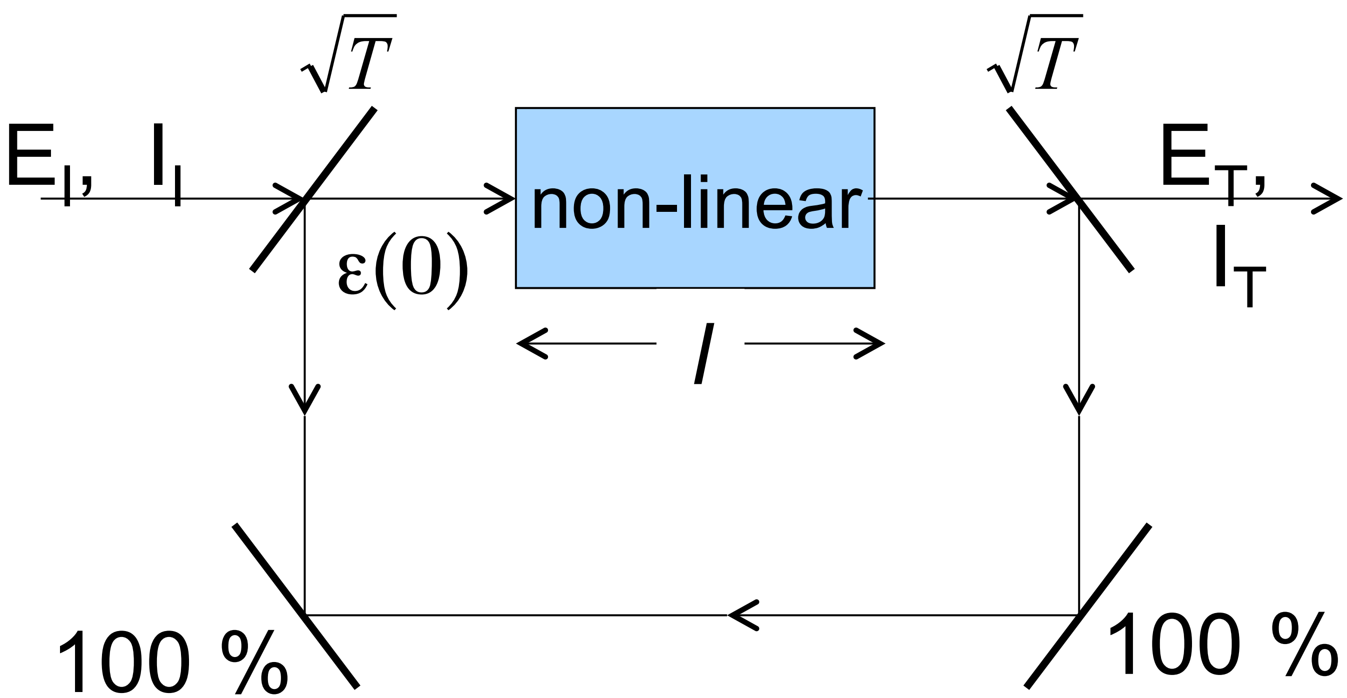
- At small intensities, the absorption due to the element is high and the output is low.

- As the intensity is increase beyond I_s, the absorption decreases and the output goes to high.

The field inside the cavity comes from the addition of the drive and what is already there

$$\text{Let } \varepsilon_{n+1}(0) = \sqrt{T} E_I + \mathrm{Re}^{-\alpha l} \; e^{iKL} \varepsilon_n(0)$$

Where e $_{n+1}$ is the electric field after the *n+1* path around the cavity, *L* is the round-trip length, a is the absorption coefficient and *R=1-T* the mirror reflectivity



- At steady state the electric field inside the cavity must be constant so that $\varepsilon_{n+1}(0) = \varepsilon_n(0) = \varepsilon_0$

$$\therefore \quad \varepsilon_0 = \sqrt{T}E_1 + \mathrm{Re}^{-\alpha l} e^{iKL} \varepsilon_0$$

rearanging this gives: $\varepsilon_0 = \dfrac{\sqrt{T}E_I}{(1 - \mathrm{Re}^{-\alpha l + iKL})}$

- The output field is given by the mirror transmittance times the internal electric field at a distance l.

$$E_T = \sqrt{T}\varepsilon(l) = \sqrt{T}\varepsilon_o e^{(-\alpha + iK)l}$$

- the amplitude transmission function is:

$$\frac{E_T}{\mathrm{E_I}} = \frac{Te^{iK(l-L)}}{e^{\alpha l - iKL} - R}$$

Absorptive Bistability

A saturable absorber, at resonance has an absorption coefficient which is a non-linear function of I:

$$\alpha = \frac{\alpha_o}{1 + I / I_s}$$

assuming that $\alpha l \ll 1$, gives on resonance:

$$\frac{E_T}{E_I} = \frac{1}{1 + \alpha l / T}$$

$$E_I = E_T\left[1+\frac{\alpha_o l/T}{1+I_T/I_sT}\right] \quad \text{with} \quad I = \frac{I_T}{T}$$

The ratio of losses: atomic losses per round trip (αl) to cavity losses per round trip (T) is the Cooperativity

$$C = \frac{\alpha_o l}{T} = \frac{\sigma_0 \rho l}{T} = \frac{\sigma_0 N}{Area_{\text{mode}}}\frac{1}{T}$$

$$C = \frac{g^2}{\kappa\gamma}N$$

The steady state for normalized input y and output x fields:

On resonance :

field:

$$y = x\left(1 + \frac{2C}{1+x^2}\right)$$

intensity:

$$Y = X\left(1 + \frac{2C}{1+X}\right)^2$$

For low intensity, the input field and the output field are linearly related,

$$y = x\,(1+2C)\ ;\ x/y=1/(1+2C) \text{ goes as } 1/N$$

For the intensity $Y=y^2$; $X=x^2$

$$Y=X(1+2C)^2\ ;\ X/Y=1/(1+2C)^2 \text{ goes as } 1/N^2$$

For very high field and intensity,

$$y = x\ ;\ Y=X+4C$$

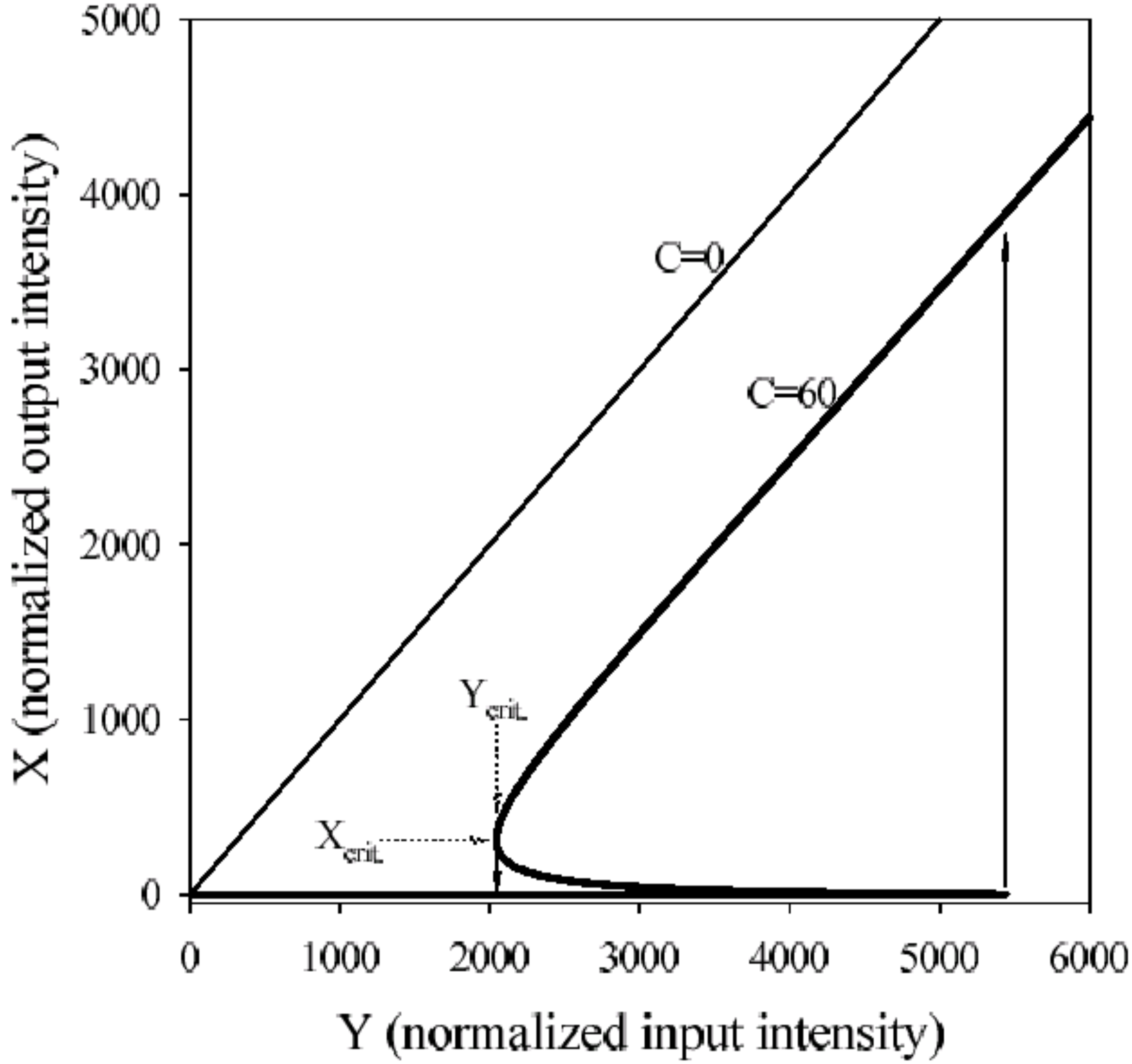
Almost an empty cavity

At intermediate intensity, there can be saturation (denominator of 1+X).

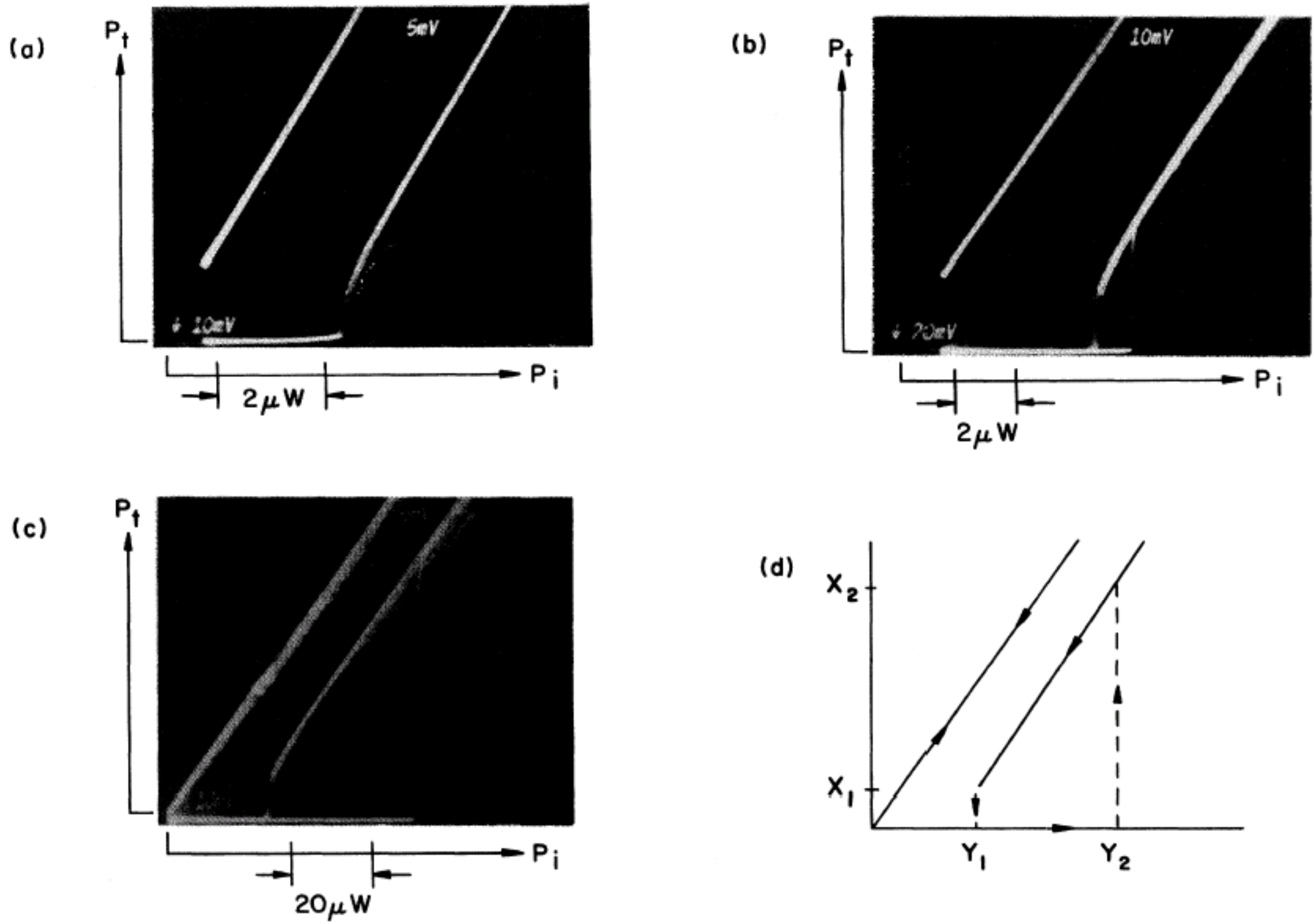
It happens in this simple model for the case of C>4. C (Cooperativity) is the negative of the laser pump paramenter.

C is a figure of merit.
It is the ratio of the atomic losses to the cavity losses or also can be read as the ratio between the good coupling (g) and the bad couplings (κ,γ), it is a ratio of areas.

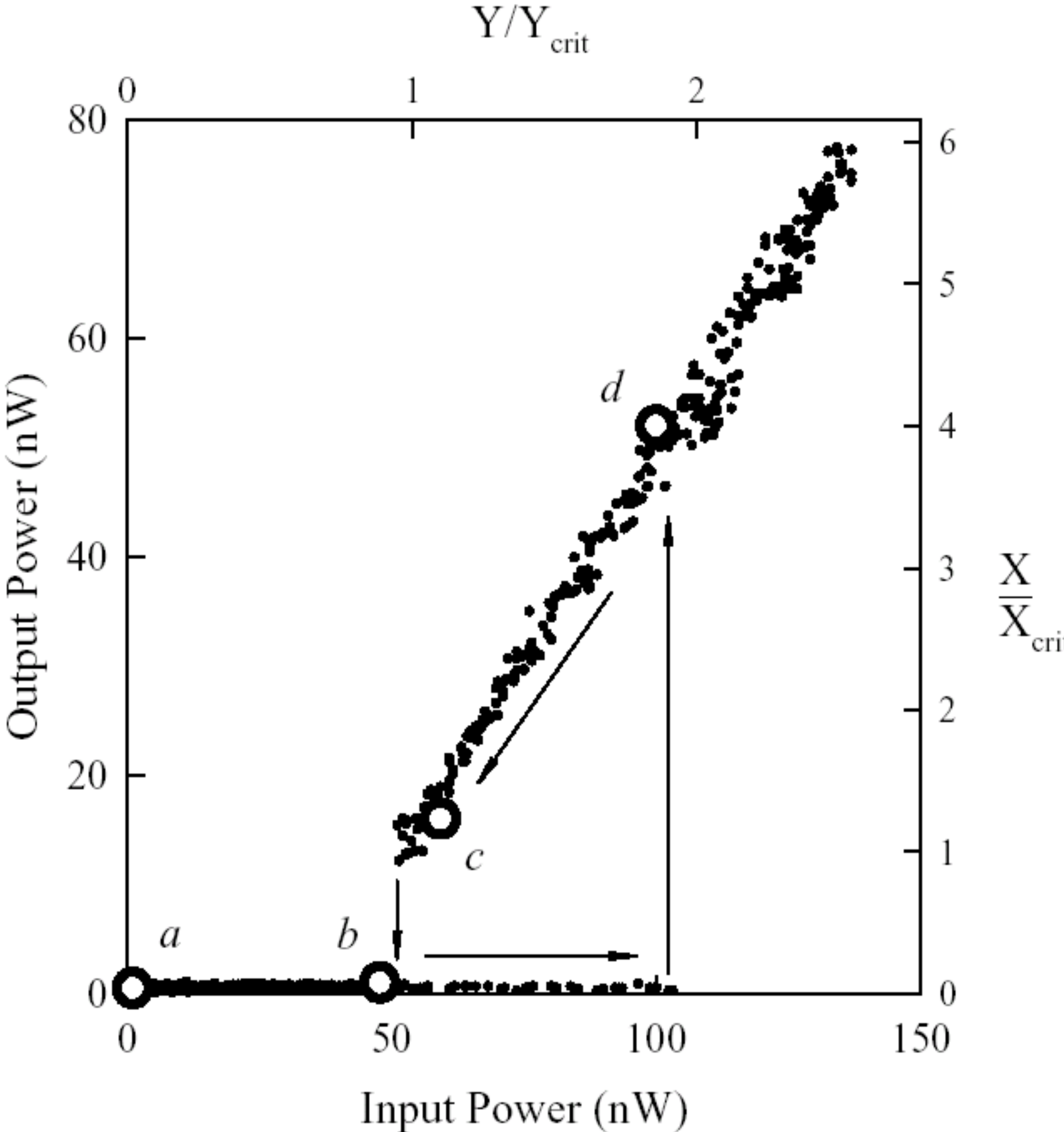
Input-Output response of the atoms-cavity system for two different cooperativities C=0 is with no atoms, C=60 has plenty of atoms, with a drive that can saturate them and we recover the linear relationship with unit slope between Y and X.



Increasing the number of atoms in the cavity:

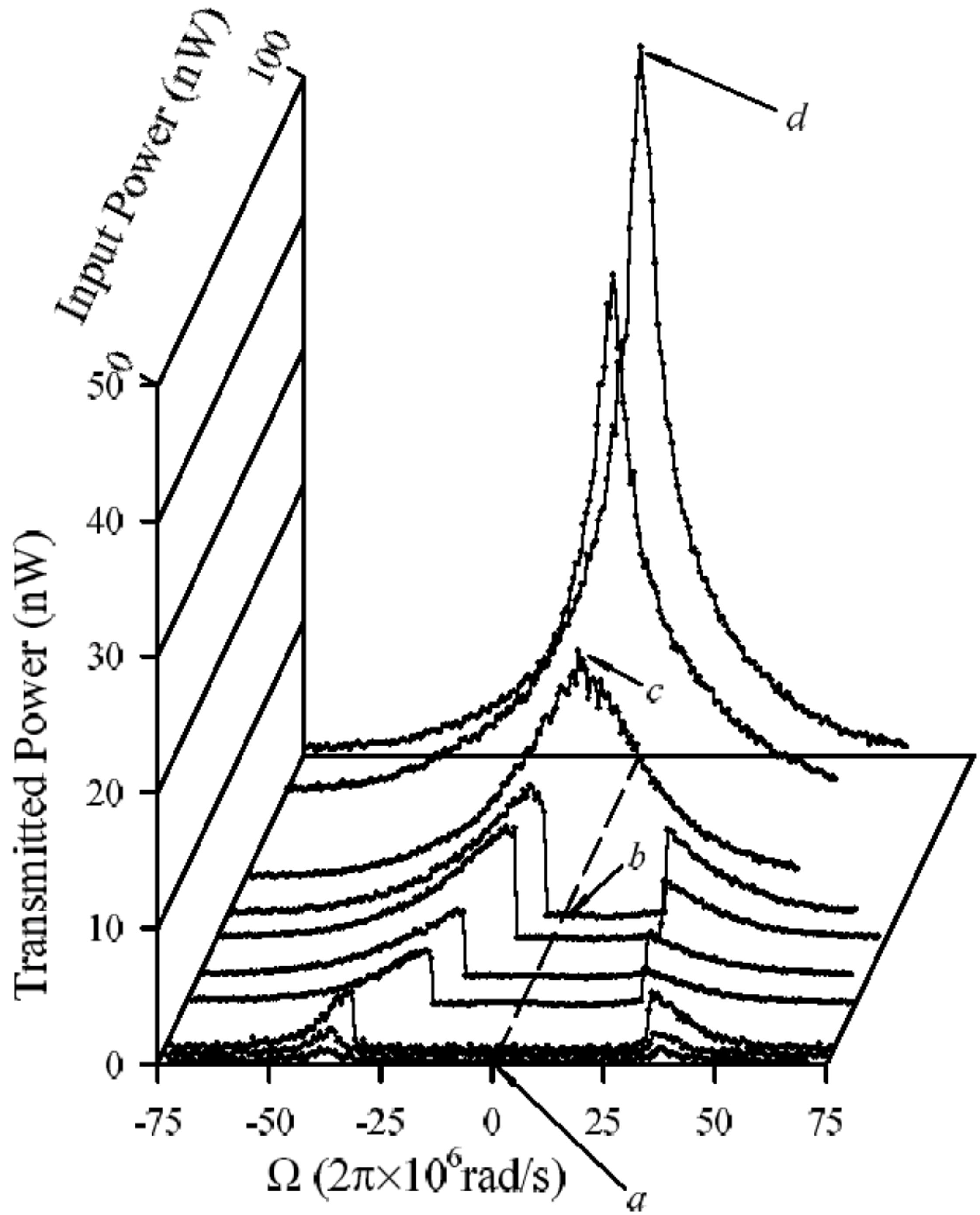


Transmission spectra at arbitrary intensity

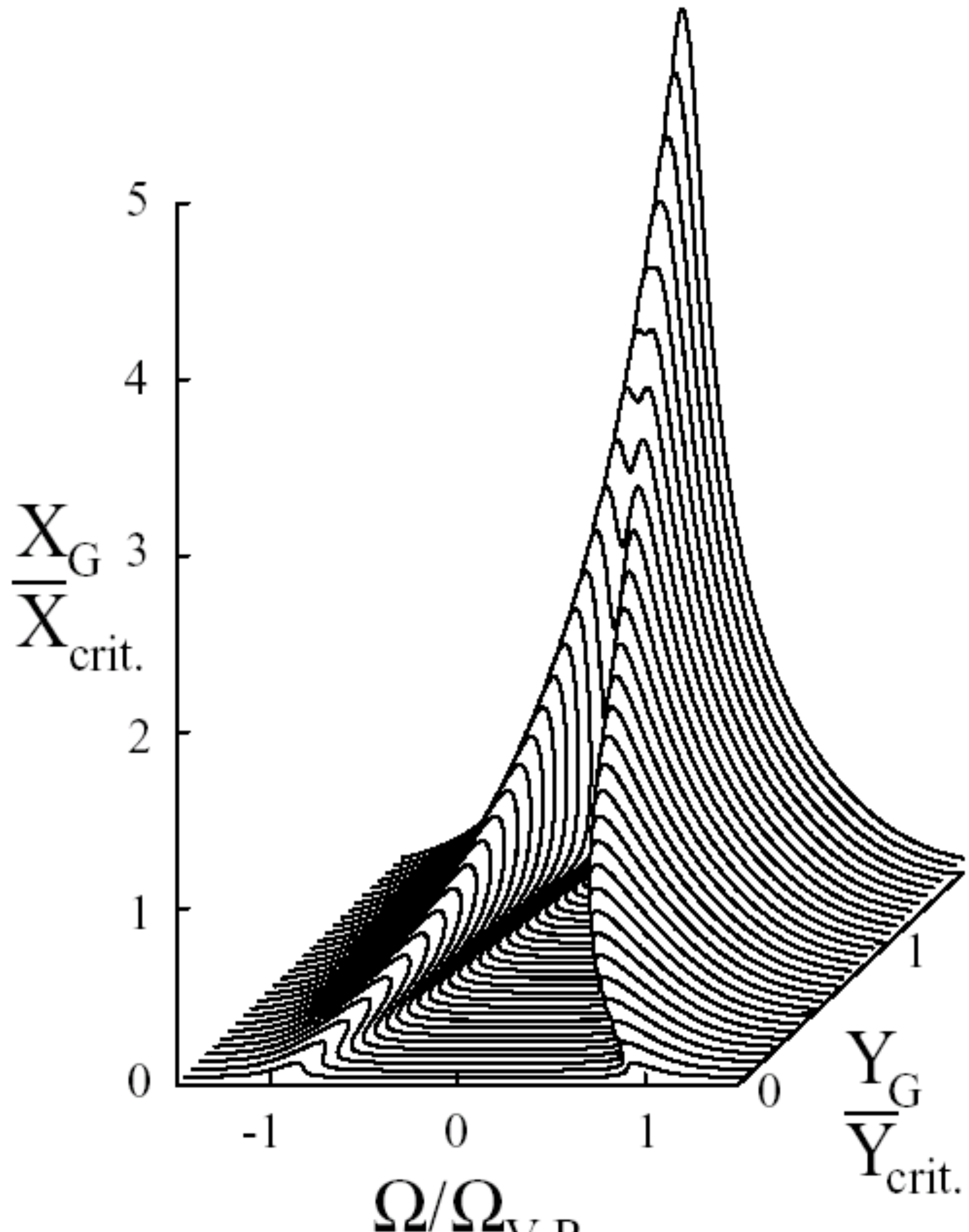


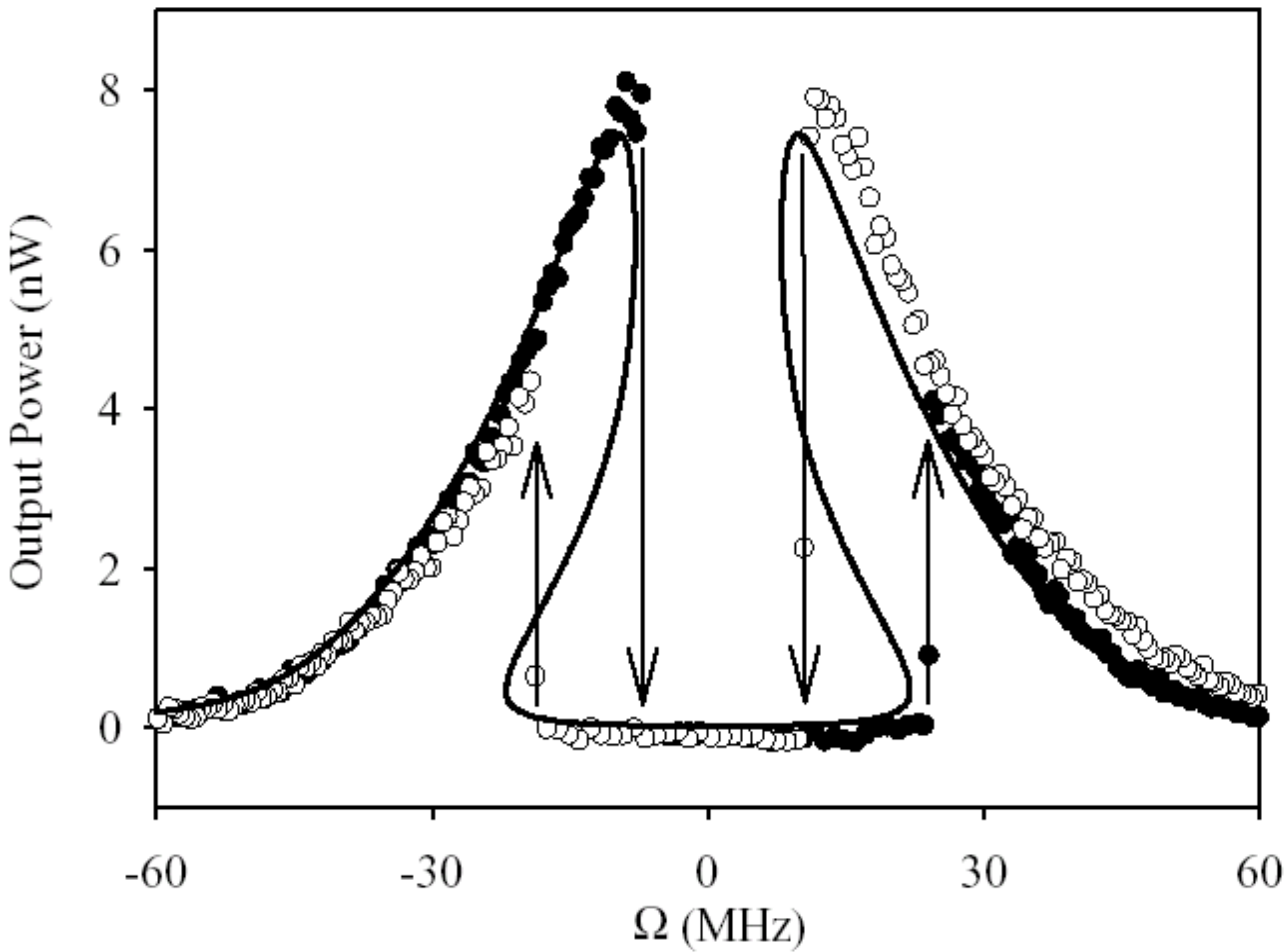
Y/Y_crit
0
1
2
80
60
40
20
0
Output Power (nW)
6
5
4
3
2
1
0
X/X_crit
a
b
c
d
0
50
100
150
Input Power (nW)

Transmission spectra for different intensities.



Theory





Hysteresis for a frequency scan of the light from the coupled atoms-cavity system.

The quantum model

Quantum Hamiltonian for N atoms

$$\hat{H} = \hat{H}_1 + \hat{H}_1 + \hat{H}_2 + \hat{H}_3 + \hat{H}_4 + \hat{H}_5 \ ,$$

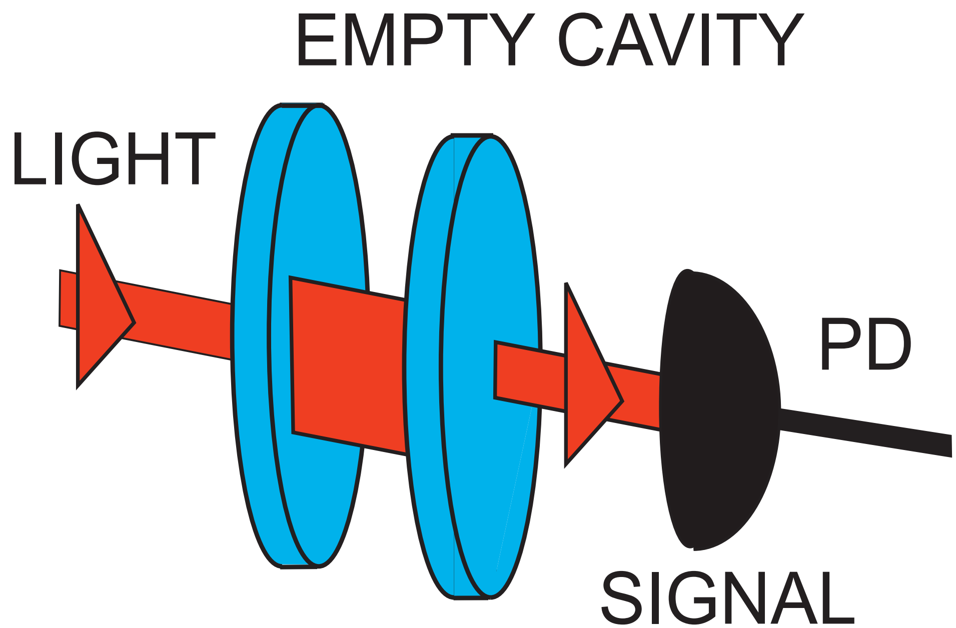
$$\hat{H}_1 = \hbar\omega_c \hat{a}^\dagger \hat{a} + \frac{1}{2}\hbar\omega_a \sum_{j=1}^{N} \hat{\sigma}_j^z \ ,$$ Free atoms free field

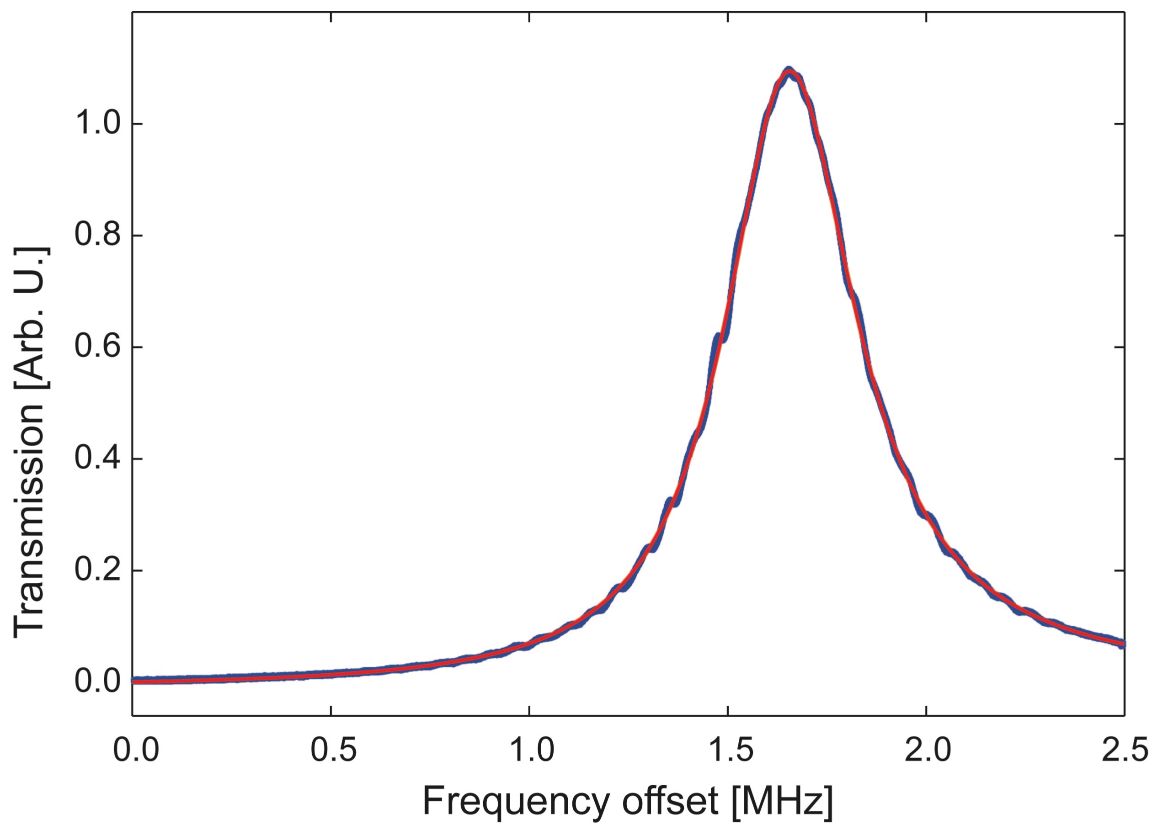
J.C $$\hat{H}_2 = i\hbar \sum_{j=1}^{N} g_j \left(\hat{a}^\dagger \hat{\sigma}_j^- e^{-i\vec{k}\cdot\vec{r}_j} - \hat{a}\hat{\sigma}_j^+ e^{i\vec{k}\cdot\vec{r}_j} \right)$$ Interaction

$$\hat{H}_3 = \sum_{j=1}^{N} \left(\hat{\Gamma}_A \hat{\sigma}_j^+ + \hat{\Gamma}_A^\dagger \hat{\sigma}_j^- \right) \ ,$$ Atomic decay

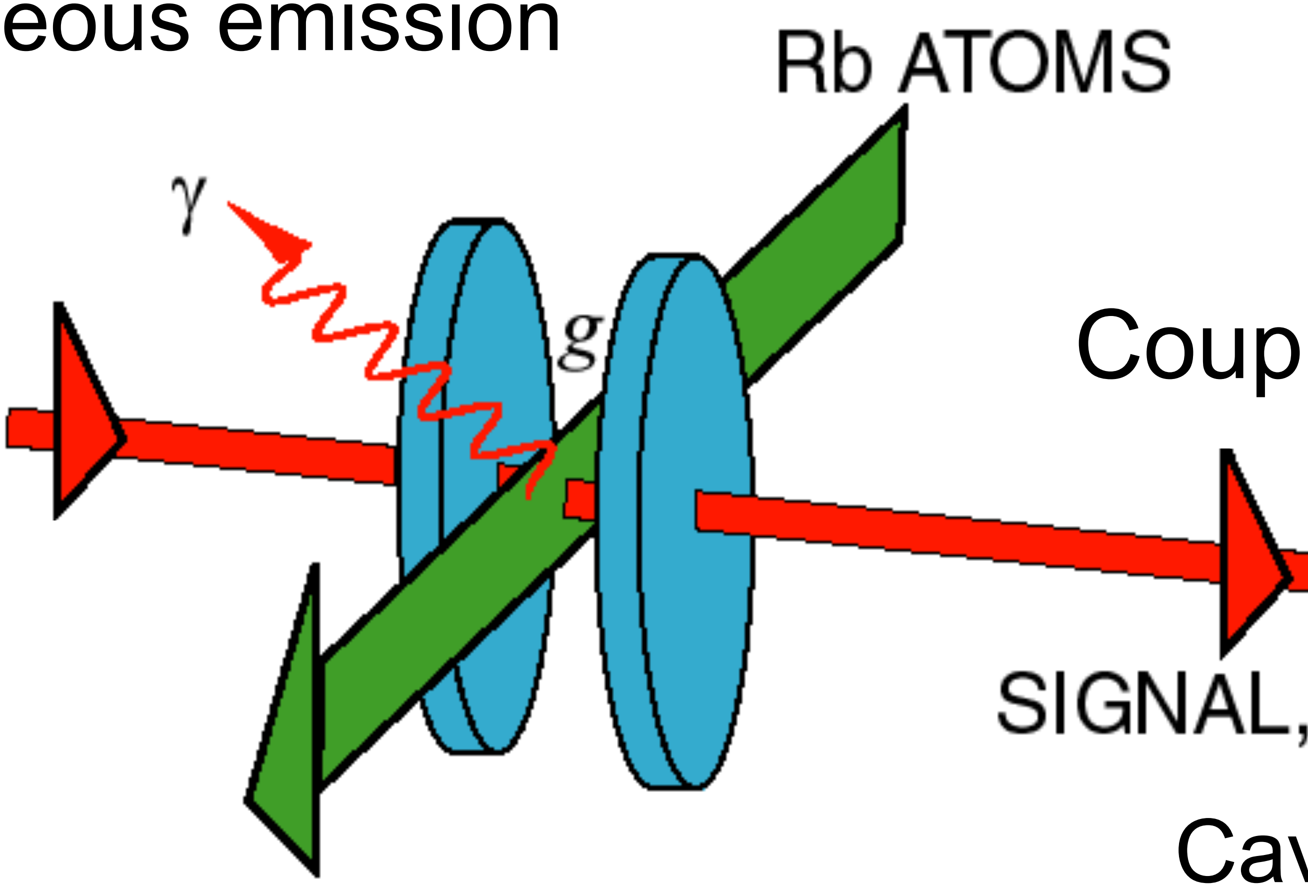
$$\hat{H}_4 = \hat{\Gamma}_F \hat{a}^\dagger + \hat{\Gamma}_F^\dagger \hat{a} \ ,$$ Cavity decay

$$\hat{H}_5 = i\hbar \left(\hat{a}^\dagger \mathcal{E} e^{-i\omega_l t} - \hat{a}\mathcal{E}^* e^{i\omega_l t} \right) \ .$$ Drive


EMPTY CAVITY
LIGHT
PD
SIGNAL


Transmission [Arb. U.]
0.0
0.2
0.4
0.6
0.8
1.0
0.0
0.5
1.0
1.5
2.0
2.5
Frequency offset [MHz]

Spontaneous emission



Coupling

Cavity decay

Cooperativity for one atom: C_1

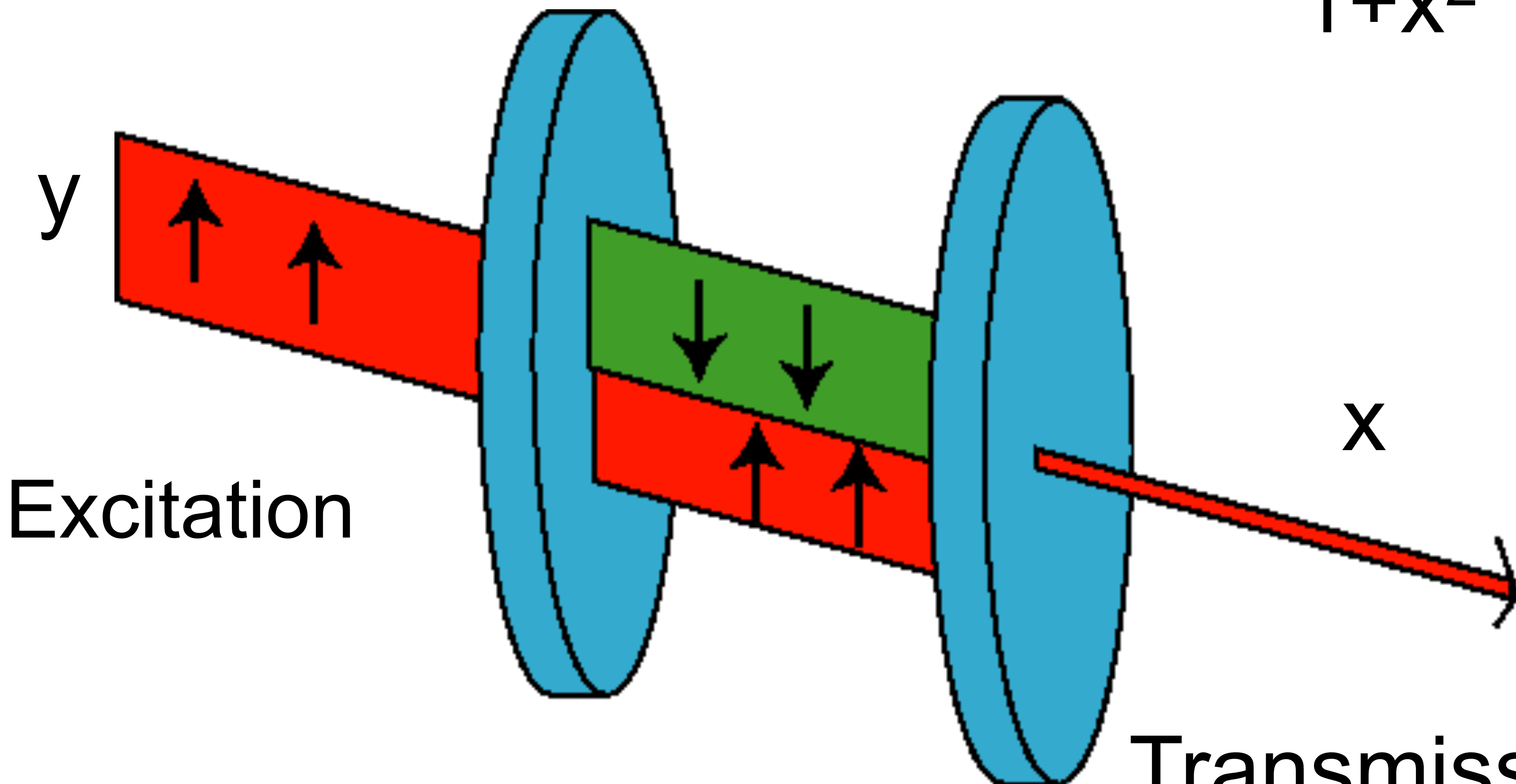
$$C_1 = \frac{g^2}{\kappa\gamma} \qquad C = C_1 N$$

Cooperativity for N atoms: C

$$g \approx \kappa \approx \gamma$$

Steady State

Atomic polarization: $\dfrac{-2Cx}{1+x^2}$

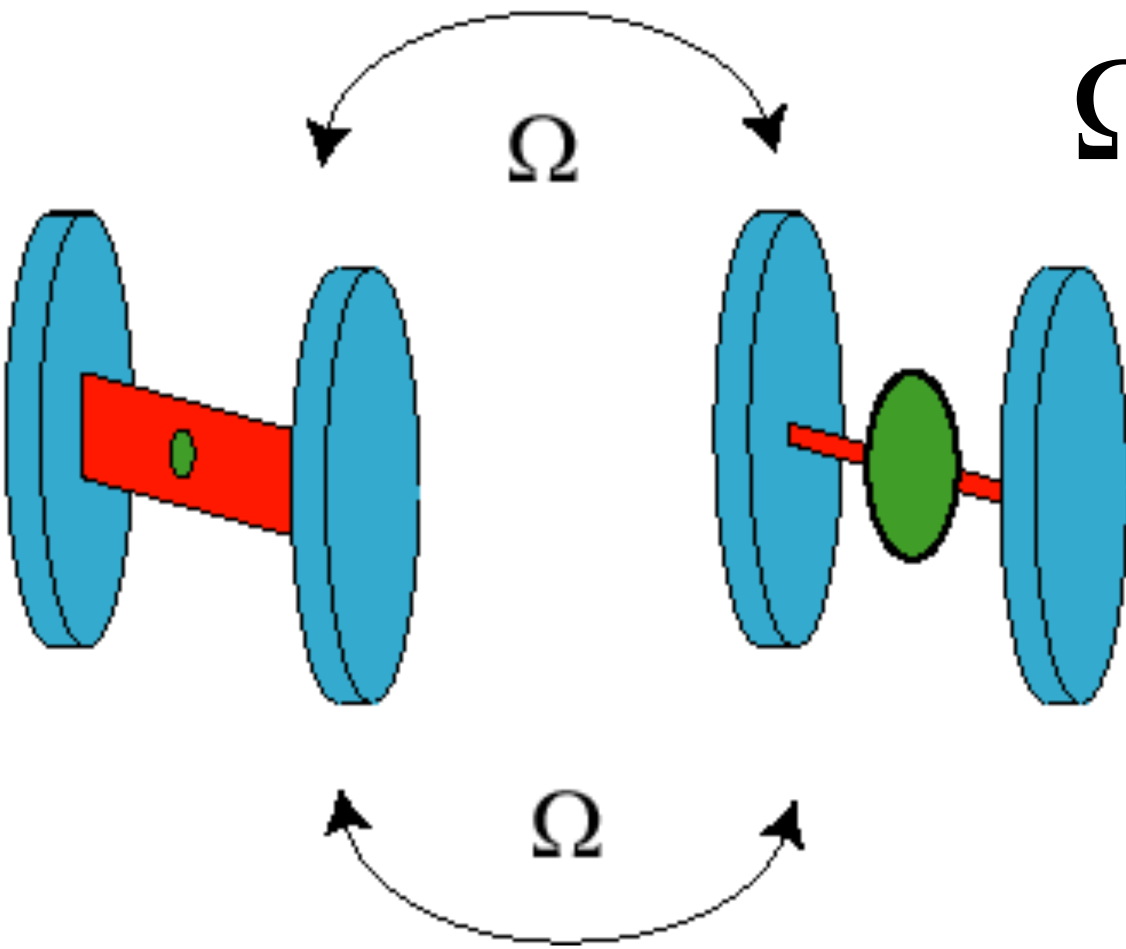


Transmission
$x/y = 1/(1+2C)$

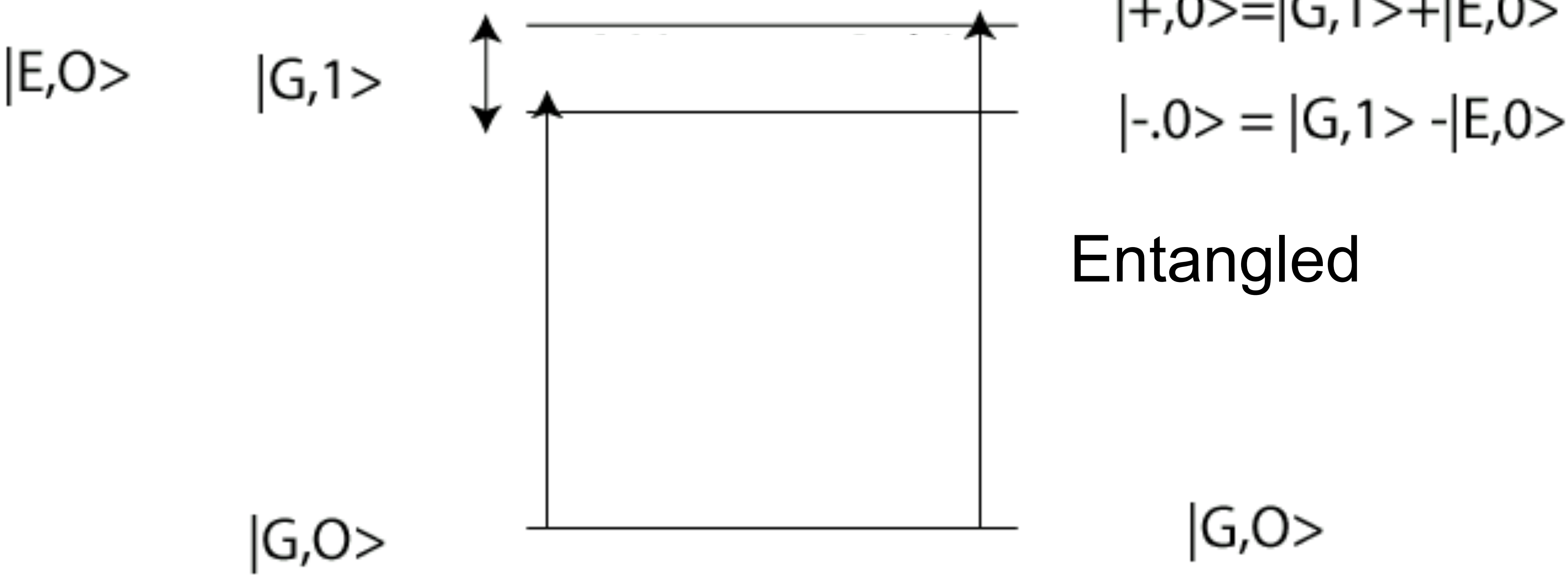
Jaynes Cummings Dynamics
Rabi Oscillations

Exchange of excitation for *N* atoms:

$$\Omega \approx g\sqrt{N}$$



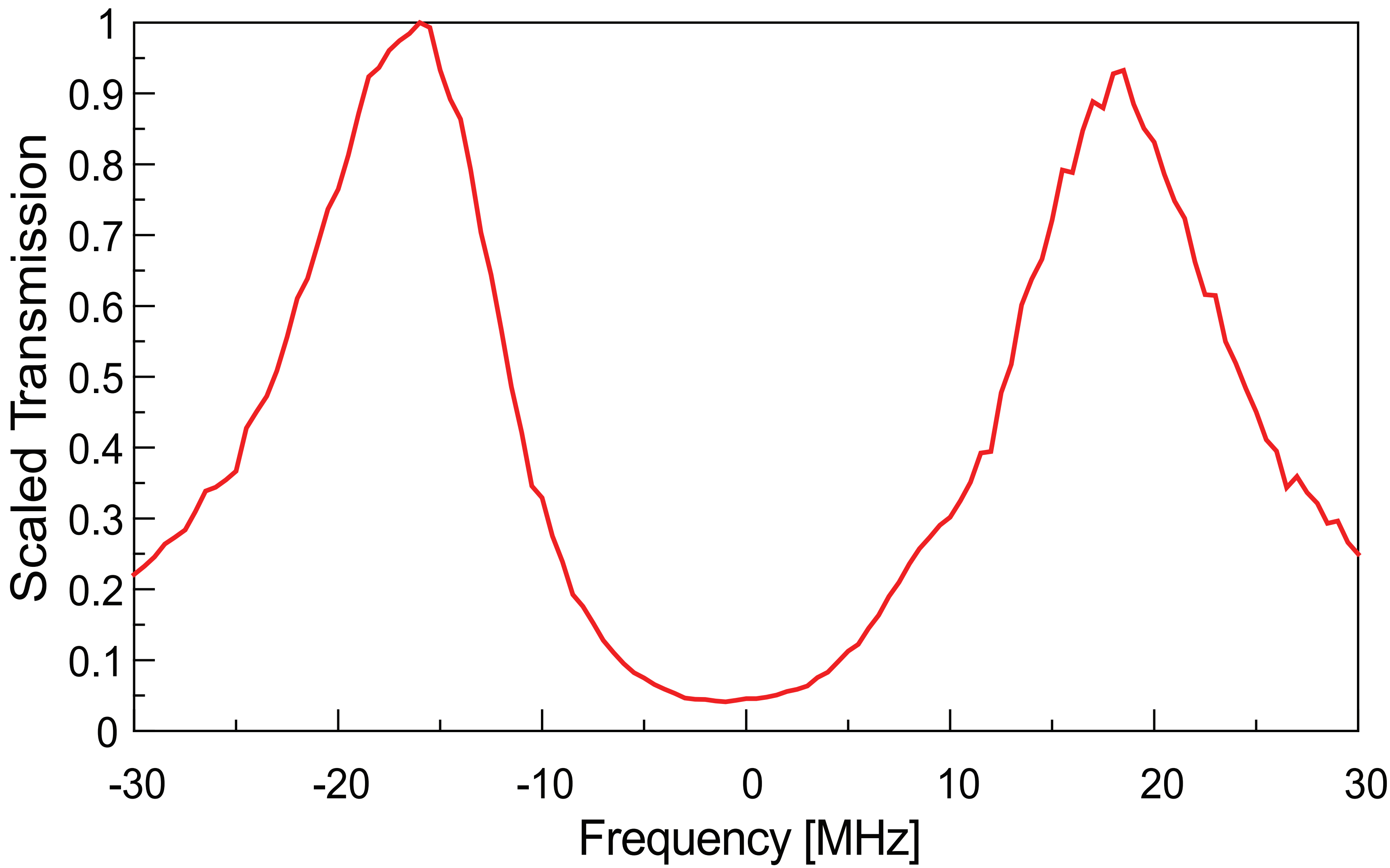
2g Vacuum Rabi Splitting

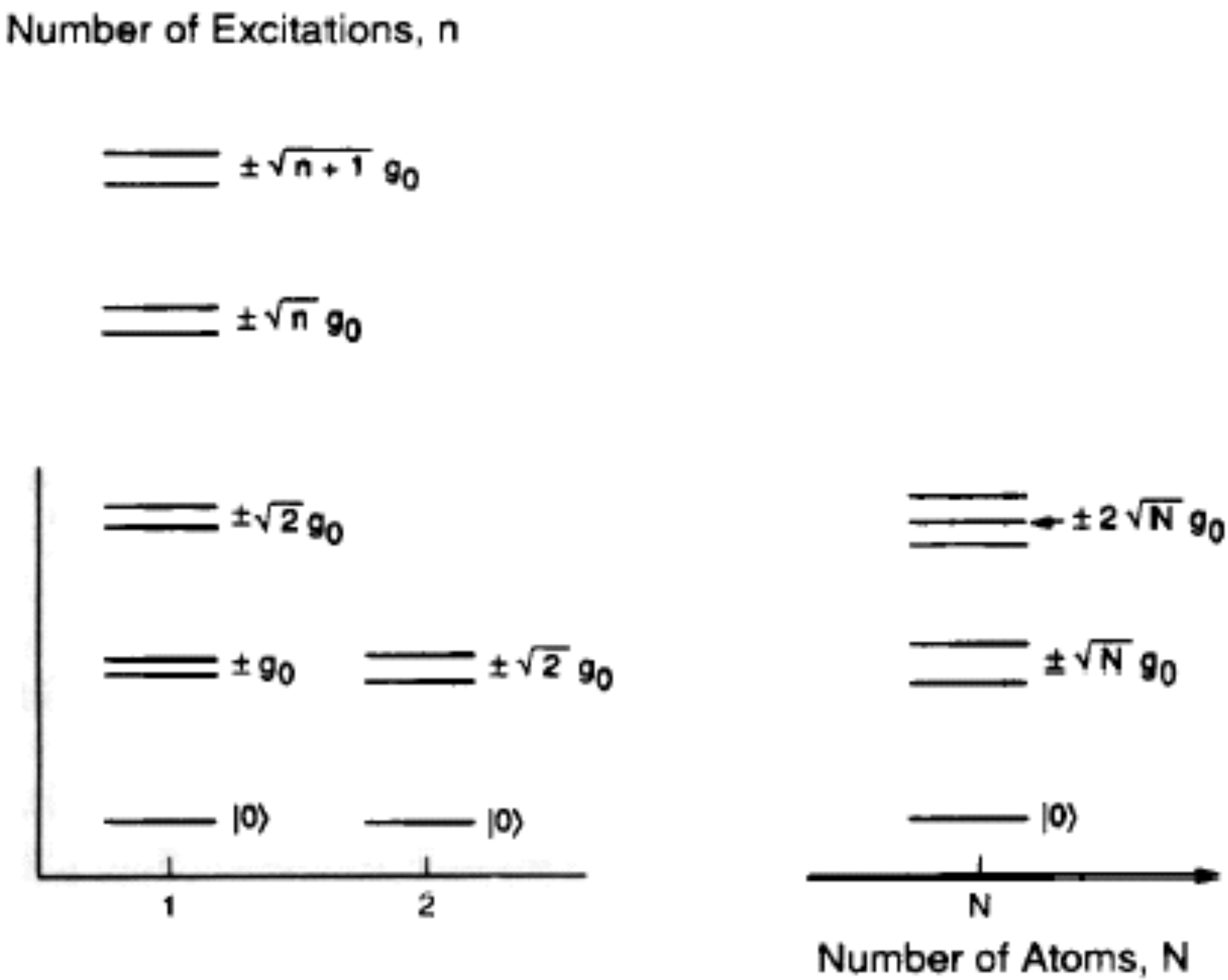


Not coupled

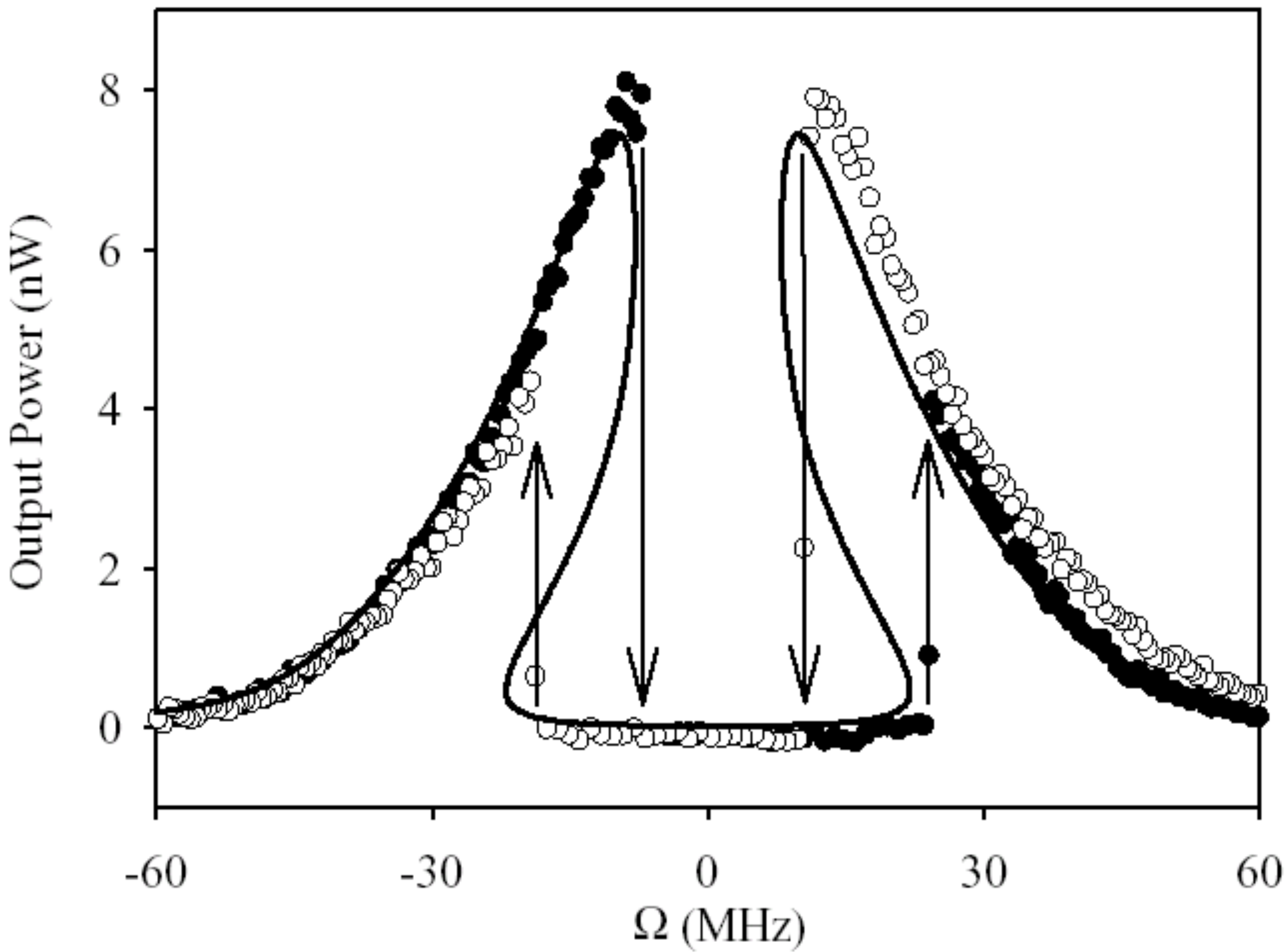
Two normal modes

Transmission doublet different from the Fabry Perot resonance



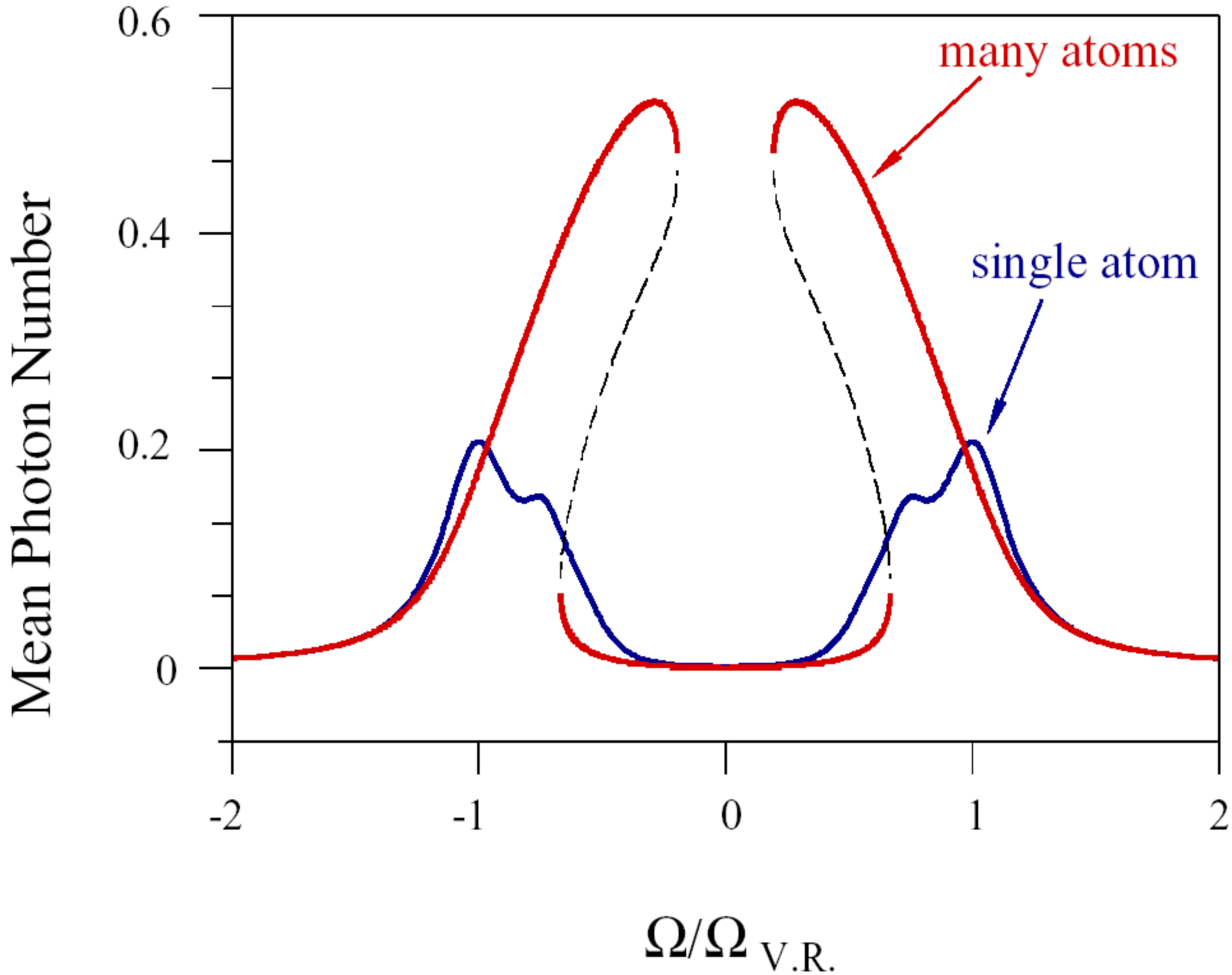


Number of Excitations, n
$\pm\sqrt{n+1}\,g_0$
$\pm\sqrt{n}\,g_0$
$\pm\sqrt{2}\,g_0$
$\pm g_0$
$\pm\sqrt{2}\,g_0$
$|0\rangle$
$|0\rangle$
1
2
$\pm 2\sqrt{N}\,g_0$
$\pm\sqrt{N}\,g_0$
$|0\rangle$
N
Number of Atoms, N



Hysteresis for a frequency scan of the light from the coupled atoms-cavity system.

Many atoms solved with Maxwell Bloch equations.
Single atom solved with the full Hamiltonian, no decorrelation. The system does not show hysteresis.



nature physics

LETTERS

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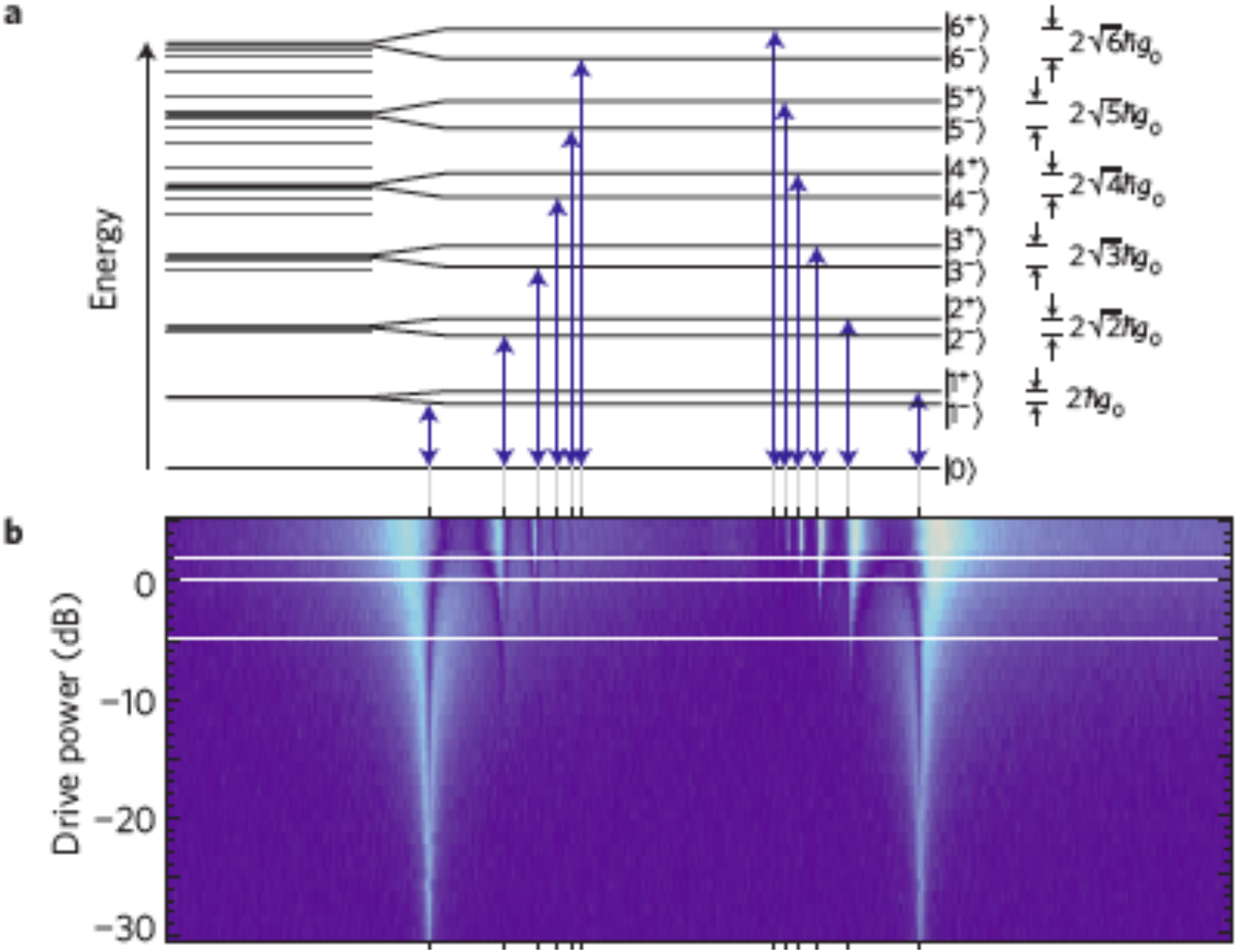
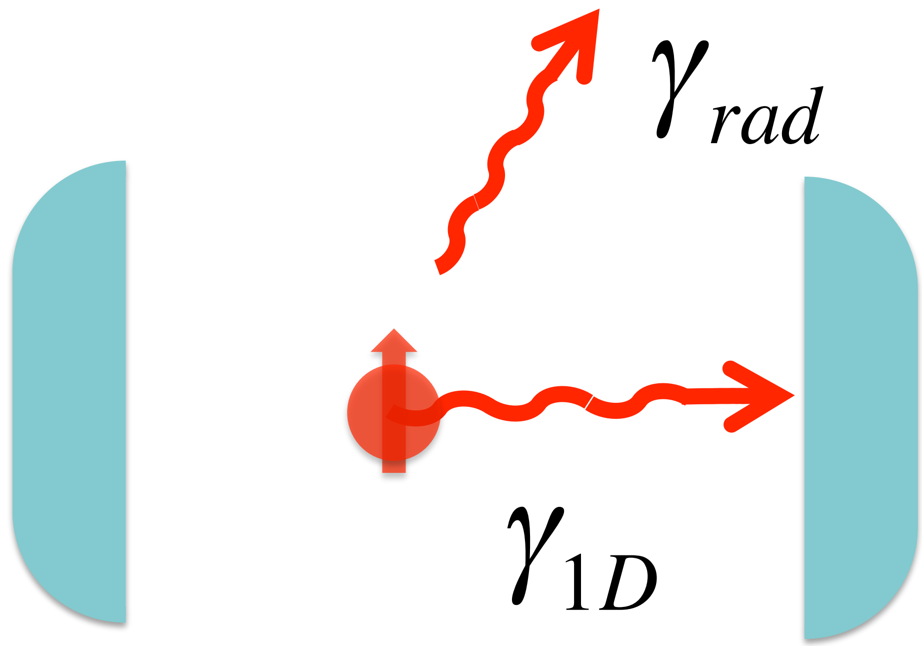
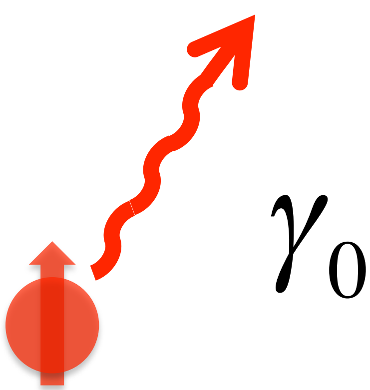
Nonlinear response of the vacuum Rabi resonance

Lev S. Bishop[1], J. M. Chow[1], Jens Koch[1], A. A. Houck[1], M. H. Devoret[1], E. Thuneberg[2], S. M. Girvin[1] and R. J. Schoelkopf[1]*


a
Energy
|6+⟩
|6−⟩
|5+⟩
|5−⟩
|4+⟩
|4−⟩
|3+⟩
|3−⟩
|2+⟩
|2−⟩
|1+⟩
|1−⟩
|0⟩
2√6ħg0
2√5ħg0
2√4ħg0
2√3ħg0
2√2ħg0
2ħg0
b
Drive power (dB)
0
−10
−20
−30

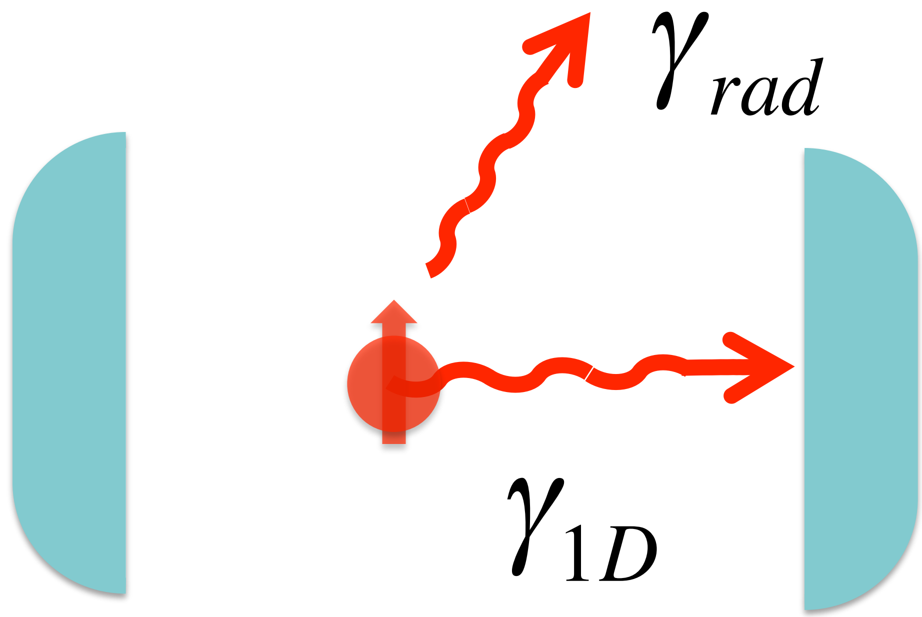
Coupling Enhancement

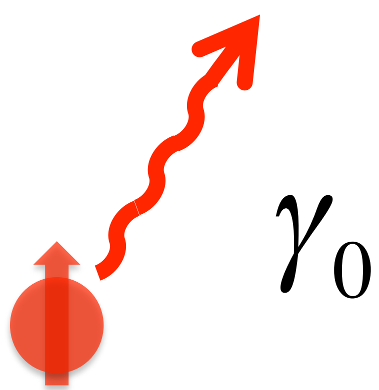




$$\alpha = \frac{\gamma_{1D}}{\gamma_0}$$

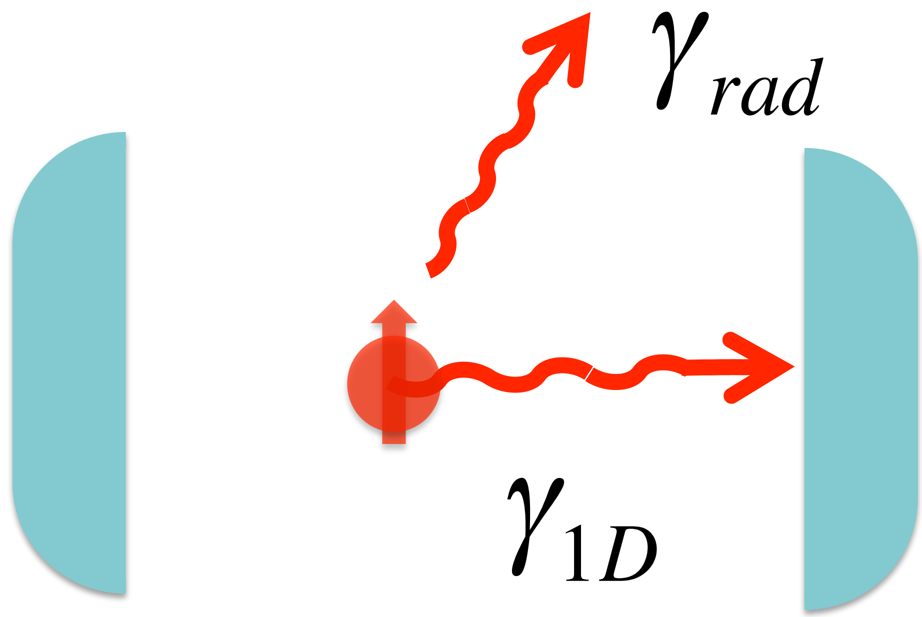
Coupling Efficiency

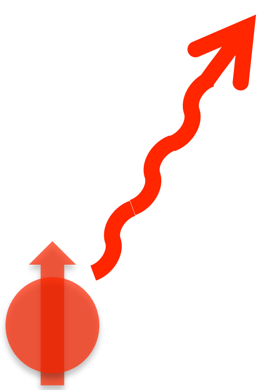




$$\beta = \frac{\gamma_{1D}}{\gamma_{Tot}} \quad ; \quad \gamma_{Tot} = \gamma_{1D} + \gamma_{rad}$$

Purcell Factor

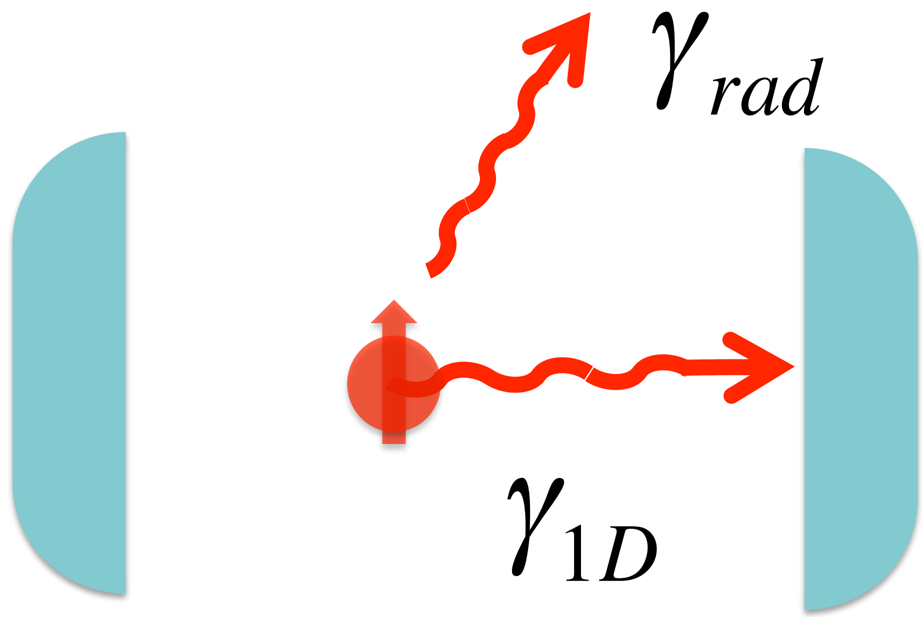


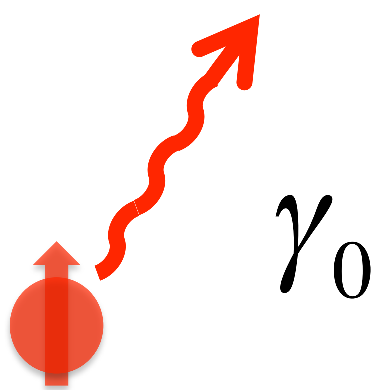


$$F_P = \frac{\gamma_{tot}}{\gamma_0} = \frac{\alpha}{\beta}$$

$$\gamma_{Tot} = \gamma_{1D} + \gamma_{rad}$$

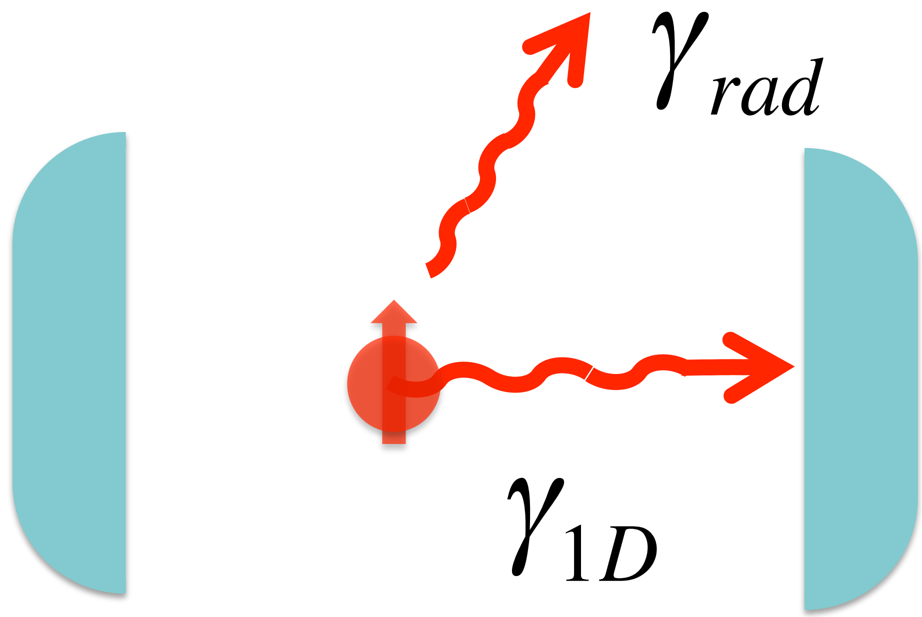
Cooperativity

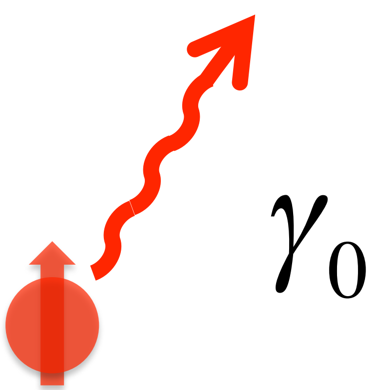




$$C_1 = \frac{\beta}{(1-\beta)} = \frac{\gamma_{1D}}{\gamma_{rad}}$$

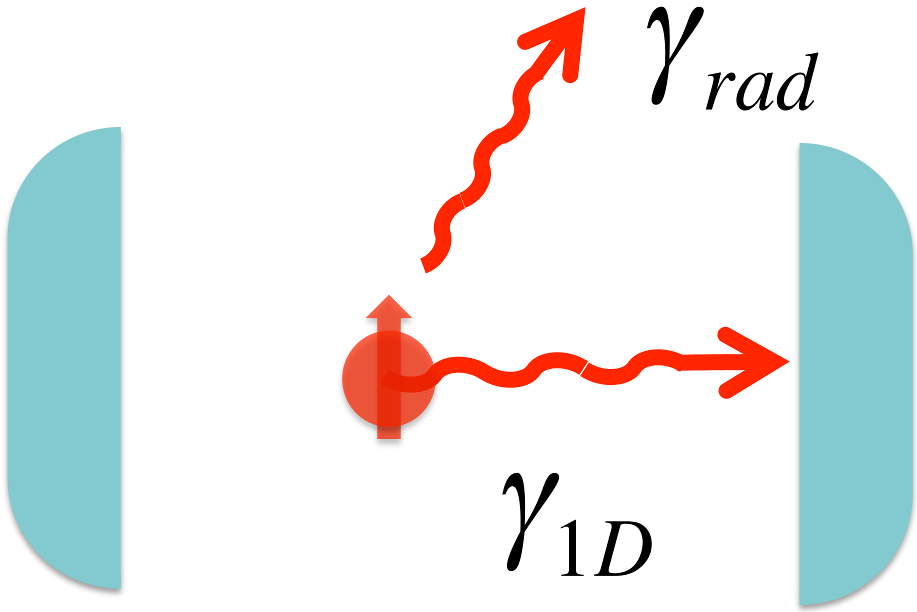
Cooperativity

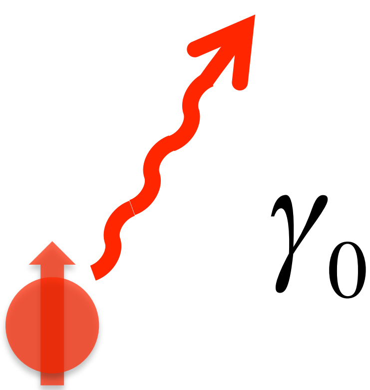




$$C_1 = \frac{g^2}{\kappa \gamma_0}$$

Cooperativity





$$C_1 = \frac{\sigma_0}{Area_{\text{mode}}}\frac{1}{T}$$

The coupling enhancement α is proportional to the total number of photons emitted into the cavity mode,

The coupling efficiency β is the percentage of photons emitted into the mode relative to the total number of emitted photons.

The cooperativity is the ratio between the photons going into the mode and those emitted out to other modes. It is the inverse of the number of atoms that are necessary to observe non-linear effects in the cavity.

Some Implementations

- Rydbergs on Superconducting cavities (Microwaves)
- Alkali atoms on Optical Cavities (Optical)
- Quantum dots on microcavities (Optical)
- Trapped ions and vibrational mode (phonons)
- Circuit QED Superconducting qbits on microwave resonators (Microwaves)
- Polaritons on optical microcavities (photons)

The cooperativity has become the figure of merit for many quantum optics experiments, it is not limited to cavity QED.

How to choose a platform?

$$C = \frac{\sigma_0}{Area_{\mathrm{mode}}} \frac{1}{T} N \qquad\qquad C = \frac{g^2}{\kappa \gamma_0} N$$

Take the area of the mode to be $\pi(\lambda/2)^2$, and σ as $3\lambda^2/2\pi$ then C does not depend on the choise of atom

Another approach is to maximize g through a large E_0, then minimize the cavity volume V

The solutions are guided by your resources and where you can approach the ideals

Microwaves can be confined to cavities with mode areas close to the atomic cross section of the Rydberg Atoms. (Experiments led by S. Haroche)

This is more difficult in the visible for free space with atoms, but recent developments at ENS on making micrometric mirrors are helping.

Thanks