Correlation functions in optics; classical and quantum 3.

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Review Article:
Reference that includes pulsed sources:
Zheyu Jeff Ou
“Quantum Optics for Experimentalists”
World Scientific, Singapore 2017
• The main object of interest in quantum optics is the optical FIELD. That is what is quantized.

• Can we measure the FIELD of a state with an average of ONE PHOTON in it?
Homodyne detection

Interfere two fields: A local oscillator (LO) and a signal (S). The resulting photocurrent has a term proportional to the amplitude of S and also depends on the cosine of the phase difference $\phi$ between LO and S.

$$|\text{LO} \cos(\phi) + S|^2 = |\text{LO}|^2 + 2 |\text{LO}| |S| \cos(\phi) + |S|^2$$
Review of shot noise:

Shot noise happens whenever the transport of energy is through a finite number of discreet particles. For example, electric charge $e$ (Schottky 1918). If the number of particles is small and it follows a Poisson distribution (random independent events), it can be the dominant noise.
• The mean of a Poisson distribution is \( n \)
• The variance of a Poisson distribution \( n \)
• The signal to noise ratio \( n^{1/2} \)
• A Poisson distribution with \( n \) large approximates a Gaussian.
• The current spectral density \( (i) \) of noise is: \((2e|i|)^{1/2}\) with units of \([A/\text{Hz}^{1/2}]\).
• The power of the noise depends on the detection bandwith and the Resistance \( R \): \( P(\nu)=R2e|i|\Delta \nu \).
Review of Coherent States $|\alpha\rangle$

The coherent state $|\alpha\rangle$ is the eigenstate of the annihilation operator:

$$\hat{a} |\alpha\rangle = \alpha |\alpha\rangle$$

Its amplitude (complex): $\alpha$

Its mean squared: $\alpha^* \alpha = |\alpha|^2$

Its uncertainty: $1/2$

They are states with the minimum uncertainty allowed by quantum mechanics. Equal on both quadratures.
Relation with the harmonic oscillators:
Quadratures of the electromagnetic field

\[ E_R = \left( \frac{\hbar \omega}{2 \varepsilon_0 V} \right)^{1/2} \cos(\theta) X \quad \text{and} \quad E_I = \left( \frac{\hbar \omega}{2 \varepsilon_0 V} \right)^{1/2} \sin(\theta) X \]

\[ H = \hbar \omega \left( P^2 + X^2 \right), \quad \text{with} \quad [X, P] \equiv XP - PX = \frac{i}{2} I \]

\[ (X - \langle X \rangle) |\alpha\rangle = -i (P - \langle P \rangle) |\alpha\rangle \]

\[ (X + iP) |\alpha\rangle = \langle X + iP \rangle |\alpha\rangle \]

States of minimum uncertainty:
\[ \langle \alpha | (X - \langle X \rangle)^2 + (P - \langle P \rangle)^2 |\alpha\rangle = 1/2 \]

Relation with Fock states (Poisson)

\[ P(n) = |\langle n | \alpha \rangle|^2 = e^{-\langle n \rangle} \frac{\langle n \rangle^n}{n!} \]
Perfect detector \( i(t) = |\alpha + \Delta \alpha|^2 \)

\[
i(t) = |\alpha|^2 + 2 \alpha \Delta \alpha + |\Delta \alpha|^2 \quad ; \quad <\alpha^* \alpha> = n
\]

DC \( \sim n \)  \quad \text{Shot noise} \sim n^{1/2} \quad \text{neglect.}
Correlation functions tell us something about the fluctuations.

Correlations have classical bounds.

They are conditional measurements.

Can we use them to measure the field associated with a FLUCTUATION of one photon?
Mach Zehnder Interferometer Wave-Wave Correlation

\[ g^{(1)}(\tau) = \frac{\langle E^*(t)E(t+\tau) \rangle}{\langle I(t) \rangle} \]

Spectrum of the signal:

\[ F(\omega) = \frac{1}{2\pi} \int \exp(i\omega\tau) g^{(1)}(\tau) d\tau \]

Basis of Fourier Transform Spectroscopy
Hanbury Brown and Twiss \textit{Intensity-Intensity Correlations}

\begin{equation}
g^{(2)}(\tau) = \frac{\langle I(t)I(t+\tau) \rangle}{\langle I(t) \rangle^2}
\end{equation}

\begin{equation}
2I(t)I(t+\tau) \leq I^2(t) + I^2(t+\tau)
\end{equation}

Cauchy-Schwarz

The correlation is largest at equal time

\begin{align*}
g^{(2)}(0) &\geq 1 \\
g^{(2)}(0) &\geq g^{(2)}(\tau)
\end{align*}
Intensity correlation function measurements:

\[ g^{(2)}(\tau) = \frac{\langle :\hat{I}(t)\hat{I}(t+\tau): \rangle}{\langle \hat{I}(t) \rangle^2} \]

Gives the probability of detecting a photon at time \( t + \tau \) given that one was detected at time \( t \). This is a conditional measurement:

\[ g^{(2)}(\tau) = \frac{\langle \hat{I}(\tau) \rangle_c}{\langle \hat{I} \rangle} \]
A strategy starts to appear:

Correlation function; Conditional measurement.

Detect a photon: Prepare a conditional quantum mechanical state in our system.

The system has to have at least two photons.

Do we have enough signal to noise ratio?

\[ |LO|^2 + 2 \text{ LO S} \cos (\phi) \]

SHOT NOISE SIGNAL
How to correlate fields and intensities?
Detection of the field: Homodyne.

Conditional Measurement: Only measure when we know there is a photon.

Source: Cavity QED
The Intensity-Field correlator.

\[
H(\tau) = \frac{\langle \hat{A}(0) \hat{B}(\tau) \rangle}{\langle \hat{A} \rangle} + \sqrt{\Gamma} \xi(\tau).
\]
Condition on a Click
Measure the correlation function of the Intensity and the Field:
\[ \langle I(t) \, E(t+\tau) \rangle \]

Normalized form:
\[ h_\theta(\tau) = \frac{\langle E(\tau) \rangle_c}{\langle E \rangle} \]

From Cauchy Schwartz inequalities:
\[ 0 \leq h_0(0) - 1 \leq 2 \]
\[ \left| \bar{h}_0(\tau) - 1 \right| \leq \left| \bar{h}_0(0) - 1 \right| \]
Photocurrent average with random conditioning
Conditional photocurrent with no atoms in the cavity.
After 1 average
After 6,000 averages
After 10,000 averages
After 30,000 averages
After 65,000 averages
Flip the phase of the Mach-Zehnder by $146^\circ$
Monte Carlo simulations for weak excitation:

Atomic beam $N=11$
This is the conditional evolution of the field of a fraction of a photon \([B(t)]\) from the correlation function.

\[ h_\theta(\tau) = \frac{<E(\tau)>_c}{<E>} \]

The conditional field prepared by the click is:

\[ A(t)|0> + B(t)|1> \] with \( A(t) \approx 1 \) and \( B(t) \ll 1 \)

We measure the field of a fraction of a photon!

Fluctuations are very important.
Conditional dynamics in cavity QED at low intensity:

\[
\left| \Psi_{ss} \right\rangle = \left| 0, g \right\rangle + \lambda \left| 1, g \right\rangle - \frac{2g}{\gamma} \lambda \left| 0, e \right\rangle + \frac{\lambda^2 p q}{\sqrt{2}} \left| 2, g \right\rangle - \frac{2g \lambda^2 q}{\gamma} \left| 1, e \right\rangle 
\]

\[\lambda = \langle \hat{a} \rangle, \quad p = p(g, \kappa, \gamma) \quad \text{and} \quad q = q(g, \kappa, \gamma)\]

A photodetection conditions the state into the following non-steady state from which the system evolves.

\[\hat{a} \left| \Psi_{ss} \right\rangle \implies \left| \Psi_c (\tau) \right\rangle = \left| 0, g \right\rangle + \lambda p q \left| 1, g \right\rangle - \frac{2g \lambda q}{\gamma} \left| 0, e \right\rangle\]

\[\left| \Psi_c (\tau) \right\rangle = \left| 0, g \right\rangle + \lambda \left[ f_1 (\tau) \left| 1, g \right\rangle + f_2 (\tau) \left| 0, e \right\rangle \right] + O \left( \lambda^2 \right)\]

Field

Atomic Polarization
Regression of the field to steady state after the detection of a photon.
Detection of the Squeezing spectrum with a balanced homodyne detector (BHD).
The fluctuations of the electromagnetic field are measured by the spectrum of squeezing. Look at the noise spectrum of the photocurrent.

\[ S(\nu,0^\circ) = 4F \int_0^\infty \cos(2\pi \nu \tau)[\bar{h}_0(\tau) - 1]d\tau, \]

F is the photon flux into the correlator.
Spectrum of Squeezing from the Fourier Transform of $h_0(t)$
Monte Carlo simulations of the wave-particle correlation and the spectrum of squeezing in the low intensity limit for an atomic beam.

It has upper and a lower classical bounds.
Single quantum trajectories simulation of cavity QED system with spontaneous emission.
(i) Spectrum of squeezing obtained from the averaged (ii) $h(0,t)$ correlation function that shows the effects of spontaneous emission.
Classical $g^{(2)}$  Non-classical $h$  Squeezing

$g^{(2)}(\tau)$

$h_{0\circ}(\tau)$

$S(\nu, \theta=0) \times 10^2$

$\tau$ (ns)  $\tau$ (ns)  $\nu$ (MHz)

$N=13; 1.2n_0$
Optical Parametric Oscillator

\[ \varepsilon(2\omega) \rightarrow \hat{a}(\omega) \rightarrow \chi^{(2)} \rightarrow \hat{b}(2\omega) \rightarrow \gamma_a, \gamma_b \]
Fig. 4

Citation

Image © 2014 Optical Society of America and may be used for noncommercial purposes only. Report a copyright concern regarding this image.
Calculation of $h_\theta(\tau)$ in an OPO with the classical bounds
Maximum of $h_\theta(\tau)$ in an OPO below threshold
Quantum State Reconstruction of the Single-Photon Fock State

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FIG. 4. Experimental results: (a) raw quantum noise data for the vacuum (left) and Fock (right) states along with their histograms corresponding to the phase-randomized marginal distributions; (b) diagonal elements of the density matrix of the state measured; (c) reconstructed WF which is negative near the origin point. The measurement efficiency is 55%.
Conclusions:

- The wave-particle correlation $h_\theta(\tau)$ measures the conditional dynamics of the electromagnetic field. The Spectrum of Squeezing $S(\Omega)$ and $h_\theta(\tau)$ are Fourier Transforms of each other.
- Many applications in many other problems of quantum optics and of optics in general: microscopy, degaussification, weak measurements, quantum feedback.
- Possibility of a tomographic reconstruction of the dynamical evolution of the electromagnetic field state.
Thanks