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An introduction to light-matter interaction, from cavity QED to waveguide QED

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Thanks to Antoine Browaeys, Quentin Glorieux, and Julien Laurat for organizing this pre-doctoral school at Les Houches and inviting me.

Happy 80th birthday to Francium

The presentation will be available at:

http://www.physics.umd.edu/rgroups/amo/orozco/results/2019/Results19.htm

1. A review of Electricity and Magnetism and Polarization

Maxell’s Equations:

$$\nabla \cdot (\varepsilon_0 \mathbf{E} + \mathbf{P}) = 0,$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$

$$\nabla \times \mathbf{B} = \mu_0 \frac{\partial (\varepsilon_0 \mathbf{E} + \mathbf{P})}{\partial t},$$

$$\nabla \cdot \mathbf{B} = 0.$$

Wave Equation:

$$\nabla \times (\nabla \times \mathbf{E}) = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2\mathbf{E}$$

$$= -\frac{\partial}{\partial t}(\nabla \times \mathbf{B}) = -\mu_0\varepsilon_0\frac{\partial^2\mathbf{E}}{\partial t^2} - \mu_0\frac{\partial^2\mathbf{P}}{\partial t^2}$$

$$-\nabla(\nabla \cdot \mathbf{E}) + \nabla^2\mathbf{E} - \mu_0\varepsilon_0\frac{\partial^2\mathbf{E}}{\partial t^2} = \mu_0\frac{\partial^2\mathbf{P}}{\partial t^2}.$$

In free space, $\nabla \cdot \mathbf{E} = 0$ and $\mathbf{P} = \mathbf{0}$,

$$\nabla^2\mathbf{E} - \frac{1}{v_0^2}\frac{\partial^2\mathbf{E}}{\partial t^2} = 0$$

The slowly varying envelop equations (Towards the Maxwell Bloch equations)

$$\frac{\partial F}{\partial z} + \frac{1}{c}\frac{\partial F}{\partial t} = -\alpha P$$

Where F and P are the slowly varying envelops of the Electric field and the polarization and α is the absorption coefficient

A note about polarization

Gauss's Law in free space:

$$\nabla \cdot \vec{E} = 0$$

$$\nabla_T \cdot \vec{E} + \frac{2\pi}{\lambda} i E_Z = 0$$

If there is a "transverse gradient" in the radiation field propagating in *z*, there is a longitudinal polarization also in *z*

Polarization at the waist of an optical nanofiber

$$\nabla \cdot \vec{E} = 0$$

$$\nabla_T \cdot \vec{E} + \frac{2\pi}{\lambda} i E_Z = 0$$

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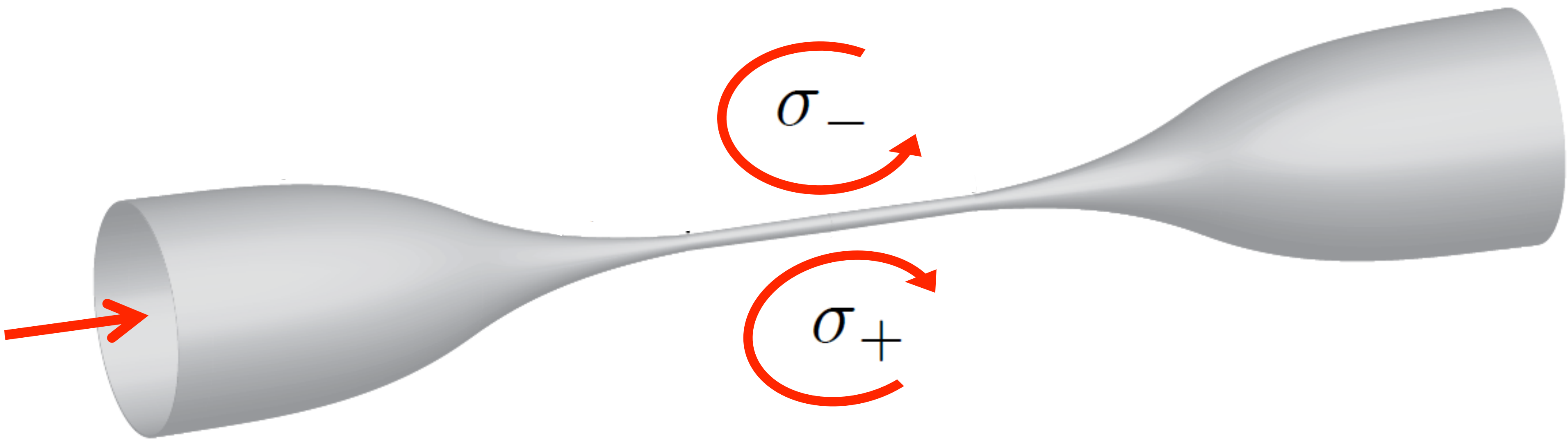

Bicycle polarization not propeller

Polarization at the waist of an optical nanofiber

$$\nabla \cdot \vec{E} = 0$$

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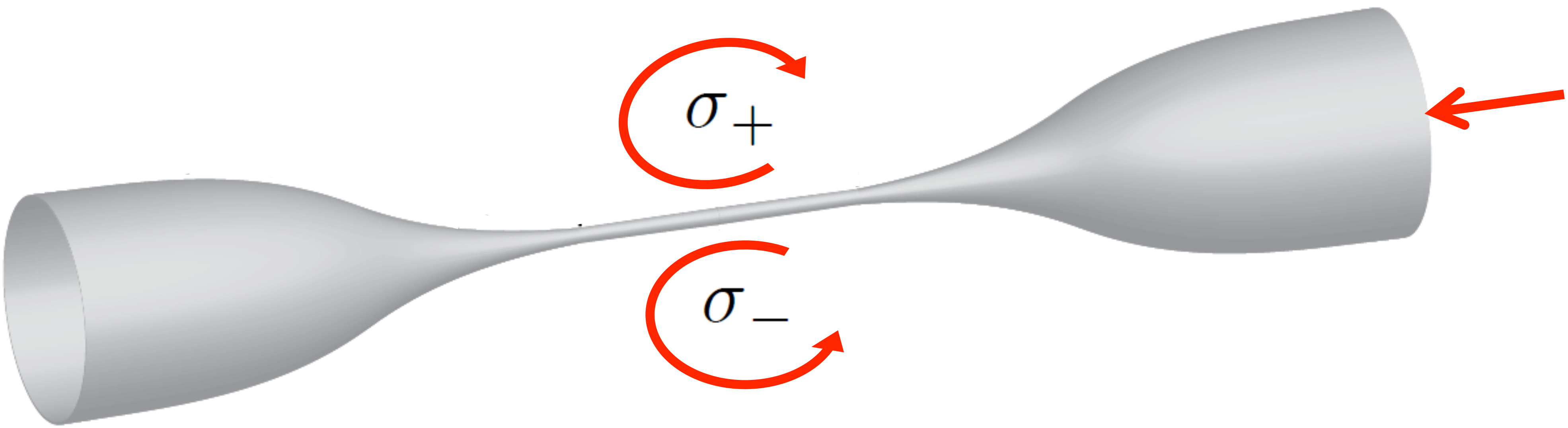
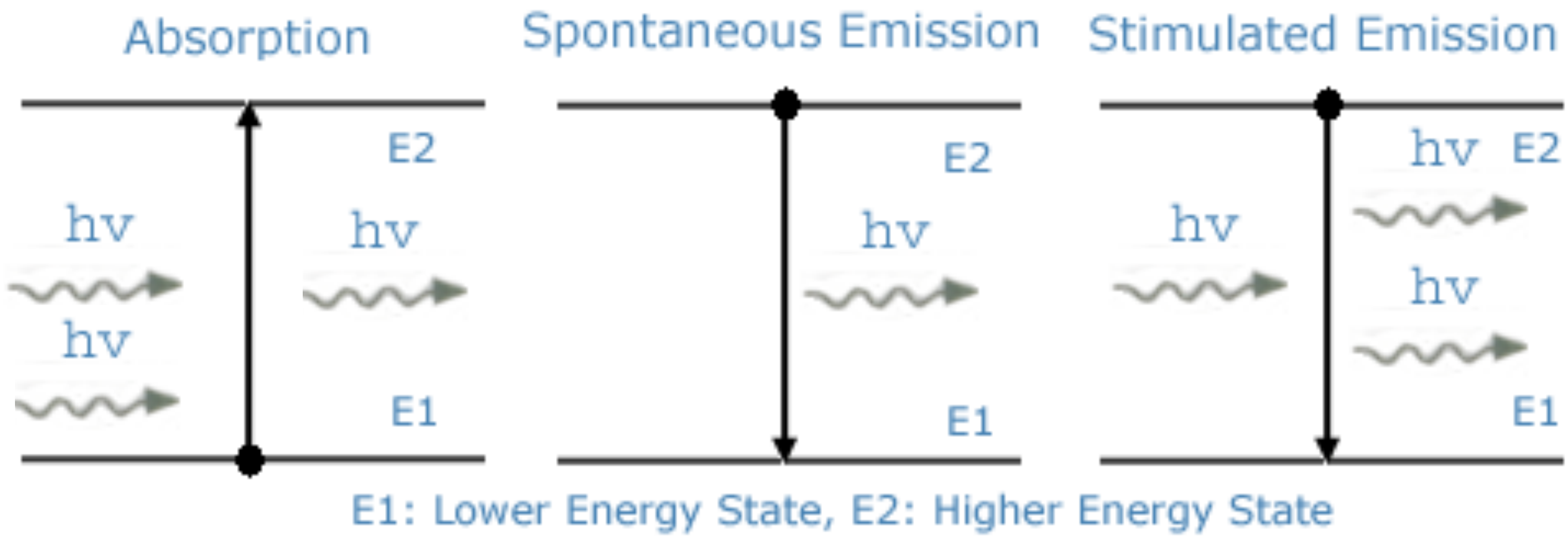


$$E_z = \pm \frac{i}{k} \nabla \cdot \mathbf{E}_\perp$$

2. One atom interacting with light in free space.



Absorption and Stimulated Emission are time reversals of each other, you can say this is the classical part. Spontaneous emission is the quantum, that is the jump.

Dipole cross section (same result for a classical dipole or from a two level atom):

$$\sigma_0 = \frac{3\lambda_0^2}{2\pi}$$

This is the "shadow" caused by a dipole on a beam of light.

Energy due to the interaction between a dipole and an electric field.

$$H = \vec{d} \cdot \vec{E}$$

The dipole matrix element between two states is fixed by the properties of the states (radial part) and the Clebsh-Gordan coefficients from the angular part of the integral. It is a few times a_0 (Bohr radius) times the electron charge e between the S ground and P first excited state in alkali atoms.

$$\vec{d} = e\left\langle 5S_{1/2} \middle| \vec{r} \middle| 5P_{3/2} \right\rangle$$

Beer-Lambert law for intensity attenuation

$$\frac{dI}{dz} = -\alpha I \quad \text{if } \alpha \text{ is resonant and independent}$$

of I, α_0 (does not saturate)

$$I = I_0 \exp(-\alpha_0 l)$$

where $\alpha_0 = \sigma_0 \rho$

and $\rho = N / V$ the density of absorbers in a length l

Rate of decay
(Fermi's golden rule)

$$\gamma_{rad} \approx \frac{2\pi}{\hbar} \rho(k) \langle H_{int} \rangle^2$$

Phase space density

Interaction

Rate of decay free space (Fermi's golden rule)

$$\gamma_0 = \frac{\omega_0^3 d^2}{\pi \varepsilon_0 \hbar c^3}$$

Where *d* is the dipole moment

Saturation intensity:
One photon every two lifetimes over the cross section of the atom (resonant)

$$I_s = \frac{\hbar\omega_0}{2\tau_0\sigma_0} = \frac{\pi}{3}\frac{\gamma_0\hbar\omega_0}{\lambda_0^2}$$

If $I=I_0$ the rate of stimulated emission is equal to the rate of spontaneous emission (Rabi Frequency Ω) and the population on the excited state 1/4.

$$\Omega = \frac{\vec{d} \bullet \vec{E}}{\hbar} = \gamma \sqrt{I / I_s}$$

$$\text{Excited Population} = \frac{1}{2} \frac{I / I_s}{1 + I / I_s}$$

3a. Cavity QED at low intensity

Optical Cavity QED

Quantum electrodynamics for pedestrians. No need for renormalization. One or a finite number of modes from the cavity.

ATOMS + CAVITY

Regimes:

Perturbative: Coupling<< Dissipation. Atomic decay suppressed or enhanced (cavity smaller than $\lambda/2$), changes in the energy levels.

Non Perturbative: Coupling>>Dissipation

Vacuum Rabi Splittings. Conditional dynamics.

Dipolar coupling between the atom and the mode of the cavity:

$$g = \frac{d \cdot E_v}{\hbar}$$

El electric field associated with one photon on average in the cavity with volume: V_{eff} is:

$$E_v = \sqrt{\frac{\hbar\omega}{2\varepsilon_0 V_{eff}}}$$

Radiation field:

$$\frac{\partial}{\partial t}\langle\hat{a}\rangle = -\kappa(1+i\theta)\langle\hat{a}\rangle + \sum_{j=1}^{N} g_j\langle\hat{\sigma}_j^-\rangle + \mathcal{E},$$

Atomic polarization:

$$\frac{\partial}{\partial t}\langle\hat{\sigma}_j^-\rangle = -\gamma_\perp(1+i\Delta)\langle\hat{\sigma}_j^-\rangle + g_j\langle\hat{a}\rangle\langle\hat{\sigma}_j^z\rangle,$$

Atomic inversion:

$$\frac{\partial}{\partial t}\langle\hat{\sigma}_j^z\rangle = -\gamma_\parallel\left(\langle\hat{\sigma}_j^z\rangle + 1\right) - 2g_j\left(\langle\hat{a}\rangle\langle\hat{\sigma}_j^+\rangle + \langle\hat{a}^\dagger\rangle\langle\hat{\sigma}_j^-\rangle\right).$$

The cavity and atomic detunings θ and Δ are defined as

$$\theta = \frac{\omega_c - \omega_l}{\kappa} \;\text{ and }\; \Delta = \frac{\omega_a - \omega_l}{\gamma_\perp}\,.$$

A first introduction to the Cooperativity

- Atomic decay rate γ
- Cavity decay rate κ
- Atom-cavity coupling rate g

$$C_1 = \frac{g^2}{\kappa\gamma} \qquad C = NC_1$$

Dimensionless input and output field normalized to the saturation intensity I_s and the transmission coefficient T of the output mirror

$$y = \frac{E_I}{\sqrt{I_s}};\ \text{and}\ x = \frac{E_T}{\sqrt{TI_s}}:$$

$$\dot{x} = \kappa\left(+2Cp + y - (i\Theta + 1)x\right)$$
$$\dot{p} = \gamma\left(-(1 + i\Delta)p + xD\right)/2$$
$$\dot{D} = \gamma\left(2(1 - D) - (x^*p + xp^*)\right)$$

$$\Theta = \frac{\omega_c - \omega_l}{\kappa}; \quad \Delta = \frac{\omega_a - \omega_l}{\gamma/2}$$

γ is the rate of spontaneous emission (energy decay)

κ is the rate of escape of the field

$\omega_{a,c,l}$ refer to atom, cavity, laser

Low intensity $x<<1$: with $D=0$, resonant $\Delta=0$ and $\Theta=0$ weakly driven.

Two coupled oscillators

$$\dot{x} = \kappa(-x + 2Cp + y)$$

Two coupled oscillators

$$\dot{p} = \gamma(-p - x)$$

Steady state

$$y = x - 2Cp$$

$$p = -x$$

$$y = x(1 + 2C)$$

$$\kappa >> \gamma \quad \dot{p} = -\gamma(1 + 2C)p - \gamma y$$

$$\gamma >> \kappa \quad \dot{x} = -\kappa(1 + 2C)x + \kappa y$$

Enhanced emission

Steady state with detuning and at all intensities:

$$y = x\left(1 + \frac{2C}{1+\Delta^2+|x|^2}\right) + ix\left(\theta - \frac{2C\Delta}{1+\Delta^2+|x|^2}\right)$$

Dispersive limit when $\Theta=0$ and $\Delta >> 1$:

$$y = -ix\frac{2C\Delta}{1+\Delta^2+|x|^2}$$

Steady state with detuning and at all intensities:

$$y = x\left(1 + \frac{2C}{1+\Delta^2+|x|^2}\right) + ix\left(\theta - \frac{2C\Delta}{1+\Delta^2+|x|^2}\right)$$

The transmission spectrum:

for $\omega_c = \omega_a \quad \Omega = (\gamma/2)\Delta = \kappa\Theta \quad \Omega_{V.R.} = g\sqrt{N} = \sqrt{C\kappa\gamma}$

$$x = y\frac{\kappa(\gamma_\perp + i\Omega)}{(\kappa + i\Omega)(\gamma_\perp + i\Omega) + \Omega^2_{V.R.}/(1+\gamma_\perp^2|x|^2/(\gamma_\perp^2+\Omega^2))}$$

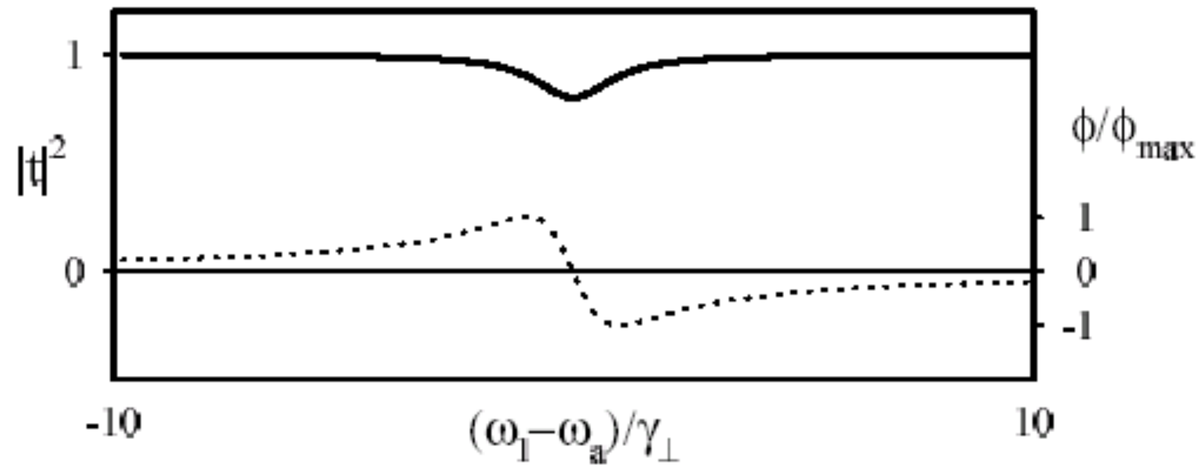
Two coupled oscillators

$$\frac{x}{y} = \frac{A}{i\Omega - \Omega_1} + \frac{B}{i\Omega - \Omega_2} \,, \qquad A = \kappa\frac{\gamma_\perp + \Omega_1}{\Omega_1 - \Omega_2} \,, \qquad B = \kappa\frac{\gamma_\perp + \Omega_2}{\Omega_2 - \Omega_1} \,,$$

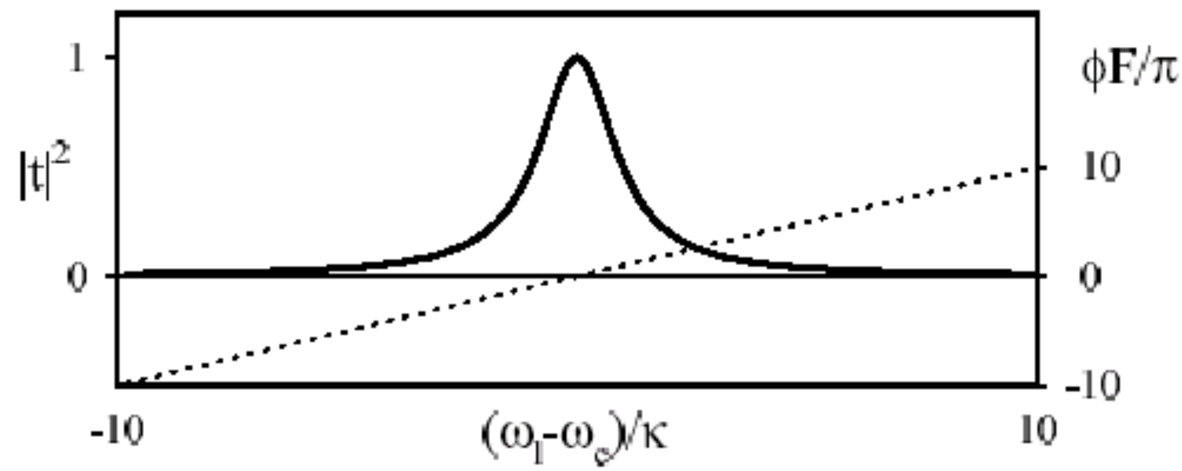
$$\Omega_{1,2} = -\frac{\kappa + \gamma_\perp}{2} \pm i\sqrt{-\Big(\frac{\kappa - \gamma_\perp}{2}\Big)^2 + \frac{\Omega^2_{V.R.}}{1 + \gamma_\perp^2|x|^2/(\gamma_\perp^2 + \Omega^2)}} \,.$$

Cavity mode and atomic polarization

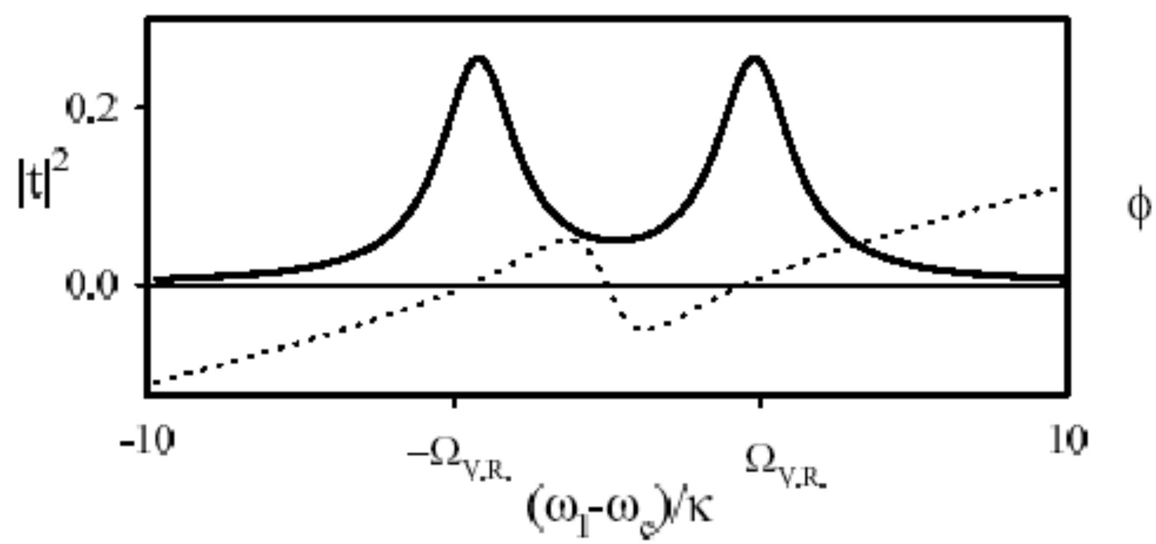
Atomic absorption



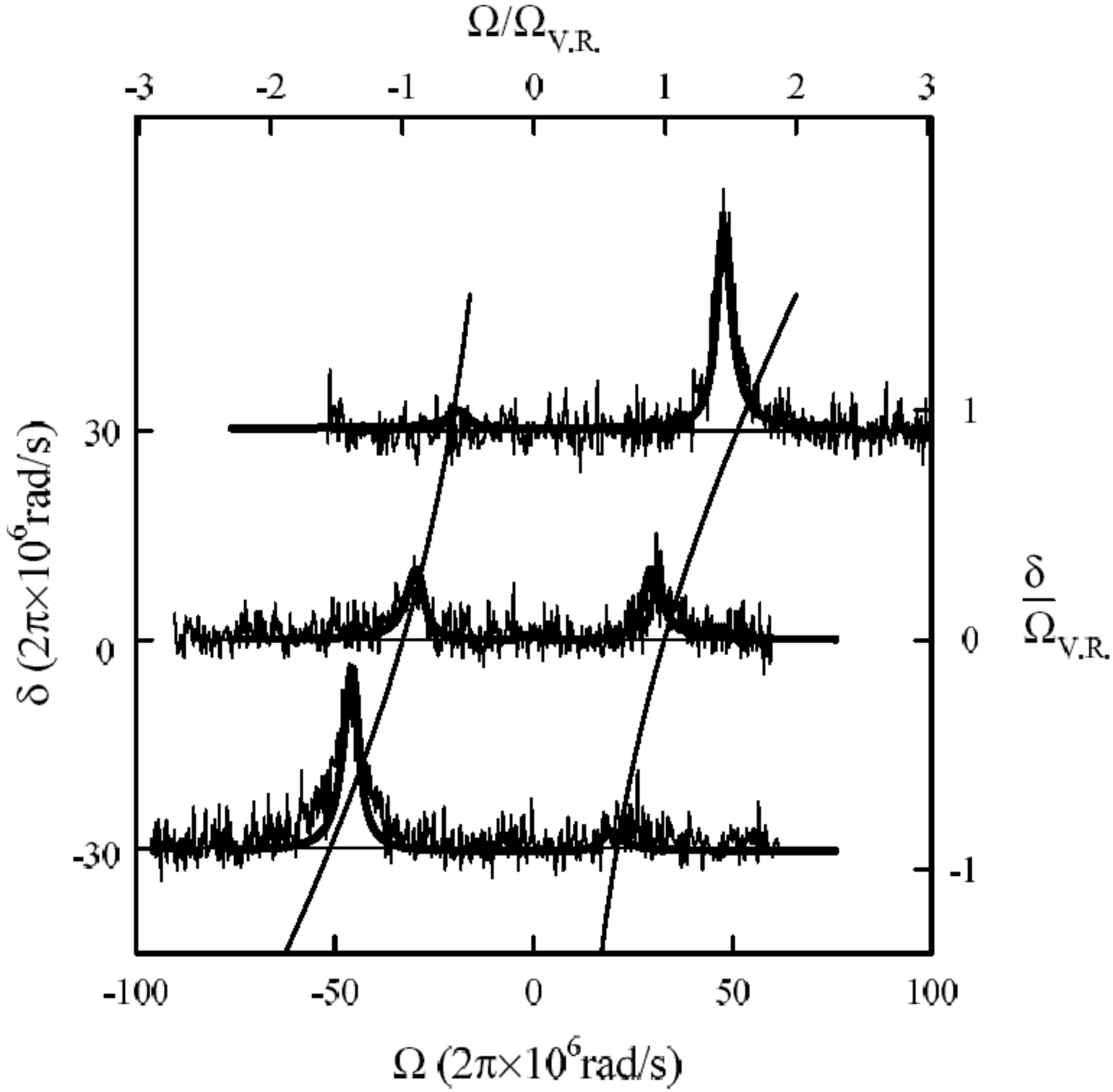
Fabry Perot



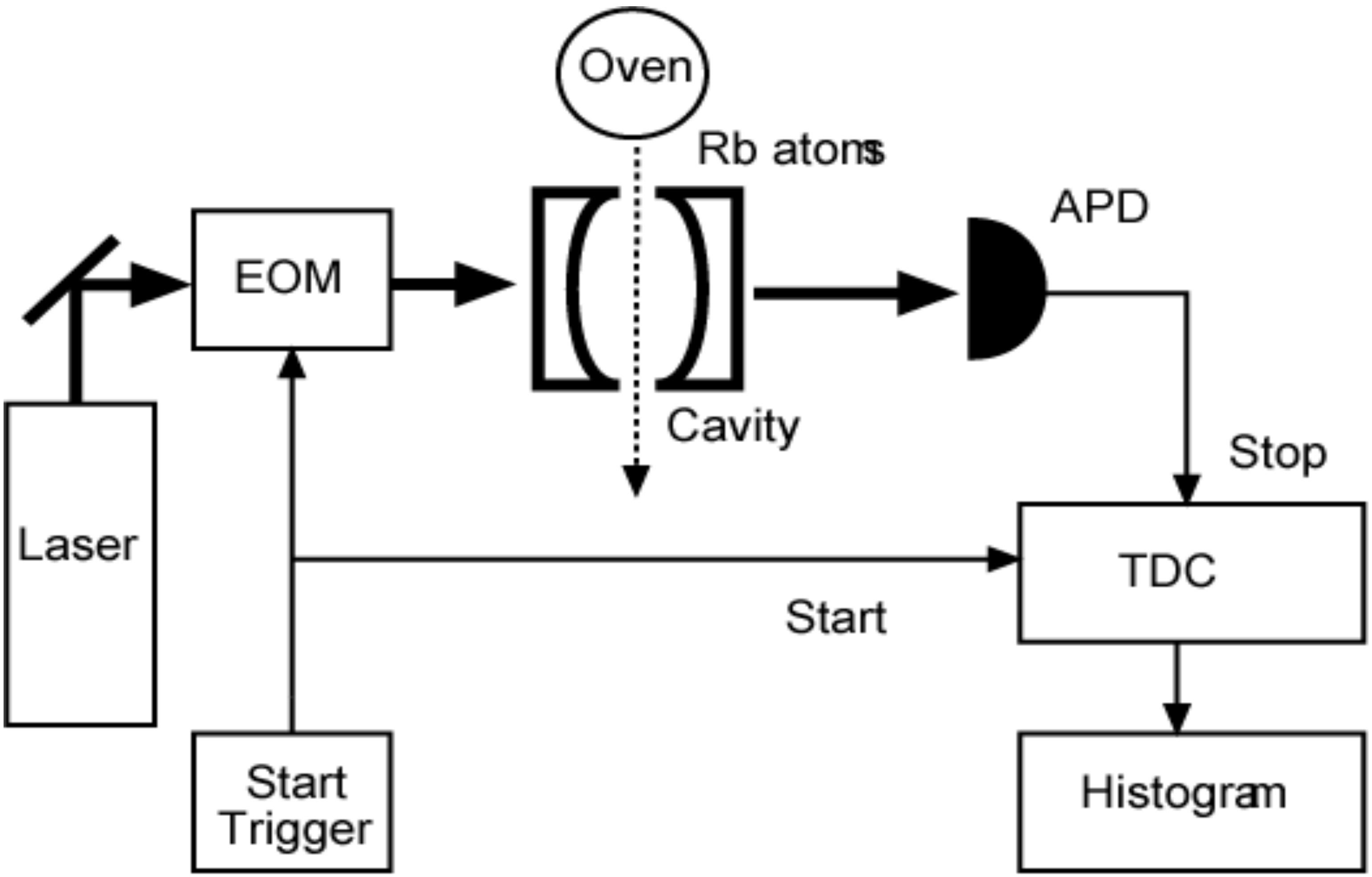
Coupled modes



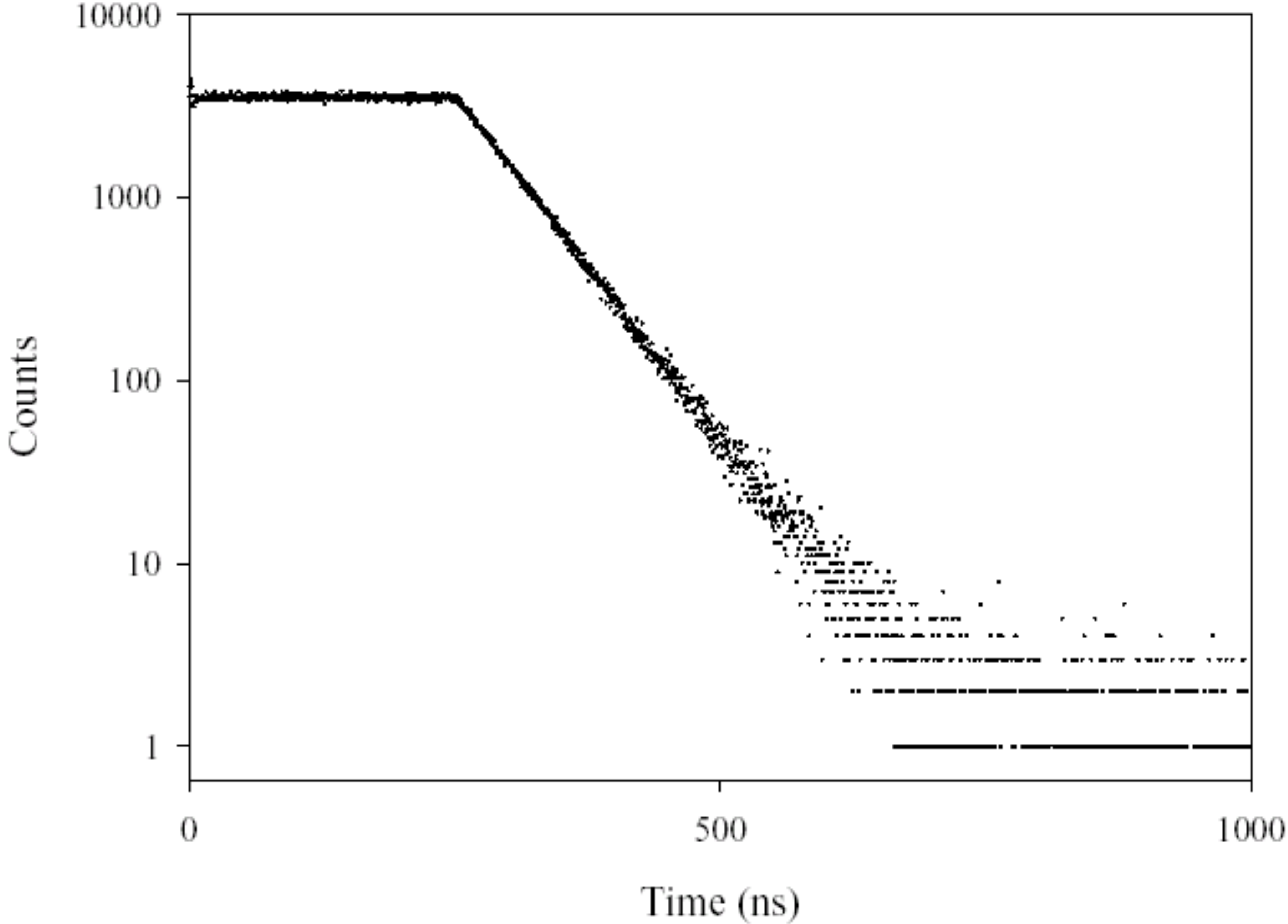
Transmission spectrum at low intensity for varying atomic detunings.



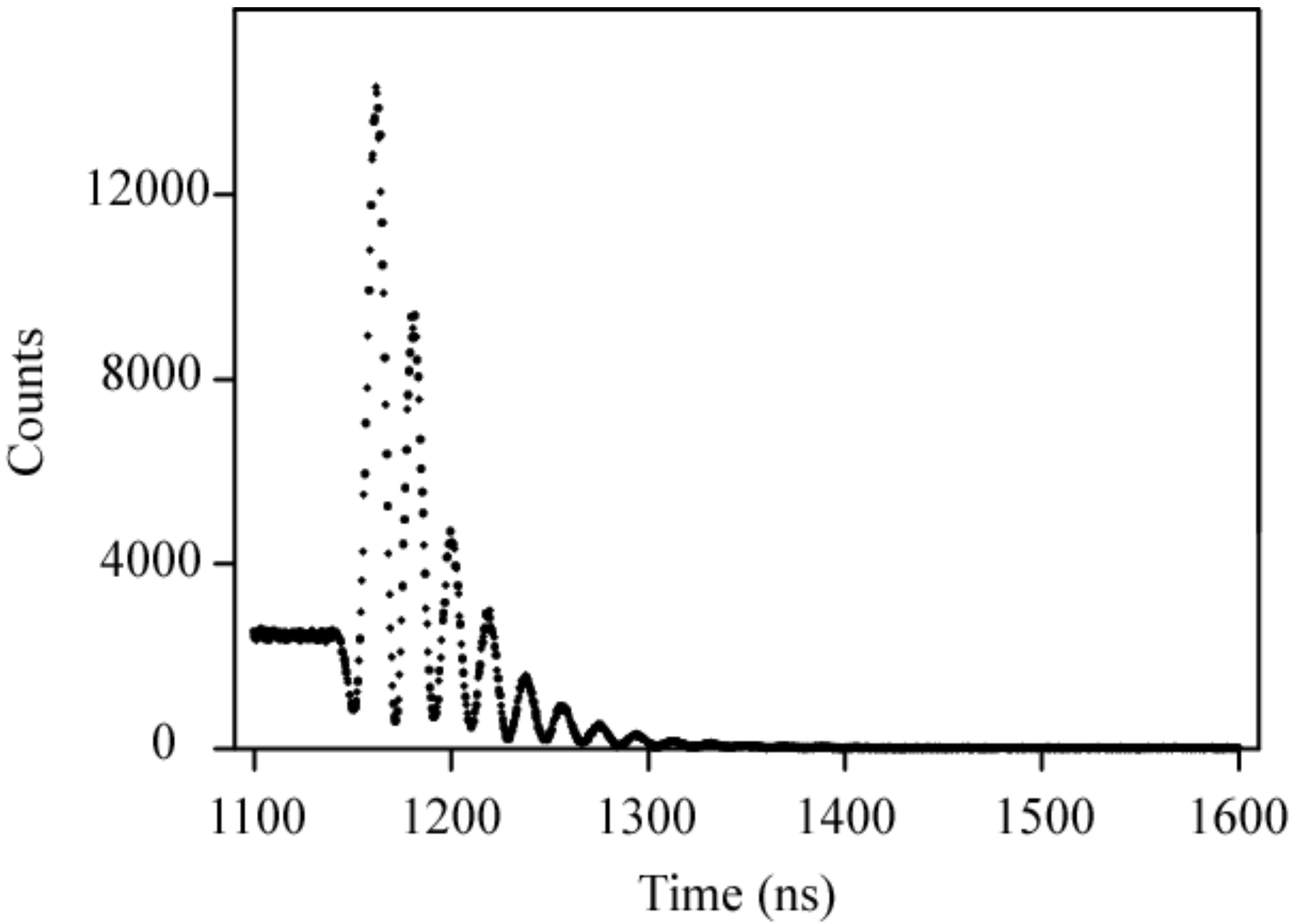
Study the system dynamics classically by providing a step function.



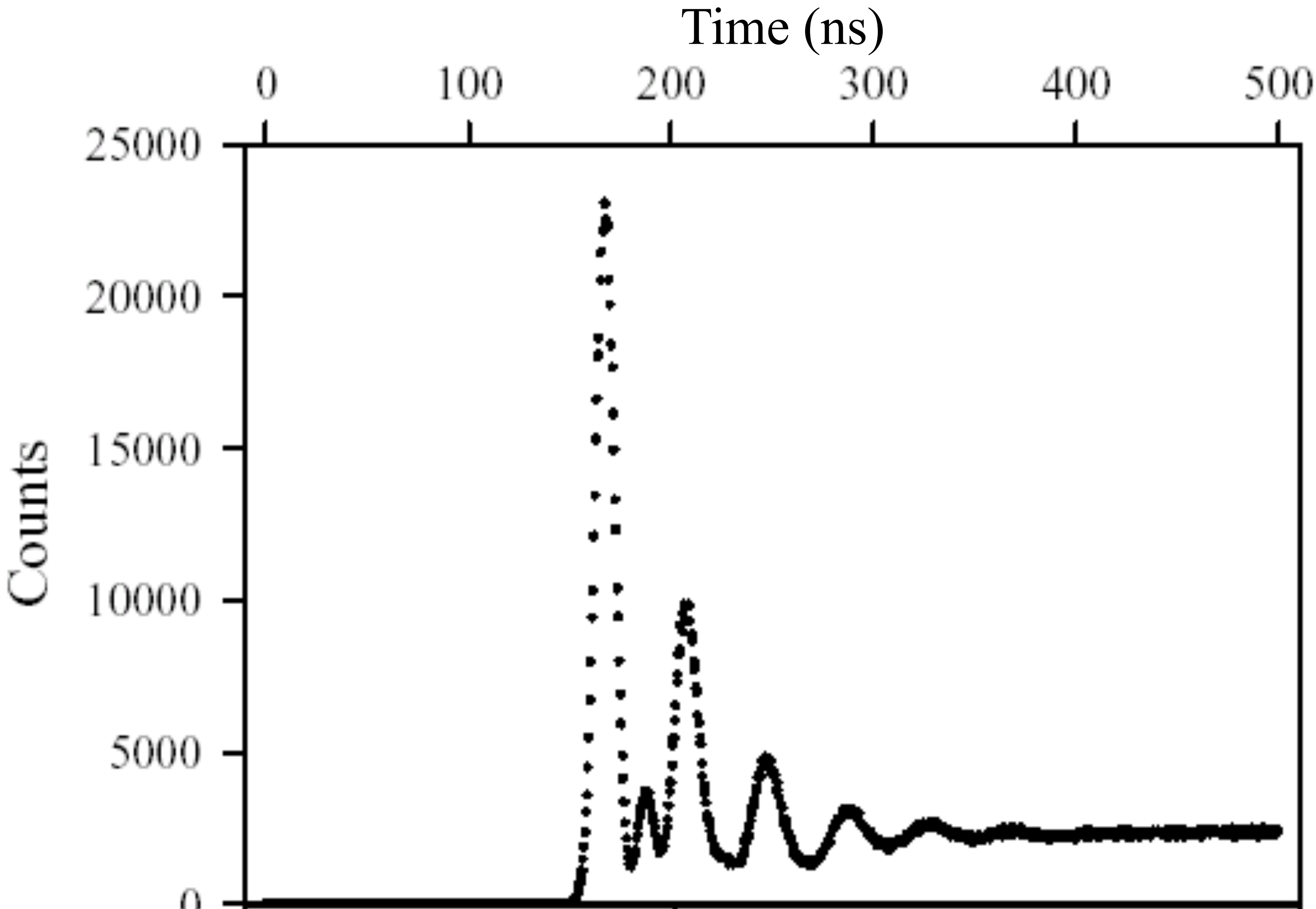
Decay of the empty cavity



Response to step down excitation



Response to step up excitation



Thanks