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**TEST**

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# Correlation functions in optics and quantum optics, 3

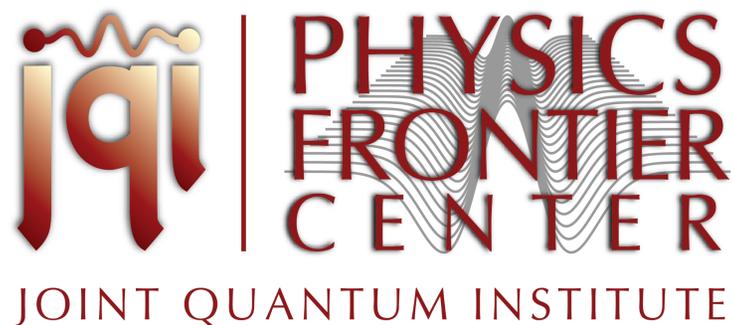
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July 2018

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The slides of the course are available at:

<http://www.physics.umd.edu/rgroups/amo/orozco/results/2018/Results18.htm>



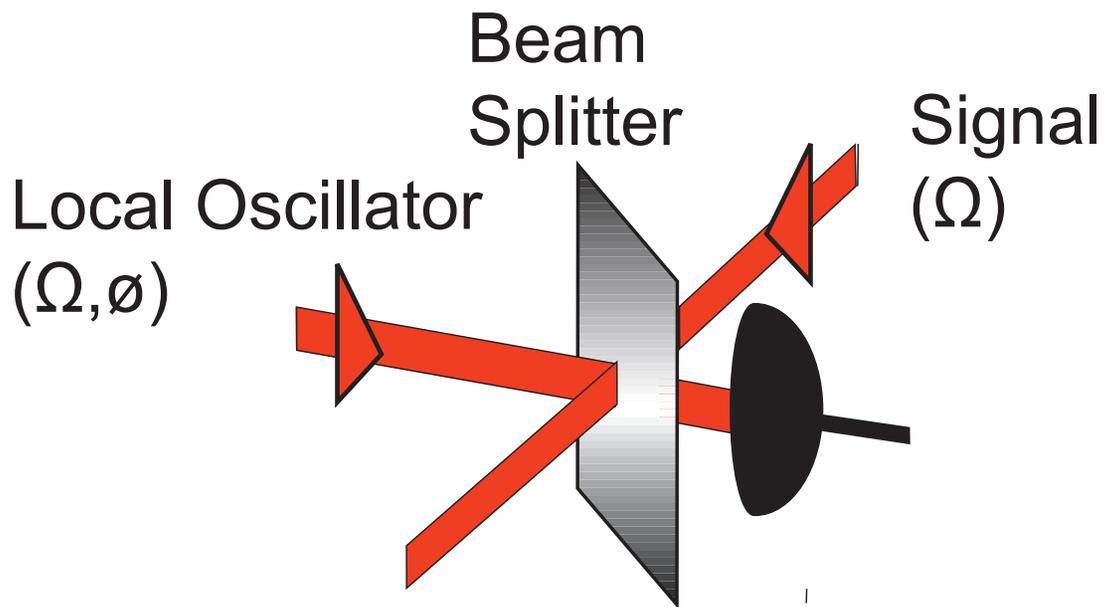
## Review Article:

H. J. Carmichael, G. T. Foster, L. A. Orozco, J. E. Reiner, and P. R. Rice, “Intensity-Field Correlations of Non-Classical Light”.

Progress in Optics, Vol. 46, 355-403, Edited by E. Wolf Elsevier, Amsterdam 2004.



# Homodyne detection



$$\text{Photocurrent} \sim |\text{L.O.} \cos(\phi) + S|^2$$

Interfere two fields: A local oscillator (LO) and a signal (S). The resulting photocurrent has a term proportional to the amplitude of S and also depends on the cosine of the phase difference  $\phi$  between LO and S.

$$|\text{LO} \cos(\phi) + S|^2 = |\text{LO}|^2 + 2 \text{LO} S \cos(\phi) + |S|^2$$

## Review of shot noise :

Shot noise happens whenever the transport of energy is through a finite number of discrete particles. For example, electric charge  $e$  (Schottky 1918). If the number of particles is small and it follows a Poisson distribution (random independent events), it can be the dominant noise.

- The mean of a Poisson distribution is  $n$
- The variance of a Poisson distribution  $n$
- The signal to noise ratio  $n^{1/2}$
- A Poisson distribution with  $n$  large approximates a Gaussian.
- The current spectral density ( $i$ ) of noise is:  $(2e|i|)^{1/2}$  with units of  $[A/Hz^{1/2}]$ .
- The power of the noise depends on the detection bandwidth and the Resistance  $R$ :  

$$P(\nu) = R 2e|i| \Delta\nu.$$

## Review of Coherent States $|\alpha\rangle$

The coherent state  $|\alpha\rangle$  is the eigenstate of the annihilation operator:

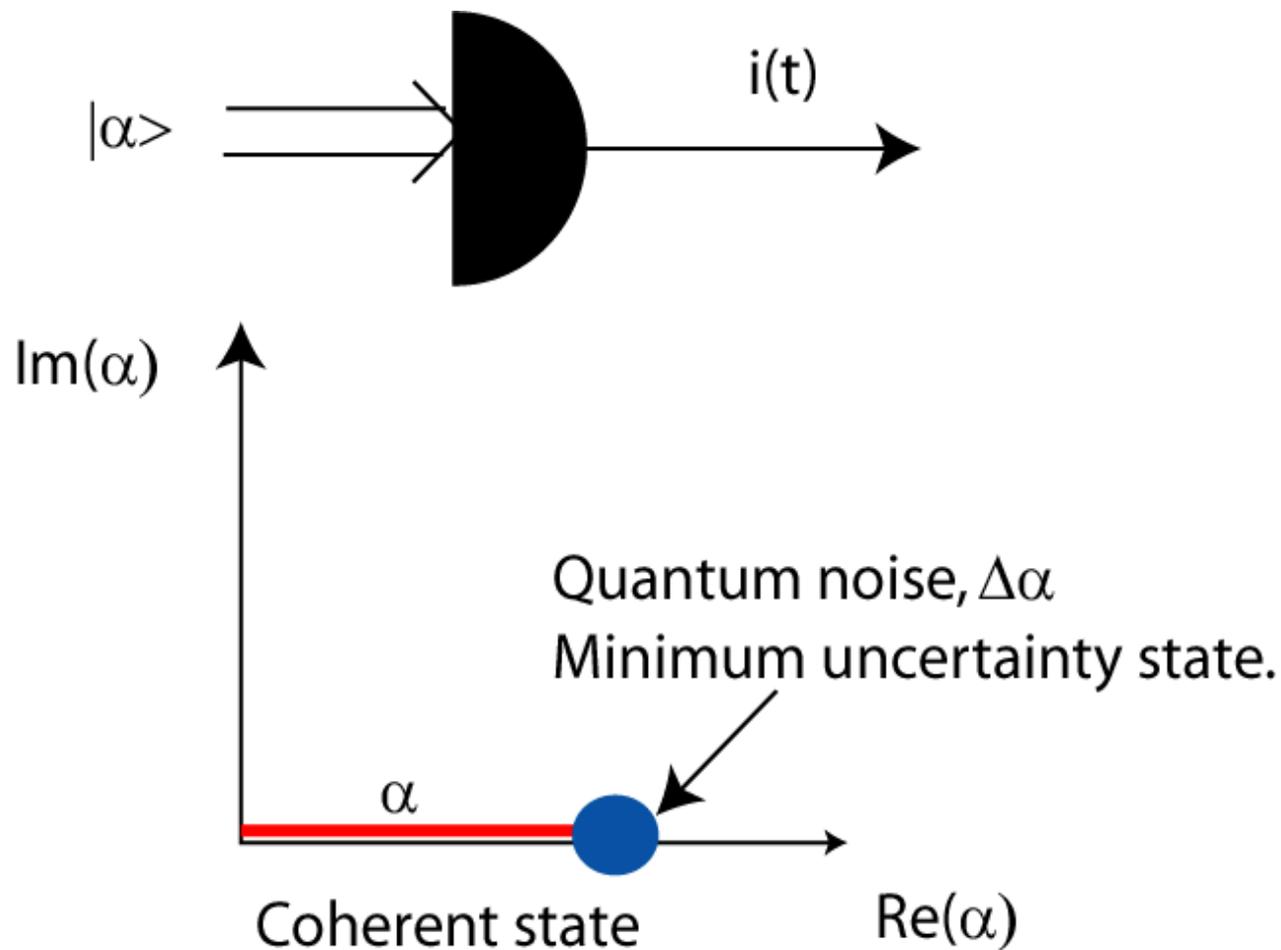
$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$$

Its amplitude (complex):  $\alpha$

Its mean squared:  $\alpha^* \alpha = |\alpha|^2$

Its uncertainty:  $1/2$

They are states with the minimum uncertainty allowed by quantum mechanics. Equal on both quadratures



Perfect detector  $i(t) = |\alpha + \Delta\alpha|^2$

$$i(t) = |\alpha|^2 + 2 \alpha \Delta\alpha + |\Delta\alpha|^2 ; \quad \langle \alpha^* \alpha \rangle = n$$

DC  $\sim n$     Shot noise  $\sim n^{1/2}$     neglect.

Correlation functions tell us something about the fluctuations.

Correlations have classical bounds.

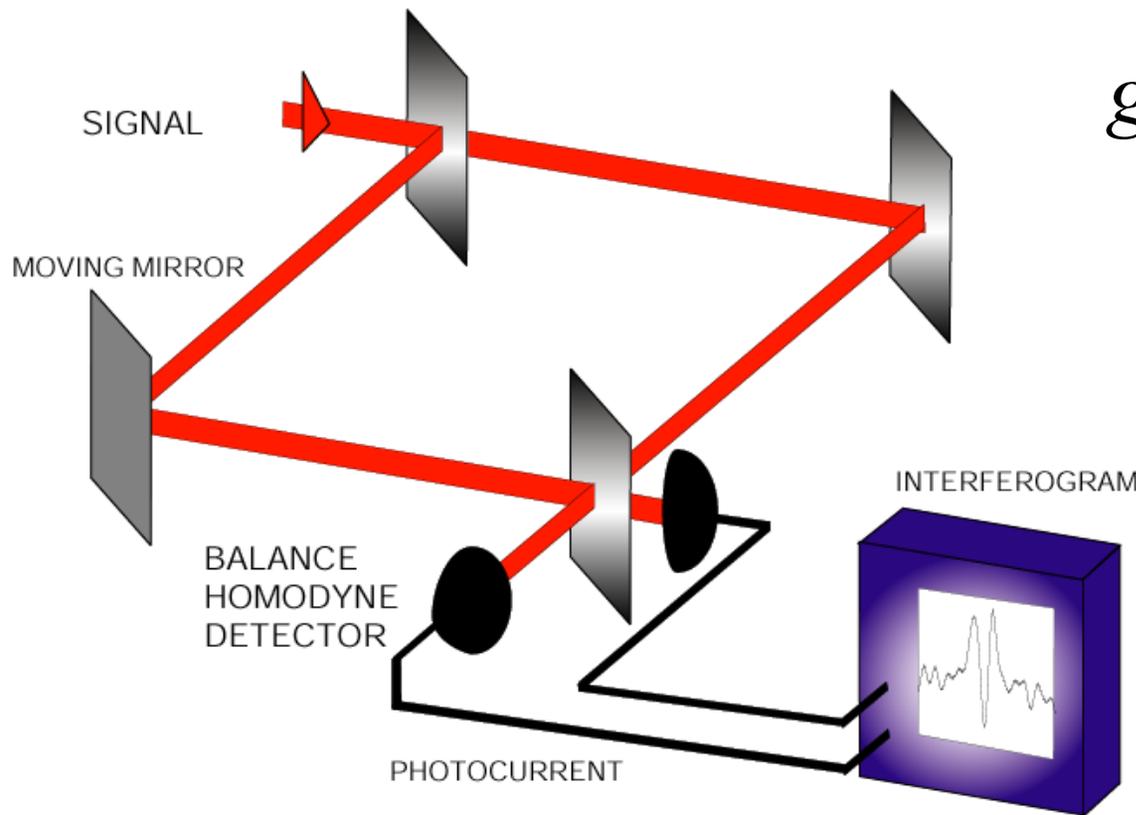
They are conditional measurements.

# Mach Zehnder Interferometer Wave-Wave Correlation

$$g^{(1)}(\tau) = \frac{\langle E^*(t)E(t+\tau) \rangle}{\langle I(t) \rangle}$$

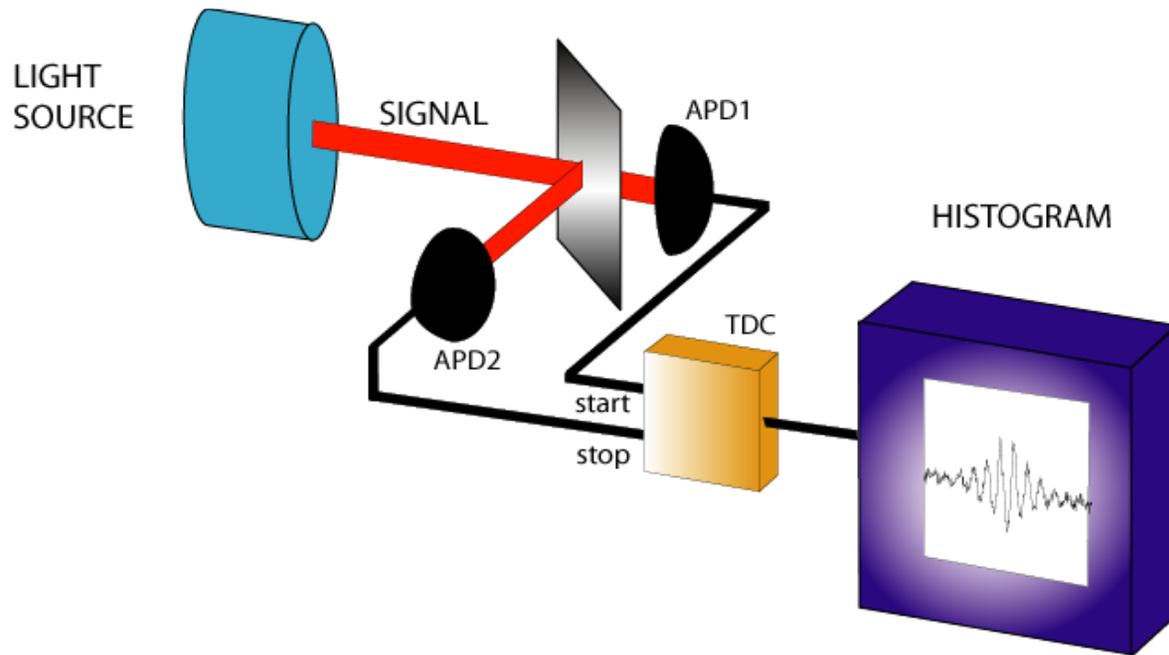
Spectrum of the signal:

$$F(\omega) = \frac{1}{2\pi} \int \exp(i\omega\tau) g^{(1)}(\tau) d\tau$$



Basis of Fourier Transform Spectroscopy.  
The correlation gives the linewidth of an atomic transition.

# Hanbury Brown and Twiss Intensity-Intensity Correlations



$$g^{(2)}(\tau) = \frac{\langle I(t)I(t+\tau) \rangle}{\langle I(t) \rangle^2}$$

Cauchy-Schwarz

$$2I(t)I(t+\tau) \leq I^2(t) + I^2(t+\tau)$$

The correlation is largest at equal time

$$g^{(2)}(0) \geq 1$$

$$g^{(2)}(0) \geq g^{(2)}(\tau)$$

# Intensity correlation function measurements:

$$g^{(2)}(\tau) = \frac{\langle : \hat{I}(t) \hat{I}(t + \tau) : \rangle}{\langle \hat{I}(t) \rangle^2}$$

Gives the probability of detecting a photon at time  $t + \tau$  given that one was detected at time  $t$ . This is a conditional measurement:

$$g^{(2)}(\tau) = \frac{\langle \hat{I}(\tau) \rangle_c}{\langle \hat{I} \rangle}$$

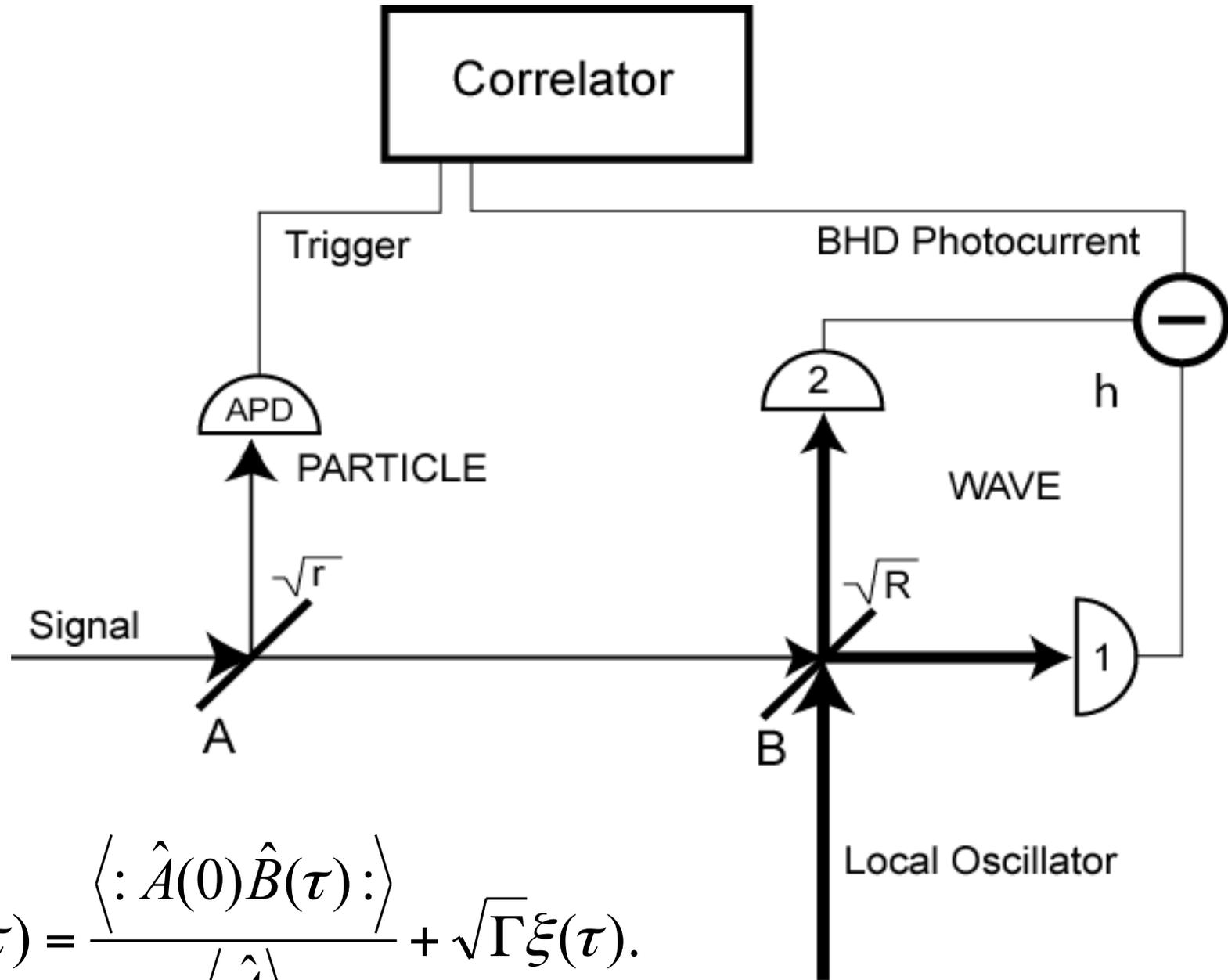
How to correlate fields  
and intensities?

Detection of the field: Homodyne detection

Source, has to have at least two photons

Conditional Measurement: Only measure when we know there is a photon.

# The Intensity-Field correlator.



$$H(\tau) = \frac{\langle : \hat{A}(0) \hat{B}(\tau) : \rangle}{\langle \hat{A} \rangle} + \sqrt{\Gamma} \xi(\tau).$$

## Condition on a Click

Measure the correlation function of the Intensity and the Field:

$$\langle I(t) E(t+\tau) \rangle$$

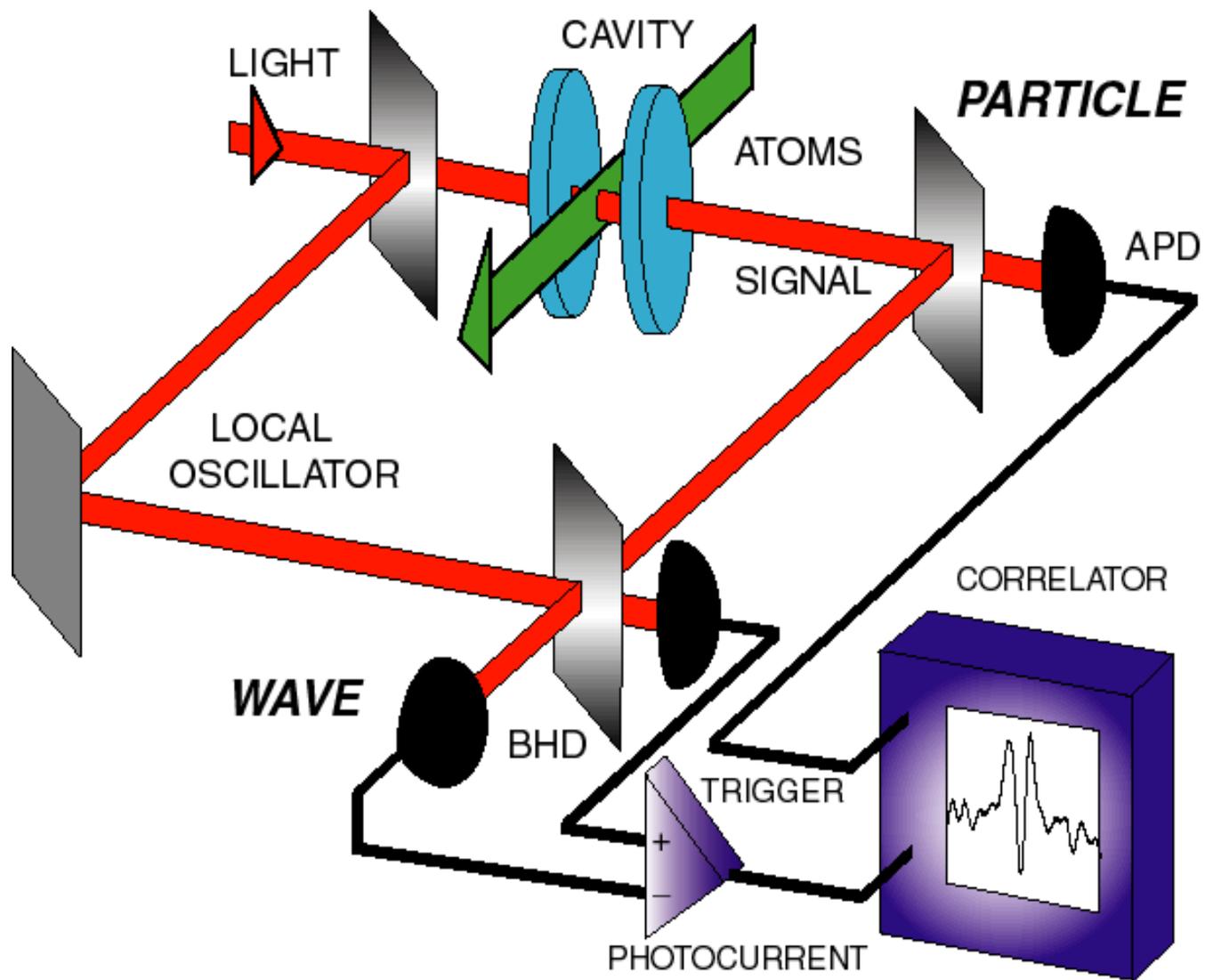
Normalized form:

$$h_{\theta}(\tau) = \langle E(\tau) \rangle_c / \langle E \rangle$$

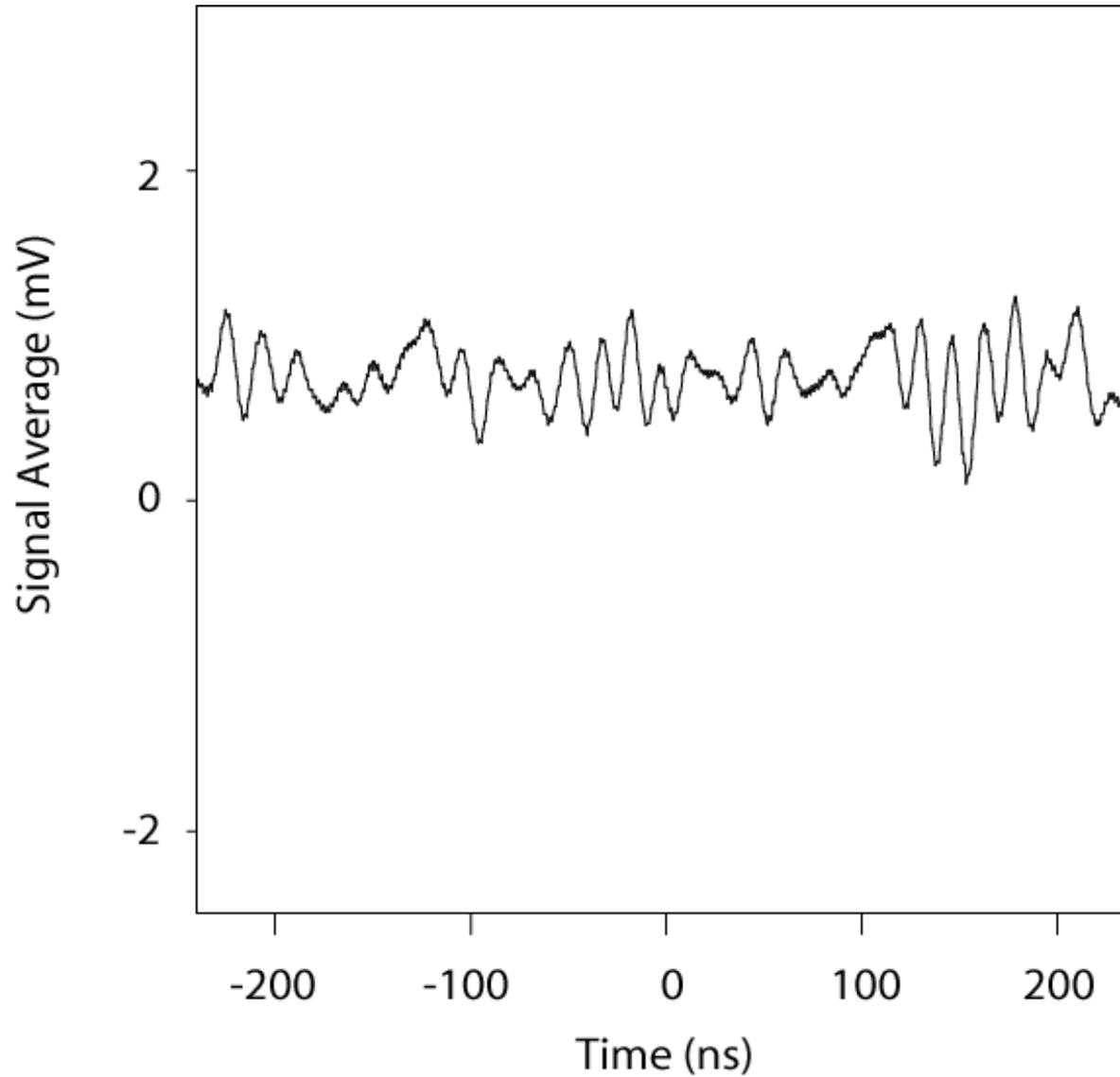
From Cauchy Schwartz inequalities:

$$0 \leq \bar{h}_0(0) - 1 \leq 2$$

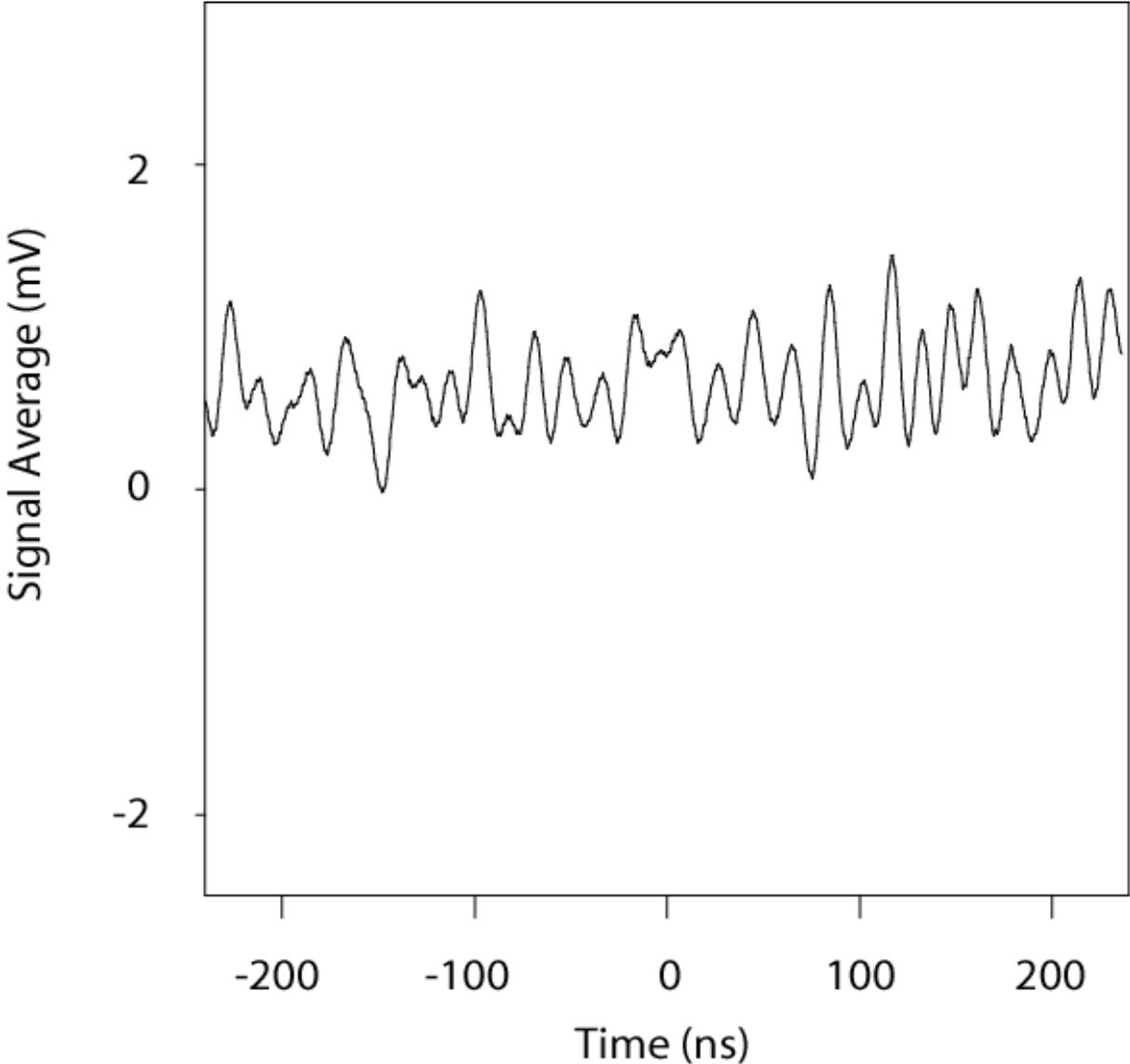
$$|\bar{h}_0(\tau) - 1| \leq |\bar{h}_0(0) - 1|$$

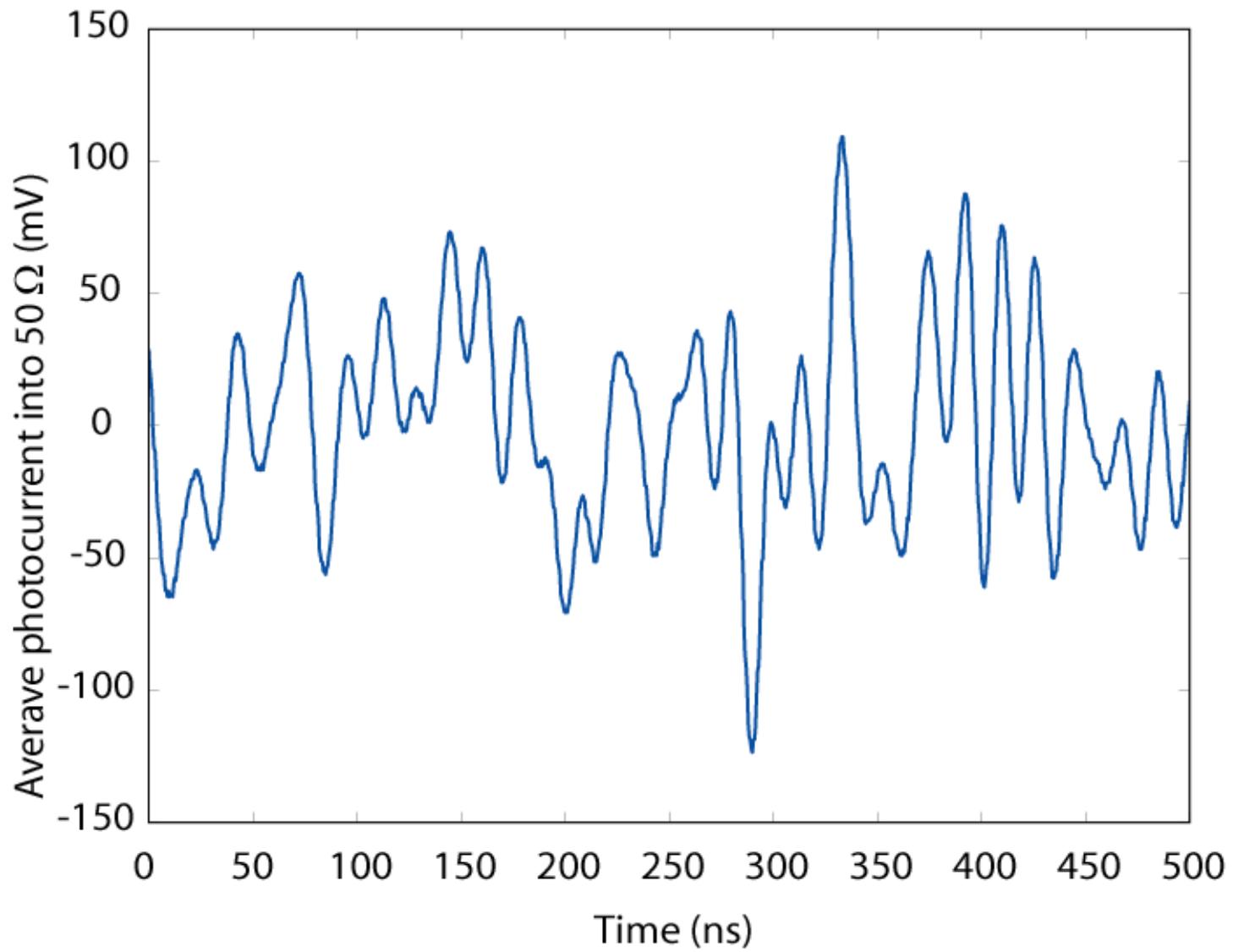


# Photocurrent average with random conditioning

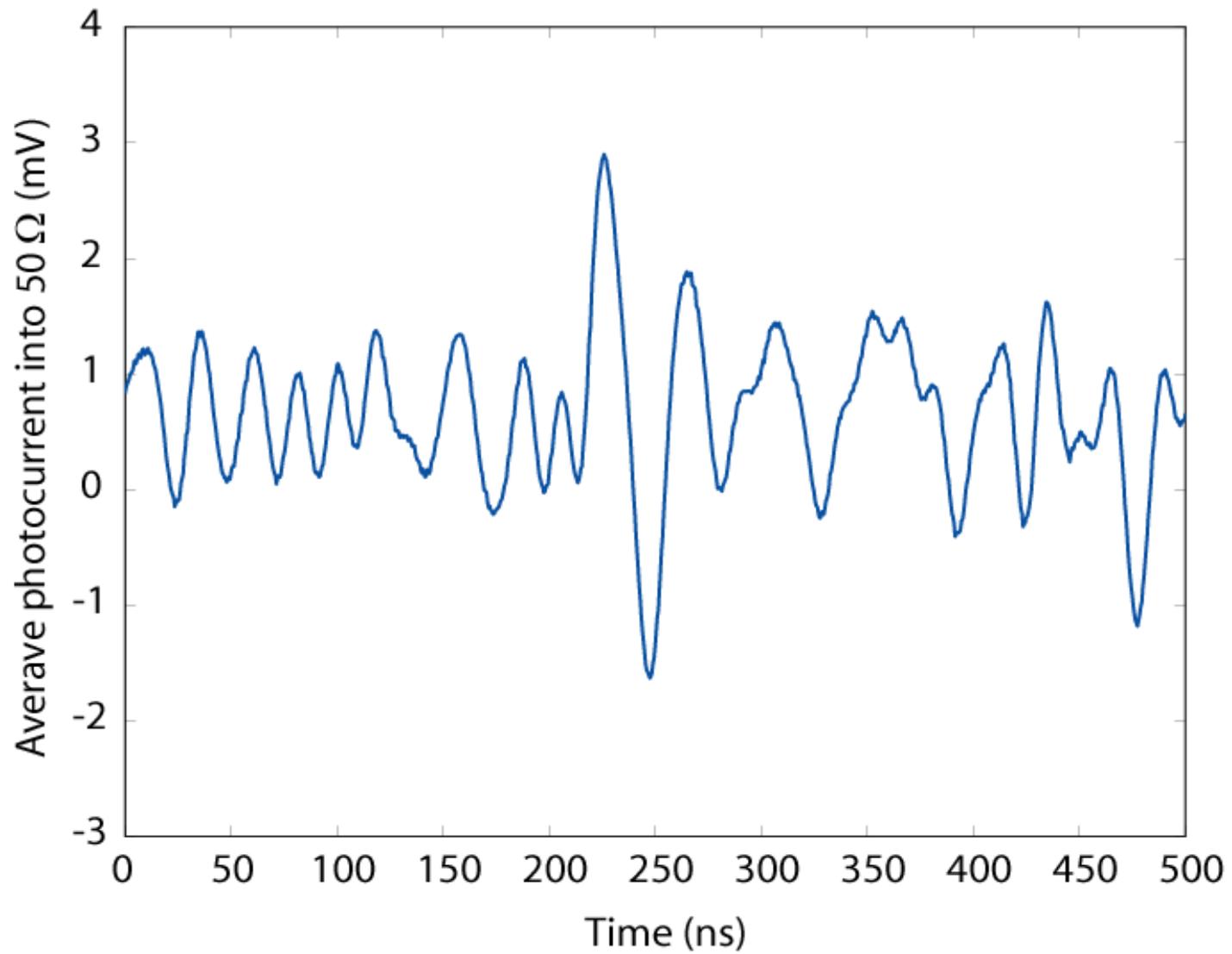


# Conditional photocurrent with no atoms in the cavity.

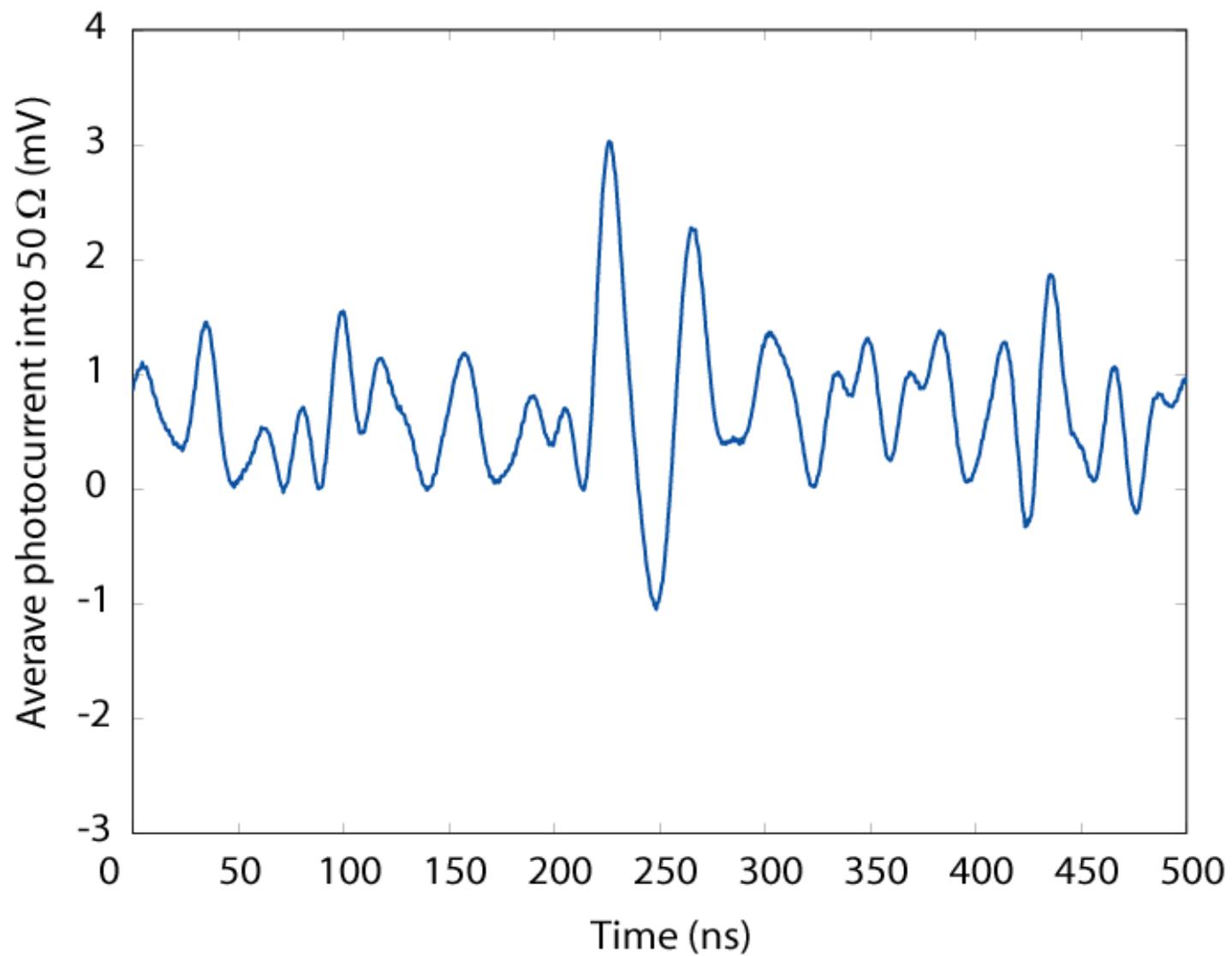




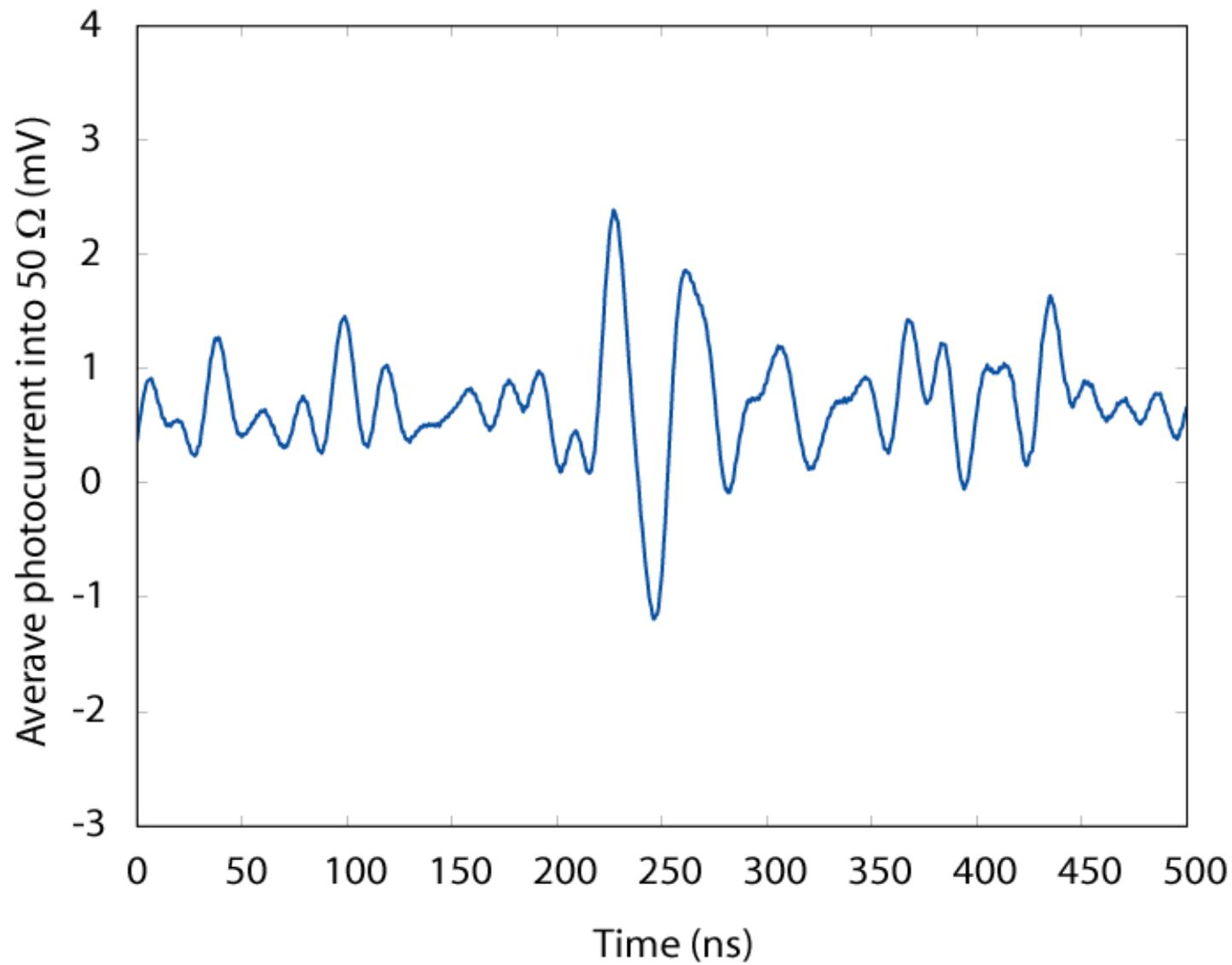
**After 1 average**



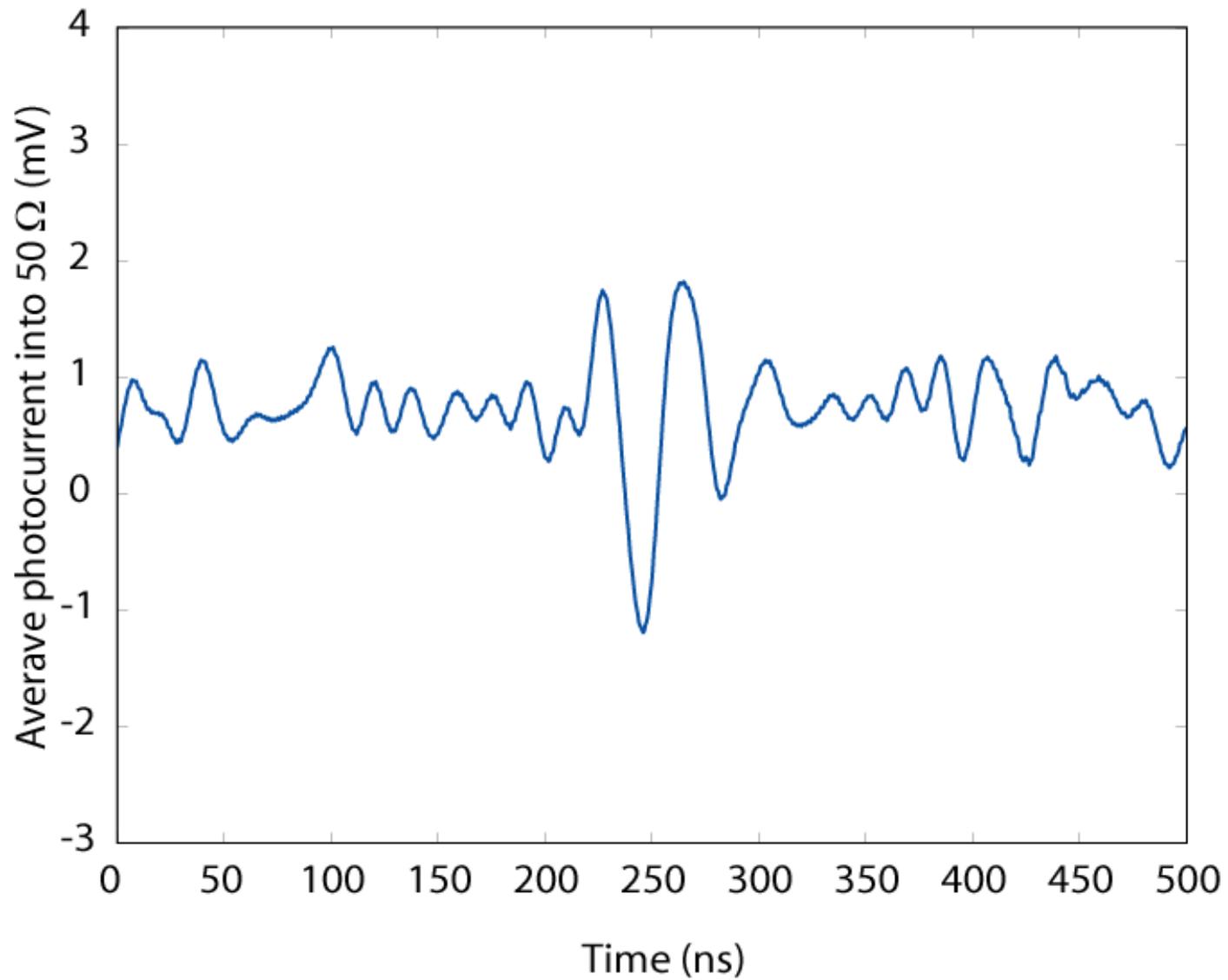
**After 6,000 averages**



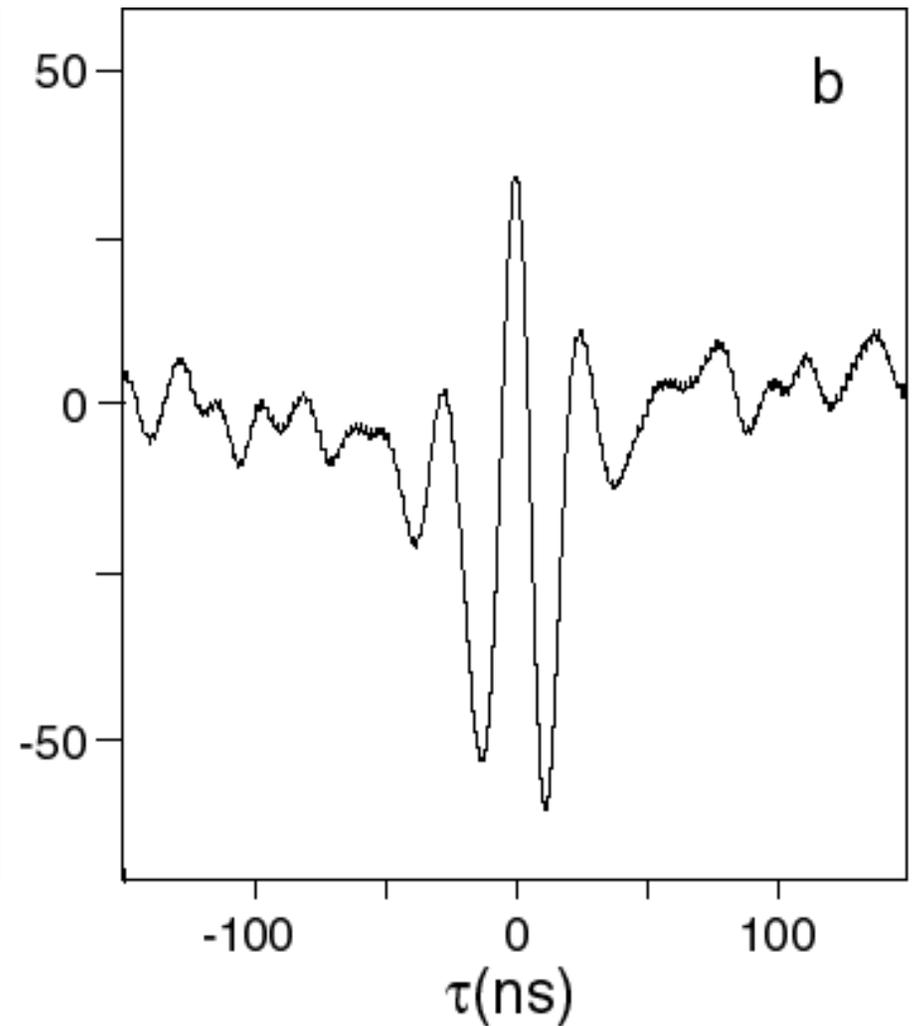
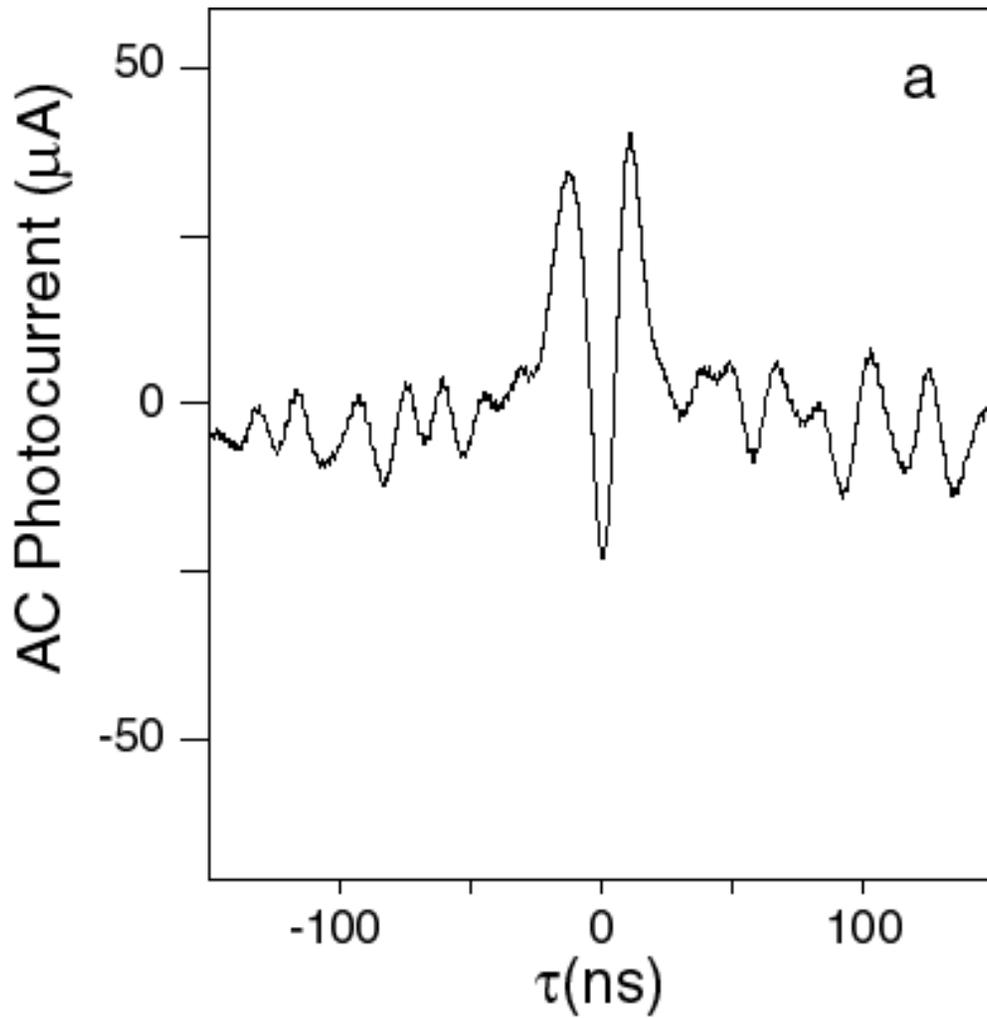
**After 10,000 averages**



**After 30,000 averages**

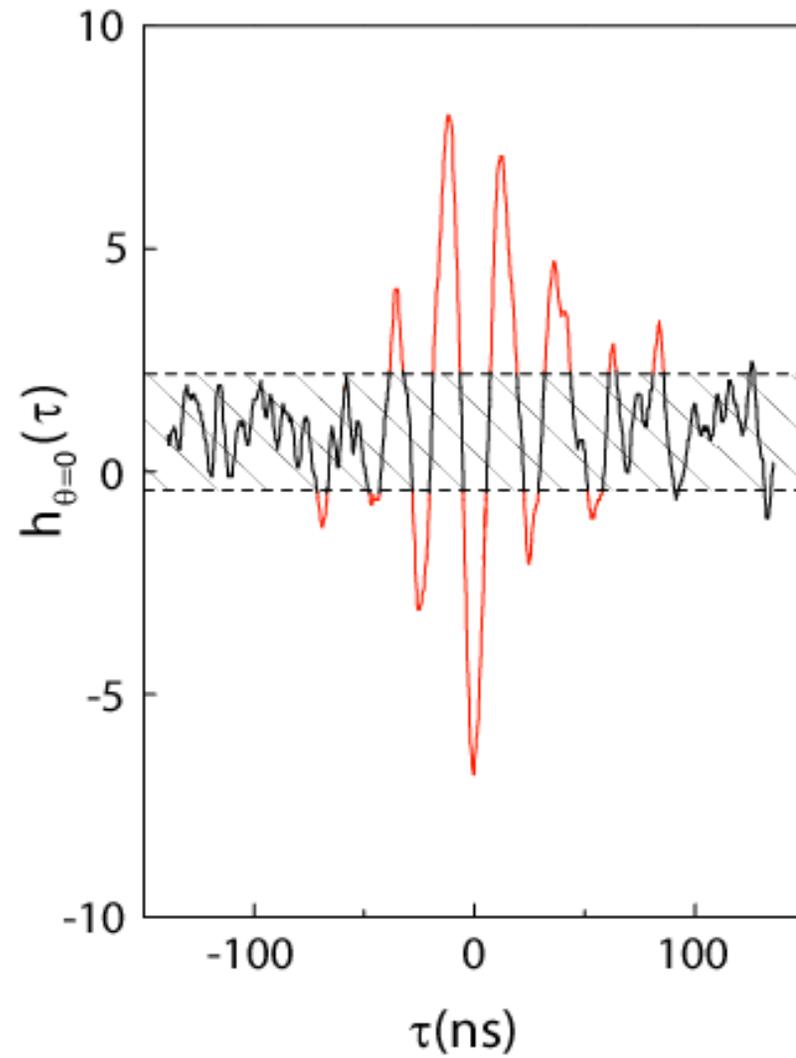


**After 65,000 averages**



Flip the phase of the Mach-Zehnder by  $146^\circ$

# Monte Carlo simulations for weak excitation:



Atomic beam  $N=11$

This is the conditional evolution of the field of a fraction of a photon  $[B(t)]$  from the correlation function.

$$h_{\theta}(\tau) = \langle E(\tau) \rangle_c / \langle E \rangle$$

The conditional field prepared by the click is:

$$A(t)|0\rangle + B(t)|1\rangle \text{ with } A(t) \approx 1 \text{ and } B(t) \ll 1$$

We measure the field of a fraction of a photon!

Fluctuations are very important.

# Conditional dynamics in cavity QED at low intensity:

$$|\Psi_{ss}\rangle = |0, g\rangle + \lambda|1, g\rangle - \frac{2g}{\gamma}\lambda|0, e\rangle + \frac{\lambda^2 pq}{\sqrt{2}}|2, g\rangle - \frac{2g\lambda^2 q}{\gamma}|1, e\rangle$$

$$\lambda = \langle \hat{a} \rangle, \quad p = p(g, \kappa, \gamma) \text{ and } q = q(g, \kappa, \gamma)$$

A photodetection conditions the state into the following non-steady state from which the system evolves.

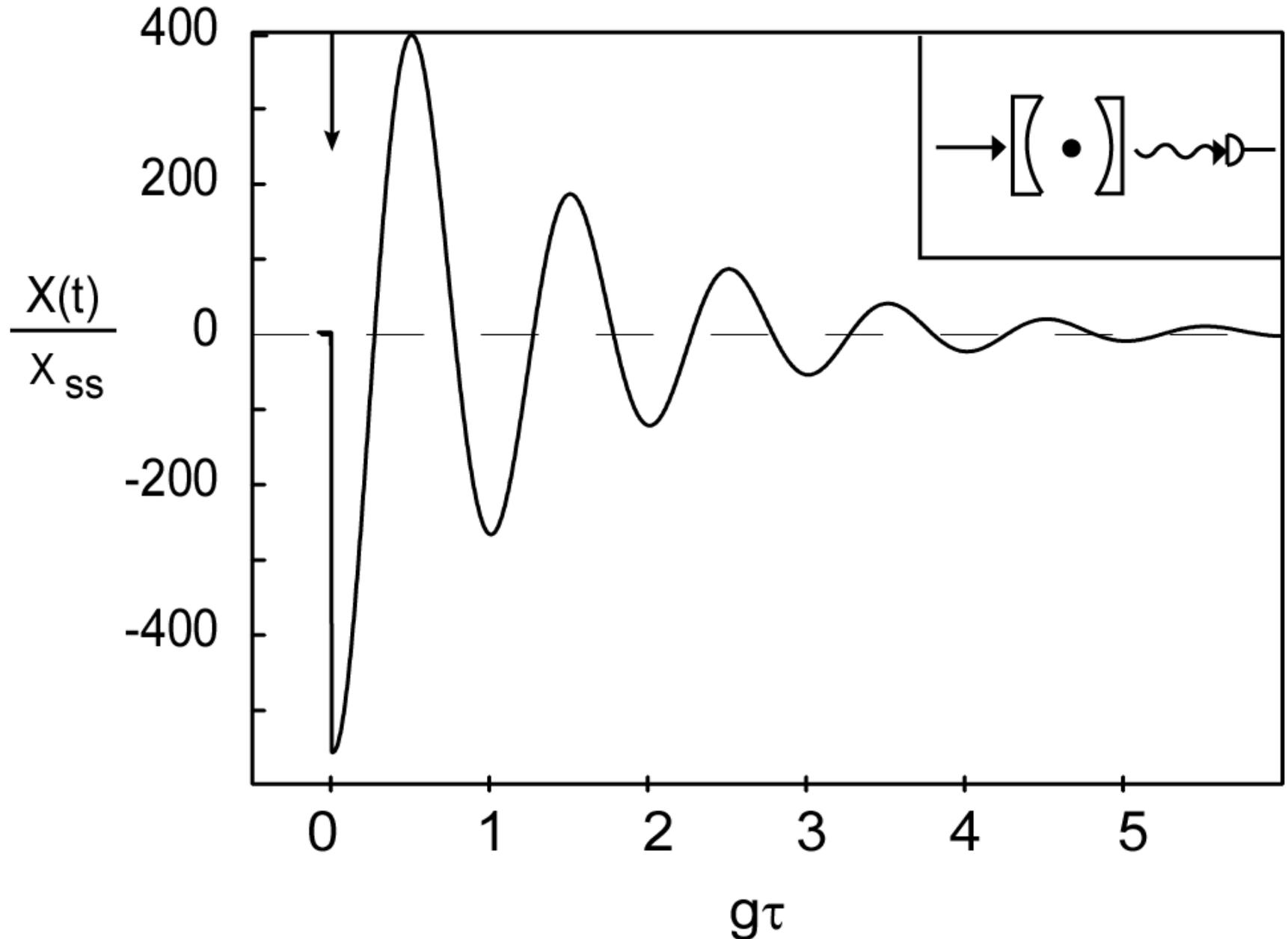
$$\hat{a}|\Psi_{ss}\rangle \Rightarrow |\Psi_c(\tau)\rangle = |0, g\rangle + \lambda pq|1, g\rangle - \frac{2g\lambda q}{\gamma}|0, e\rangle$$

$$|\Psi_c(\tau)\rangle = |0, g\rangle + \lambda [f_1(\tau)|1, g\rangle + f_2(\tau)|0, e\rangle] + O(\lambda^2)$$

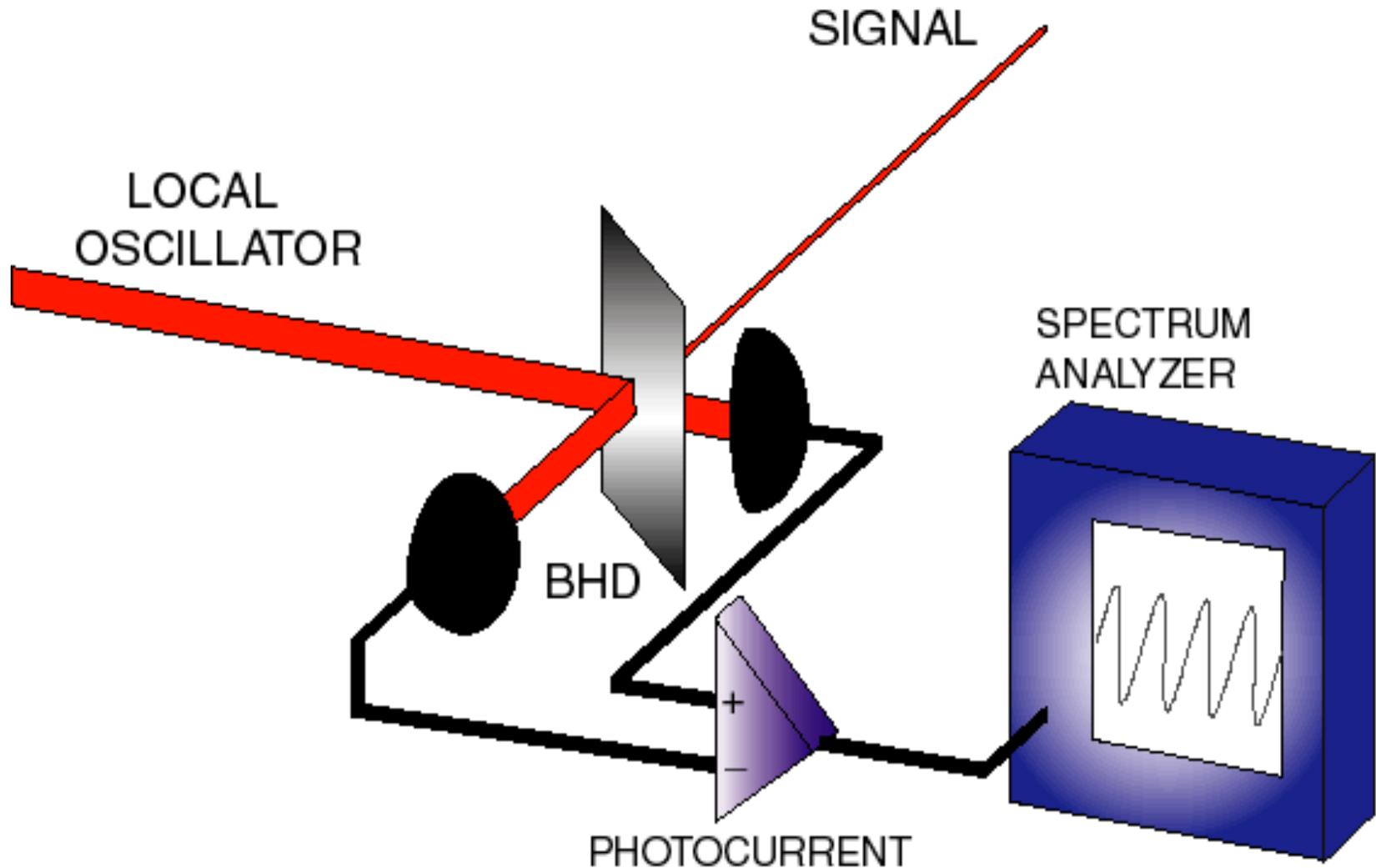
Field

Atomic Polarization

# Regression of the field to steady state after the detection of a photon.



# Detection of the Squeezing spectrum with a balanced homodyne detector (BHD).

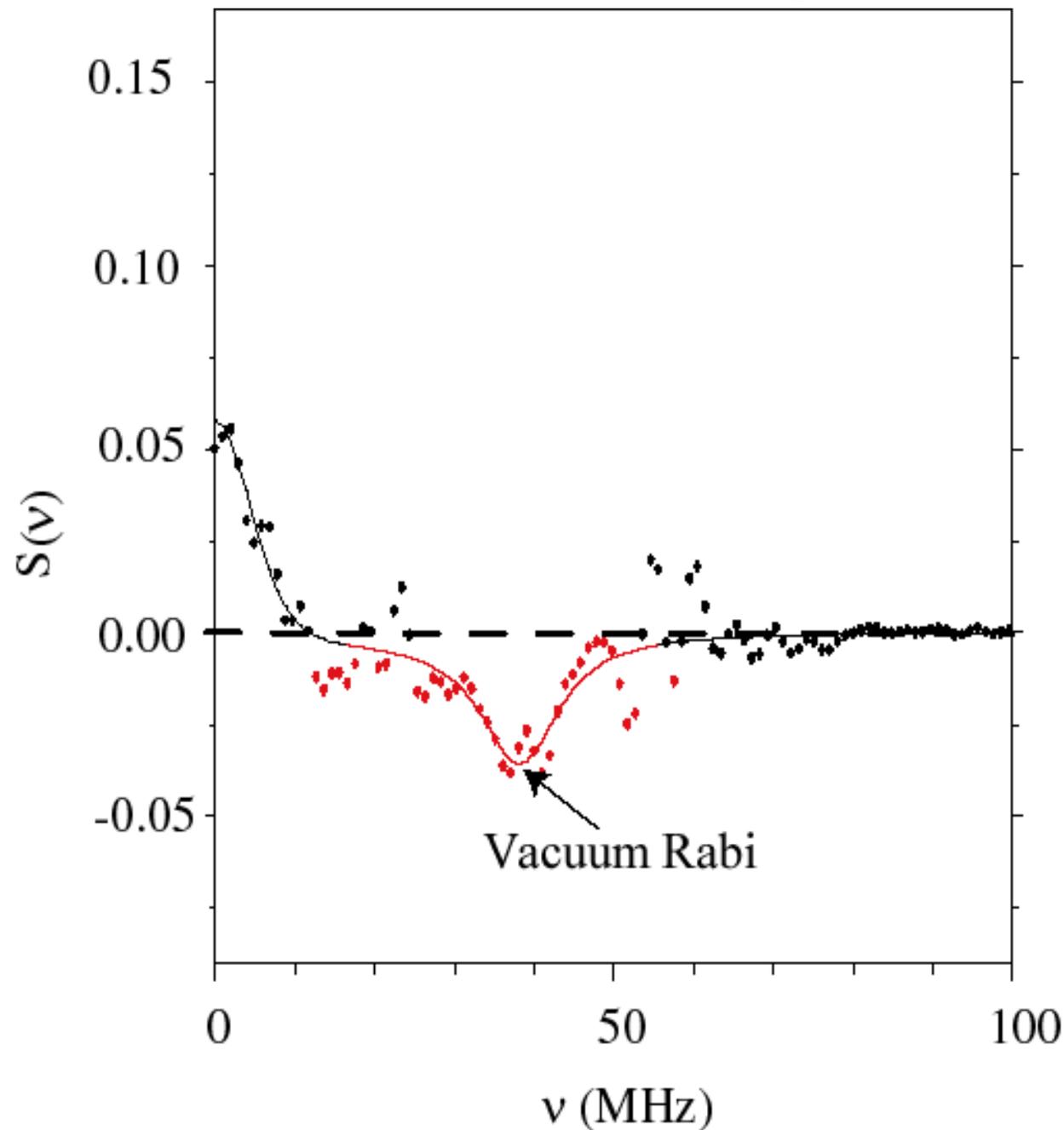


The fluctuations of the electromagnetic field are measured by the spectrum of squeezing. Look at the noise spectrum of the photocurrent.

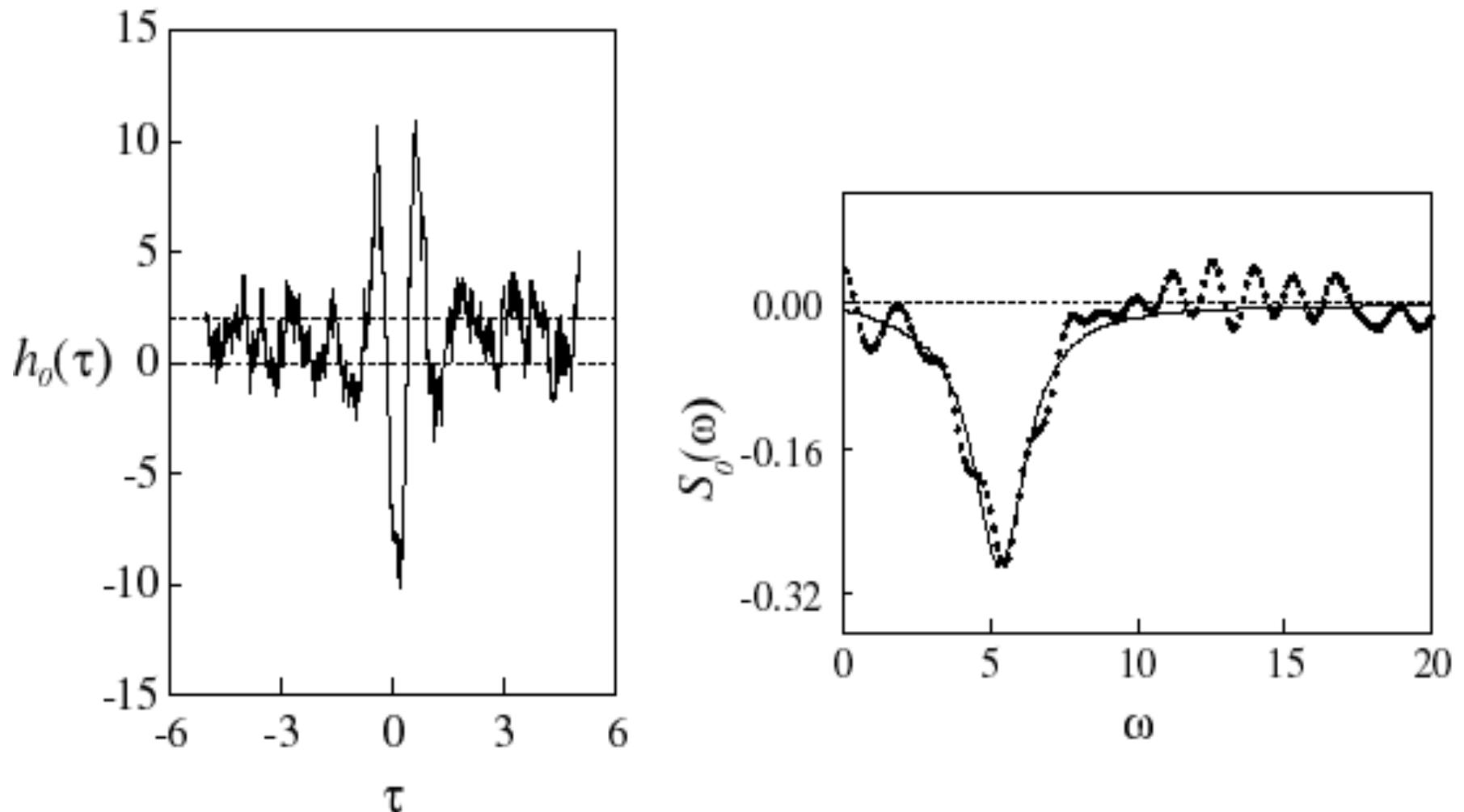
$$S(\nu, 0^\circ) = 4F \int_0^\infty \cos(2\pi\nu\tau) [\bar{h}_0(\tau) - 1] d\tau,$$

F is the photon flux into the correlator.

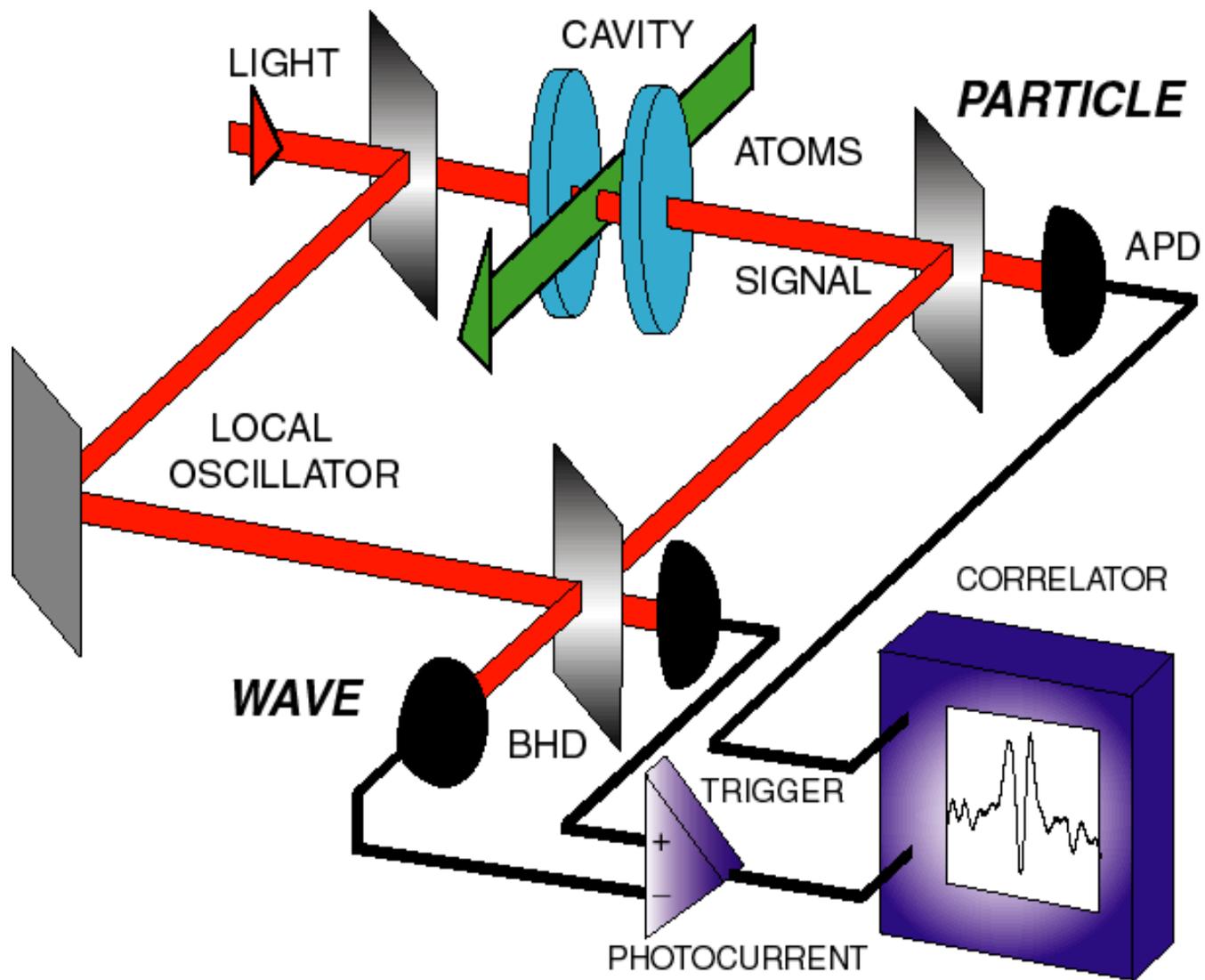
# Spectrum of Squeezing from the Fourier Transform of $h_0(t)$

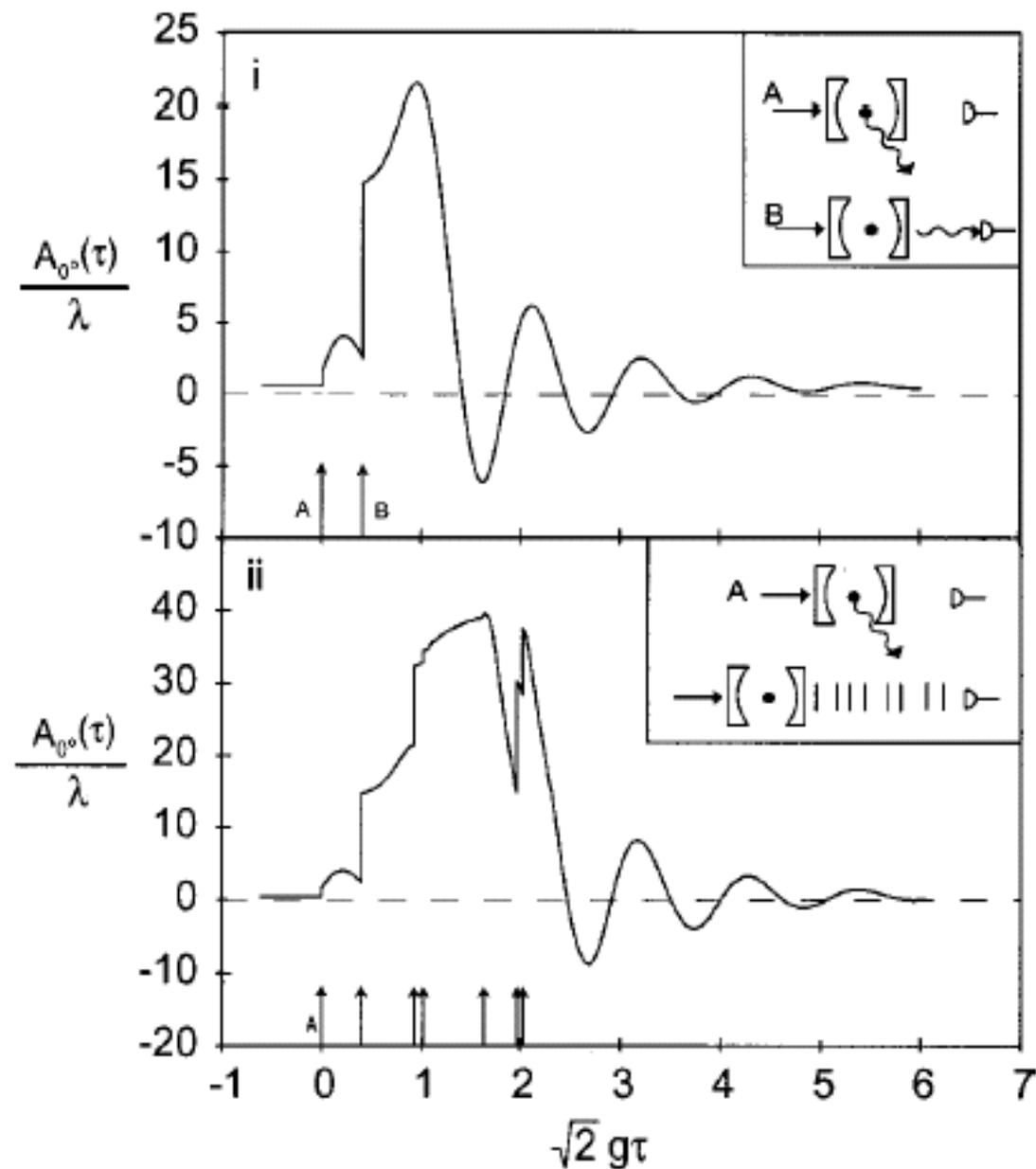


Monte Carlo simulations of the wave-particle correlation and the spectrum of squeezing in the low intensity limit for an atomic beam.

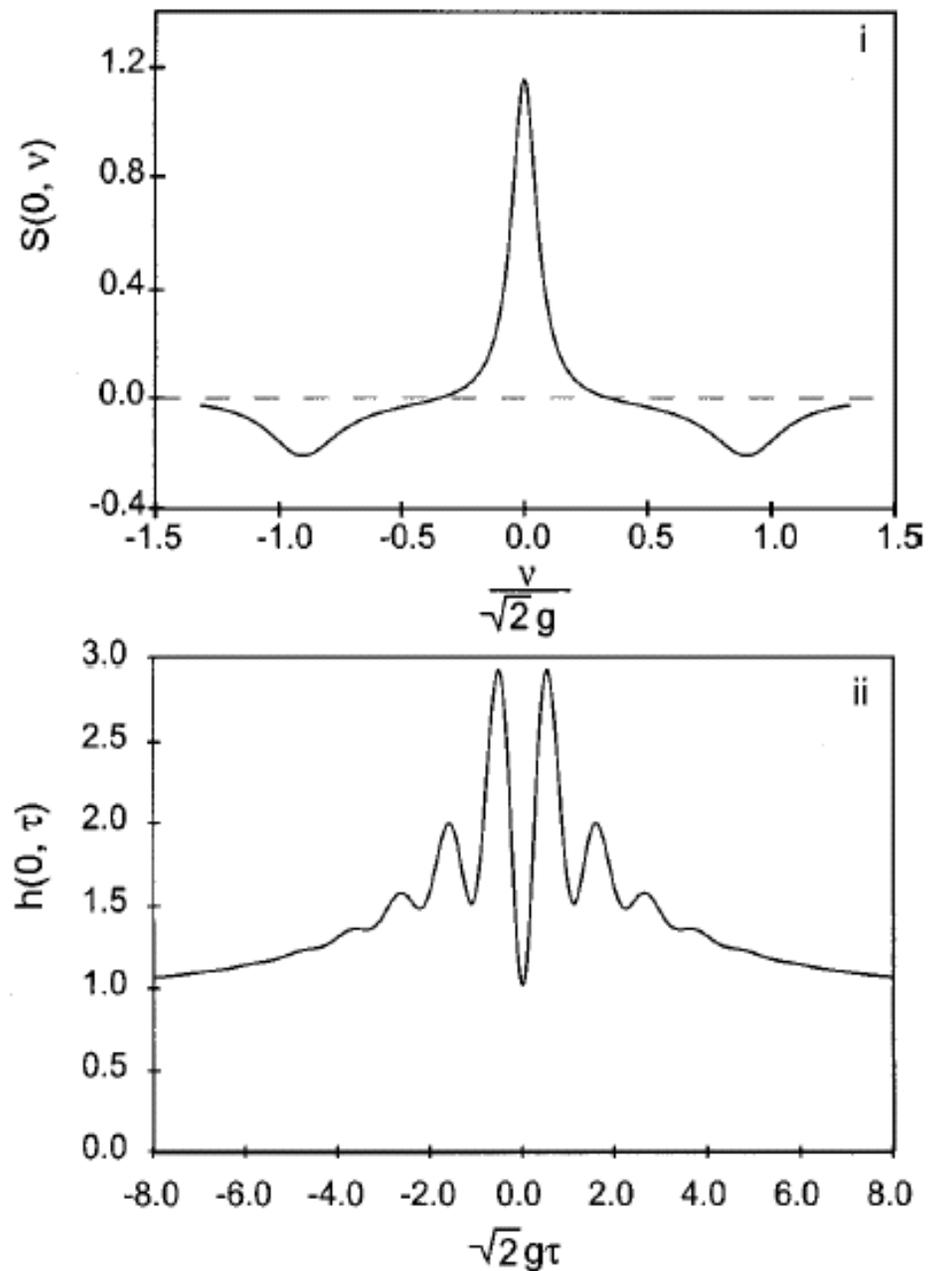


It has upper and a lower classical bounds



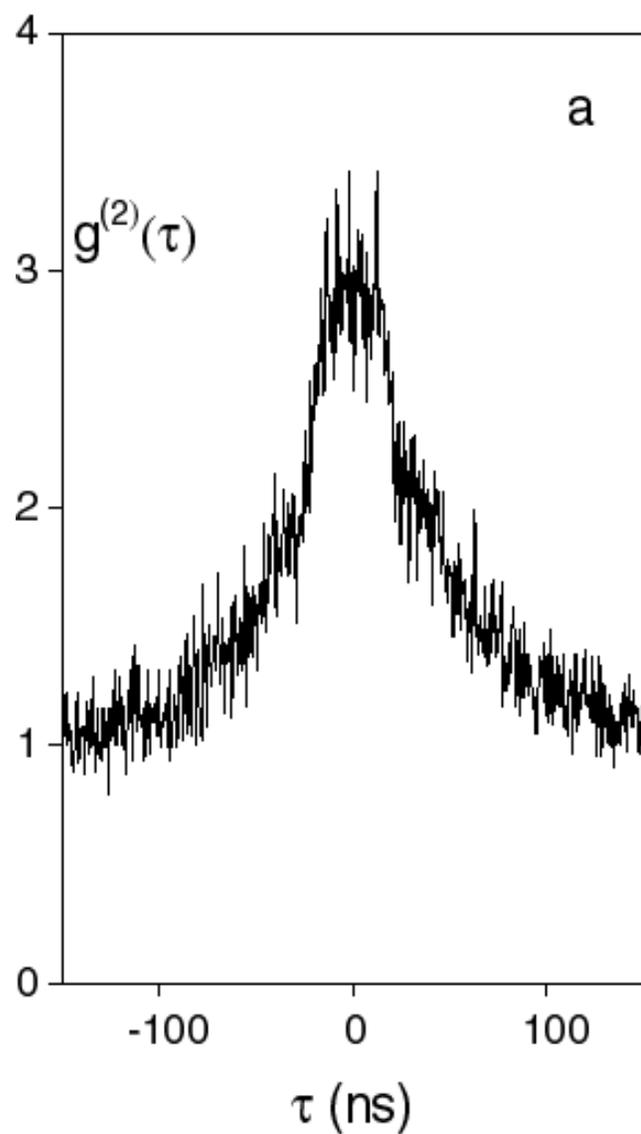


Single quantum trajectories simulation of cavity QED system with spontaneous emission.

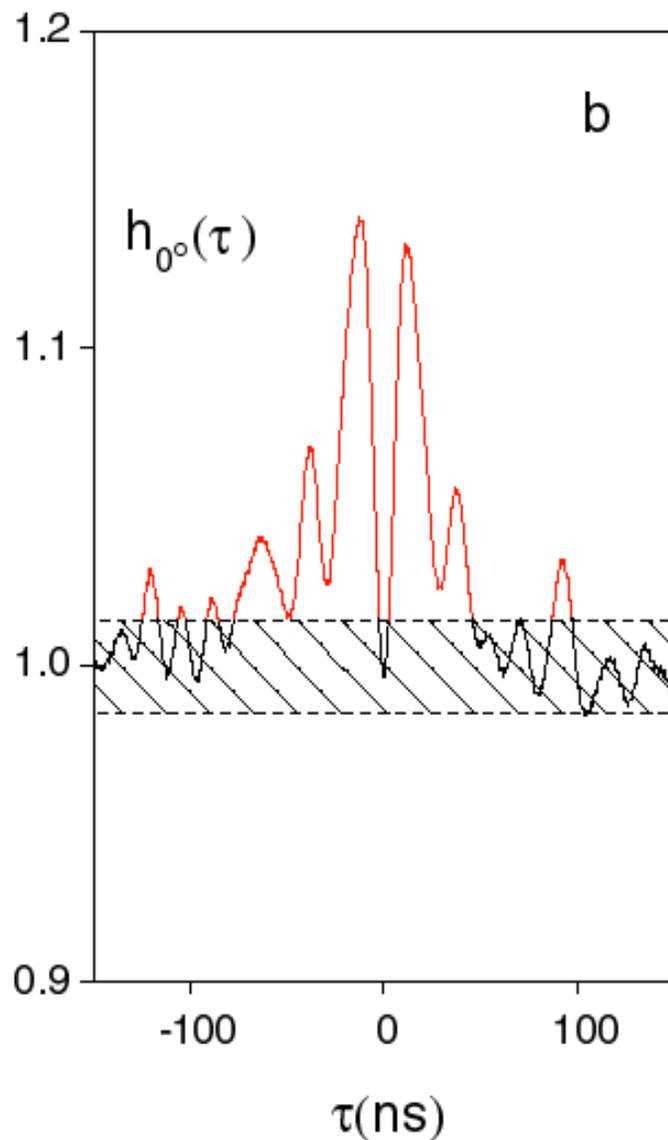


(i) Spectrum of squeezing obtained from the averaged (ii)  $h(0,t)$  correlation function that shows the effects of spontaneous emission.

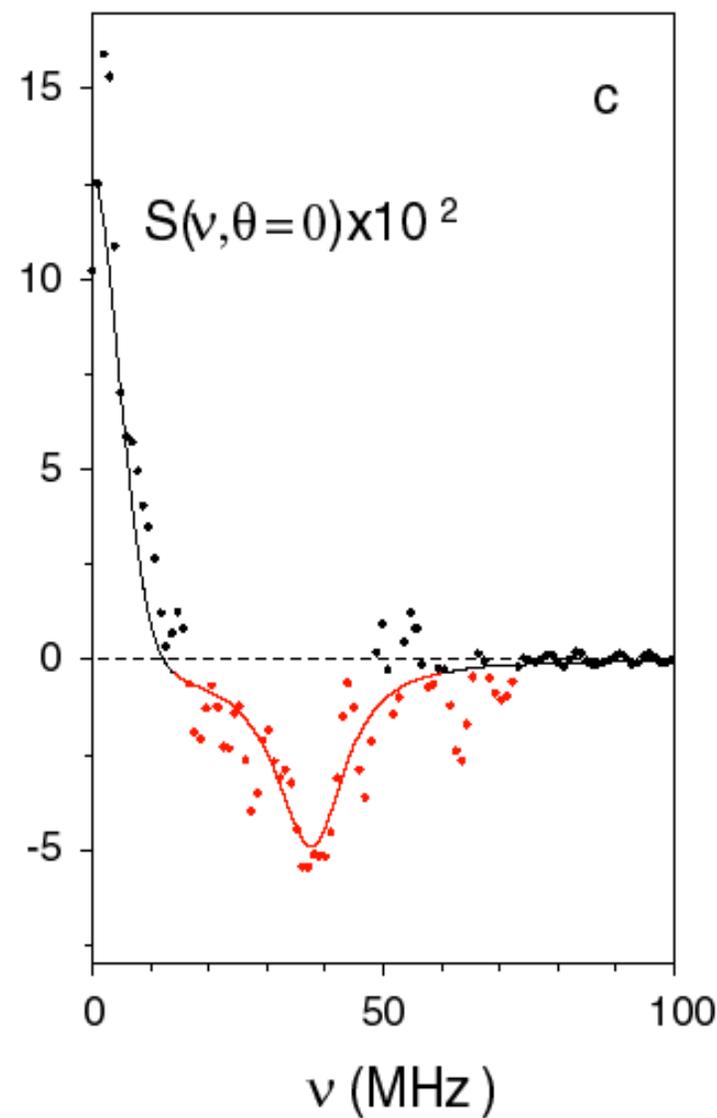
Classical  $g^{(2)}$



Non-classical  $h$

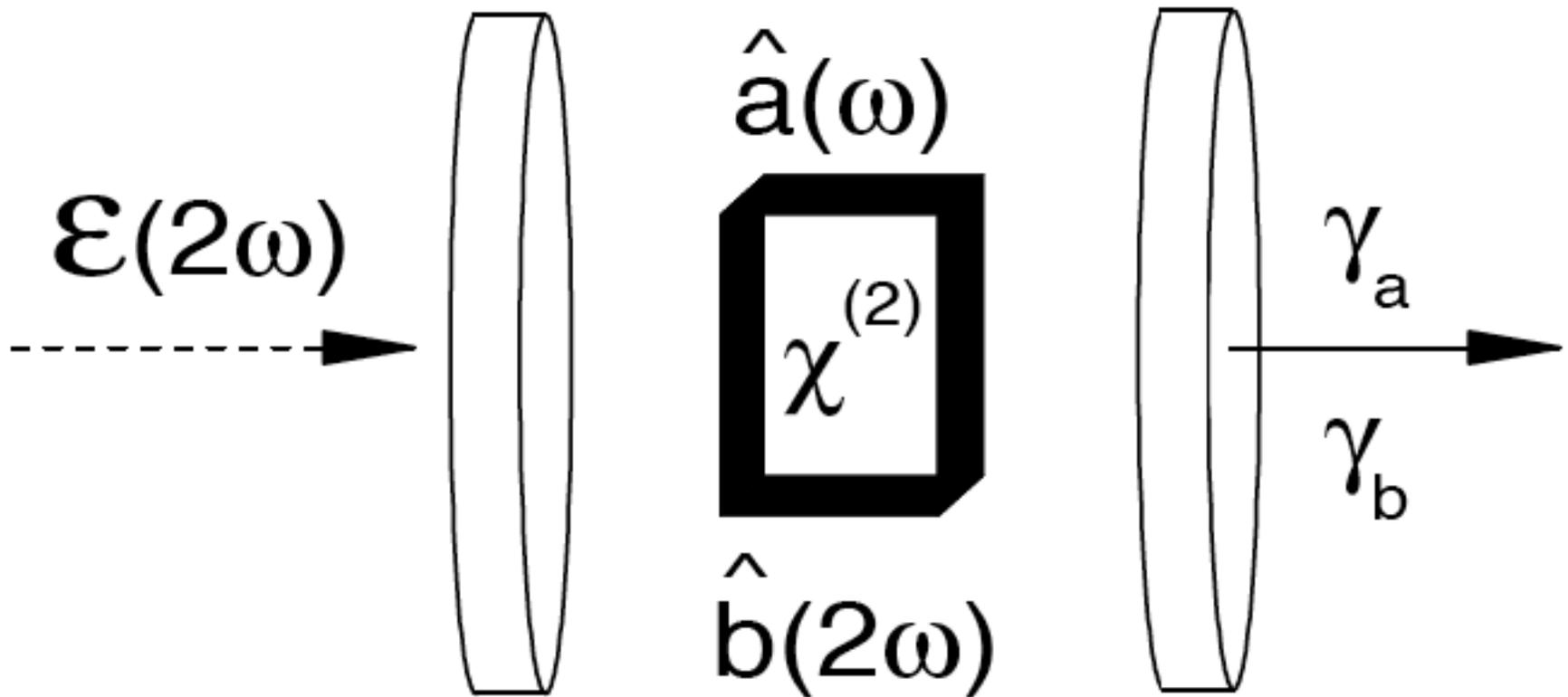


Squeezing



$N=13; 1.2n_0$

# Optical Parametric Oscillator



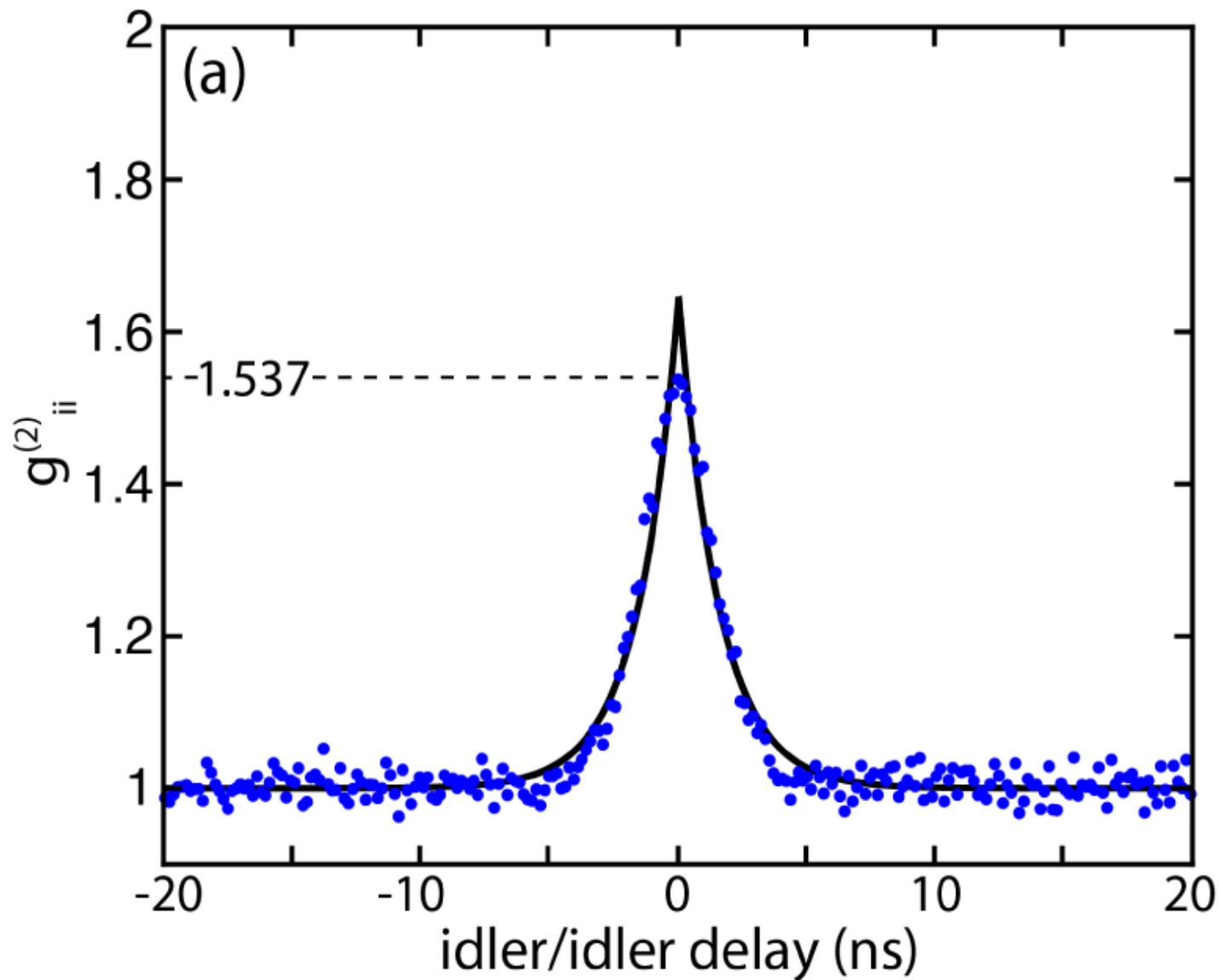


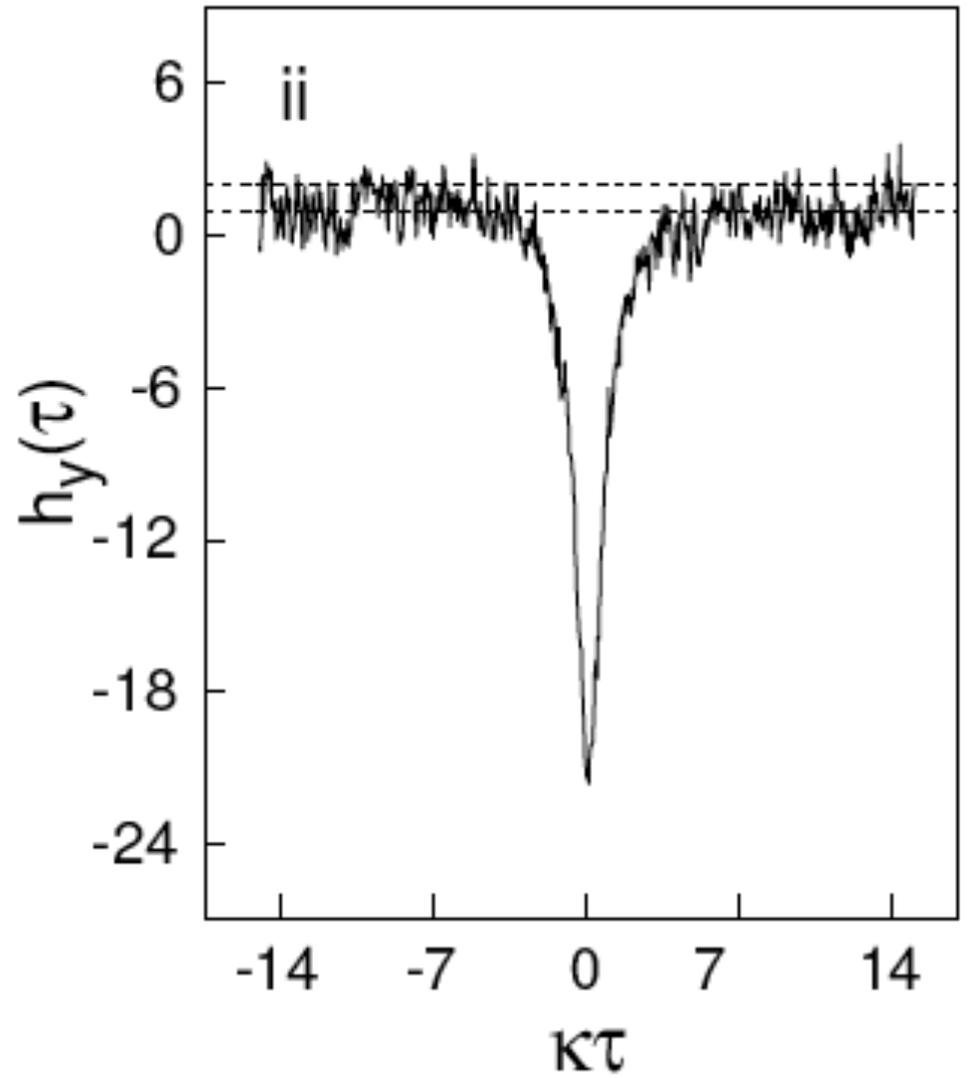
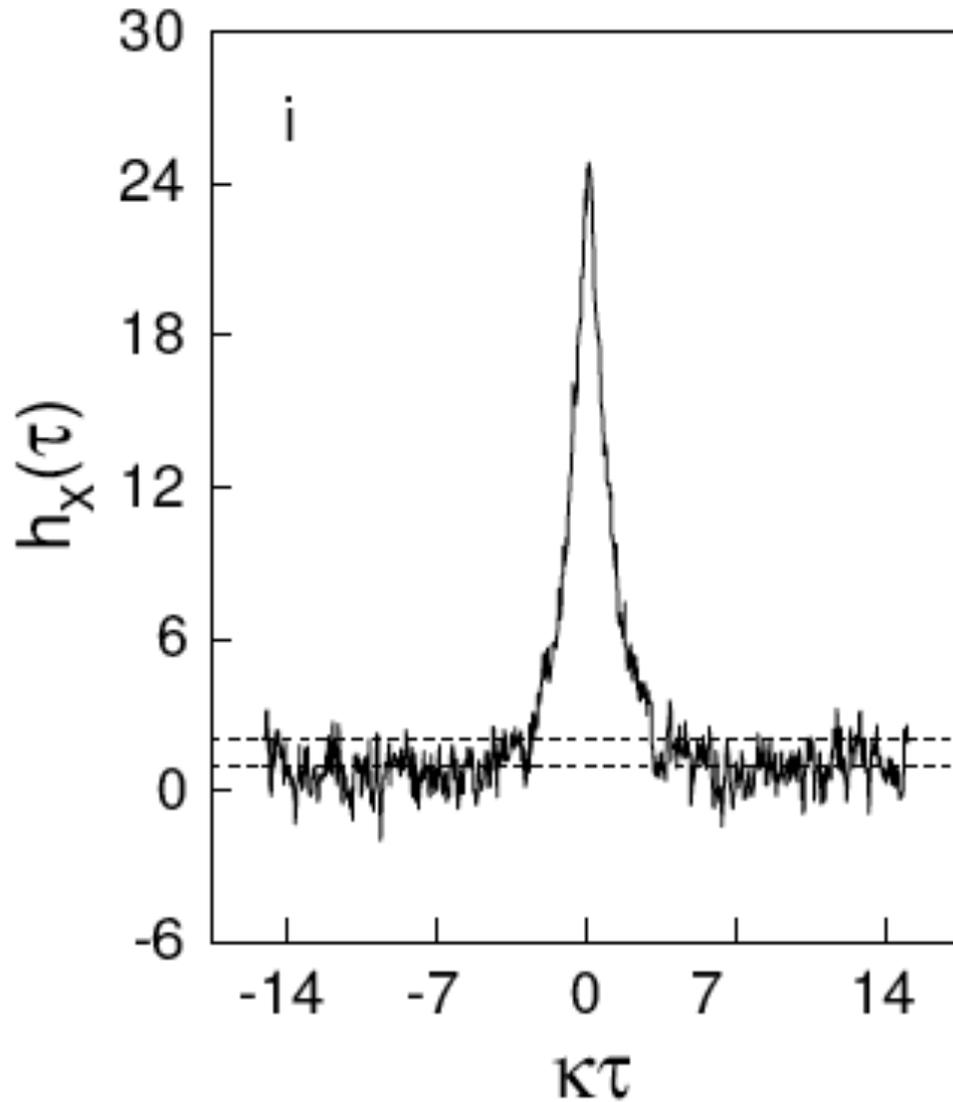
Fig. 4

**Citation**

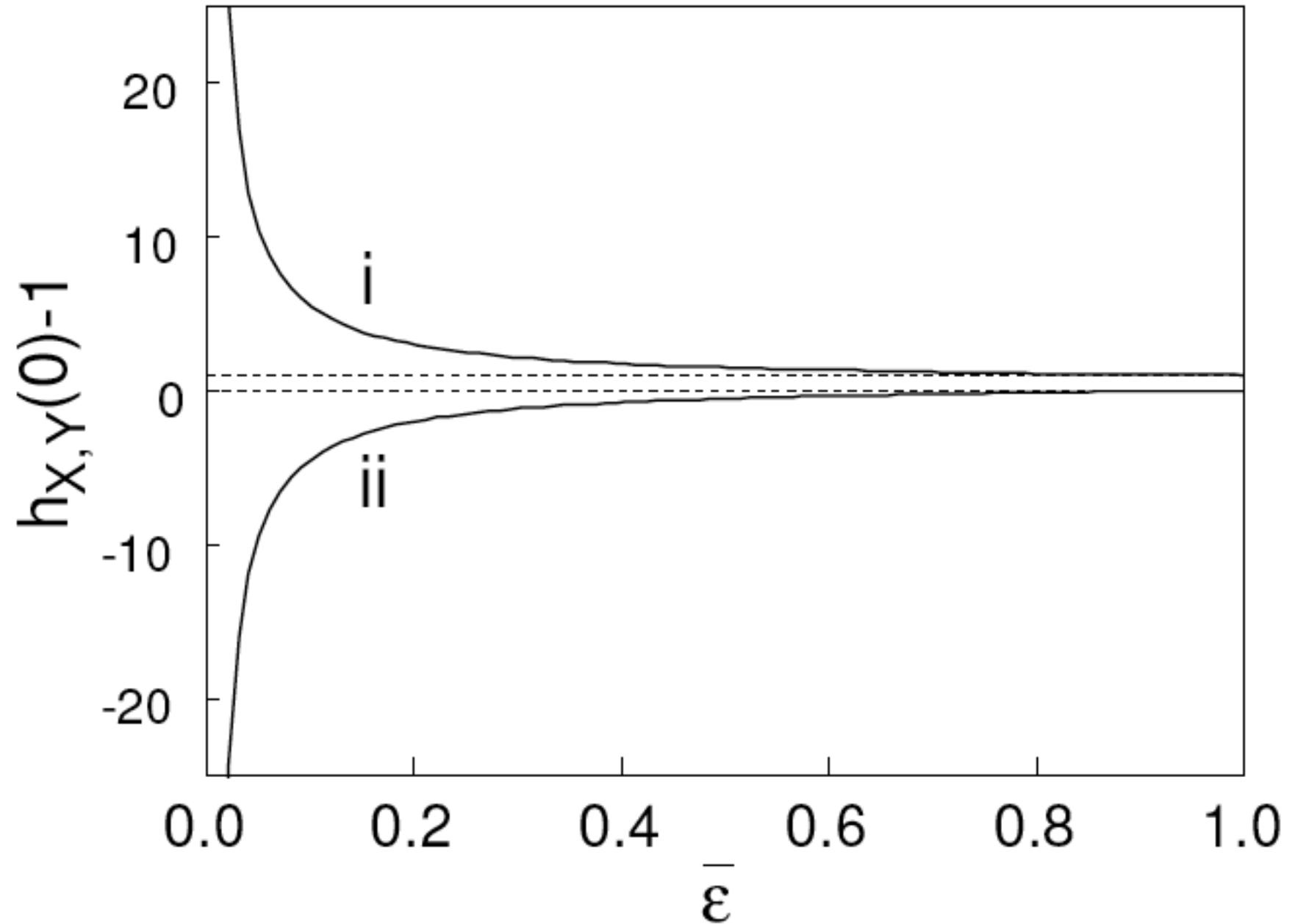
Christian Reimer, Lucia Caspani, Matteo Clerici, Marcello Ferrera, Michael Kues, Marco Peccianti, Alessia Pasquazi, Luca Razzari, Brent E. Little, Sai T. Chu, David J. Moss, Roberto Morandotti, "Integrated frequency comb source of heralded single photons," *Opt. Express* **22**, 6535-6546 (2014);

<https://www.osapublishing.org/oe/abstract.cfm?uri=oe-22-6-6535>

# Calculation of $h_{\theta}(\tau)$ in an OPO with the classical bounds



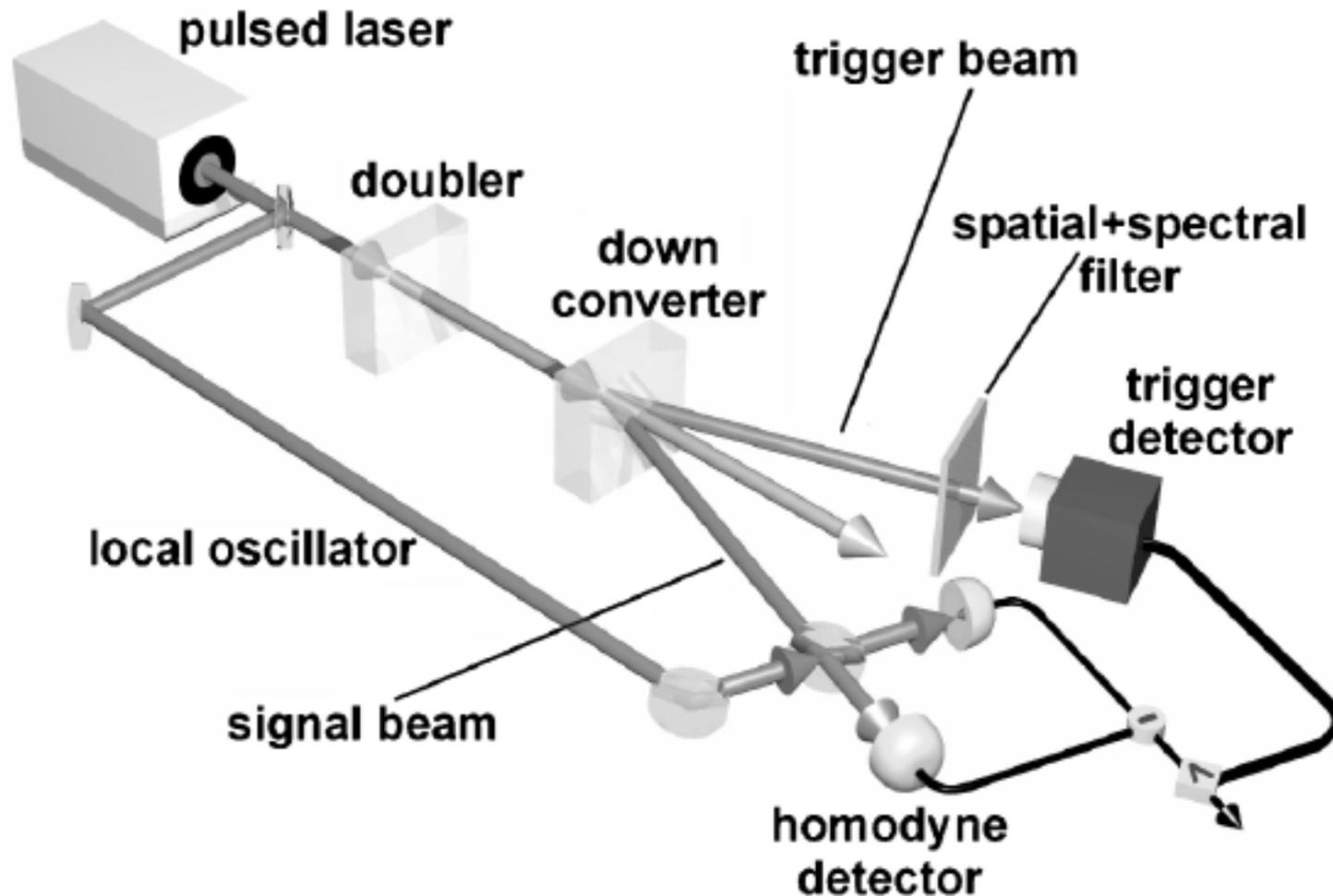
# Maximum of $h_{\theta}(\tau)$ in an OPO below threshold



# Quantum State Reconstruction of the Single-Photon Fock State

A. I. Lvovsky,\* H. Hansen, T. Aichele, O. Benson, J. Mlynek,<sup>†</sup> and S. Schiller<sup>‡</sup>

Phys. Rev. Lett. 87, 050402 (2001)



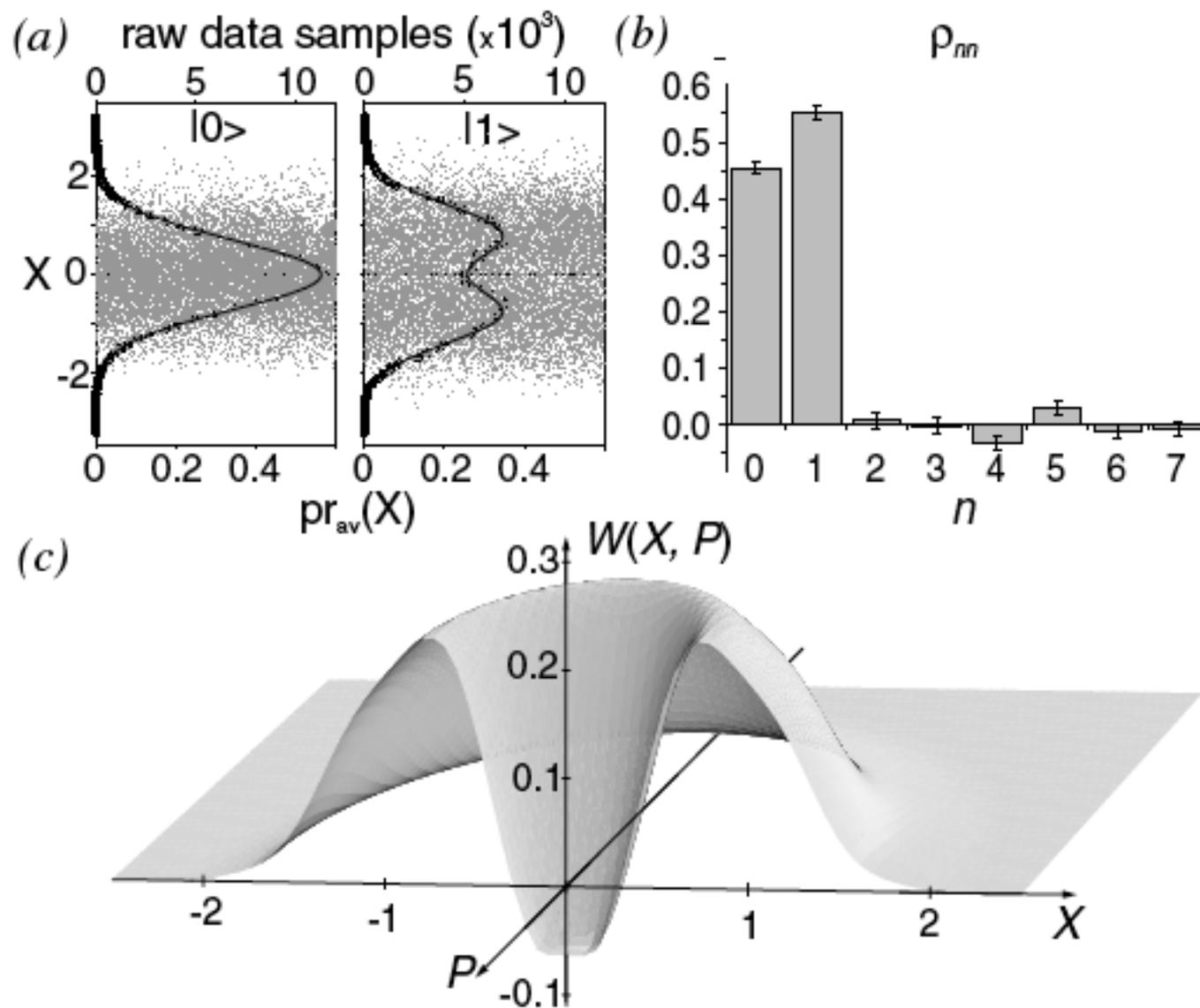


FIG. 4. Experimental results: (a) raw quantum noise data for the vacuum (left) and Fock (right) states along with their histograms corresponding to the phase-randomized marginal distributions; (b) diagonal elements of the density matrix of the state measured; (c) reconstructed WF which is negative near the origin point. The measurement efficiency is 55%.

## Non-Gaussian Statistics from Individual Pulses of Squeezed Light

Jérôme Wenger,\* Rosa Tualle-Brouri, and Philippe Grangier

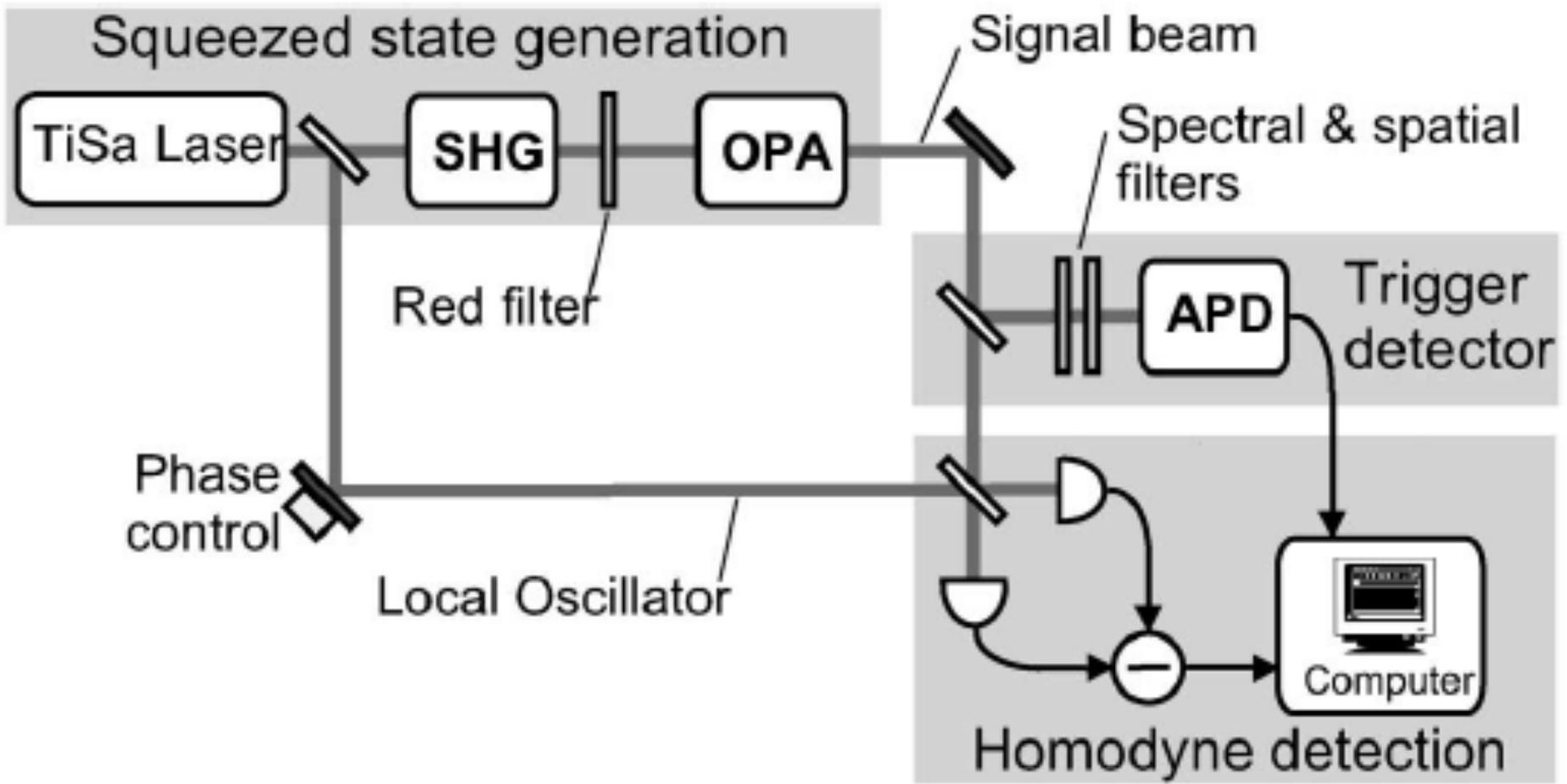
*Laboratoire Charles Fabry de l'Institut d'Optique, CNRS UMR 8501, F-91403 Orsay, France*

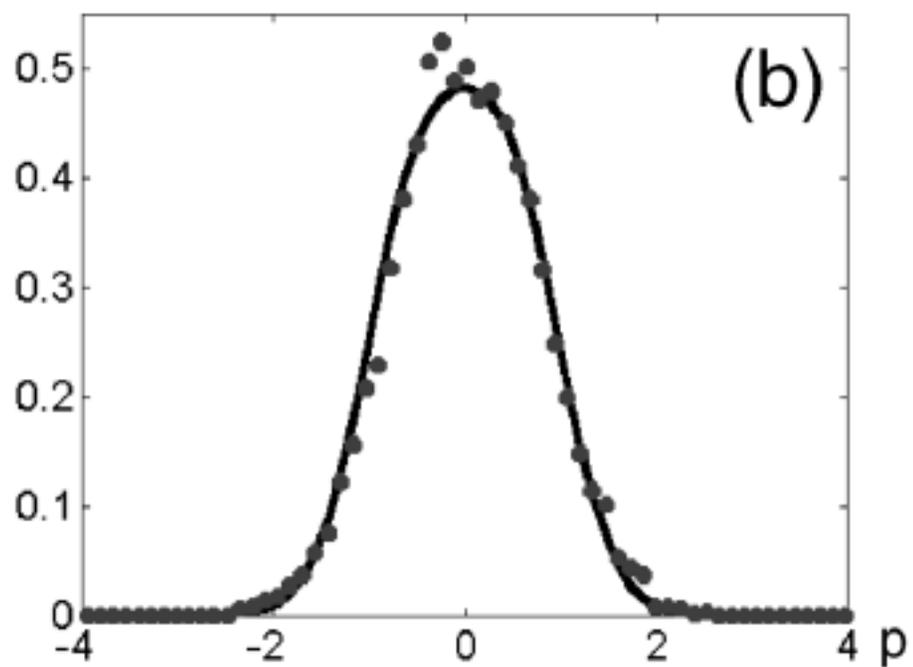
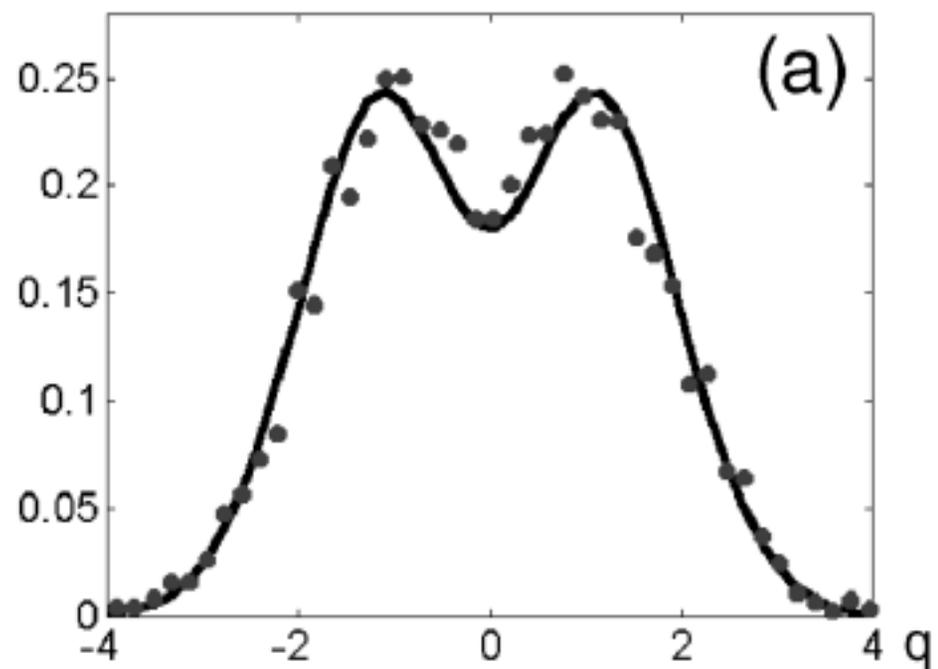
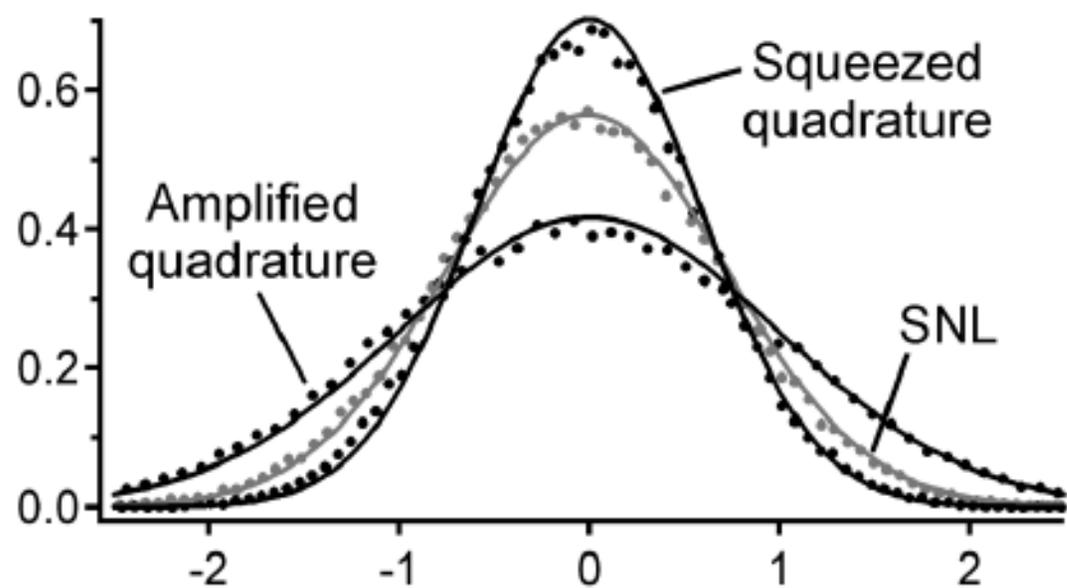
(Received 20 November 2003; published 13 April 2004)

We describe the observation of a “degaussification” protocol that maps individual pulses of squeezed light onto non-Gaussian states. This effect is obtained by sending a small fraction of the squeezed vacuum beam onto an avalanche photodiode, and by conditioning the single-shot homodyne detection of the remaining state upon the photon-counting events. The experimental data provide clear evidence of phase-dependent non-Gaussian statistics. This protocol is closely related to the first step of an entanglement distillation procedure for continuous variables.

DOI: 10.1103/PhysRevLett.92.153601

PACS numbers: 42.50.Dv, 03.67.–a, 03.65.Wj

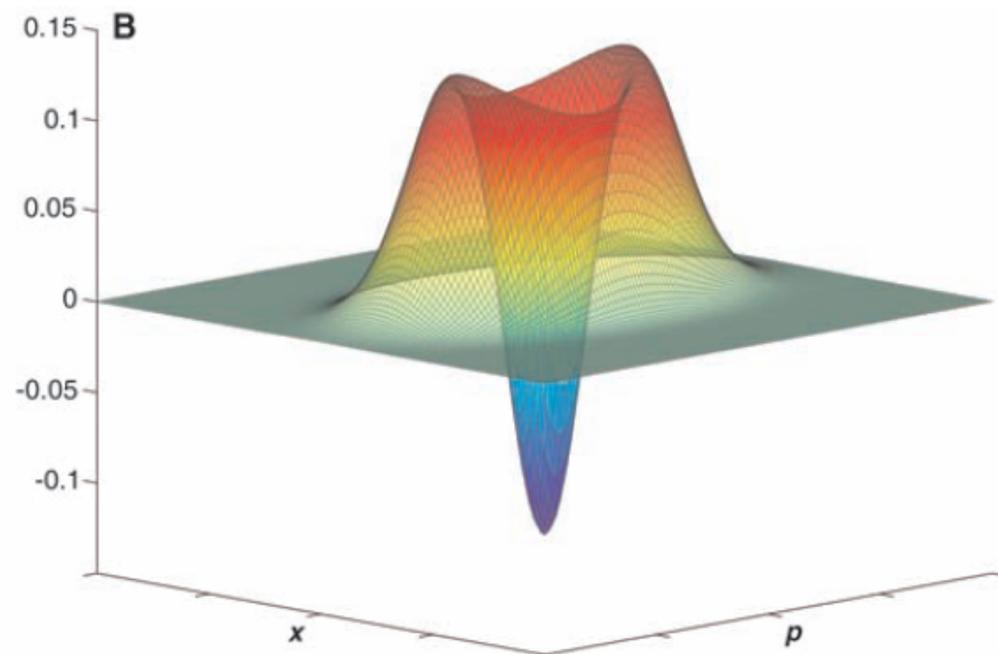
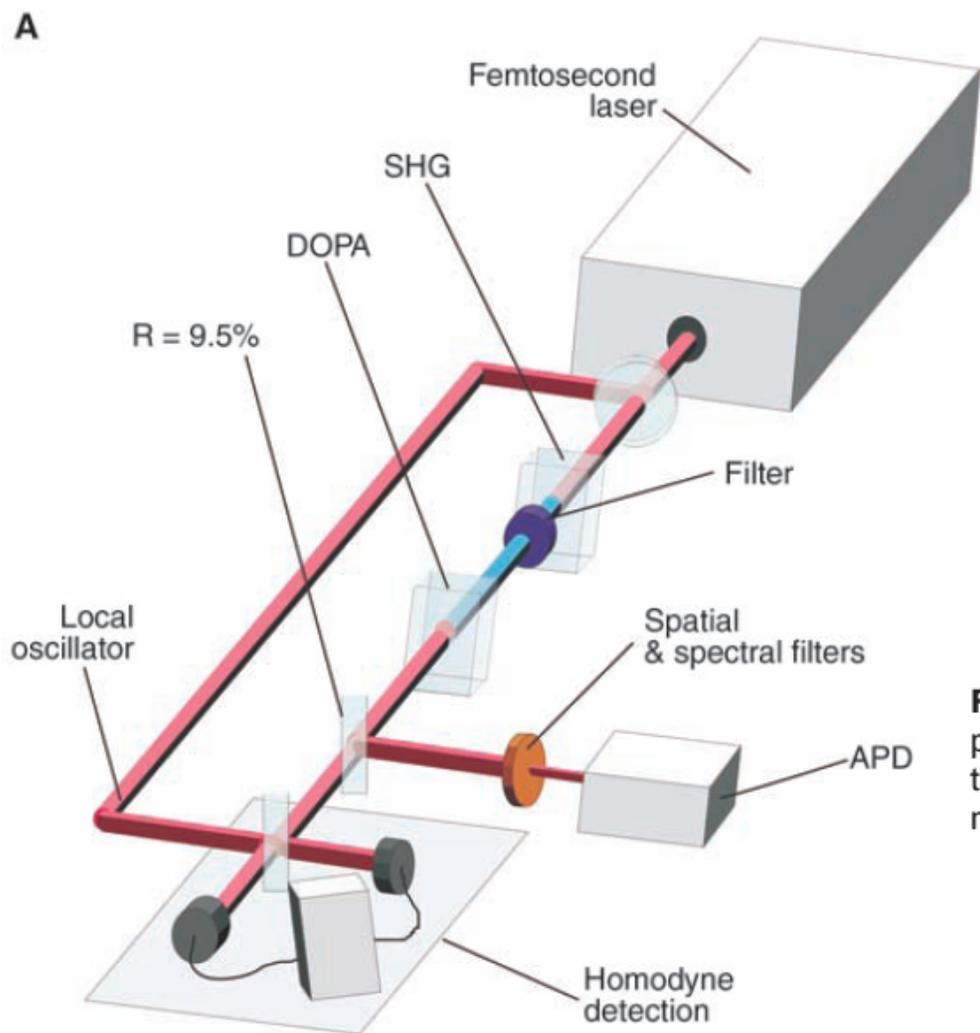




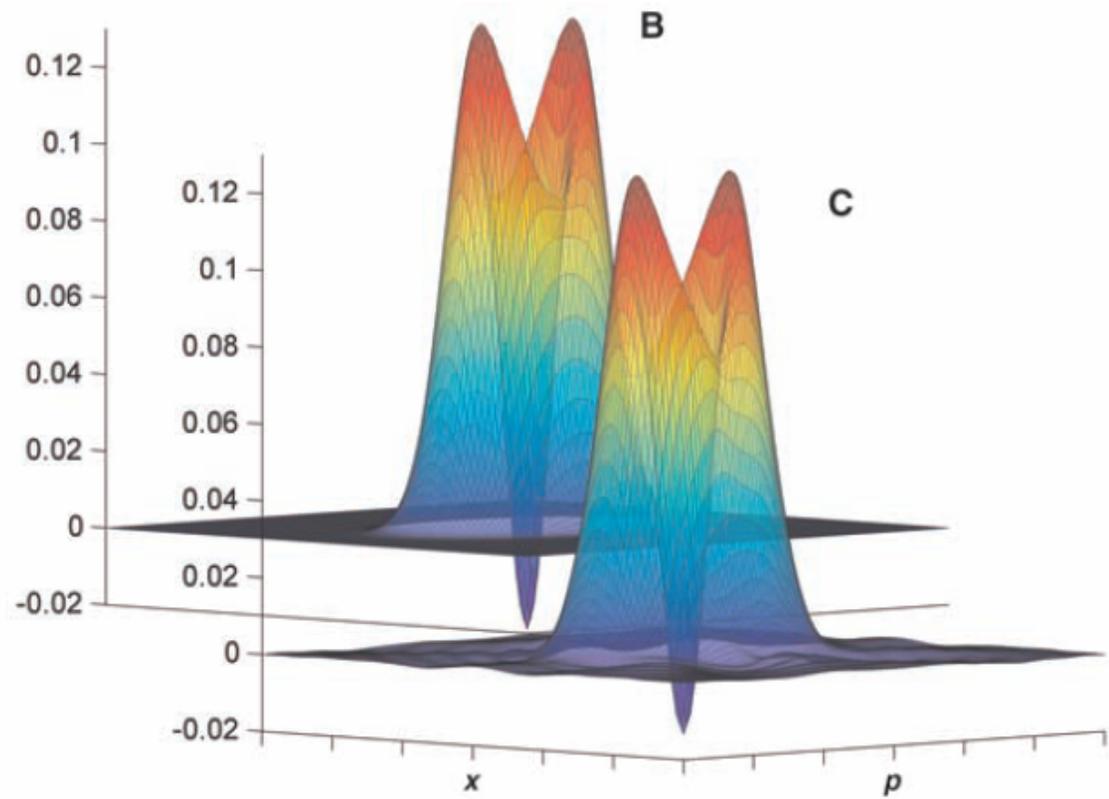
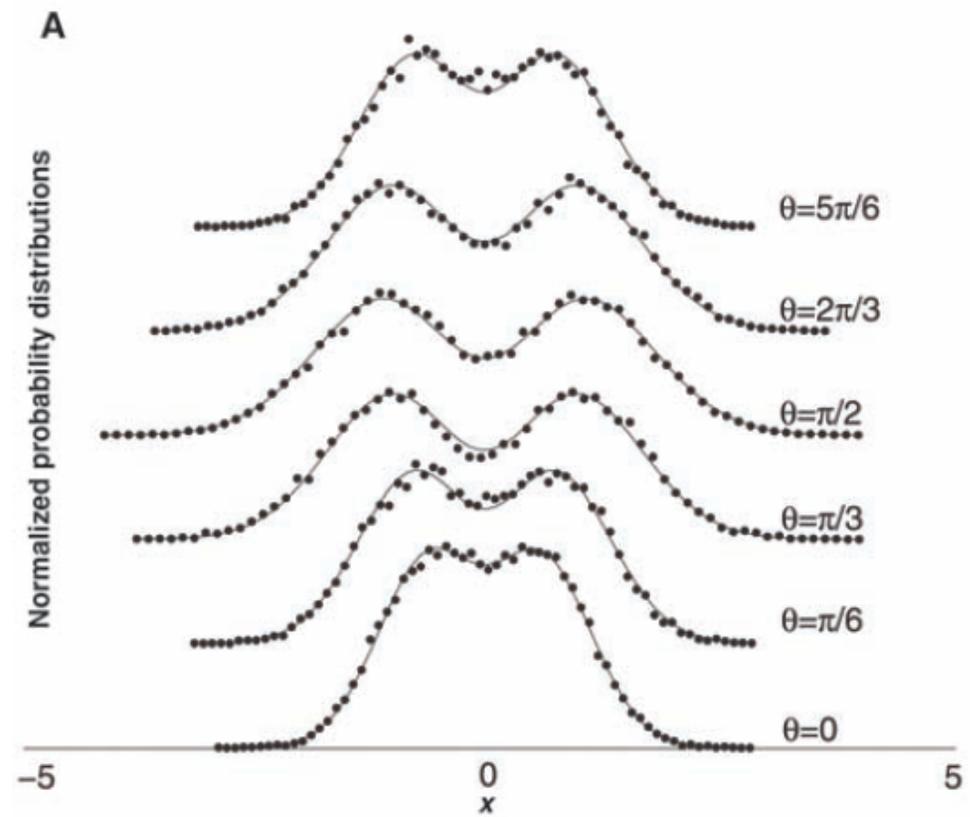
# Generating Optical Schrödinger Kittens for Quantum Information Processing

Alexei Ourjoutsev, Rosa Tualle-Brouri, Julien Laurat, Philippe Grangier\*

We present a detailed experimental analysis of a free-propagating light pulse prepared in a “Schrödinger kitten” state, which is defined as a quantum superposition of “classical” coherent states with small amplitudes. This kitten state is generated by subtracting one photon from a squeezed vacuum beam, and it clearly presents a negative Wigner function. The predicted influence of the experimental parameters is in excellent agreement with the experimental results. The amplitude of the coherent states can be amplified to transform our “Schrödinger kittens” into bigger Schrödinger cats, providing an essential tool for quantum information processing.



**Fig. 2. (A)** Experimental setup and **(B)** reconstructed Wigner function of the photon-subtracted squeezed vacuum ("Schrödinger's kitten") propagating in the experiment ( $s = 0.56$ , corrected for homodyne losses). SHG, second harmonic generation crystal.



## Conclusions:

- The wave-particle correlation  $h_{\theta}(\tau)$  measures the conditional dynamics of the electromagnetic field. The Spectrum of Squeezing  $S(\Omega)$  and  $h_{\theta}(\tau)$  are Fourier Transforms of each other.
- Many applications in many other problems of quantum optics and of optics in general: microscopy, degaussification, weak measurements, quantum feedback.
- Possibility of a tomographic reconstruction of the dynamical evolution of the electromagnetic field state.

**End of Third Part**

谢谢