Correlation functions in optics; classical and quantum 3.

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NIST

Review Article:
H. J. Carmichael, G. T. Foster, L. A. Orozco, J. E. Reiner, and P. R. Rice, “Intensity-Field Correlations of Non-Classical Light”. Progress in Optics, Vol. 46, 355-403, Edited by E. Wolf Elsevier, Amsterdam 2004.

Reference that includes pulsed sources:

Zheyu Jeff Ou

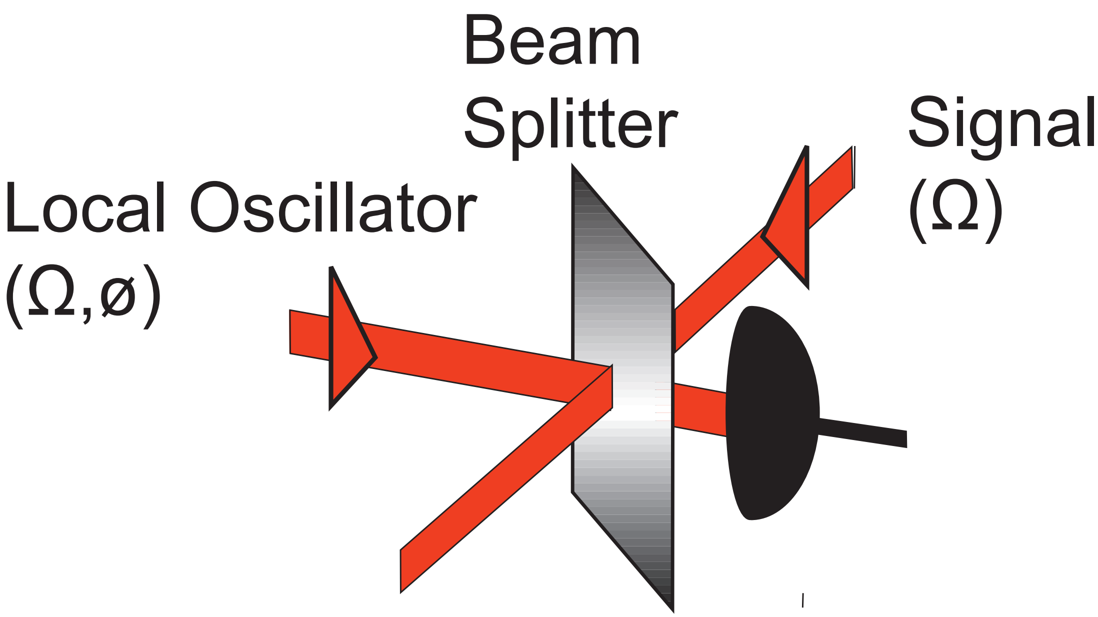
"*Quantum Optics for Experimentalists*"

World Scientific, Singapore 2017

- The main object of interest in quantum optics is the optical FIELD. That is what is quantized.

- Can we measure the FIELD of a state with an average of ONE PHOTON in it?

Homodyne detection



Photocurrent ~ $|\text{L.O.} \cos(ø)+S|^2$

Interfere two fields: A local oscillator (LO) and a signal (S). The resulting photocurrent has a term proportional to the amplitude of S and also depends on the cosine of the phase difference ϕ between LO and S.

$$|LO \cos(\phi) + S|^2 = |LO|^2 + 2\, LO\, S \cos(\phi) + |S|^2$$

Review of shot noise :

Shot noise happens whenever the transport of energy is through a finite number of discreet particles. For example, electric charge *e* (Schottky 1918). If the number of particles is small and it follows a Poisson distribution (random independent events), it can be the dominant noise.

- The mean of a Poisson distribution is n
- The variance of a Poisson distribution n
- The signal to noise ratio $n^{1/2}$
- A Poisson distribution with n large approximates a Gaussian.
- The current spectral density (i) of noise is: $(2e|i|)^{1/2}$ with units of [A/Hz$^{1/2}$].
- The power of the noise depends on the detection bandwith and the Resistance R: $P(\nu)=R2e|i|\Delta\nu$.

Review of Coherent States $|\alpha>$

The coherent state $|\alpha>$ is the eigenstate of the annihilation operator:

$$\hat{a}\left|\alpha\right\rangle = \alpha\left|\alpha\right\rangle$$

Its amplitude (complex): α
Its mean squared: $\alpha^{*}\alpha=|\alpha|^{2}$
Its uncertainty: 1/2

They are states with the minimum uncertainty allowed by quantum mechanics. Equal on both quadratures

Relation with the harmonic oscillaros:

Quadratures of the electromagnetic field

$$E_{\mathrm{R}} = \left(\frac{\hbar\omega}{2\epsilon_0 V}\right)^{1/2} \cos(\theta) X \qquad \text{and} \qquad E_{\mathrm{I}} = \left(\frac{\hbar\omega}{2\epsilon_0 V}\right)^{1/2} \sin(\theta) X$$

$$H = \hbar\omega\left(P^2 + X^2\right), \qquad \text{with} \qquad [X, P] \equiv XP - PX = \frac{i}{2} I$$

$$(X - \langle X \rangle)\, |\alpha\rangle = -i\,(P - \langle P \rangle)\, |\alpha\rangle$$

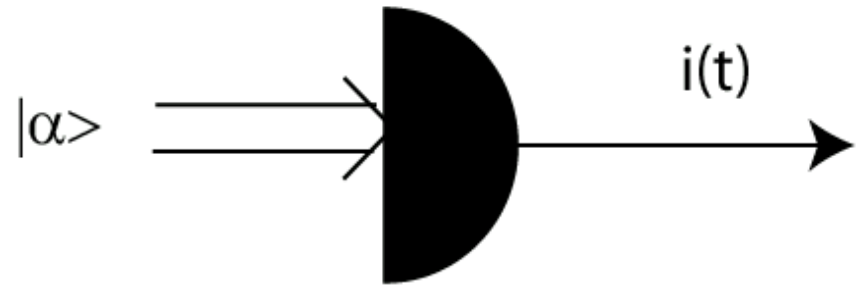
$$(X + iP)\, |\alpha\rangle = \langle X + iP \rangle\, |\alpha\rangle$$

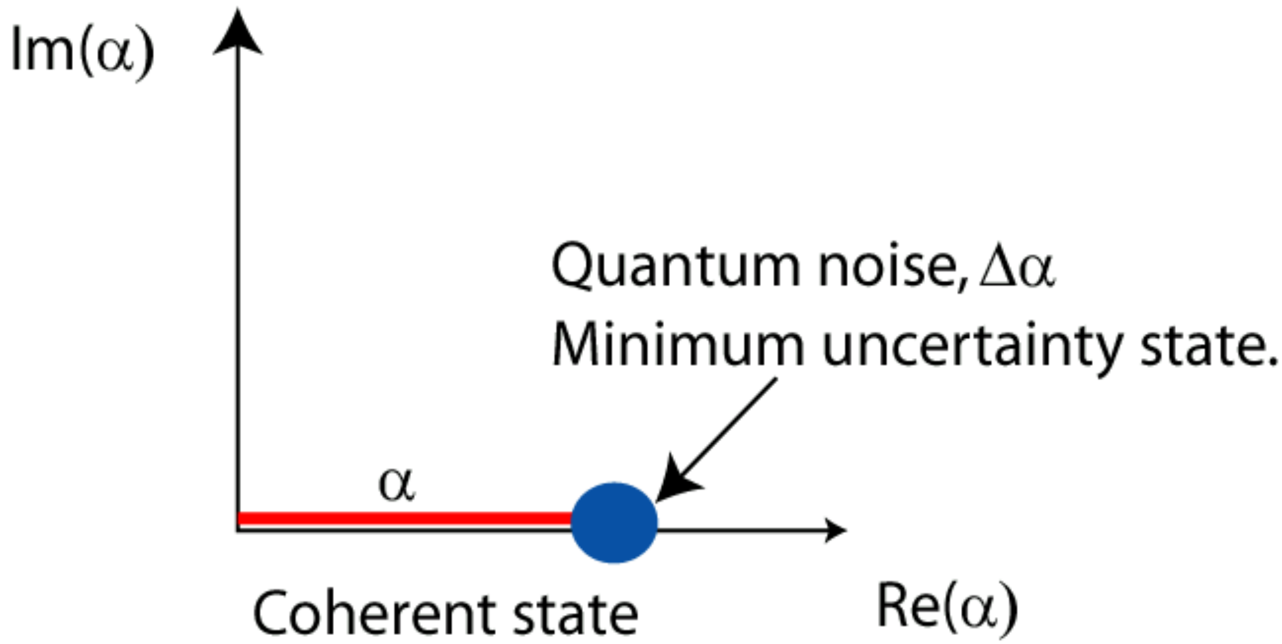
States of minimum uncertainty :

$$\langle \alpha |\, (X - \langle X \rangle)^2 + (P - \langle P \rangle)^2\, |\alpha\rangle = 1/2$$

Relation with Fock states (Poisson)

$$P(n) = |\langle n | \alpha \rangle|^2 = e^{-\langle n \rangle} \frac{\langle n \rangle^n}{n!}$$





Perfect detector $i(t) = |\alpha + \Delta\alpha|^2$

$$i(t) = |\alpha|^2 + 2\alpha\,\Delta\alpha + |\Delta\alpha|^2 \quad ; \quad \langle\alpha^*\alpha\rangle = n$$

DC ~ n   Shot noise ~ $n^{1/2}$ neglect.

Correlation functions tell us something about the fluctuations.

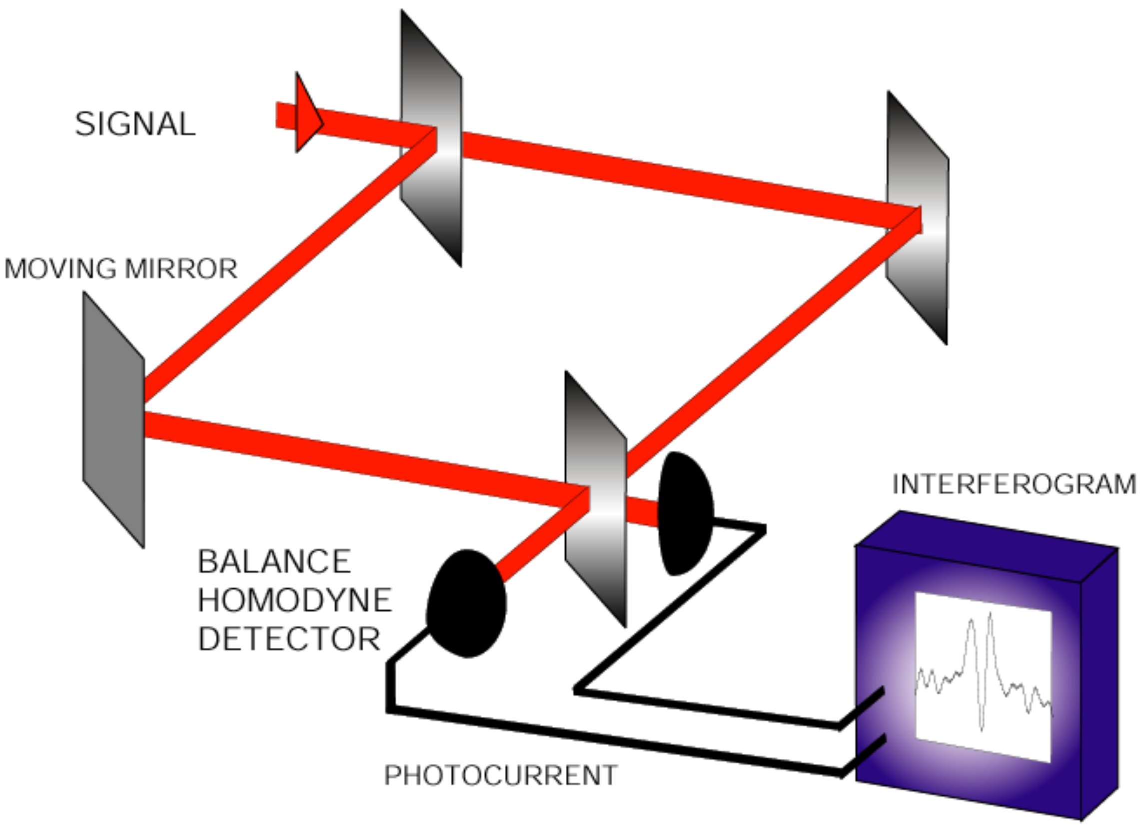
Correlations have classical bounds.

They are conditional measurements.

Can we use them to measure the field associated with a FLUCTUATION of one photon?

Mach Zehnder Interferometer Wave-Wave Correlation

$$g^{(1)}(\tau) = \frac{\langle E^*(t)E(t+\tau)\rangle}{\langle I(t)\rangle}$$

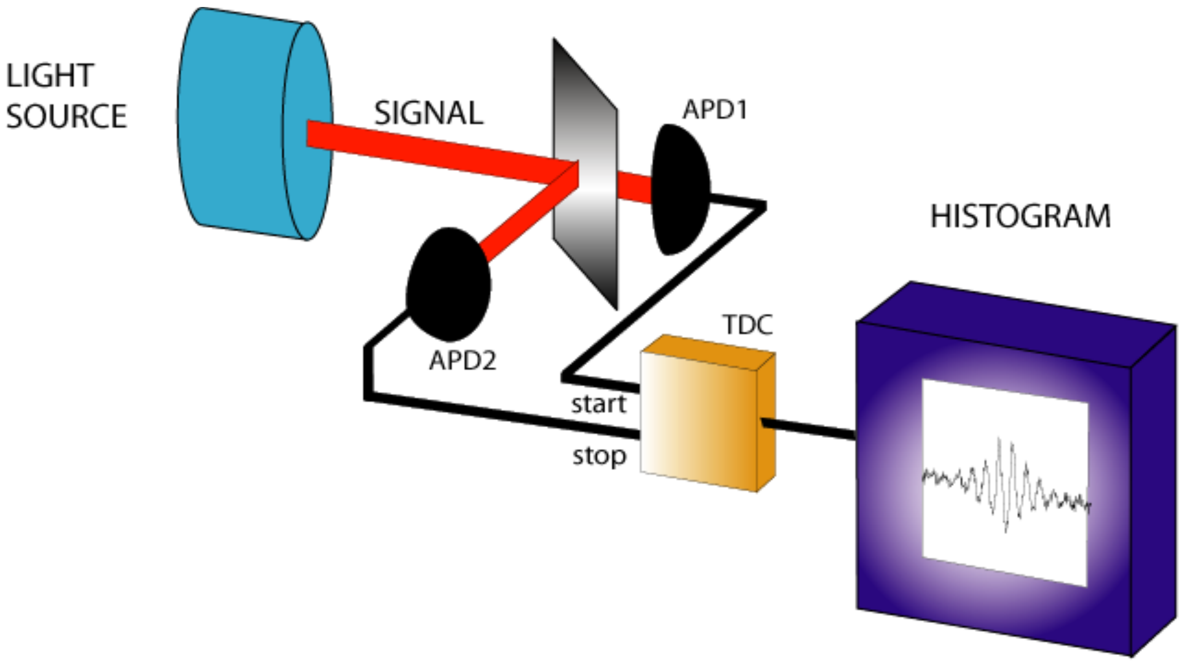


Spectrum of the signal:

$$F(\omega) = \frac{1}{2\pi}\int exp(i\omega\tau)g^{(1)}(\tau)d\tau$$

Basis of Fourier Transform Spectroscopy

Hanbury Brown and Twiss Intensity-Intensity Correlations



$$g^{(2)}(\tau) = \frac{\langle I(t)I(t+\tau)\rangle}{\langle I(t)\rangle^2}$$

Cauchy-Schwarz

$$2I(t)I(t+\tau) \le I^2(t) + I^2(t+\tau)$$

The correlation is largest at equal time

$$g^{(2)}(0) \ge 1$$

$$g^{(2)}(0) \ge g^{(2)}(\tau)$$

Intensity correlation function measurements:

$$g^{(2)}(\tau) = \frac{\left\langle : \hat{I}(t)\hat{I}(t+\tau) : \right\rangle}{\left\langle \hat{I}(t) \right\rangle^2}$$

Gives the probability of detecting a photon at time *t* + τ given that one was detected at time *t*. This is a conditional measurement:

$$g^{(2)}(\tau) = \frac{\left\langle \hat{I}(\tau) \right\rangle_c}{\left\langle \hat{I} \right\rangle}$$

A strategy starts to appear:

Correlation function; Conditional measurement.

Detect a photon: Prepare a conditional quantum mechanical state in our system.

The system has to have at least two photons.

Do we have enough signal to noise ratio?

$$\underbrace{|LO|^2}_{\text{SHOT NOISE}} \quad + \quad \underbrace{2\ LO\ S \cos(\phi)}_{\text{SIGNAL}}$$

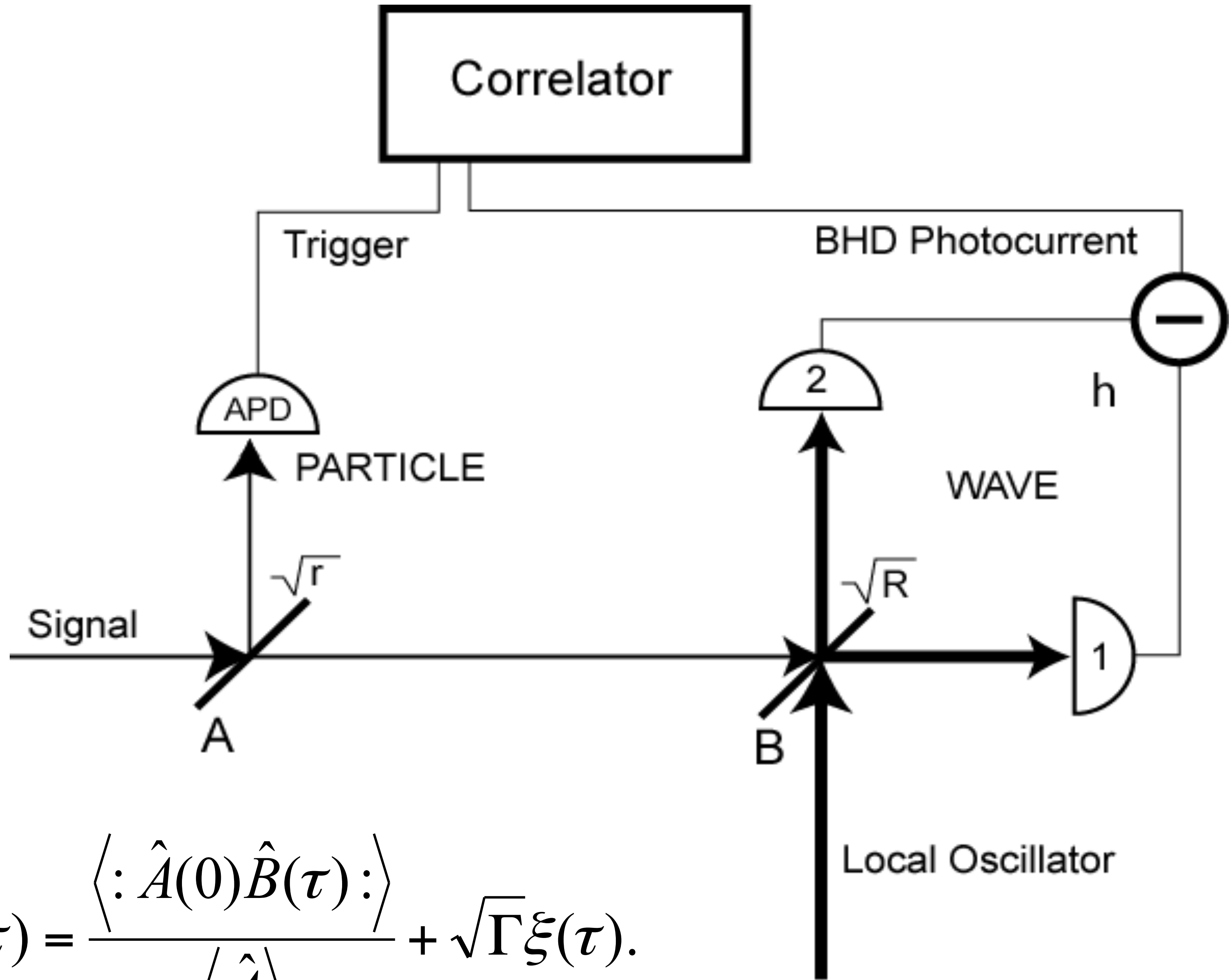
How to correlate fields and intensities?

Detection of the field: Homodyne.

Conditional Measurement: Only measure when we know there is a photon.

Source: Cavity QED

The Intensity-Field correlator.



$$\mathrm{H}(\tau) = \frac{\left\langle : \hat{A}(0)\hat{B}(\tau) : \right\rangle}{\left\langle \hat{A} \right\rangle} + \sqrt{\Gamma}\xi(\tau).$$

Condition on a Click

Measure the correlation function of the Intensity and the Field:

<I(t) E(t+τ)>

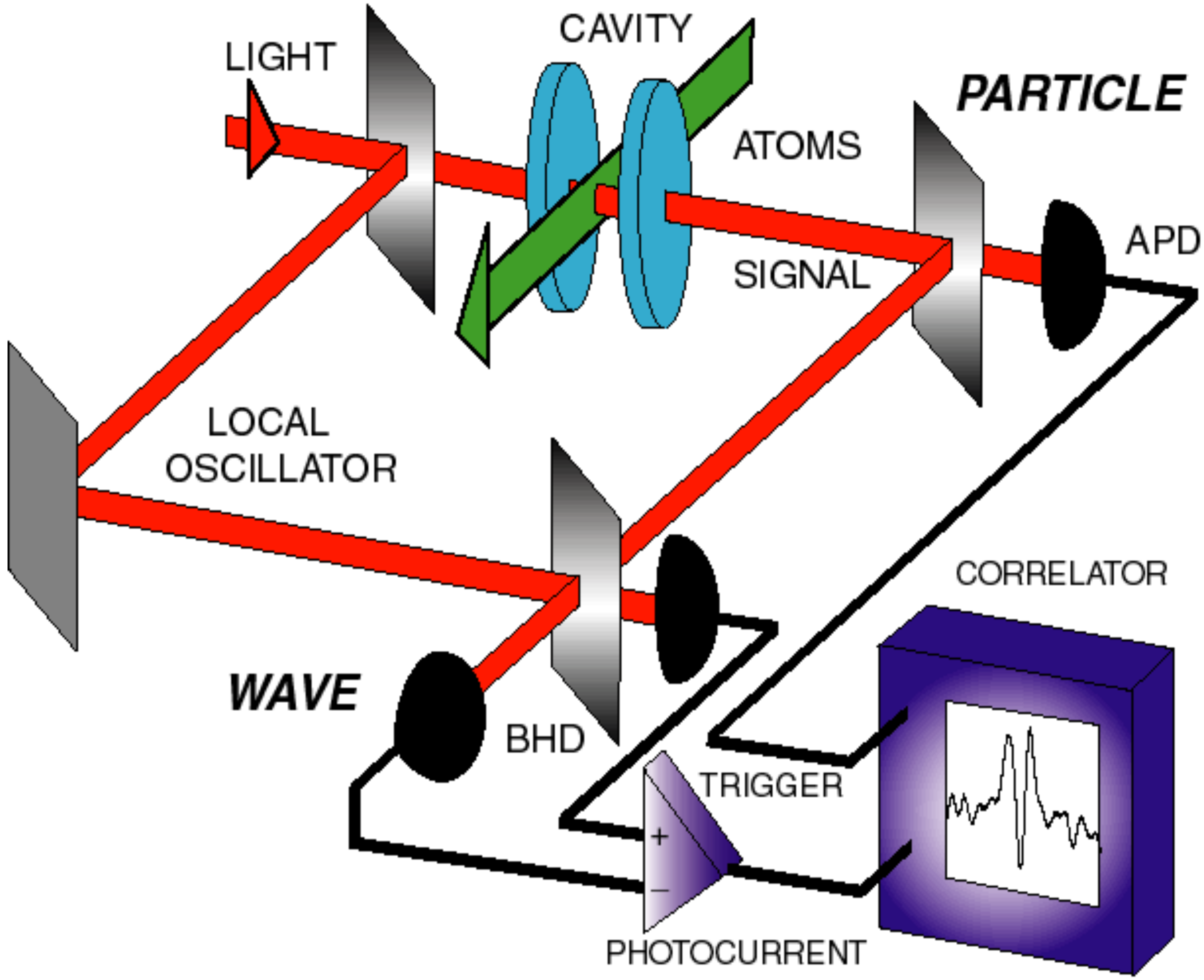
Normalized form:

$h_\theta(\tau) = \langle E(\tau)\rangle_c / \langle E \rangle$

From Cauchy Schwartz inequalities:

$$0 \le \bar{h}_0(0) - 1 \le 2$$

$$\left|\bar{h}_0(\tau) - 1\right| \le \left|\bar{h}_0(0) - 1\right|$$


LIGHT
CAVITY
PARTICLE
ATOMS
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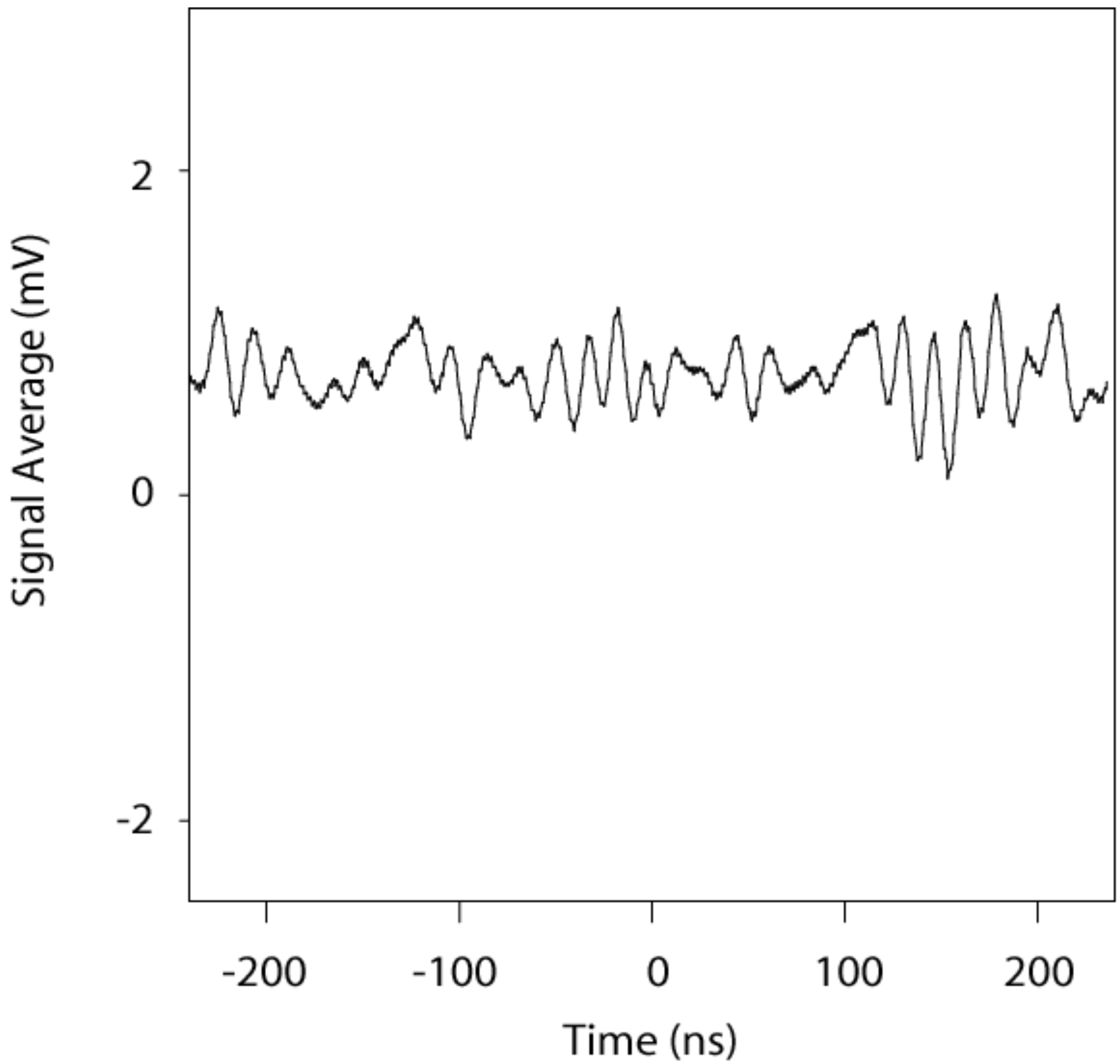
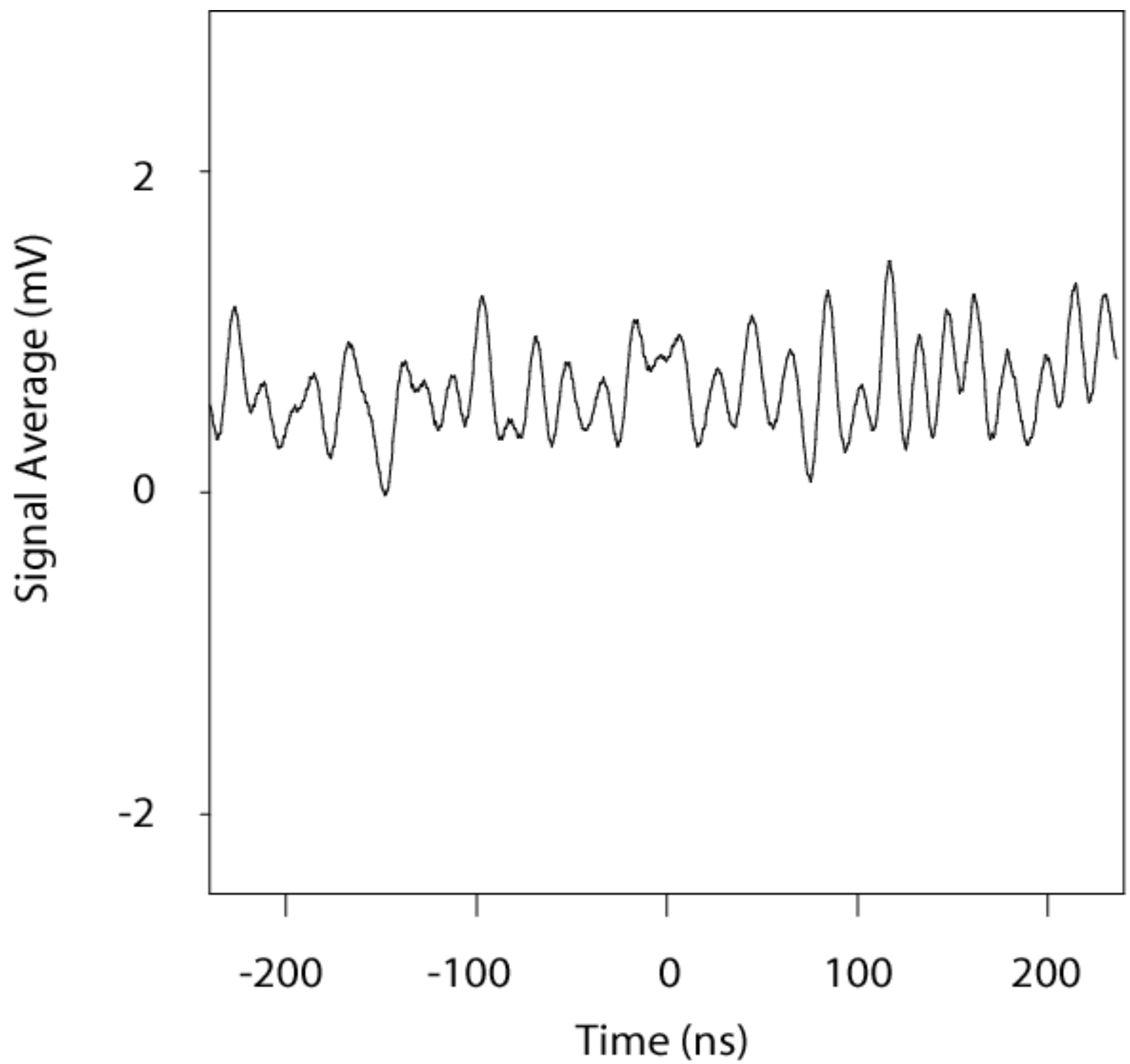
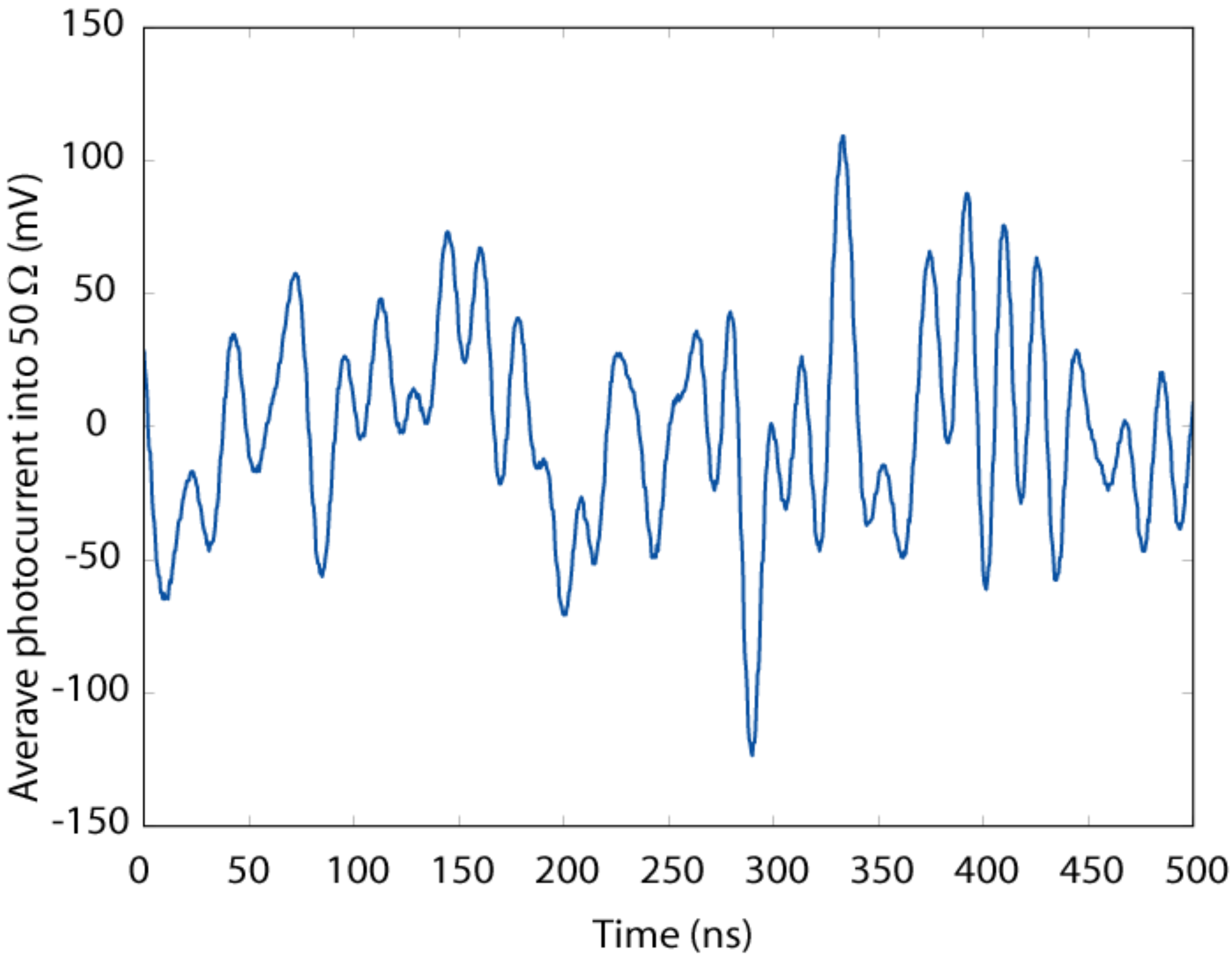
PHOTOCURRENT

Photocurrent average with random conditioning

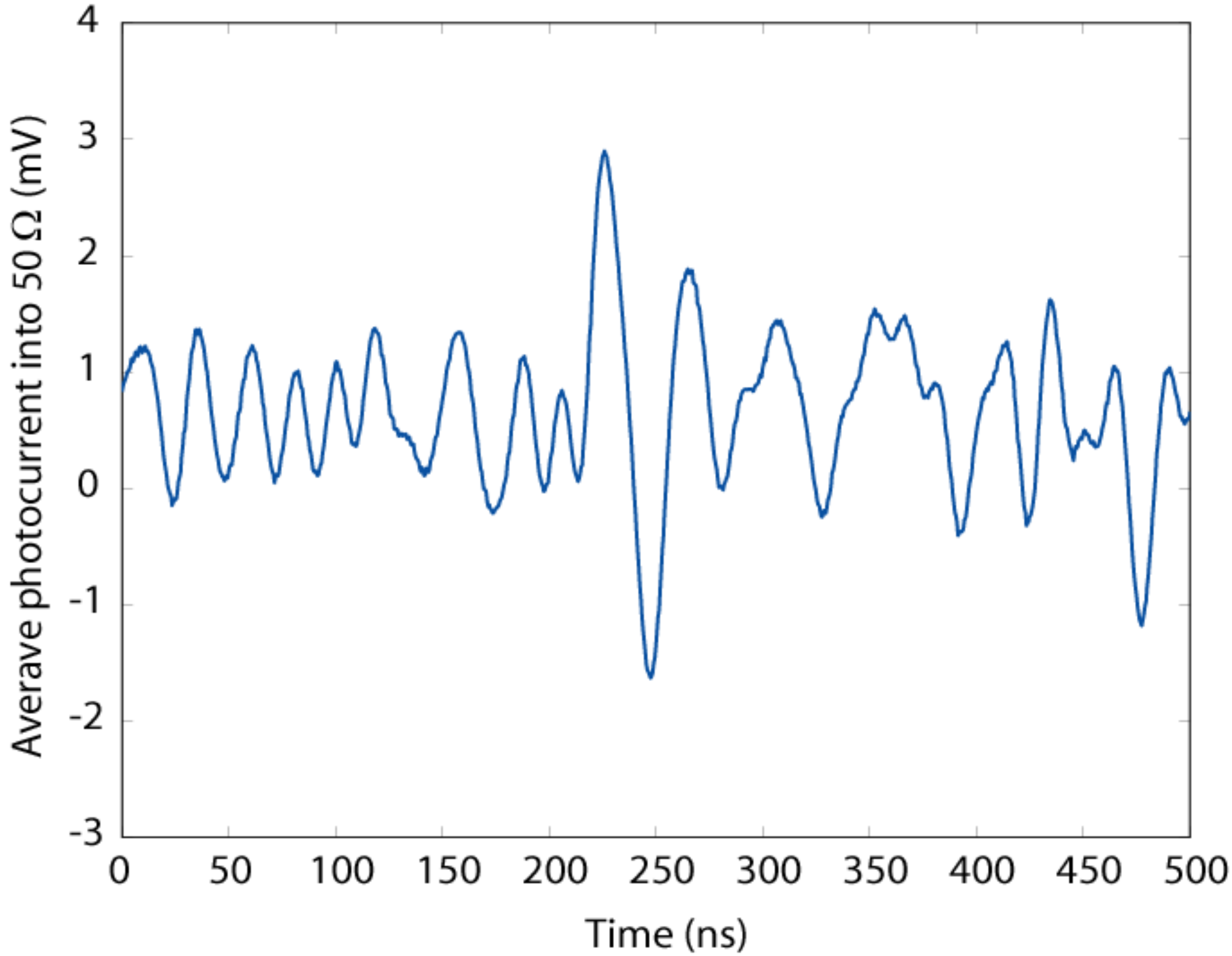


Conditional photocurrent with no atoms in the cavity.

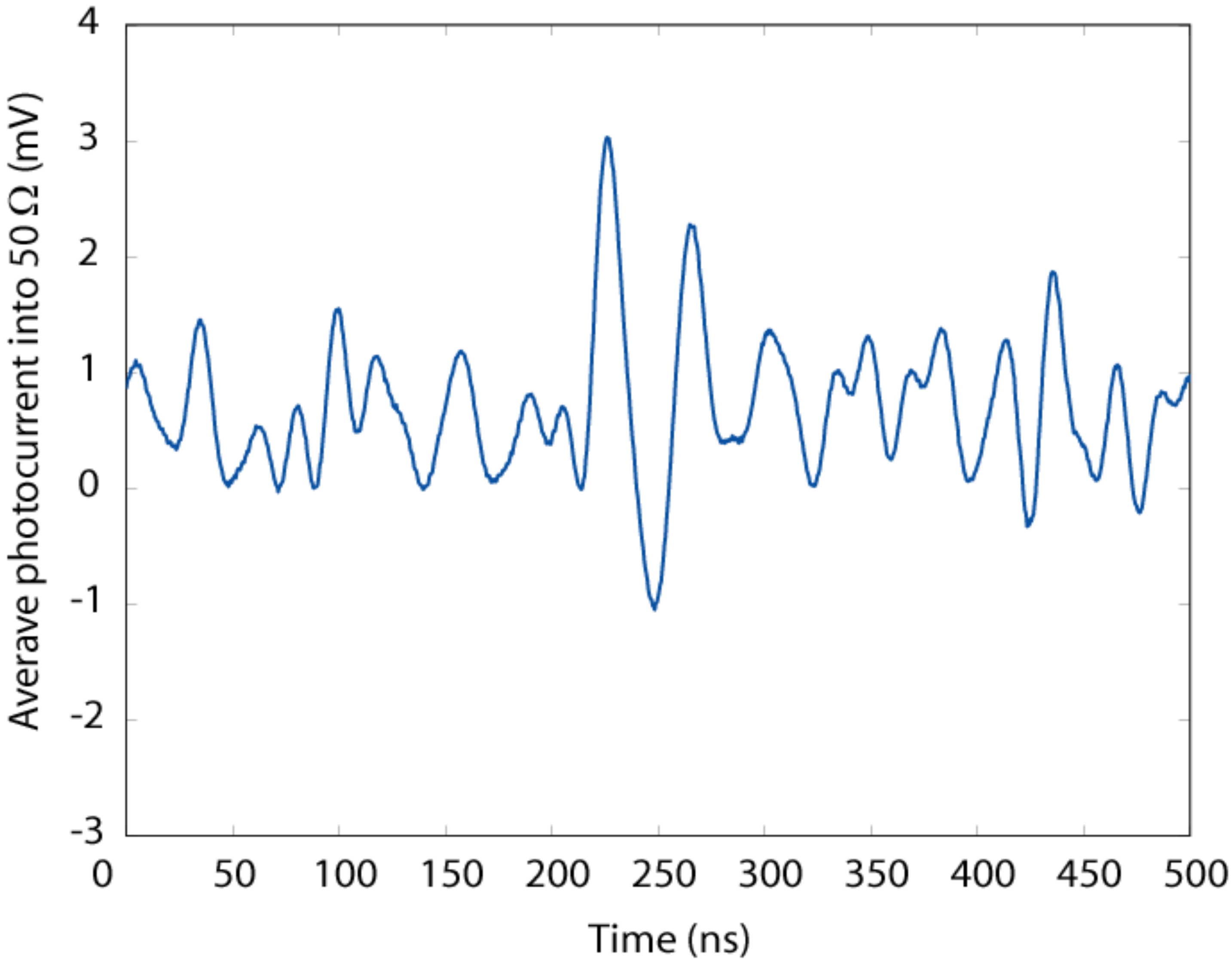




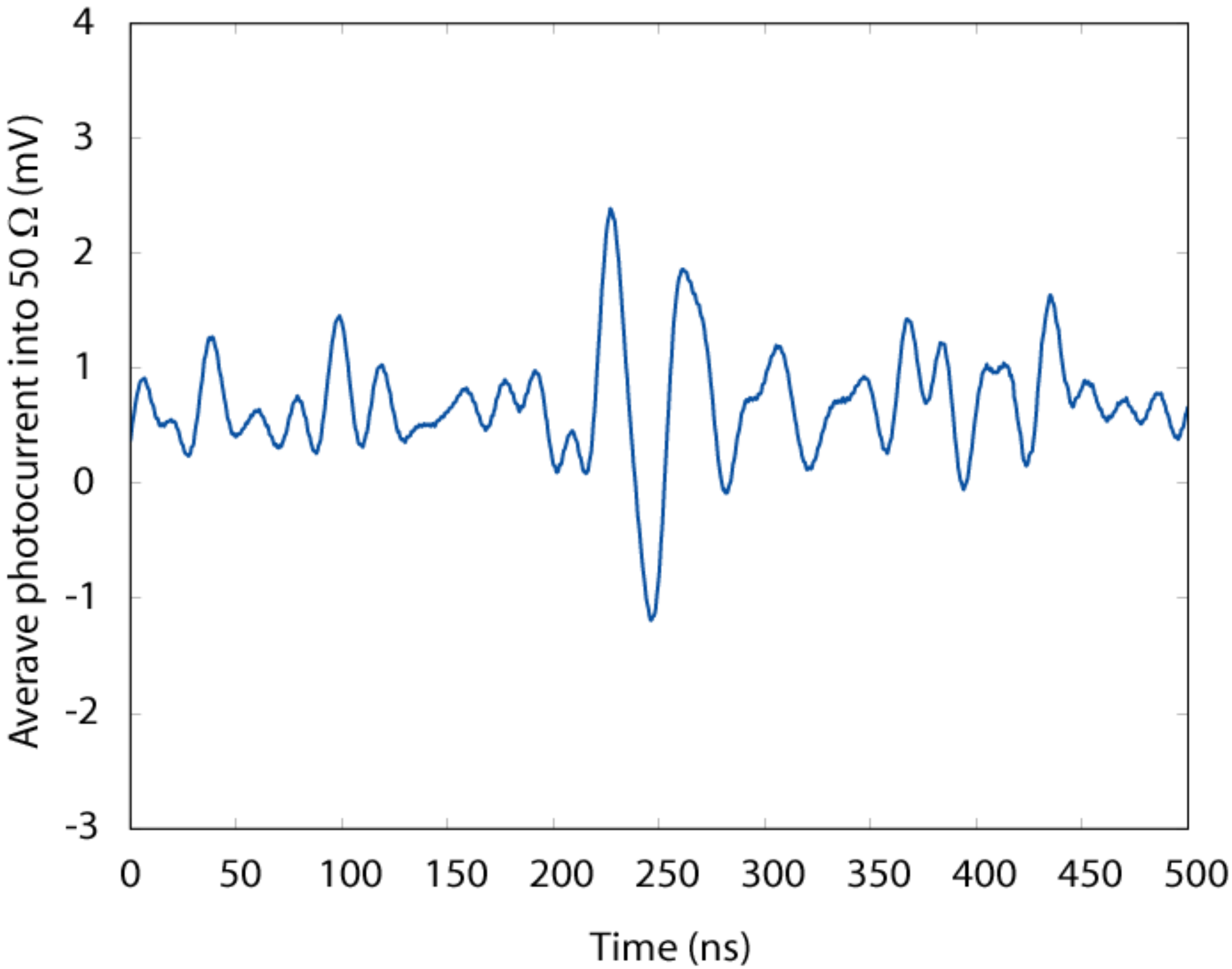
After 1 average



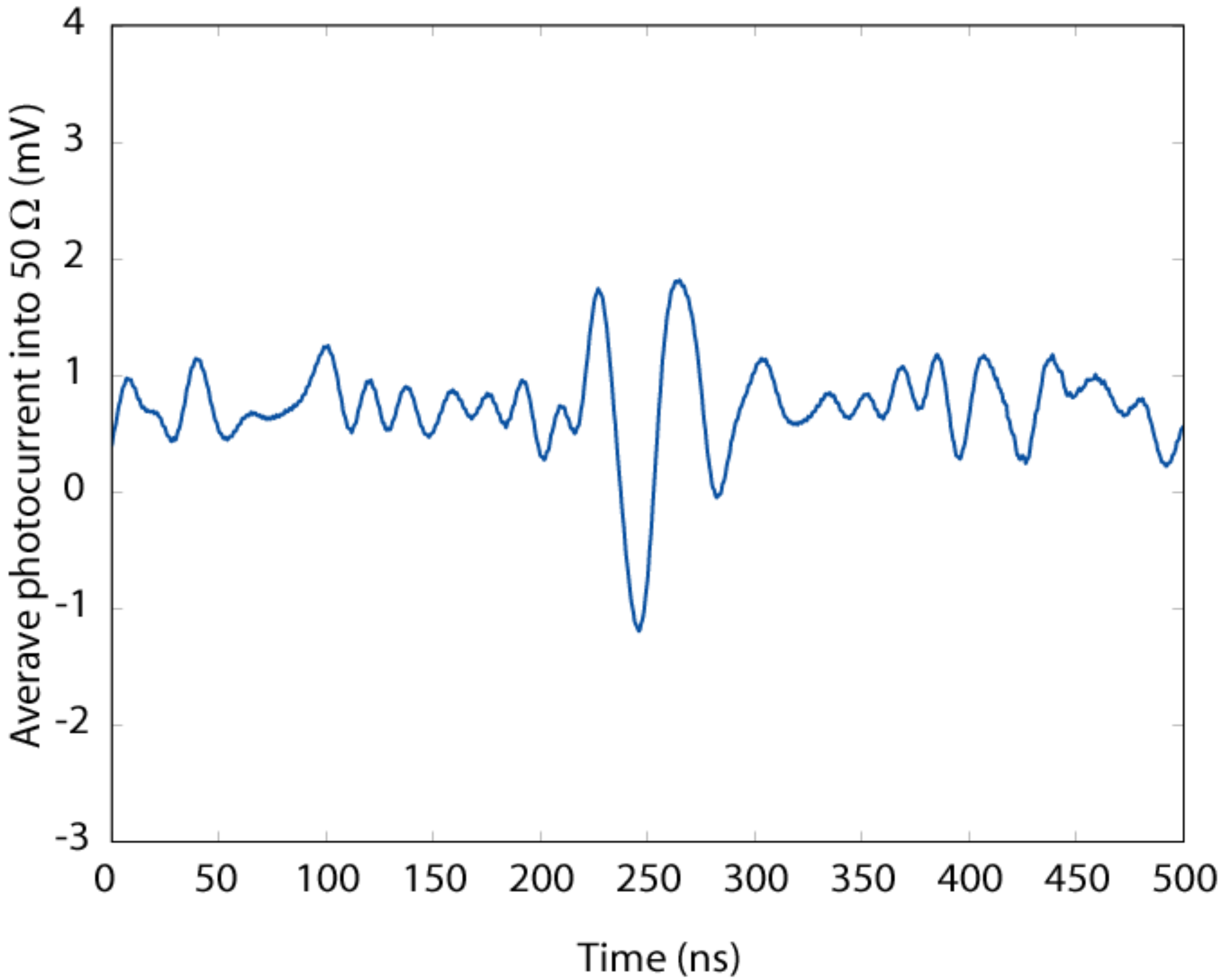
After 6,000 averages



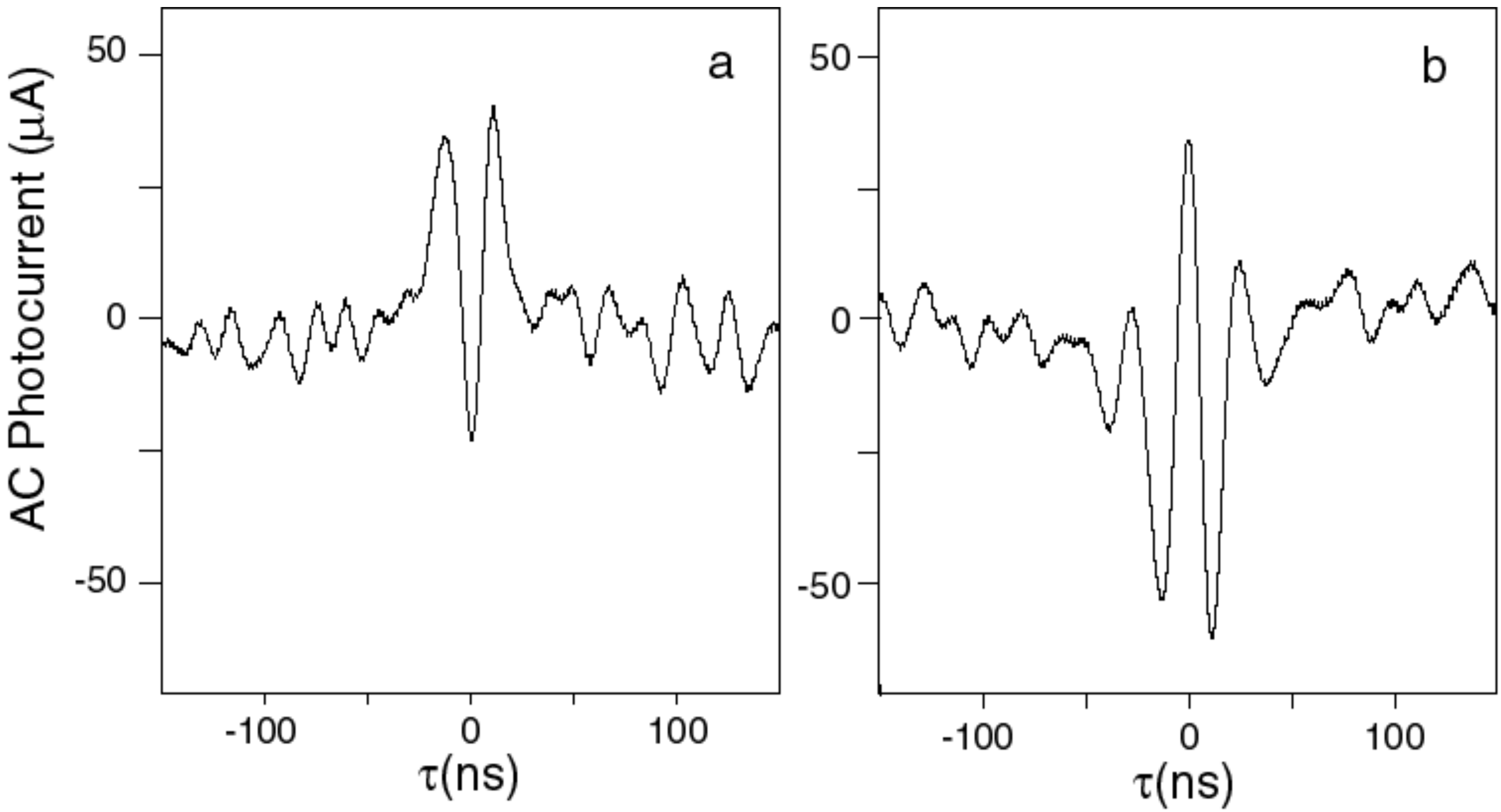
After 10,000 averages



After 30,000 averages

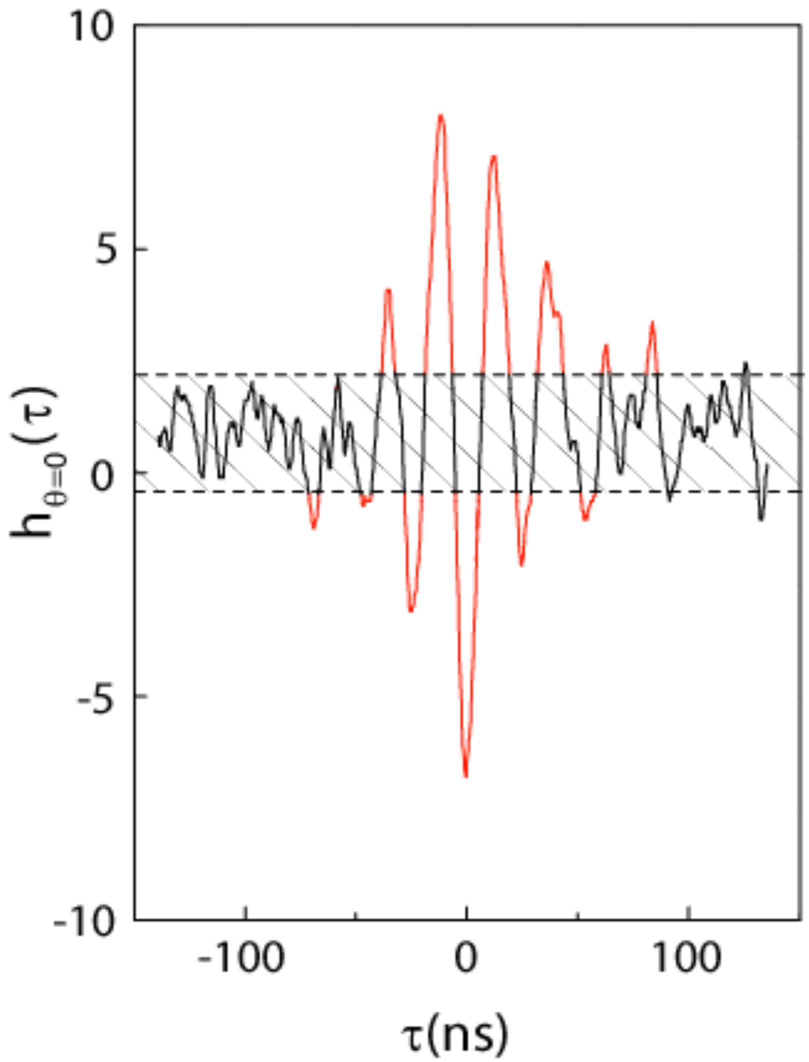


After 65,000 averages



Flip the phase of the Mach-Zehnder by 146^o

Monte Carlo simulations for weak excitation:



Atomic beam N=11

This is the conditional evolution of the field of a fraction of a photon [B(t)] from the correlation function.

$h_\theta(\tau) = \langle E(\tau)\rangle_c / \langle E \rangle$

The conditional field prepared by the click is:

$A(t)|0\rangle + B(t)|1\rangle$ with $A(t) \approx 1$ and $B(t) \ll 1$

We measure the field of a fraction of a photon!

Fluctuations are very important.

Conditional dynamics in cavity QED at low intensity:

$$\left|\Psi_{ss}\right\rangle = \left|0,g\right\rangle + \lambda\left|1,g\right\rangle - \frac{2g}{\gamma}\lambda\left|0,e\right\rangle + \frac{\lambda^2 pq}{\sqrt{2}}\left|2,g\right\rangle - \frac{2g\lambda^2 q}{\gamma}\left|1,e\right\rangle$$

$$\lambda = \langle \hat{a} \rangle, \quad p = p(g,\kappa,\gamma) \text{ and } q = q(g,\kappa,\gamma)$$

A photodetection conditions the state into the following non-steady state from which the system evolves.

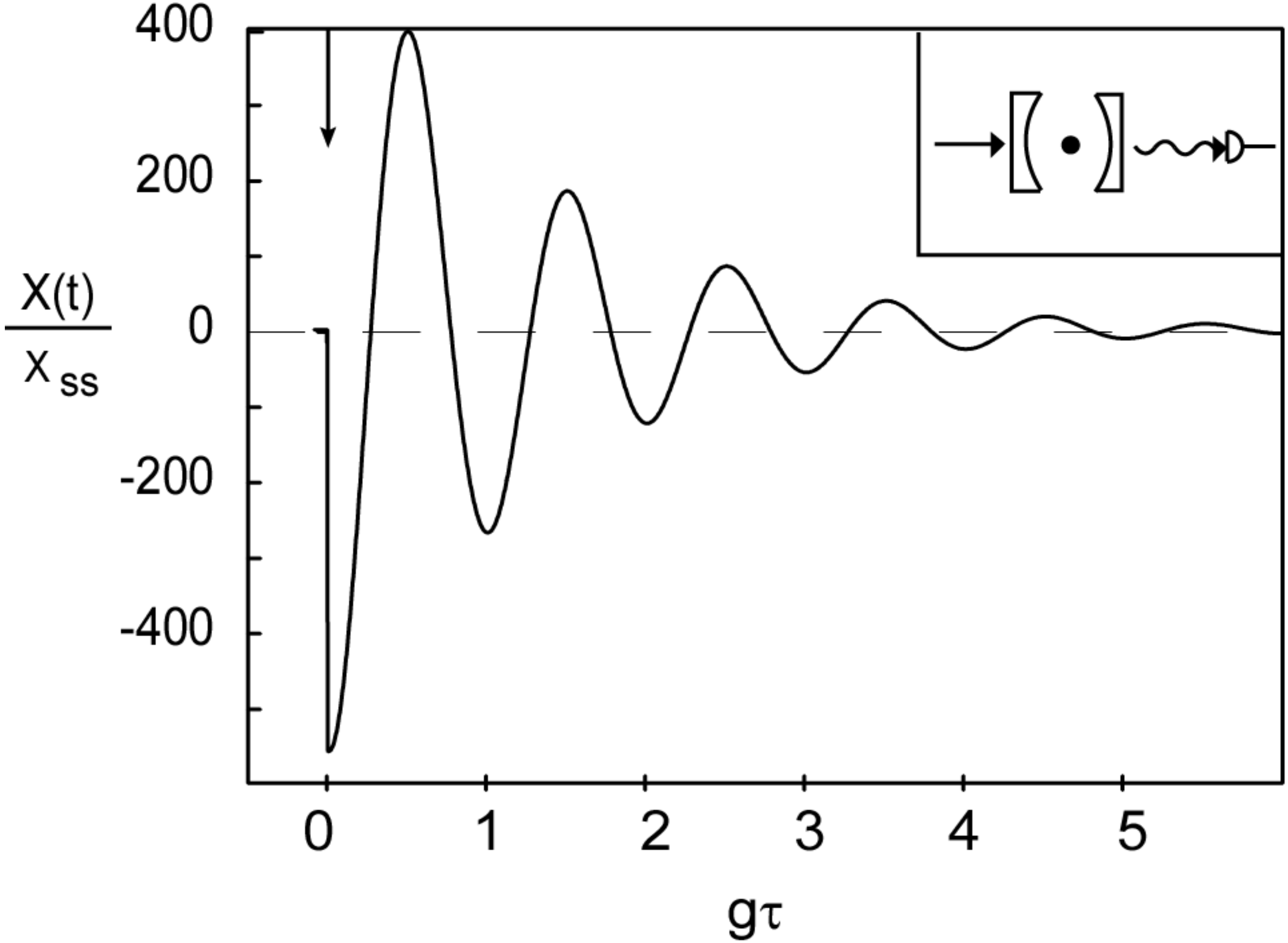
$$\hat{a}\left|\Psi_{ss}\right\rangle \Rightarrow \left|\Psi_c(\tau)\right\rangle = \left|0,g\right\rangle + \lambda pq\left|1,g\right\rangle - \frac{2g\lambda q}{\gamma}\left|0,e\right\rangle$$

$$\left|\Psi_c(\tau)\right\rangle = \left|0,g\right\rangle + \lambda\left[f_1(\tau)\left|1,g\right\rangle + f_2(\tau)\left|0,e\right\rangle\right] + O\left(\lambda^2\right)$$

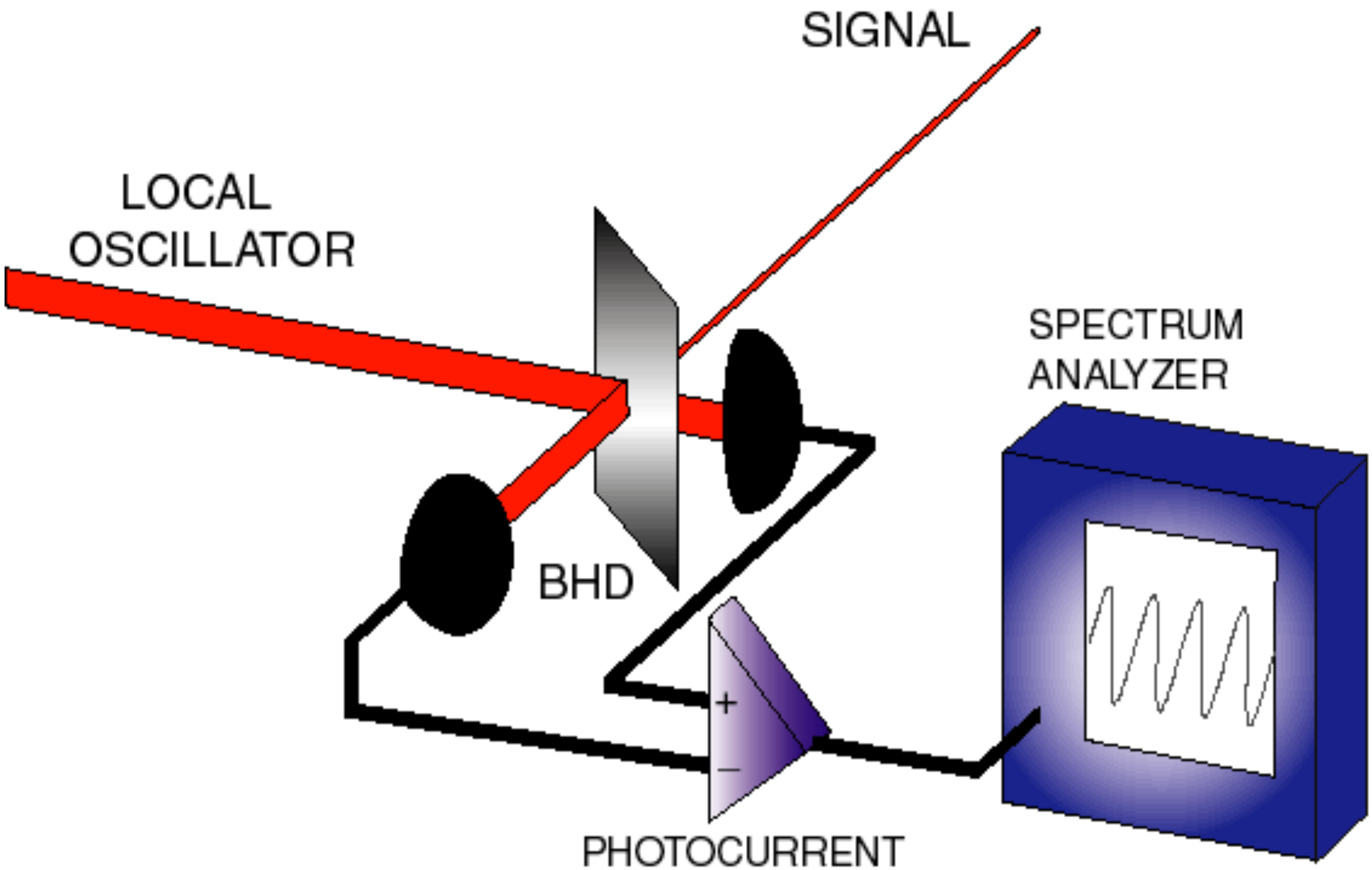
Field

Atomic Polarization

Regression of the field to steady state after the detection of a photon.



Detection of the Squeezing spectrum with a balanced homodyne detector (BHD).

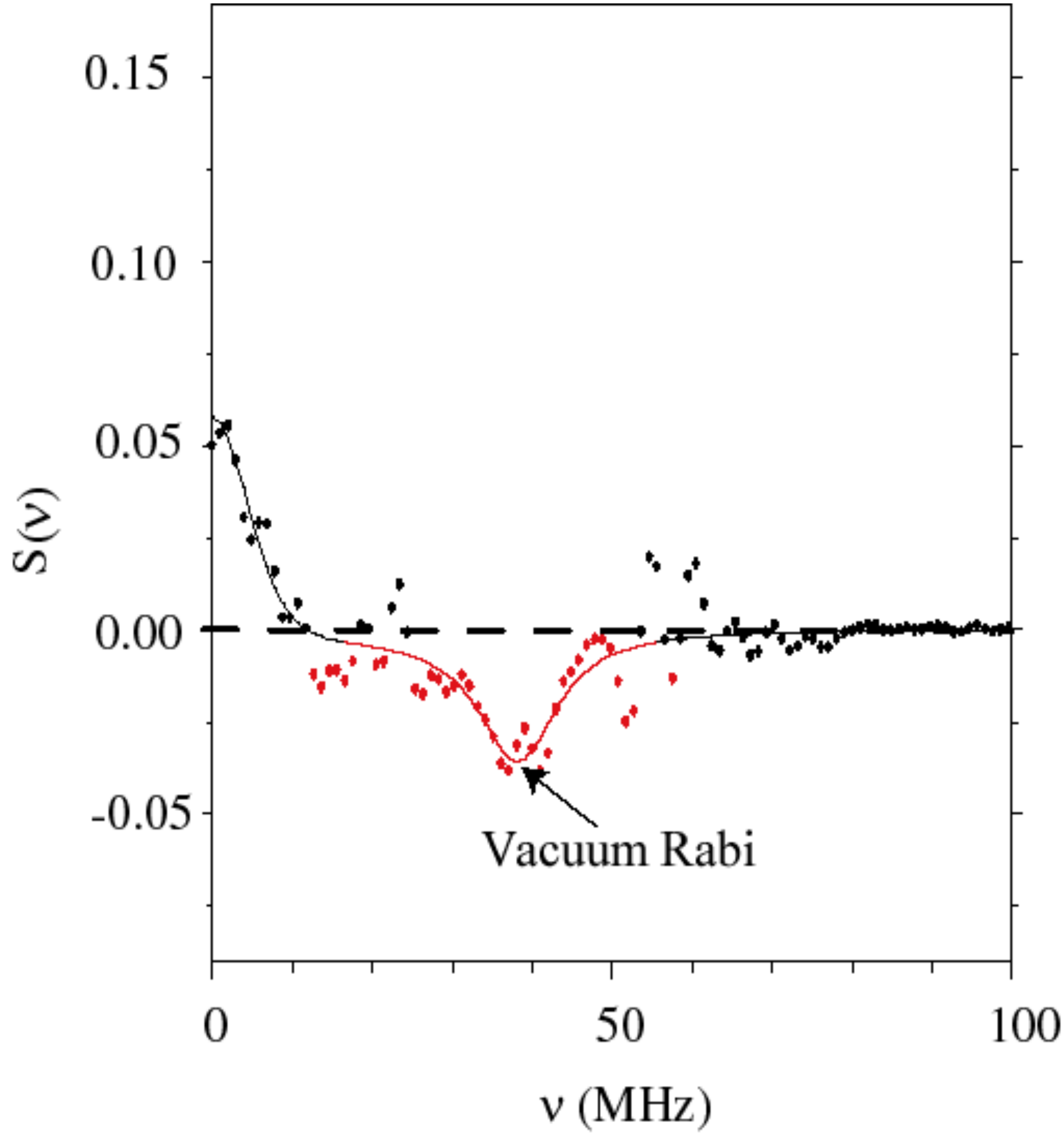


The fluctuations of the electromagnetic field are measured by the spectrum of squeezing. Look at the noise spectrum of the photocurrent.

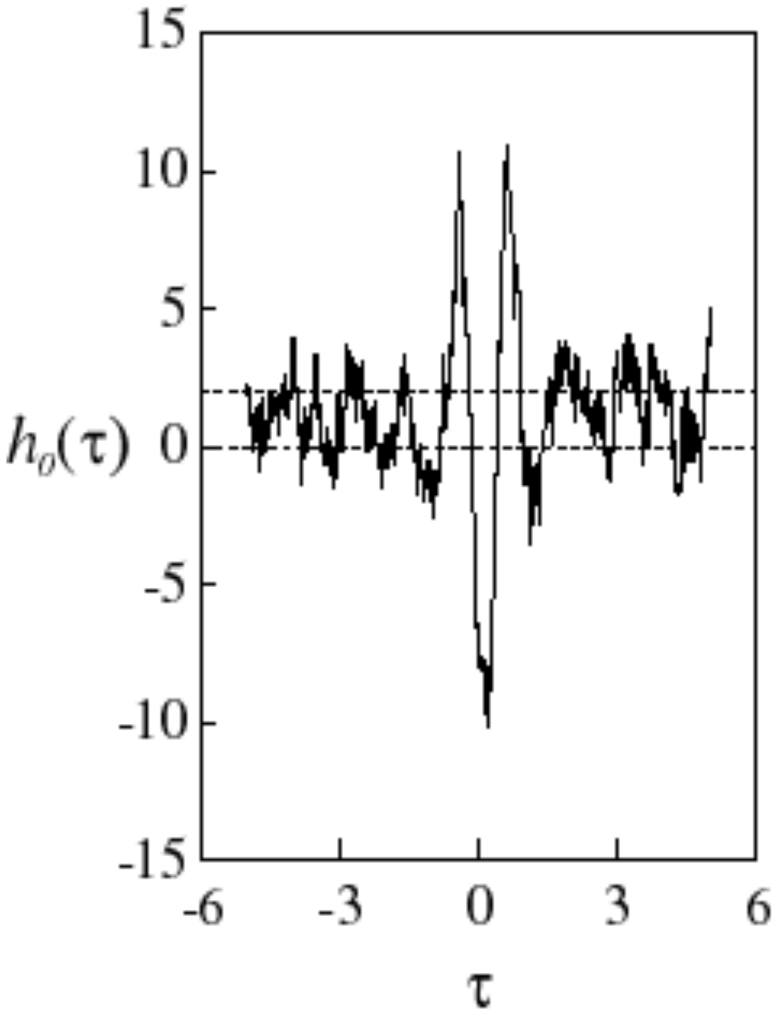
$$S(\nu, 0^\circ) = 4F \int_0^\infty \cos(2\pi\nu\tau)[\overline{h}_0(\tau) - 1]d\tau,$$

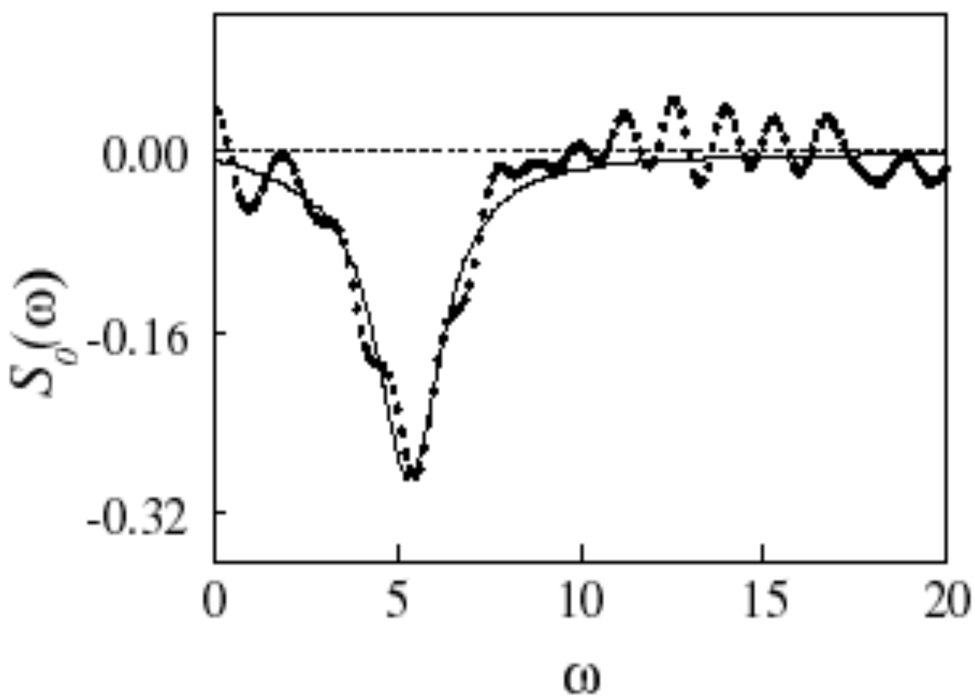
F is the photon flux into the correlator.

Spectrum of Squeezing from the Fourier Transform of $h_0(t)$

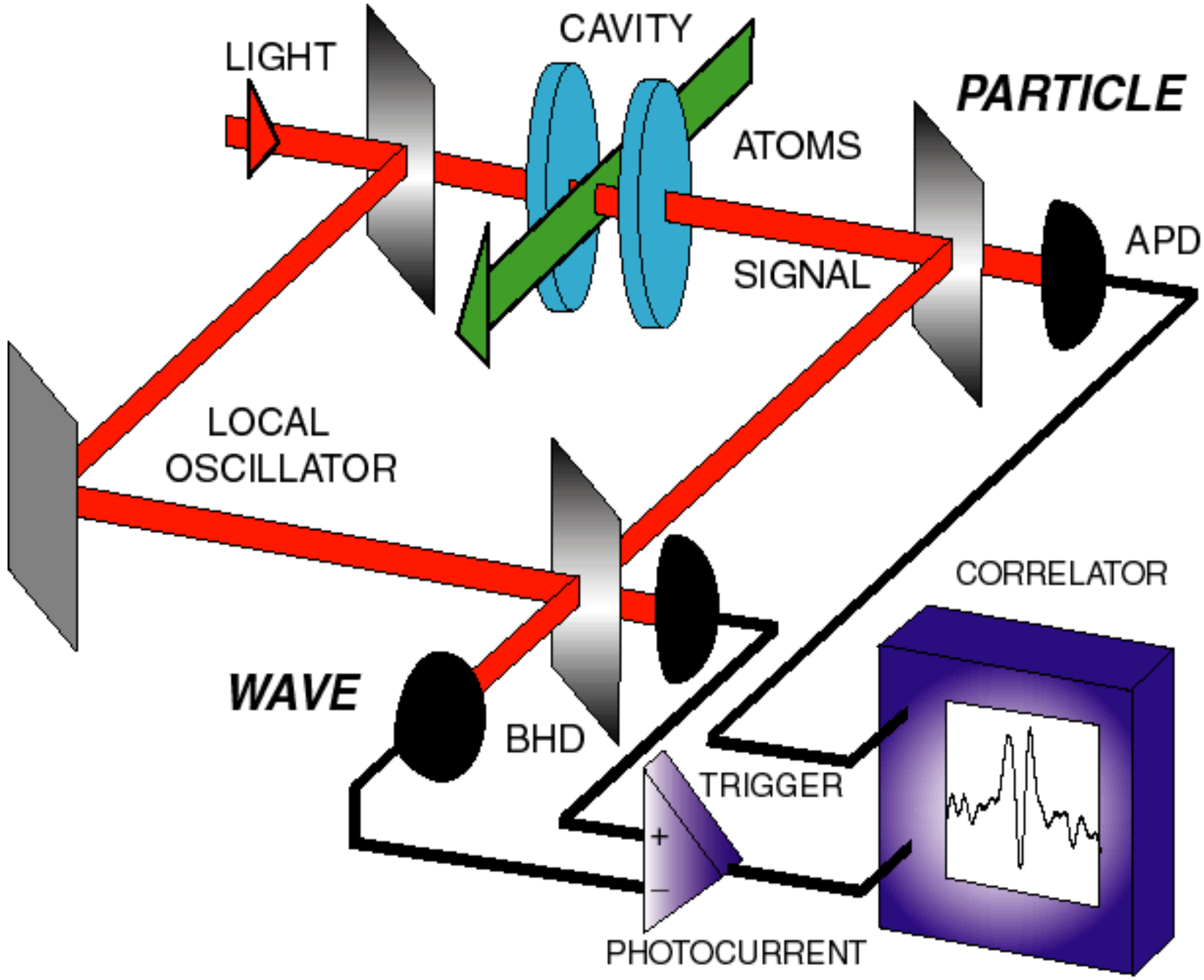


Monte Carlo simulations of the wave-particle correlation and the spectrum of squeezing in the low intensity limit for an atomic beam.





It has upper and a lower classical bounds



LIGHT
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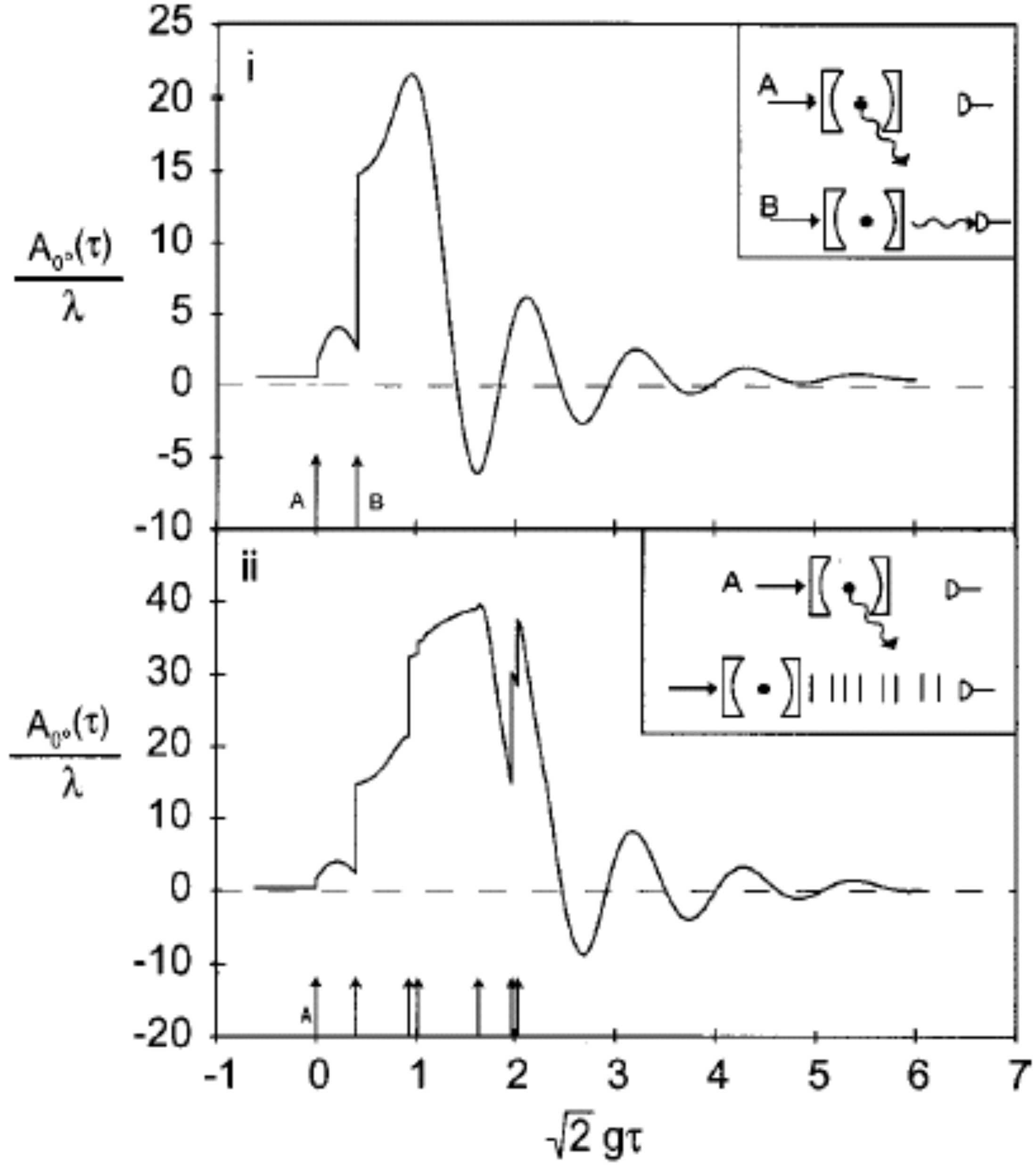
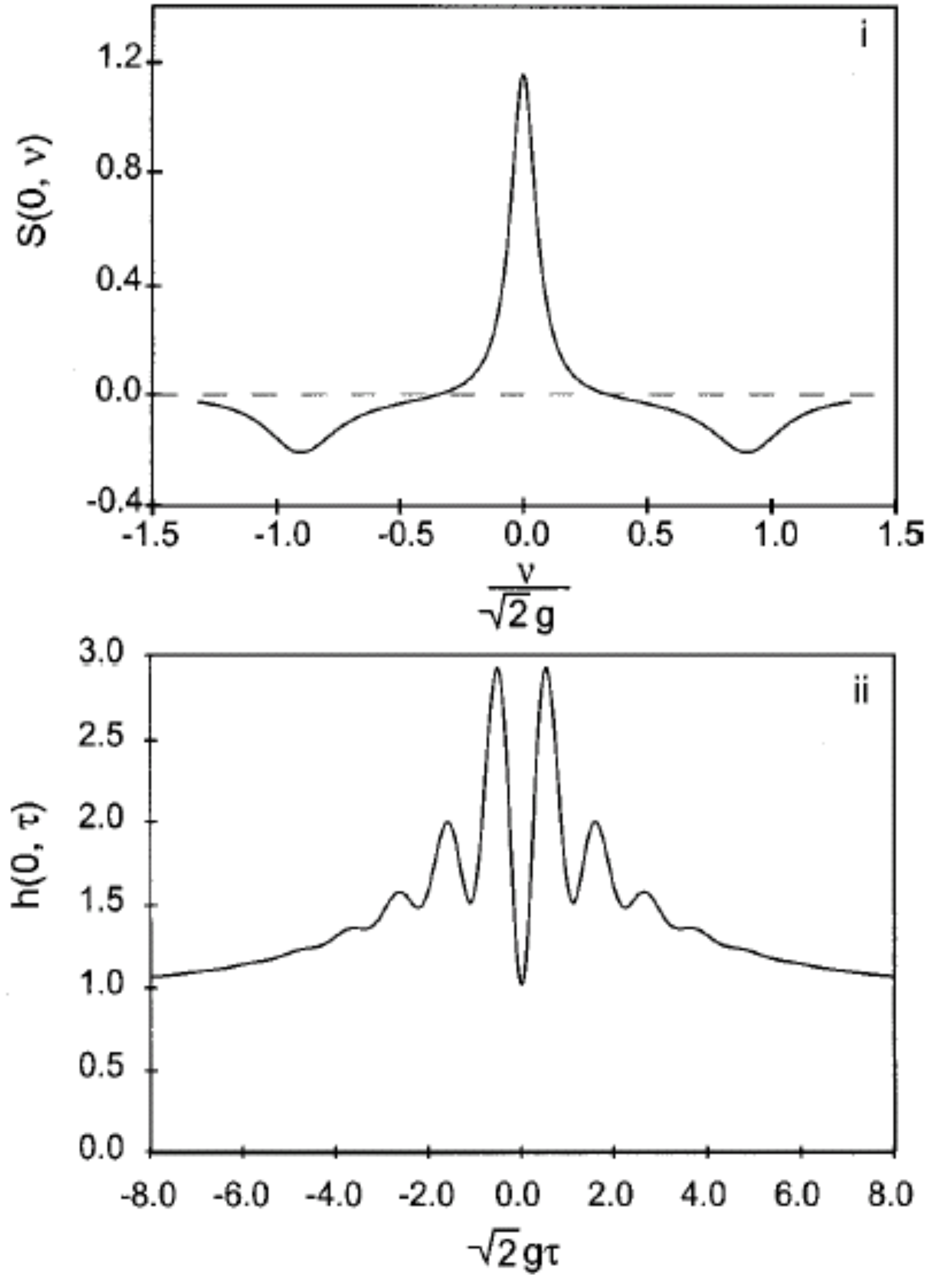
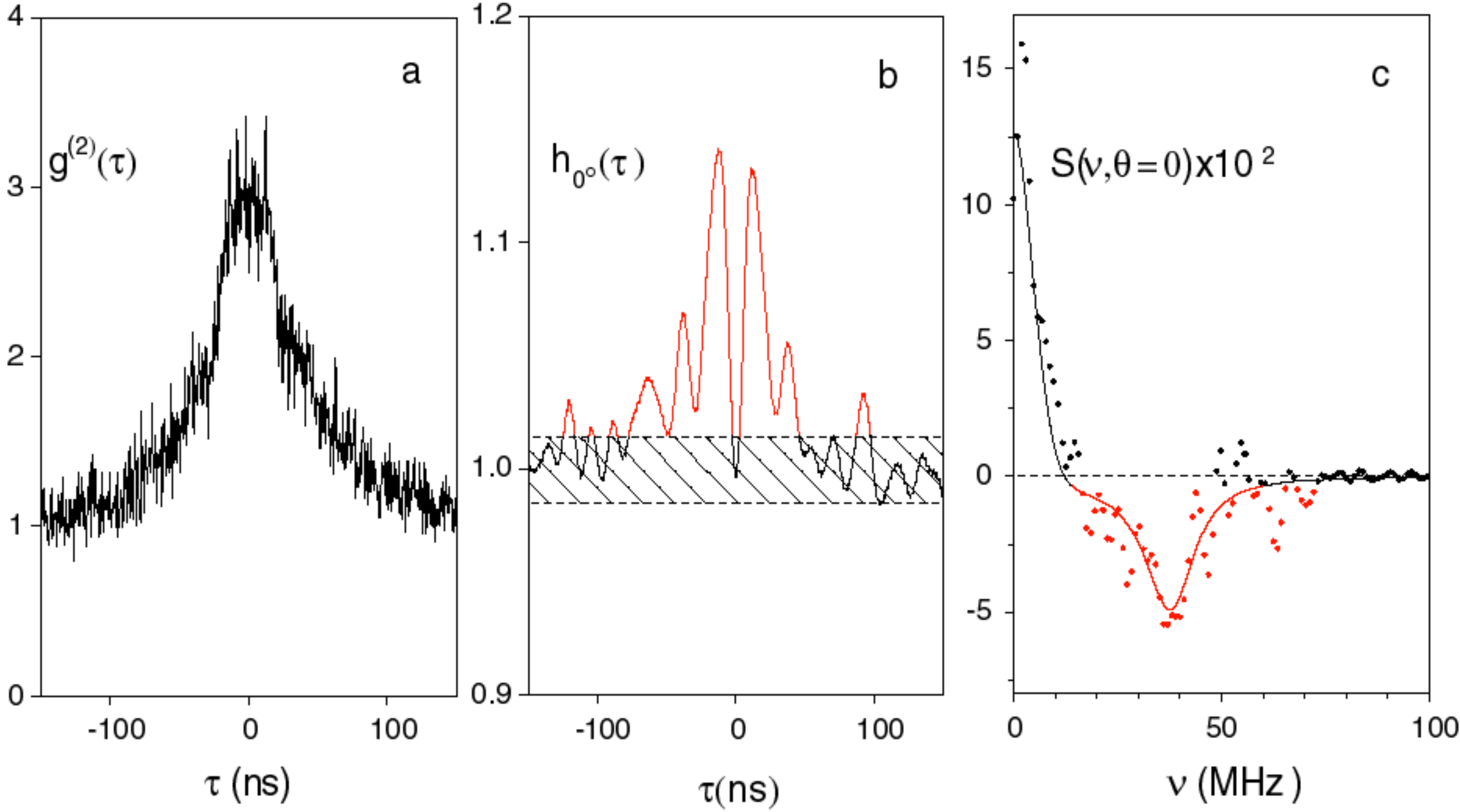


Single quantum trajectories simulation of cavity QED system with spontaneous emission.



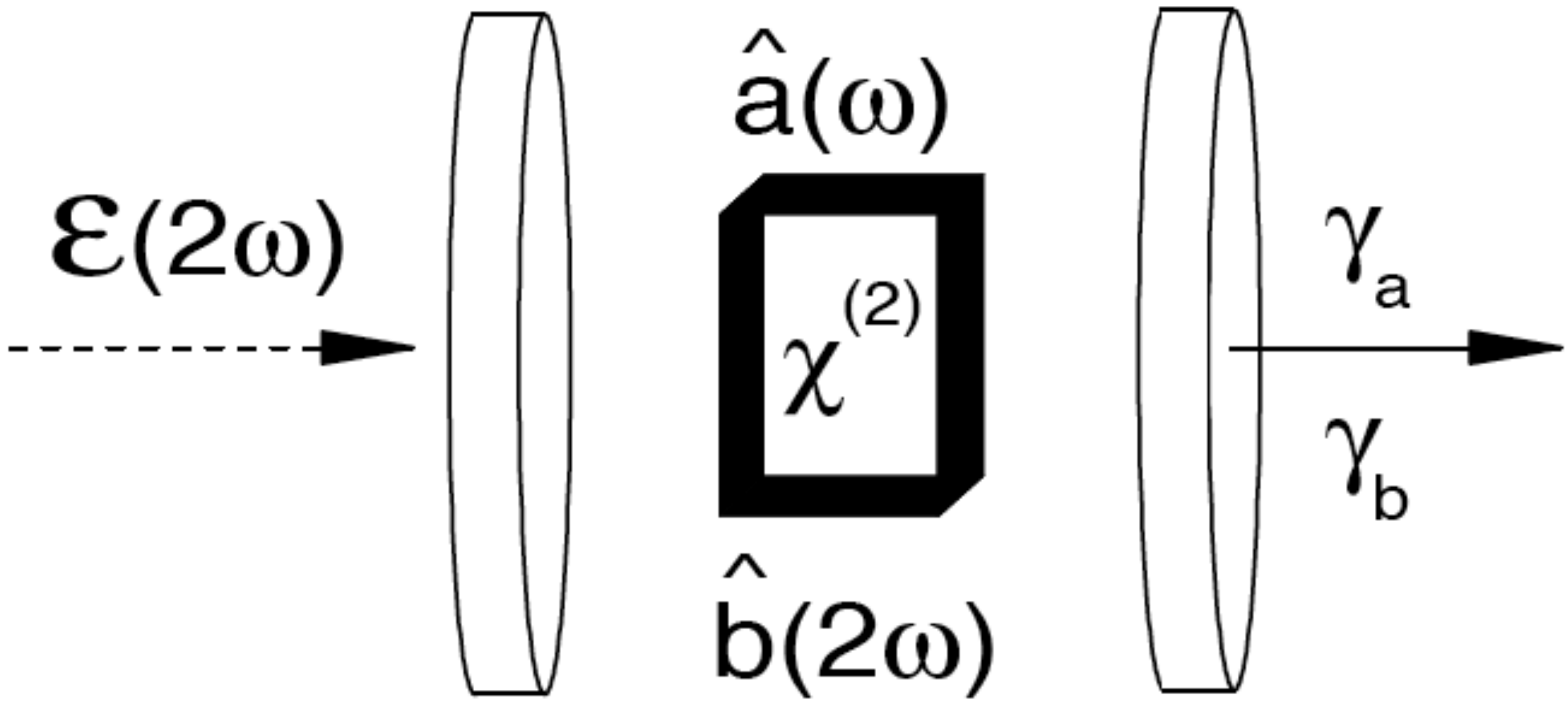
(i) Spectrum of squeezing obtained from the averaged (ii) h(0,t) correlation function that shows the effects of spontaneous emission.

Classical $g^{(2)}$ Non-classical h Squeezing



N=13; 1.2n_0

Optical Parametric Oscillator



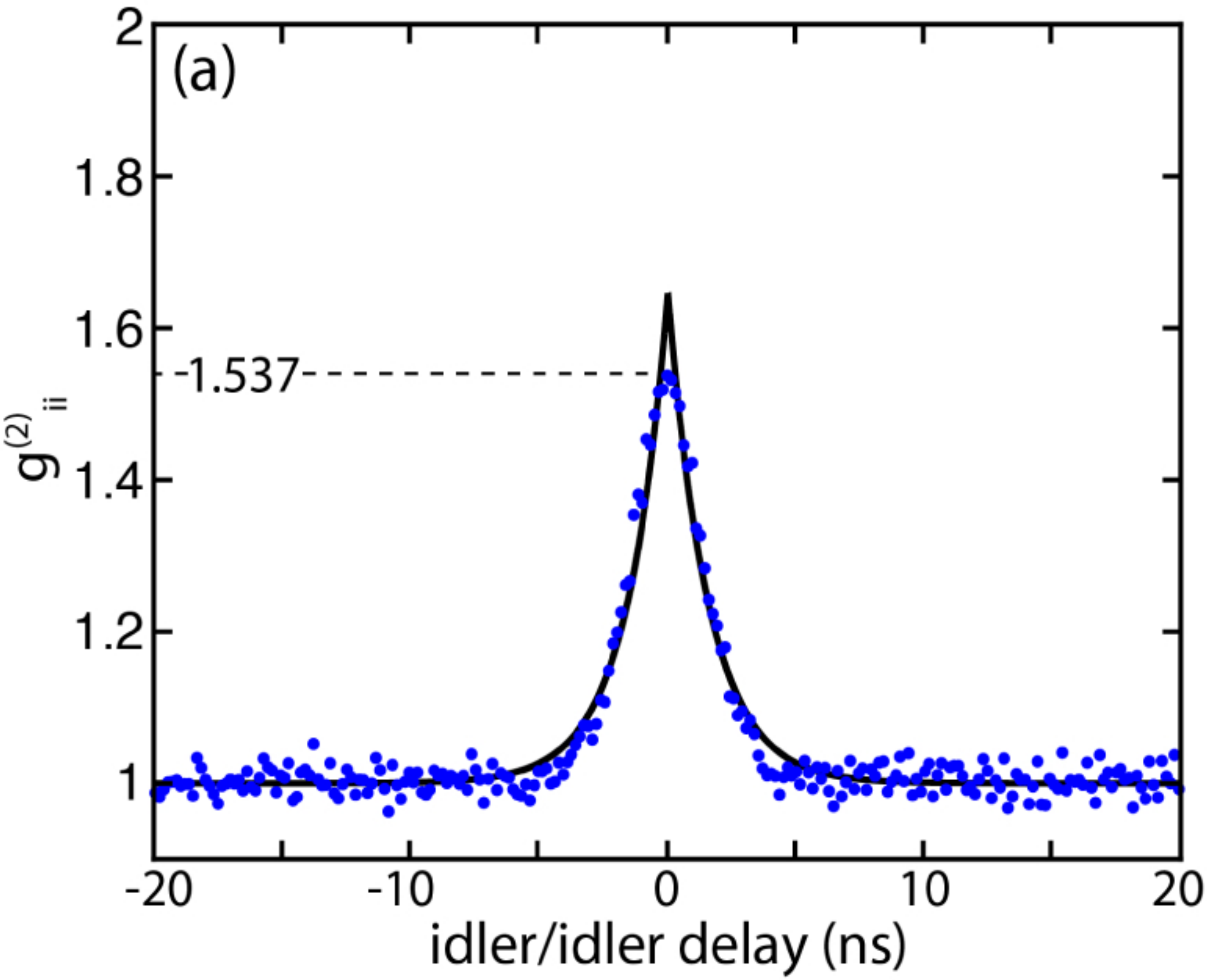
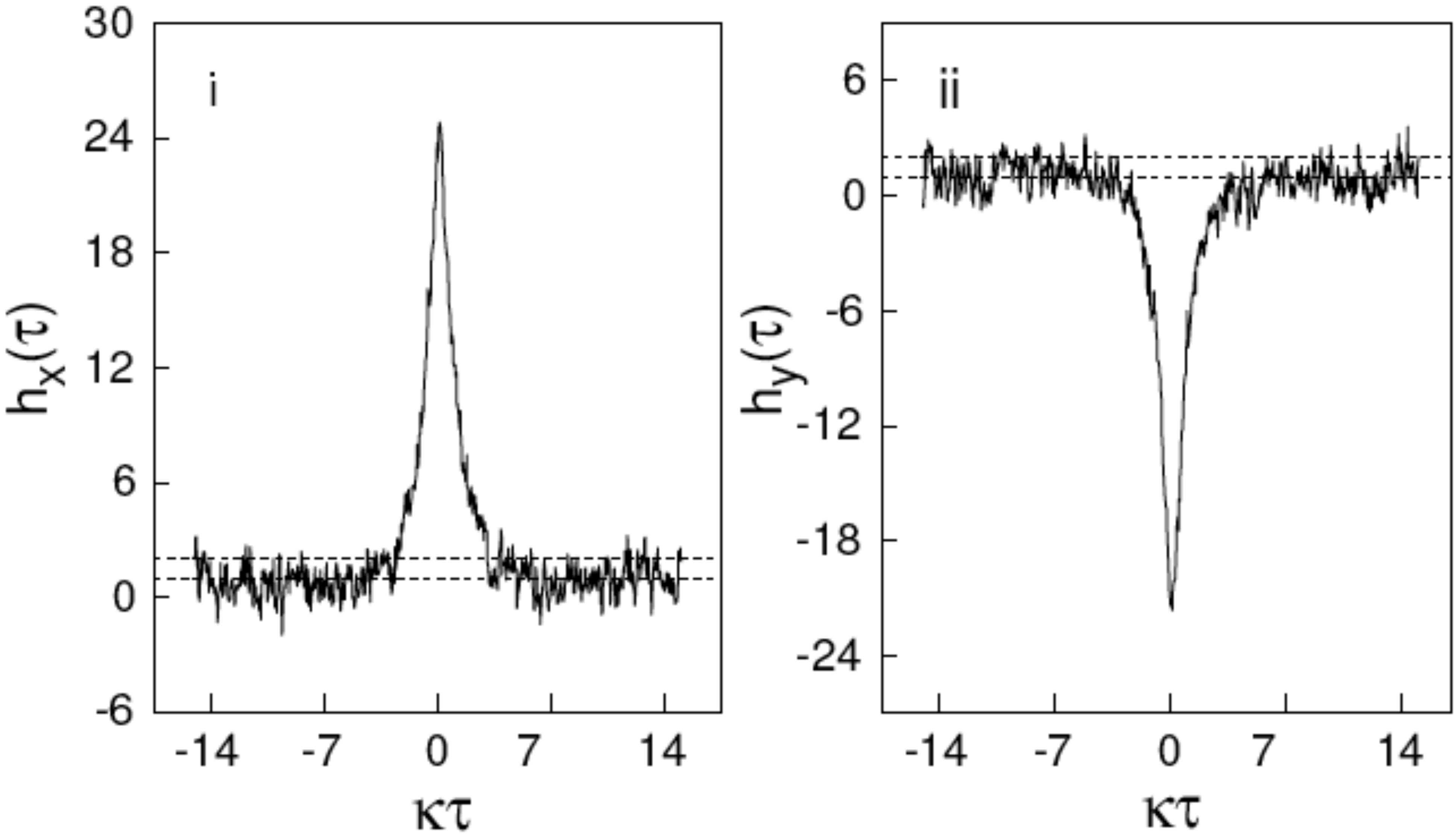


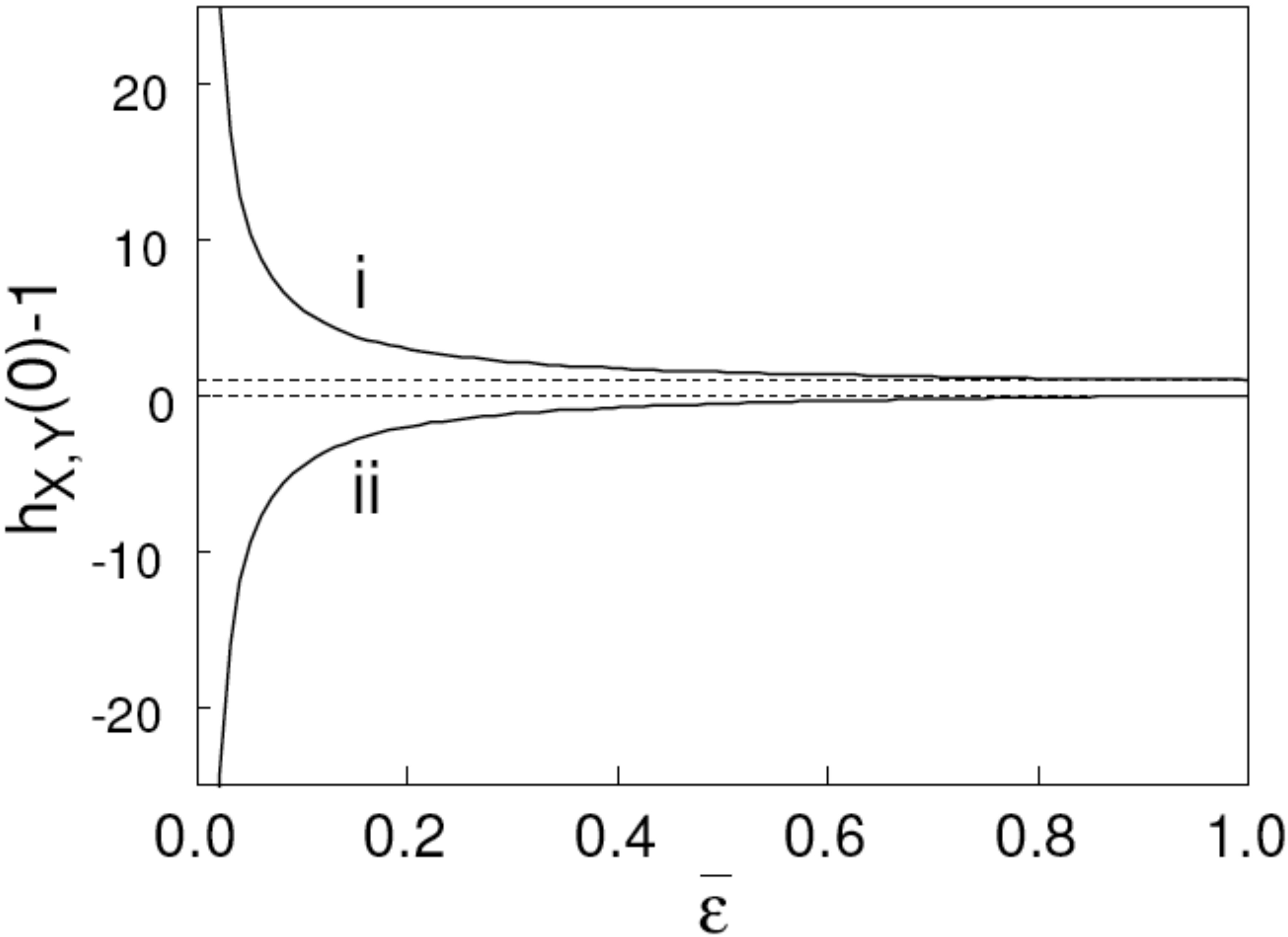
Fig. 4

Citation
Christian Reimer, Lucia Caspani, Matteo Clerici, Marcello Ferrera, Michael Kues, Marco Peccianti, Alessia Pasquazi, Luca Razzari, Brent E. Little, Sai T. Chu, David J. Moss, Roberto Morandotti, "Integrated frequency comb source of heralded single photons," Opt. Express **22**, 6535-6546 (2014);
https://www.osapublishing.org/oe/abstract.cfm?uri=oe-22-6-6535

Calculation of $h_\theta(\tau)$ in an OPO with the classical bounds



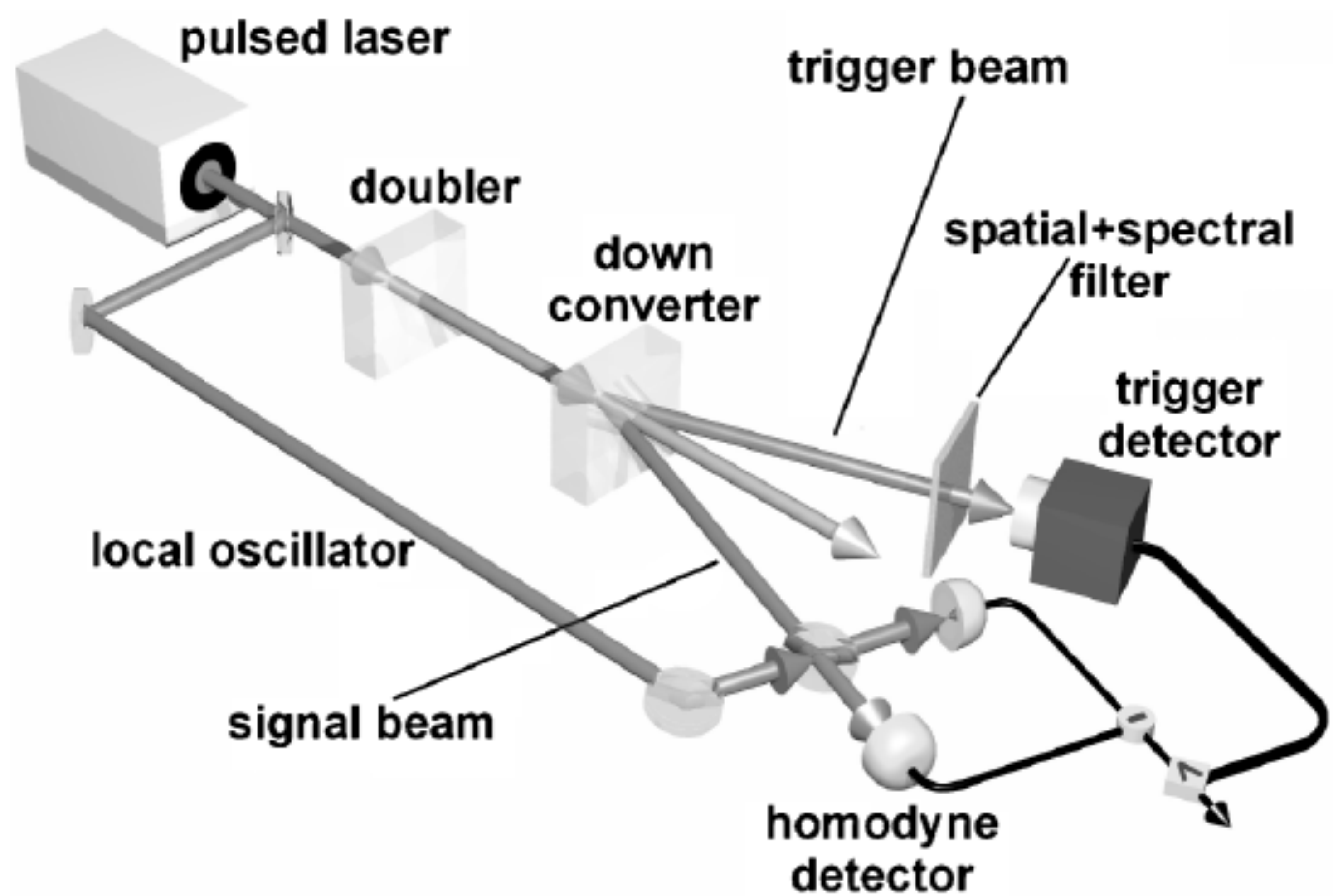
Maximum of $h_\theta(\tau)$ in an OPO below threshold



Quantum State Reconstruction of the Single-Photon Fock State

A. I. Lvovsky,* H. Hansen, T. Aichele, O. Benson, J. Mlynek,† and S. Schiller‡

Phys. Rev. Lett. 87, 050402 (2001)



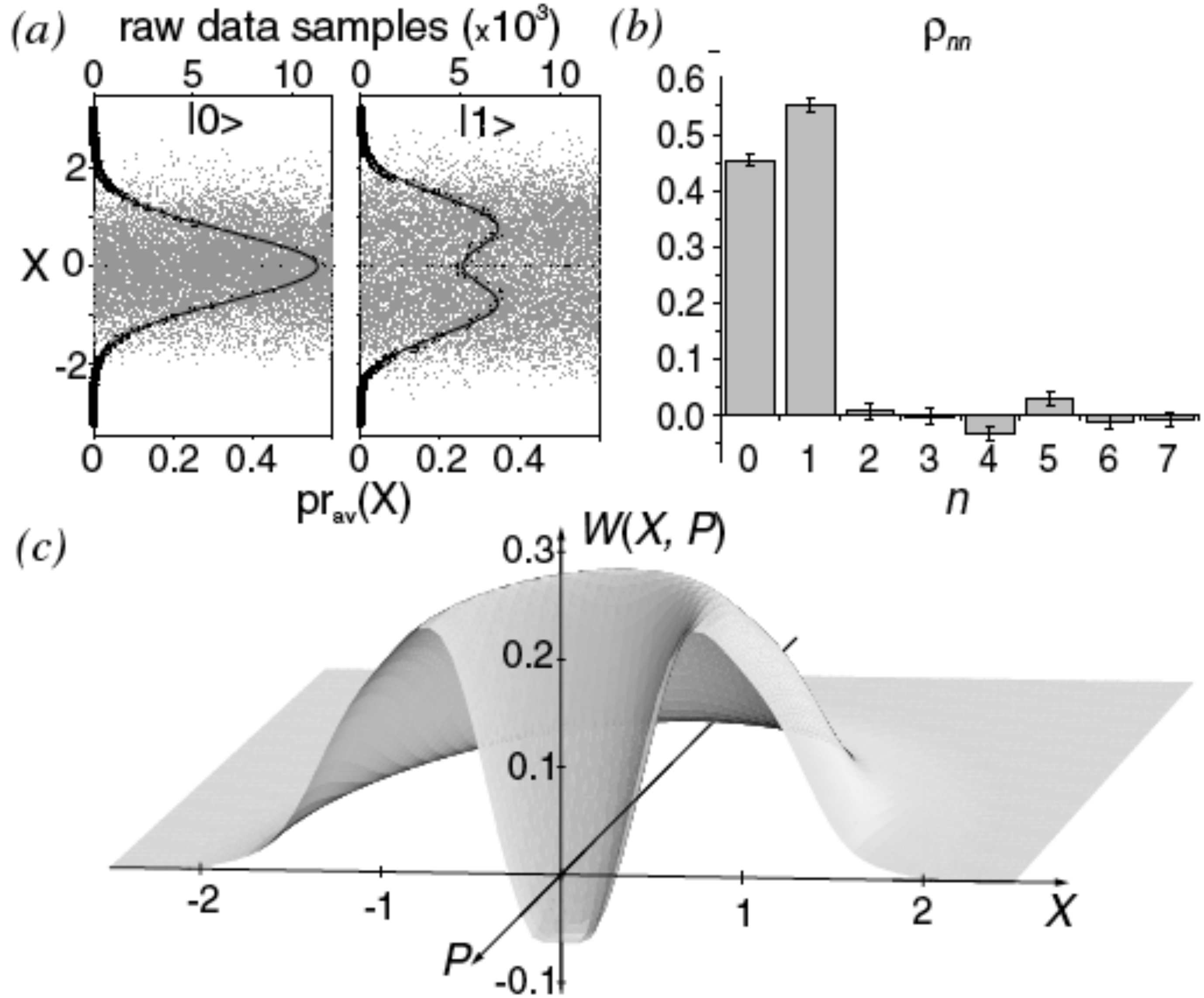


FIG. 4. Experimental results: (a) raw quantum noise data for the vacuum (left) and Fock (right) states along with their histograms corresponding to the phase-randomized marginal distributions; (b) diagonal elements of the density matrix of the state measured; (c) reconstructed WF which is negative near the origin point. The measurement efficiency is 55%.

Conclusions:

- The wave-particle correlation $h_\theta(\tau)$ measures the conditional dynamics of the electromagnetic field. The Spectrum of Squeezing $S(\Omega)$ and $h_\theta(\tau)$ are Fourier Transforms of each other.
- Many applications in many other problems of quantum optics and of optics in general: microscopy, degaussification, weak measurements, quantum feedback.
- Possibility of a tomographic reconstruction of the dynamical evolution of the electromagnetic field state.

Thanks