Cavity and Waveguide QED
Lecture 1

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NIST

0. A review of Electricity and Magnetism

Maxell’s Equations:

$$\nabla \cdot (\varepsilon_0 \mathbf{E} + \mathbf{P}) = 0,$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$

$$\nabla \times \mathbf{B} = \mu_0 \frac{\partial (\varepsilon_0 \mathbf{E} + \mathbf{P})}{\partial t},$$

$$\nabla \cdot \mathbf{B} = 0.$$

Wave Equation:

$$\nabla \times (\nabla \times \mathbf{E}) = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E}$$

$$= -\frac{\partial}{\partial t}(\nabla \times \mathbf{B}) = -\mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} - \mu_0 \frac{\partial^2 \mathbf{P}}{\partial t^2}$$

$$-\nabla(\nabla \cdot \mathbf{E}) + \nabla^2 \mathbf{E} - \mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} = \mu_0 \frac{\partial^2 \mathbf{P}}{\partial t^2}.$$

In free space, $\nabla \cdot \mathbf{E} = 0$ and $\mathbf{P} = \mathbf{0}$,

$$\nabla^2 \mathbf{E} - \frac{1}{v_0^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0$$

A note about polarization

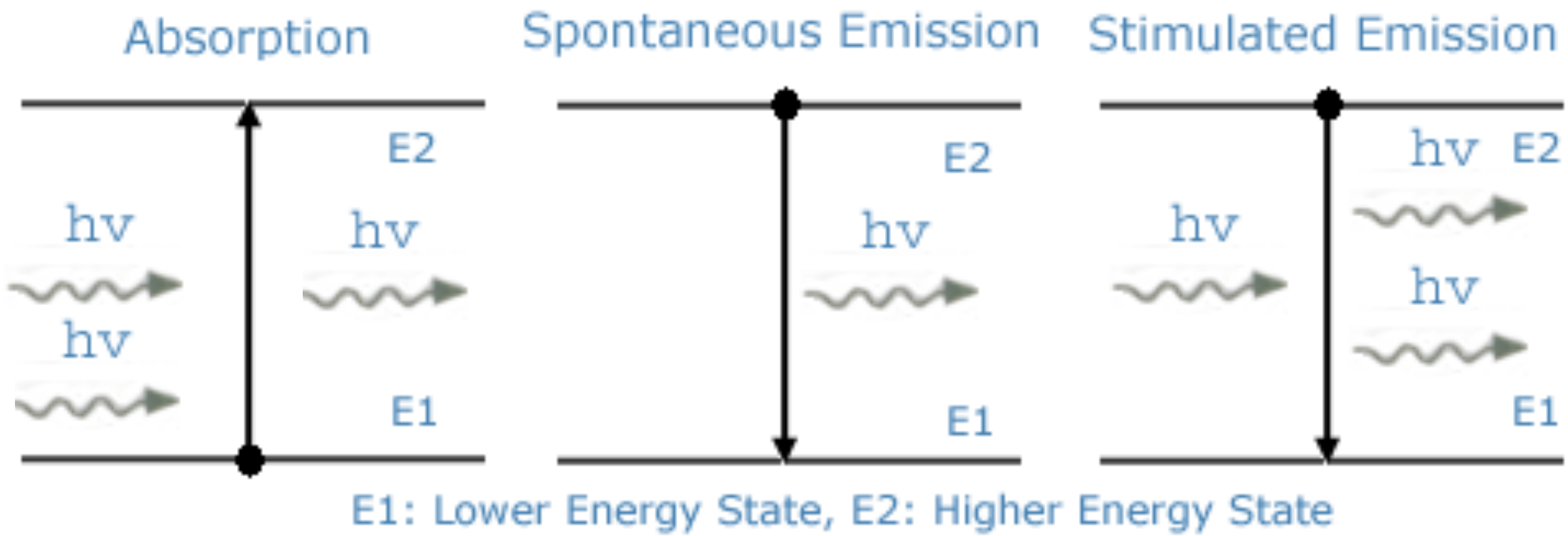
Gauss's Law in free space:

$$\nabla \cdot \vec{E} = 0$$

$$\nabla_T \cdot \vec{E} + \frac{2\pi}{\lambda} i E_Z = 0$$

If there is a "transverse gradient" in the radiation field propagating in *z*, there is a longitudinal polarization also in *z*

1. One atom interacting with light in free space.



Absorption and Stimulated Emission are time reversals of each other, you can say this is the classical part. Spontaneous emission is the quantum, that is the jump.

Dipole cross section (same result for a classical dipole or from a two level atom):

$$\sigma_0 = \frac{3\lambda_0^2}{2\pi}$$

This is the "shadow" caused by a dipole on a beam of light.

Energy due to the interaction between a dipole and an electric field.

$$H = \vec{d} \cdot \vec{E}$$

The dipole matrix element between two states is fixed by the properties of the states (radial part) and the Clebsh-Gordan coefficients from the angular part of the integral. It is a few times a_0 (Bohr radius) times the electron charge e between the S ground and P first excited state in alkali atoms.

$$\vec{d} = e\left\langle 5S_{1/2} \middle| \vec{r} \middle| 5P_{3/2} \right\rangle$$

Beer-Lambert law for intensity attenuation

$$\frac{dI}{dz} = -\alpha I \quad \text{if } \alpha \text{ is resonant and independent}$$

of I, α_0 (does not saturate)

$$I = I_0 \exp(-\alpha_0 l)$$

where $\alpha_0 = \sigma_0 \rho$

and $\rho = N / V$ the density of absorbers in a length l

Rate of decay
(Fermi's golden rule)

$$\gamma_{rad} \approx \frac{2\pi}{\hbar} \rho(k) \langle H_{int} \rangle^2$$

Phase space density   Interaction

Rate of decay free space (Fermi's golden rule)

$$\gamma_0 = \frac{\omega_0^3 d^2}{\pi \varepsilon_0 \hbar c^3}$$

Where *d* is the dipole moment

Saturation intensity:
One photon every two lifetimes over the cross section of the atom (resonant)

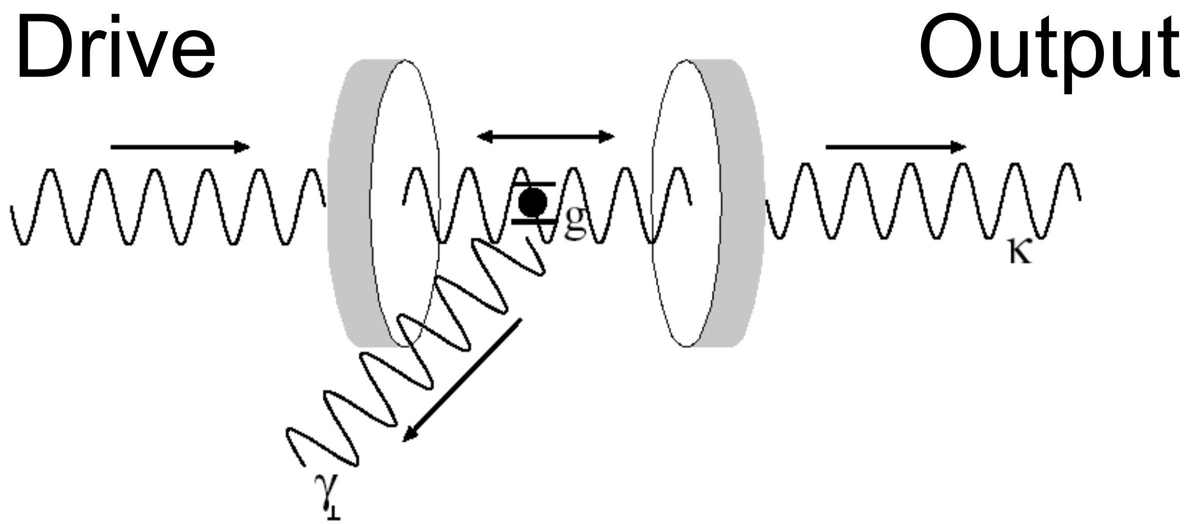
$$I_s = \frac{\hbar\omega_0}{2\tau_0\sigma_0} = \frac{\pi}{3}\frac{\gamma_0\hbar\omega_0}{\lambda_0^2}$$

If $I = I_0$ the rate of stimulated emission is equal to the rate of spontaneous emission (Rabi Frequency Ω) and the population on the excited state 1/4.

$$\Omega = \frac{\vec{d} \bullet \vec{E}}{\hbar} = \gamma \sqrt{I / I_s}$$

$$\text{Excited Population} = \frac{1}{2} \frac{I / I_s}{1 + I / I_s}$$

Coupled atoms and cavities:



Collection of N Two level atoms coupled to a single mode of the electromagnetic field (g). Driven with dissipation (atoms γ, cavity κ).

Microwaves → Micromaser → Cavity QED

Visible light → Optical Bistability → Cavity QED

Absorptive Element

A saturable absorber has an absorption coefficient which is a non-linear function of I:

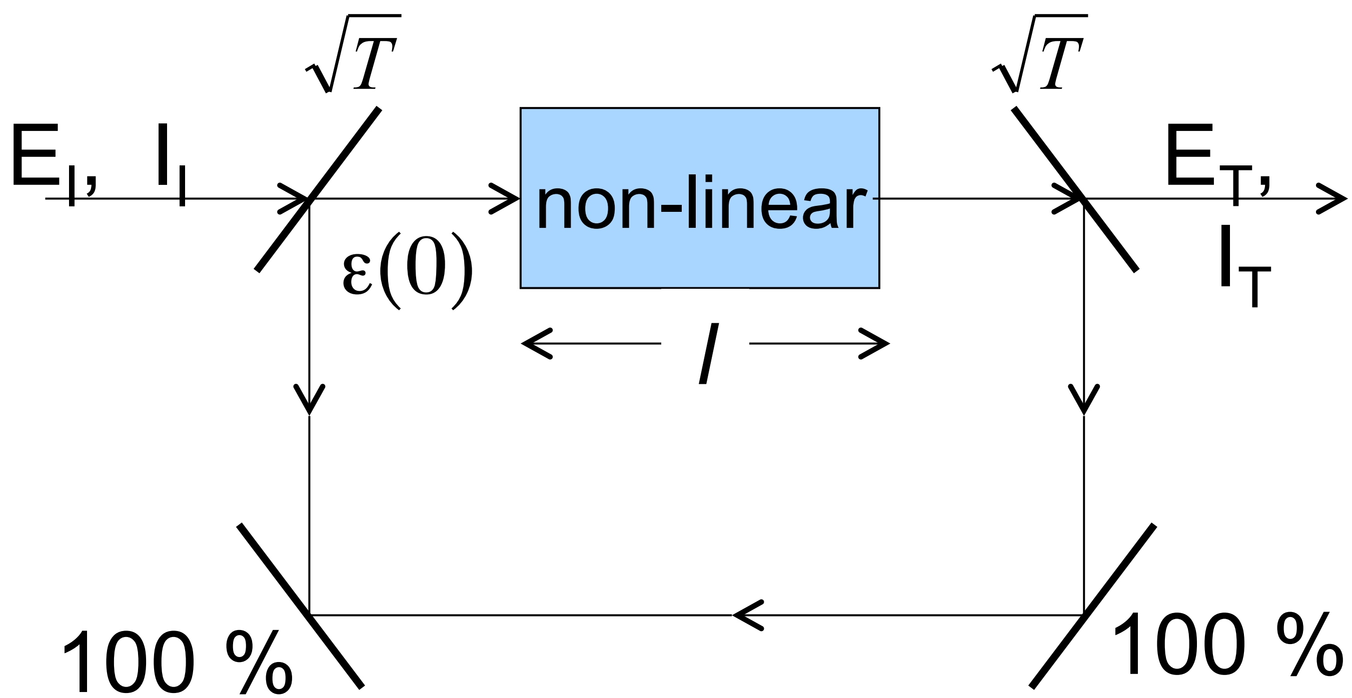
$$\alpha = \frac{\alpha_o}{1 + I / I_s}$$

The cavity is setup for resonance.
At small intensities, the absorption due to the element is high and the output is low.
As the intensity is increase beyond Is, the absorption rapidly decreases and the output goes to high.

The field inside the cavity comes from the addition of the drive and what is already there

$$\text{Let } \varepsilon_{n+1}(0) = \sqrt{T}E_I + \text{Re}^{-\alpha l} \; e^{iKL}\varepsilon_n(0)$$

Where e $_{n+1}$ is the electric field after the *n+1+* path around the cavity, *L* is the round-trip length, a is the absorption coefficient and *R=1-T* the mirror reflectivity



- At steady state the electric field inside the cavity must be constant so that $\varepsilon_{n+1}(0) = \varepsilon_n(0) = \varepsilon_0$

$$\therefore \quad \varepsilon_0 = \sqrt{T}E_1 + \mathrm{Re}^{-\alpha l}\, e^{iKL}\varepsilon_0$$

rearanging this gives: $\varepsilon_0 = \dfrac{\sqrt{T}E_I}{(1-\mathrm{Re}^{-\alpha l + iKL})}$

- The output field is given by the mirror transmittance times the internal electric field at a distance l.

$$E_T = \sqrt{T}\varepsilon(l) = \sqrt{T}\varepsilon_o e^{(-\alpha + iK)l}$$

- the amplitude transmission function is:

$$\frac{E_T}{\mathrm{E_I}} = \frac{Te^{iK(l-L)}}{e^{\alpha l - iKL} - R}$$

Absorptive Bistability

A saturable absorber, at resonance has an absorption coefficient which is a non-linear function of I:

$$\alpha = \frac{\alpha_o}{1 + I / I_s}$$

assuming that αl<<1, gives on resonance:

$$\frac{E_T}{E_I} = \frac{1}{1 + \alpha l / T}$$

$$E_I = E_T \left[1 + \frac{\alpha_o l / T}{1 + I_T / I_s T} \right] \quad \text{with } I = \frac{I_T}{T}$$

The ratio of losses: atomic losses per round trip (αl) to cavity losses per round trip (T) is the Cooperativity

$$C = \frac{\alpha_o l}{T} = \frac{\sigma_0 \rho l}{T} = \frac{\sigma_0 N}{Area_{\text{mode}}} \frac{1}{T}$$

$$y = x \left[1 + \frac{2C}{1 + x^2} \right] \quad \text{with } y = \frac{E_I}{\sqrt{I_s}}; \text{ and } x = \frac{E_T}{\sqrt{T I_s}}:$$

For low intensity, the input and the output are linearly related,

$$y = x\,(1+2C)\ ;\ x/y=1/(1+2C) \text{ goes as } 1/N$$

$$Y=y^2\ ;\ X=x^2$$

$$Y=X(1+2C)^2\ ;\ X/Y=1/(1+2C)^2 \text{ goes as } 1/N^2$$

For very high intensity,

$$y = x\ ;\ Y=X+4C$$

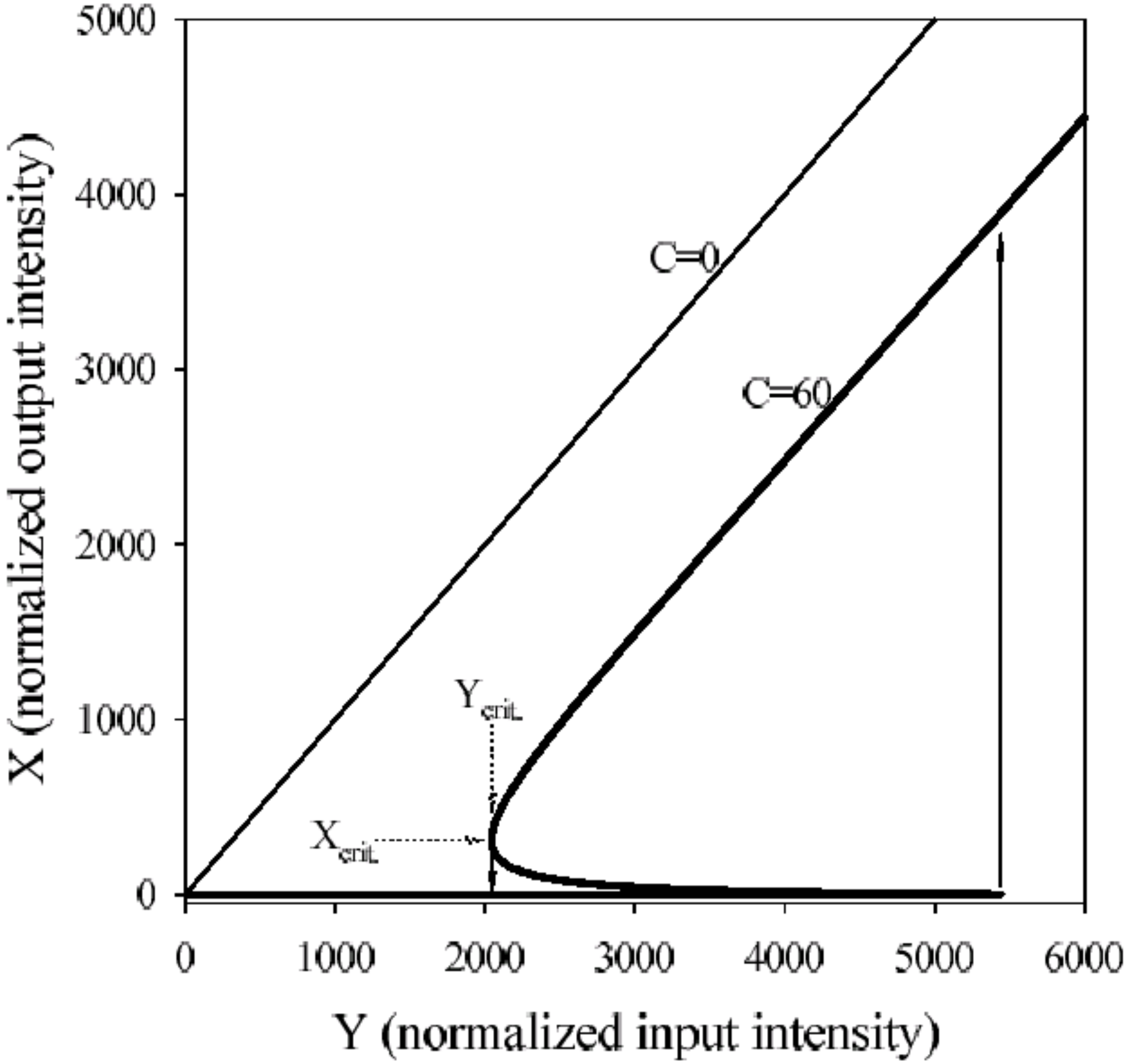
Almost an empty cavity

At intermediate intensity, there can be saturation.

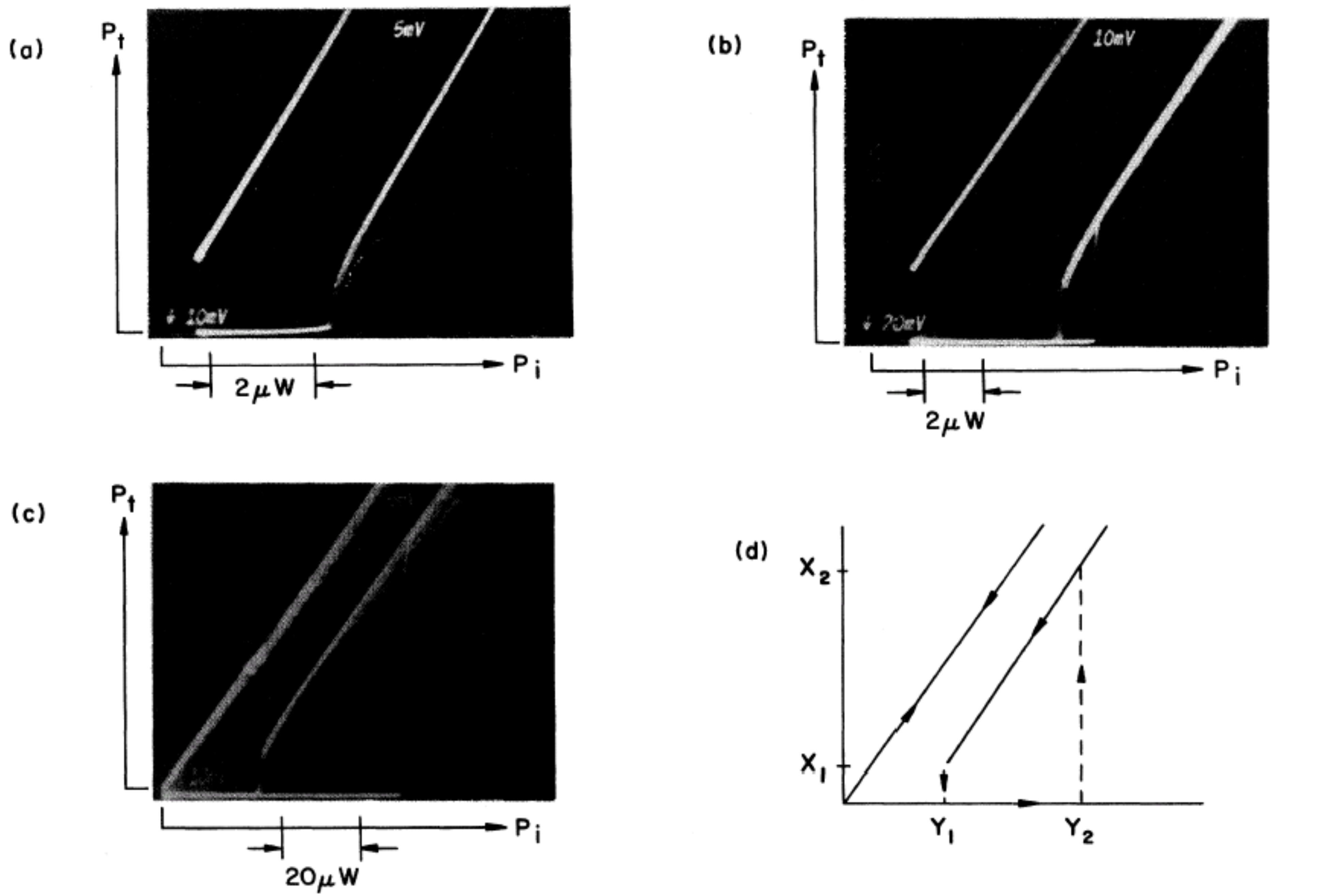
It happens in this simple model for the case of C>4. C (Cooperativity) is the negative of the laser pump paramenter.

It is the ratio of the atomic losses to the cavity losses or also can be read as the ratio between the good coupling (g) and the bad couplings (κ,γ).

Input-Output response of the atoms-cavity system for two different cooperativities C=0 is with no atoms, C=60 has plenty of atoms, with a drive that can saturate them and we recover the linear relationship with unit slope between Y and X.



Increasing the number of atoms in the cavity:



Steady state with detuning and at all intensities:

$$y = x\left(1 + \frac{2C}{1+\Delta^2+|x|^2}\right) + ix\left(\theta - \frac{2C\Delta}{1+\Delta^2+|x|^2}\right)$$

$$\Theta = \frac{\omega_c - \omega_l}{\kappa}; \quad \Delta = \frac{\omega_a - \omega_l}{\gamma/2}$$

Dispersive limit when $\Theta=0$ and $\Delta >> 1$:

$$y = -ix\frac{2C\Delta}{1+\Delta^2+|x|^2}$$

Steady state with detuning and at all intensities:

$$y = x\left(1 + \frac{2C}{1+\Delta^2+|x|^2}\right) + ix\left(\theta - \frac{2C\Delta}{1+\Delta^2+|x|^2}\right)$$

Transmission spectrum:

when $\omega_c = \omega_a \quad \Omega = (\gamma/2)\Delta = \kappa\Theta \quad \Omega_{V.R.} = g\sqrt{N} = \sqrt{C\kappa\gamma}$

$$x = y\frac{\kappa(\gamma_\perp + i\Omega)}{(\kappa+i\Omega)(\gamma_\perp+i\Omega) + \Omega_{V.R.}^2/(1+\gamma_\perp^2|x|^2/(\gamma_\perp^2+\Omega^2))}$$

Two coupled oscillators

$$\frac{x}{y} = \frac{A}{i\Omega - \Omega_1} + \frac{B}{i\Omega - \Omega_2} ,$$

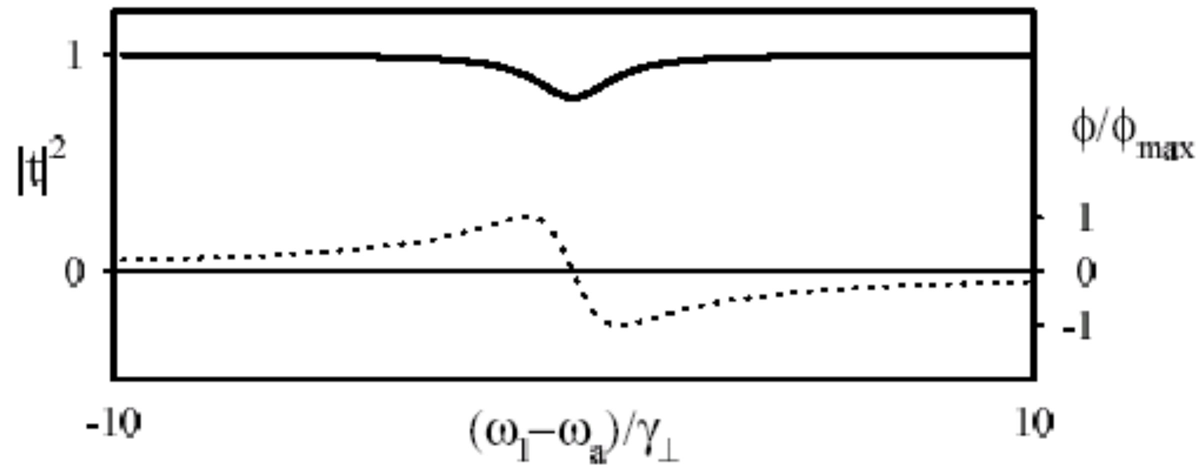
$$A = \kappa \frac{\gamma_\perp + \Omega_1}{\Omega_1 - \Omega_2} ,$$

$$B = \kappa \frac{\gamma_\perp + \Omega_2}{\Omega_2 - \Omega_1} ,$$

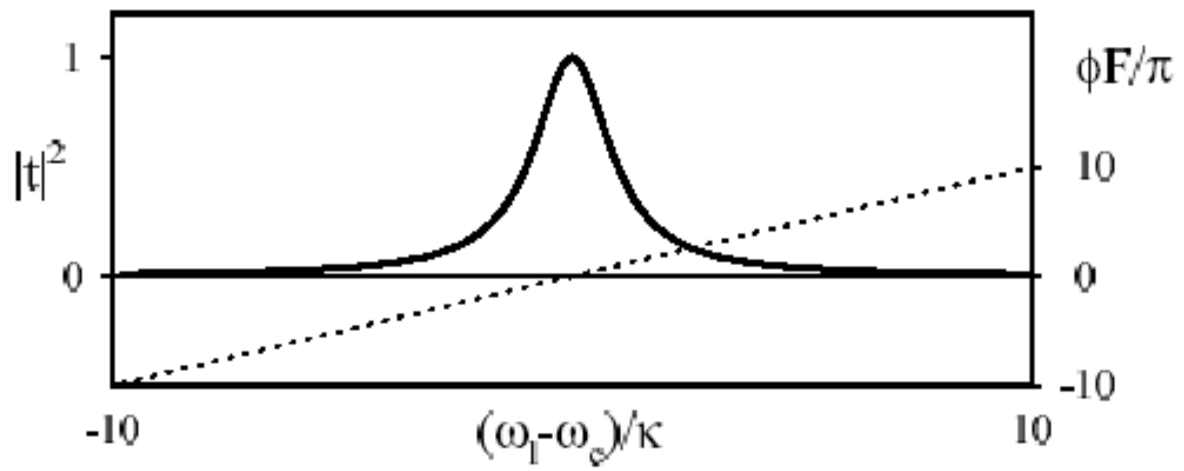
$$\Omega_{1,2} = -\frac{\kappa + \gamma_\perp}{2} \pm i \sqrt{ -\left(\frac{\kappa - \gamma_\perp}{2}\right)^2 + \frac{\Omega_{V.R.}^2}{1 + \gamma_\perp^2 |x|^2 / (\gamma_\perp^2 + \Omega^2)} } .$$

One is more cavity like, the other atom like

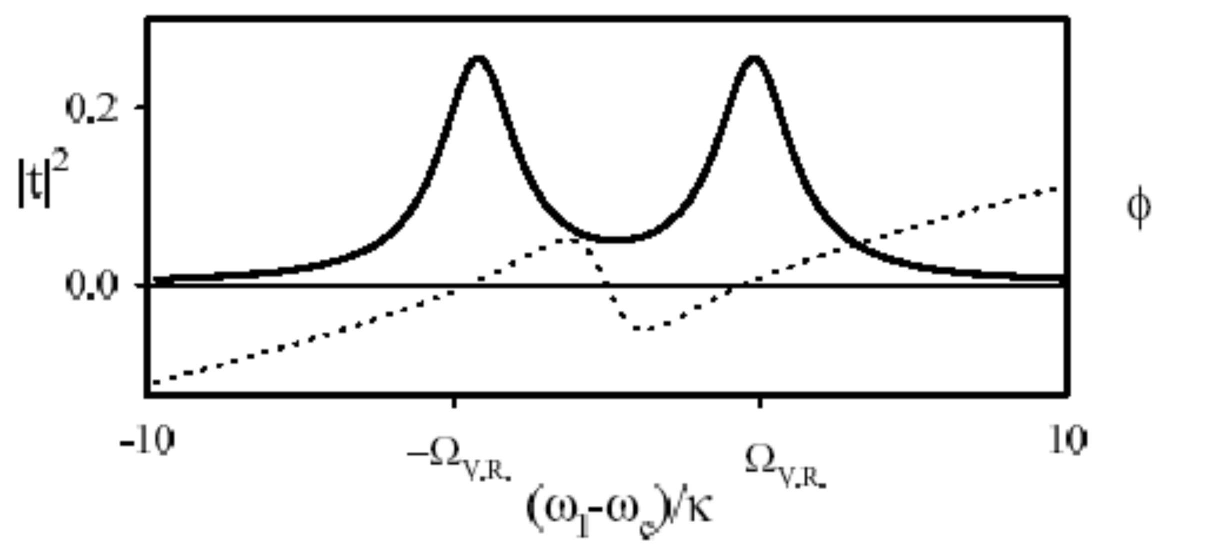
Absorption of atoms



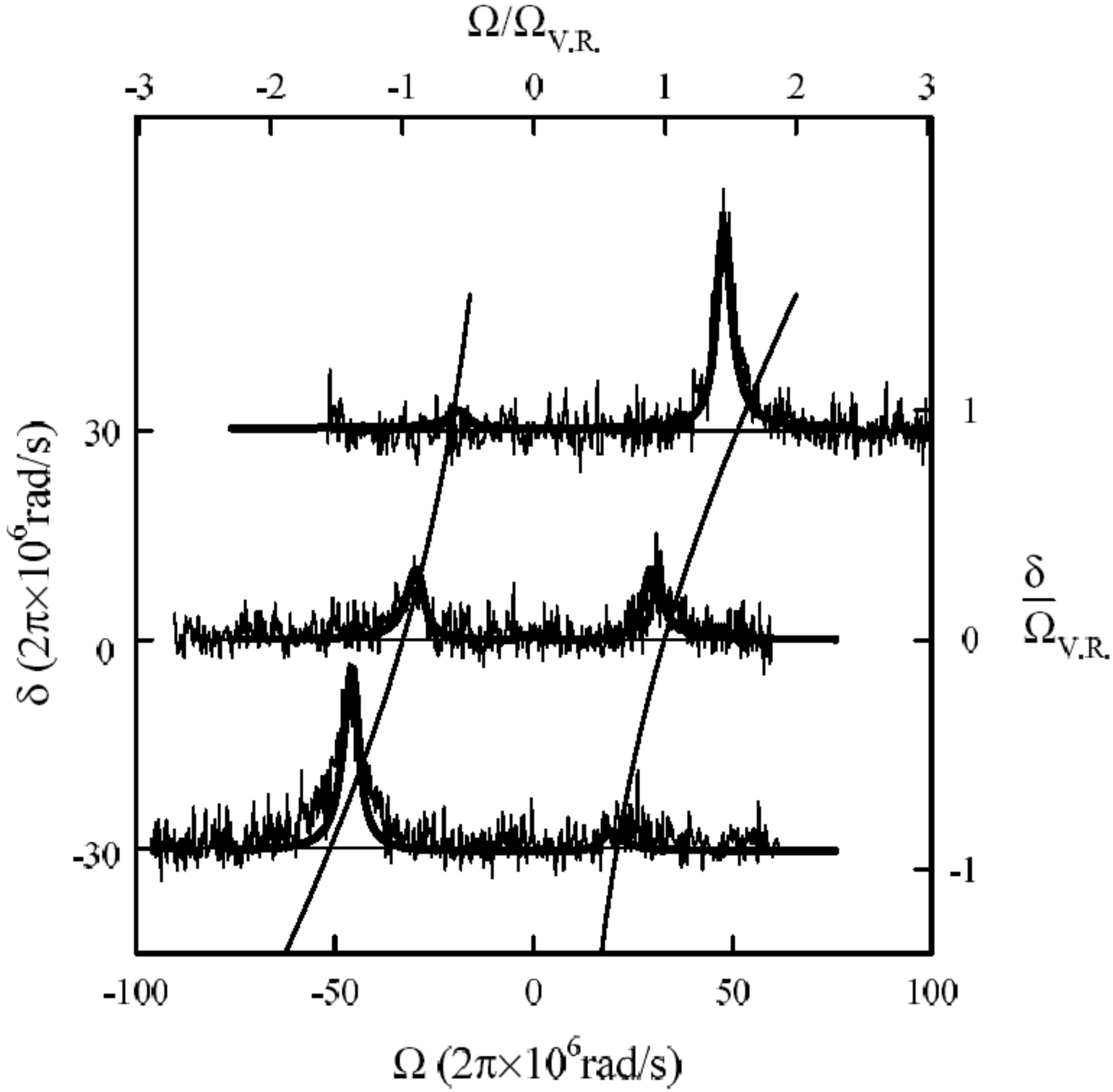
Fabry Perot fringe



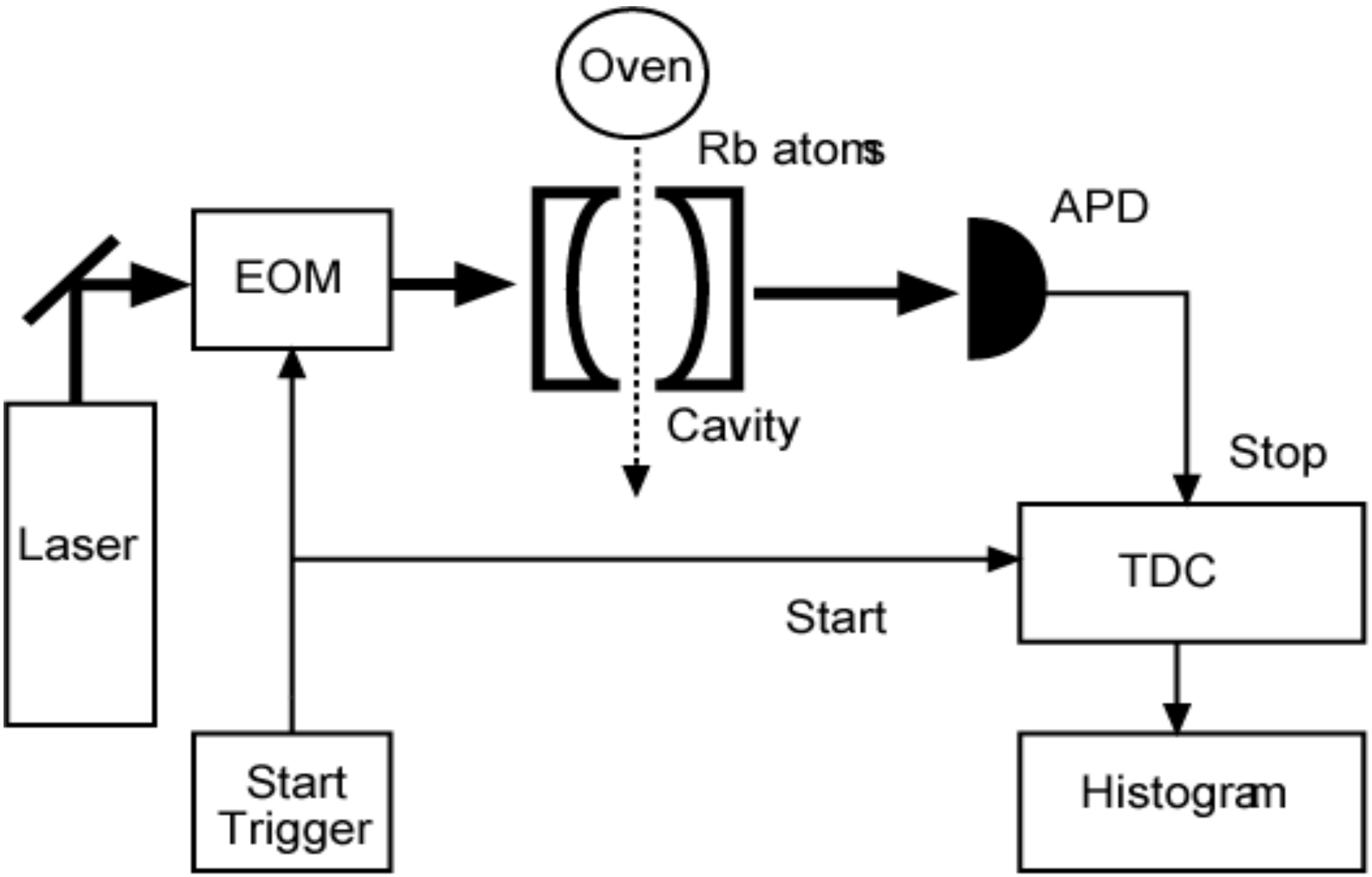
Coupled modes



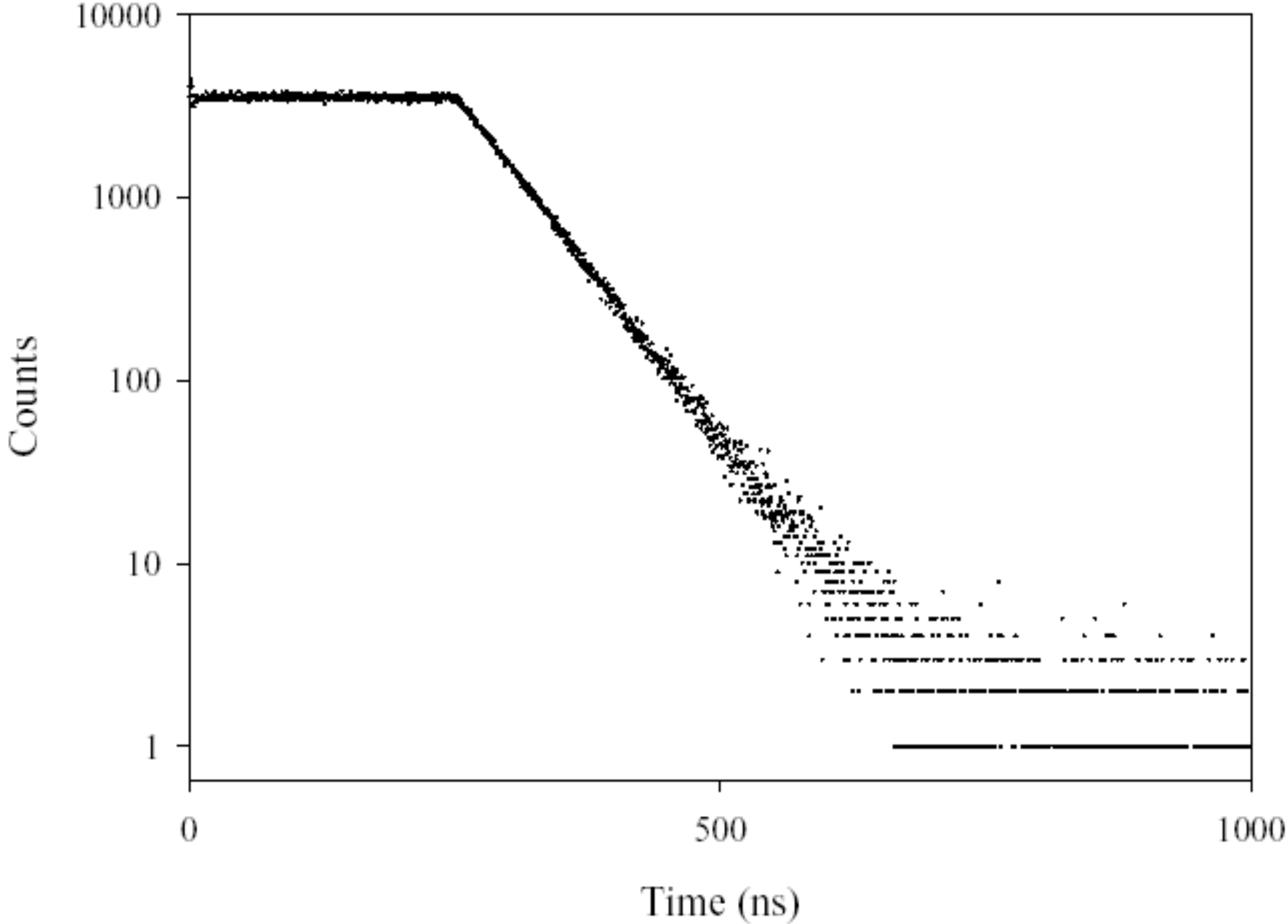
Transmission Spectra at low intensity for different atomic detunings.



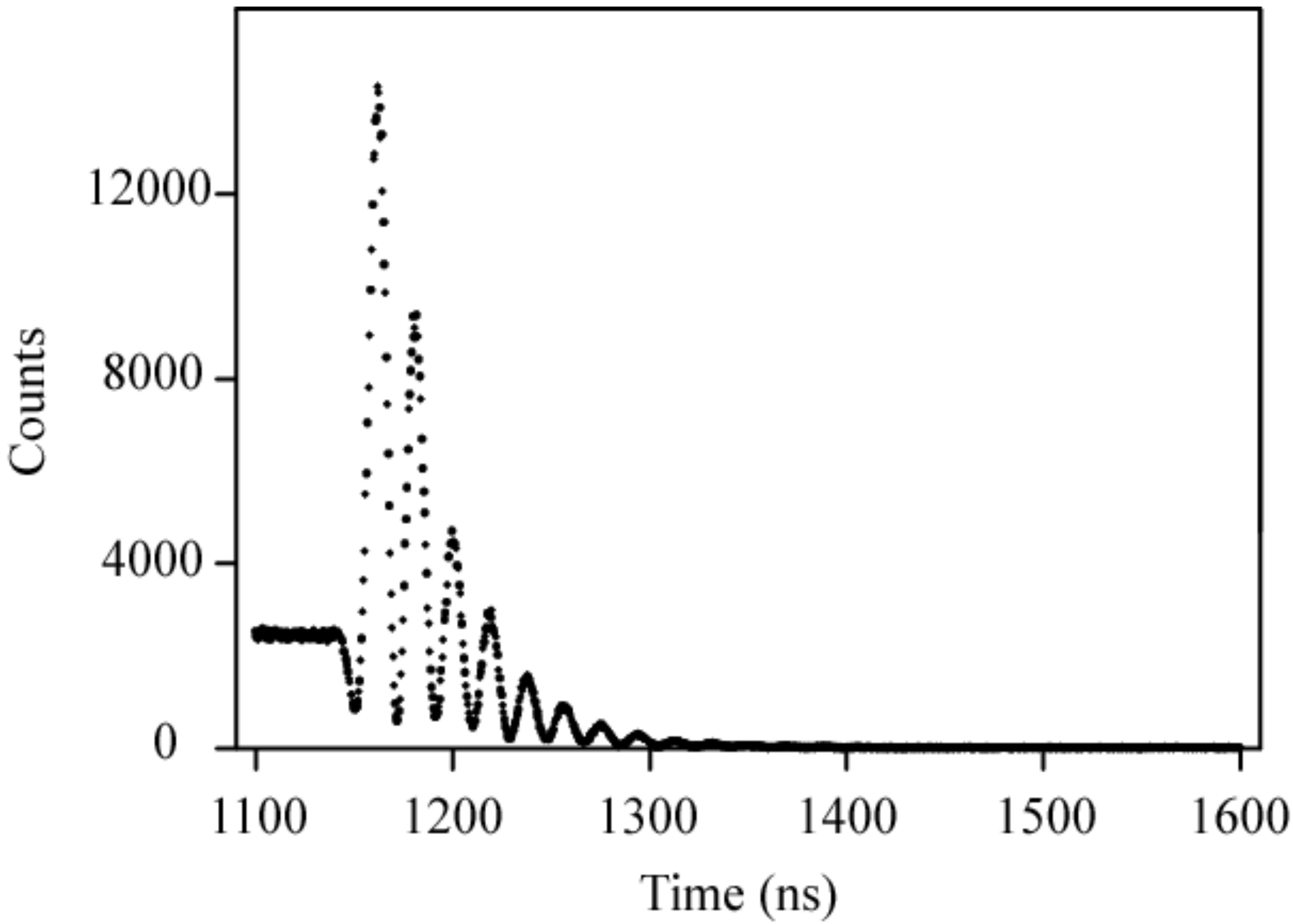
Study the system dynamics classically by providing a step function.



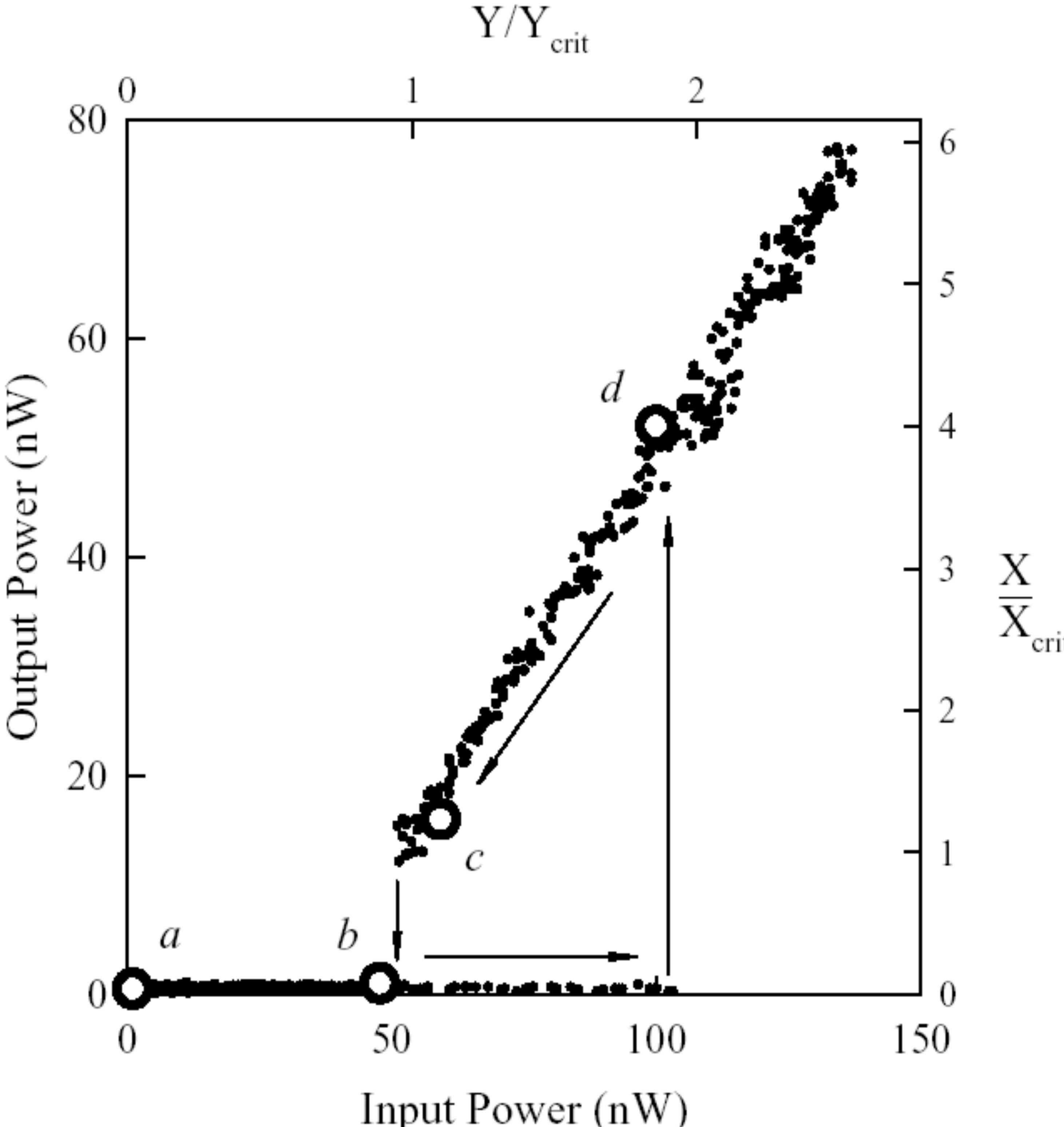
Decay of the empty cavity



Response to step down excitation

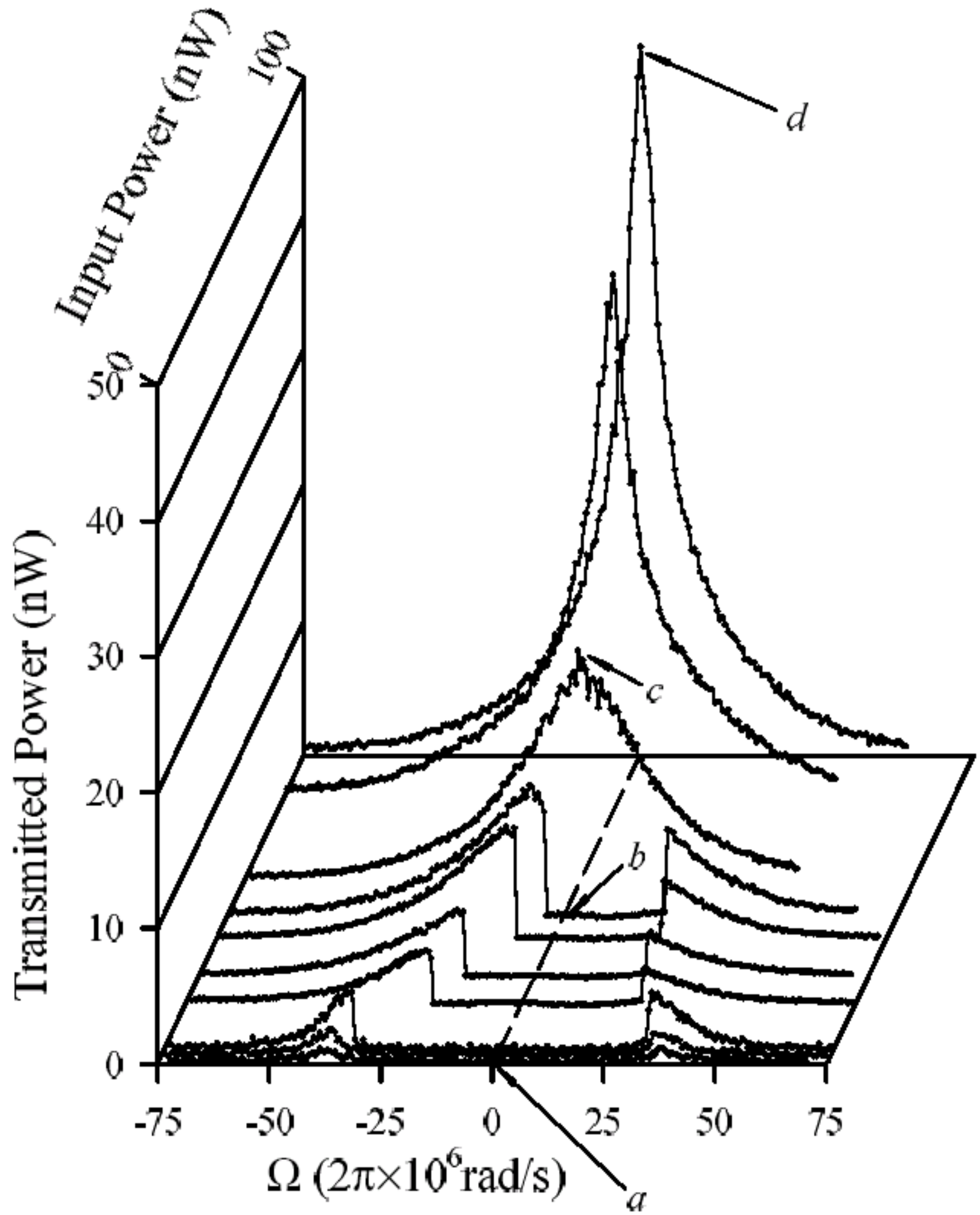


Transmission spectra at arbitrary intensity

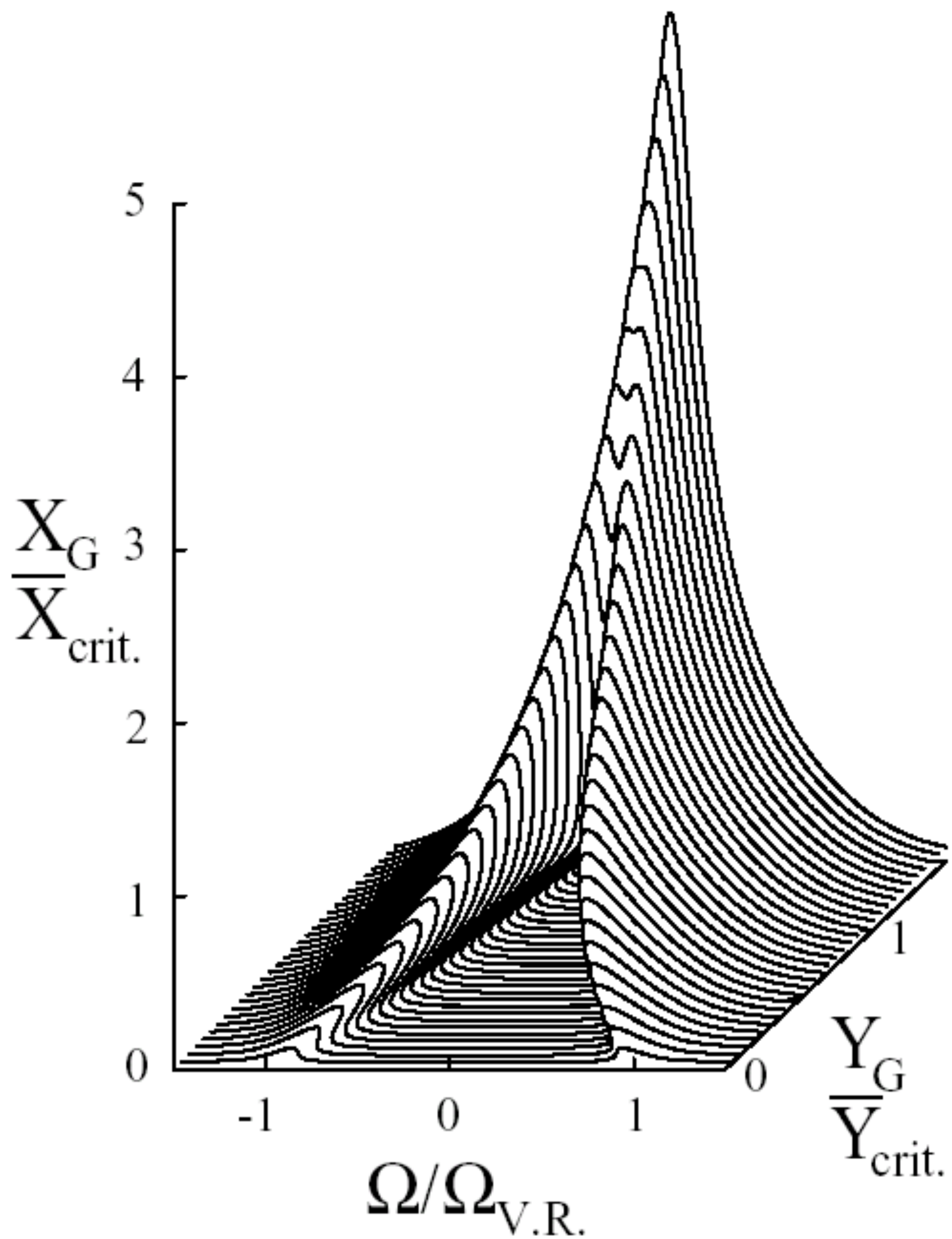


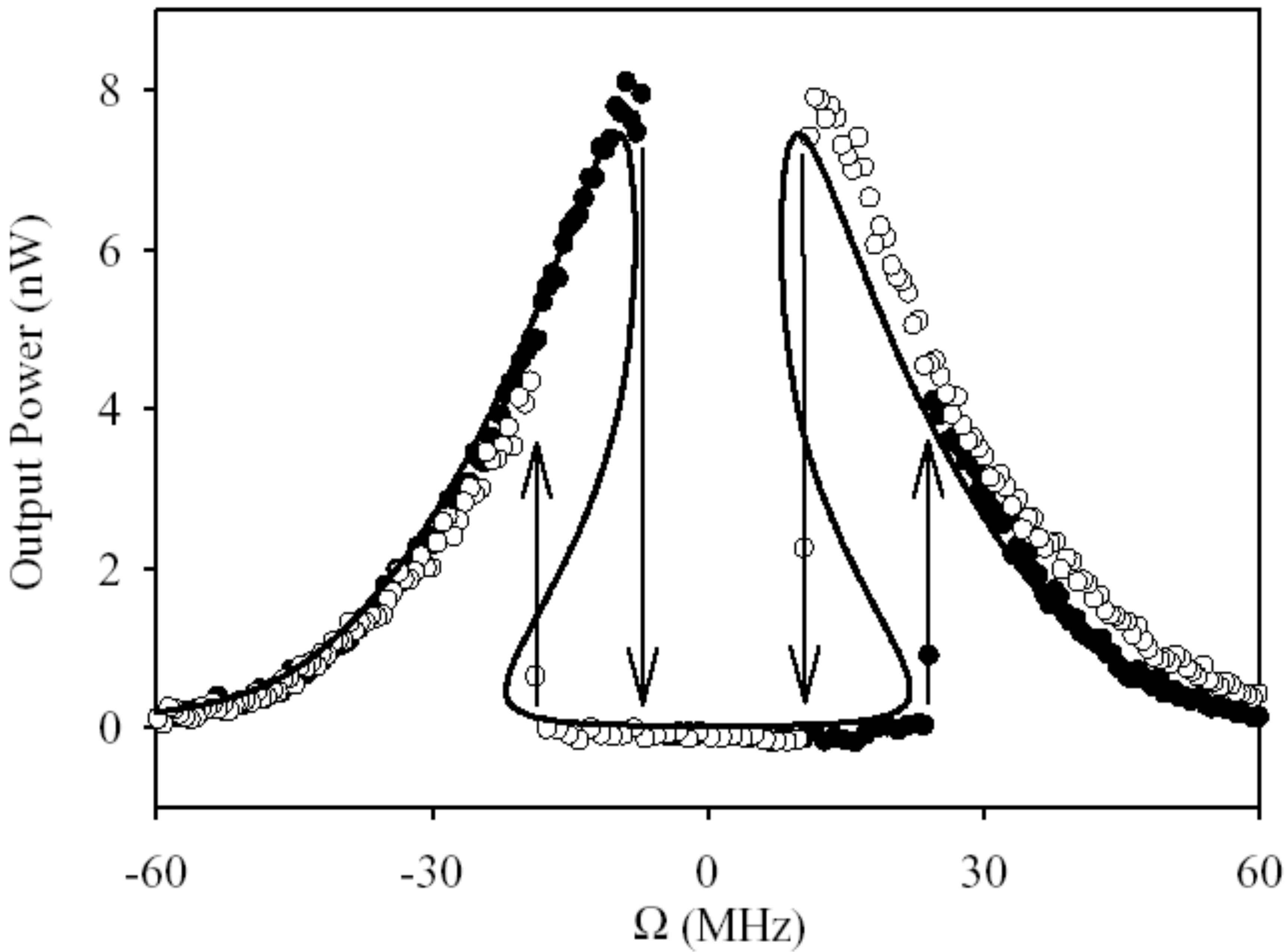
Y/Y_crit
0
1
2
80
60
40
20
0
6
5
4
3
2
1
0
Output Power (nW)
X/X_crit
a
b
c
d
0
50
100
150
Input Power (nW)

Transmission spectra for different intensities.



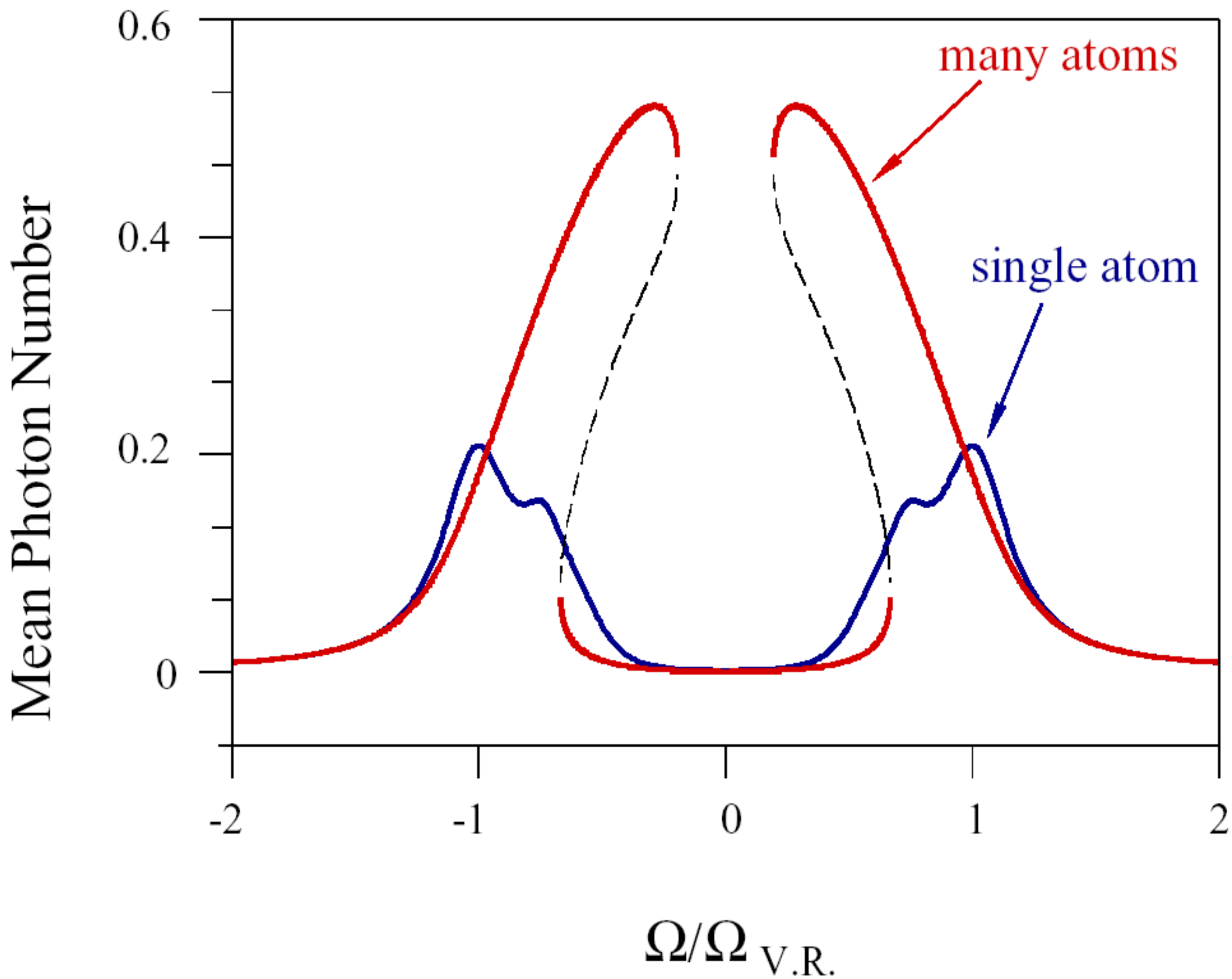
Theory





Hysteresis for a frequency scan of the light from the coupled atoms-cavity system.

Many atoms solved with Maxwell Bloch equations. Single atom solved with the full Hamiltonian, no decorrelation. The system does not show hysteresis.



The semiclassical theory: Maxwell Bloch Equations

Optical Cavity QED

Quantum electrodynamics for pedestrians. No need for renormalization. One or a finite number of modes from the cavity.

ATOMS + CAVITY

Regimes:

Perturbative: Coupling<< Dissipation. Atomic decay suppressed or enhanced (cavity smaller than $\lambda/2$), changes in the energy levels.

Non Perturbative: Coupling>>Dissipation

Vacuum Rabi Splittings. Conditional dynamics.

Dipolar coupling between the atom and the mode of the cavity:

$$g = \frac{d \cdot E_v}{\hbar}$$

El electric field associated with one photon on average in the cavity with volume: V_{eff} is:

$$E_v = \sqrt{\frac{\hbar\omega}{2\varepsilon_0 V_{eff}}}$$

Radiation field:

$$\frac{\partial}{\partial t}\langle\hat{a}\rangle = -\kappa(1+i\theta)\langle\hat{a}\rangle + \sum_{j=1}^{N} g_j\langle\hat{\sigma}_j^-\rangle + \mathcal{E},$$

Atomic polarization:

$$\frac{\partial}{\partial t}\langle\hat{\sigma}_j^-\rangle = -\gamma_\perp(1+i\Delta)\langle\hat{\sigma}_j^-\rangle + g_j\langle\hat{a}\rangle\langle\hat{\sigma}_j^z\rangle,$$

Atomic inversion:

$$\frac{\partial}{\partial t}\langle\hat{\sigma}_j^z\rangle = -\gamma_\parallel\left(\langle\hat{\sigma}_j^z\rangle + 1\right) - 2g_j\left(\langle\hat{a}\rangle\langle\hat{\sigma}_j^+\rangle + \langle\hat{a}^\dagger\rangle\langle\hat{\sigma}_j^-\rangle\right).$$

The cavity and atomic detunings θ and Δ are defined as

$$\theta = \frac{\omega_c - \omega_l}{\kappa} \;\text{ and }\; \Delta = \frac{\omega_a - \omega_l}{\gamma_\perp}\ .$$

$$\dot{x} = \kappa\left(+2Cp + y - (i\Theta + 1)x\right)$$
$$\dot{p} = \gamma\left(-(1 + i\Delta)p + xD\right)/2$$
$$\dot{D} = \gamma\left(2(1 - D) - (x^*p + xp^*)\right)$$

$$\Theta = \frac{\omega_c - \omega_l}{\kappa}; \quad \Delta = \frac{\omega_a - \omega_l}{\gamma/2}$$

γ is the rate of spontaneous emission (energy decay)

κ is the rate of escape of the field

$\omega_{a,c,l}$ refer to atom, cavity, laser

Low intensity $x<<1$: with $D=0$, resonant $\Delta=0$ and $\Theta=0$ weakly driven.

Two coupled oscillators

$$\dot{x} = \kappa(-x + 2Cp + y)$$

$$\dot{p} = \gamma(-p - x)$$

Steady state

$$y = x - 2Cp$$

$$p = -x$$

$$y = x(1 + 2C)$$

$$\kappa >> \gamma \quad \dot{p} = -\gamma(1 + 2C)p - \gamma y$$

$$\gamma >> \kappa \quad \dot{x} = -\kappa(1 + 2C)x + \kappa y$$

Enhanced emission

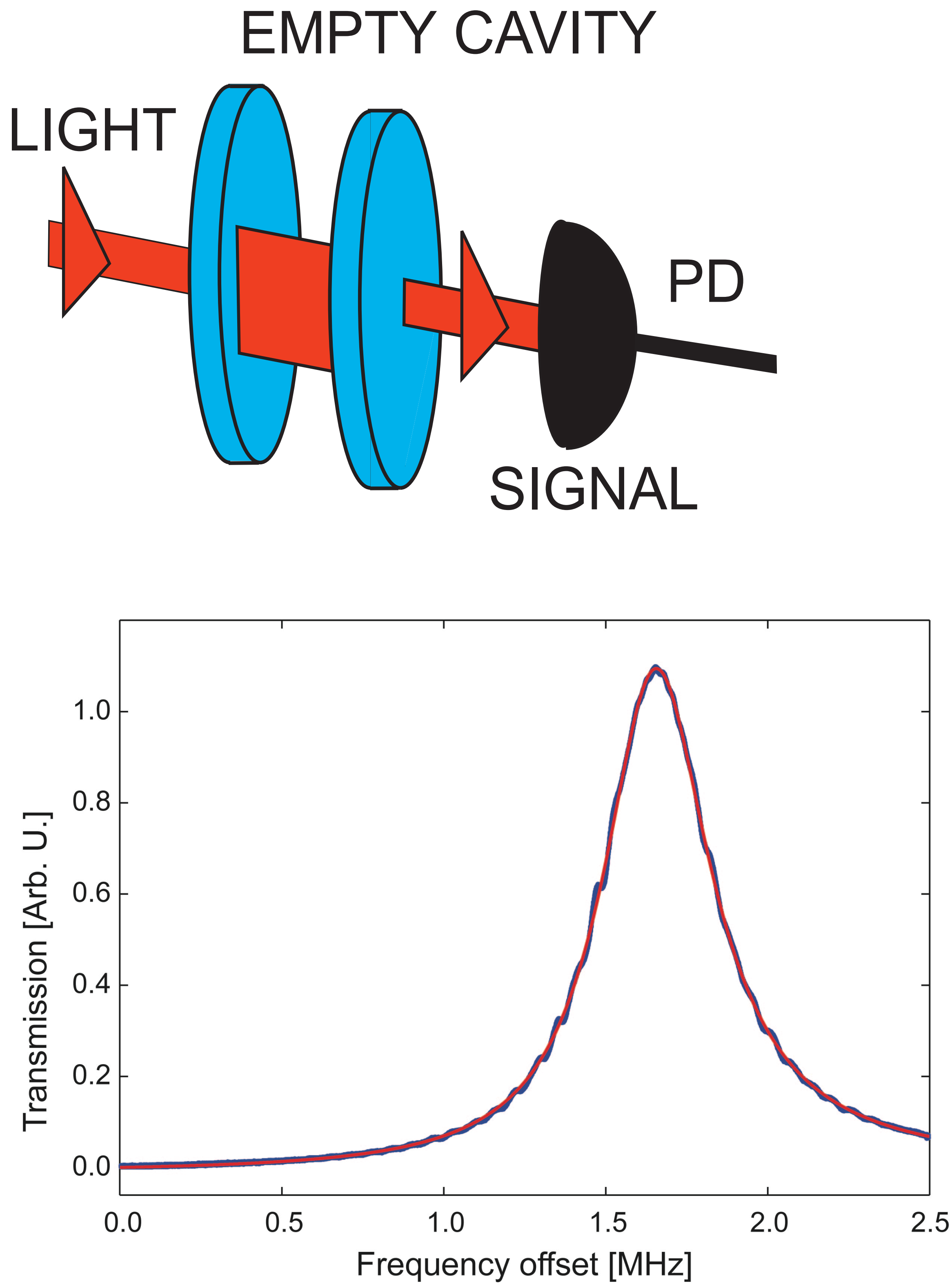
Steady state with detuning and at all intensities:

$$y = x\left(1 + \frac{2C}{1+\Delta^2+|x|^2}\right) + ix\left(\theta - \frac{2C\Delta}{1+\Delta^2+|x|^2}\right)$$

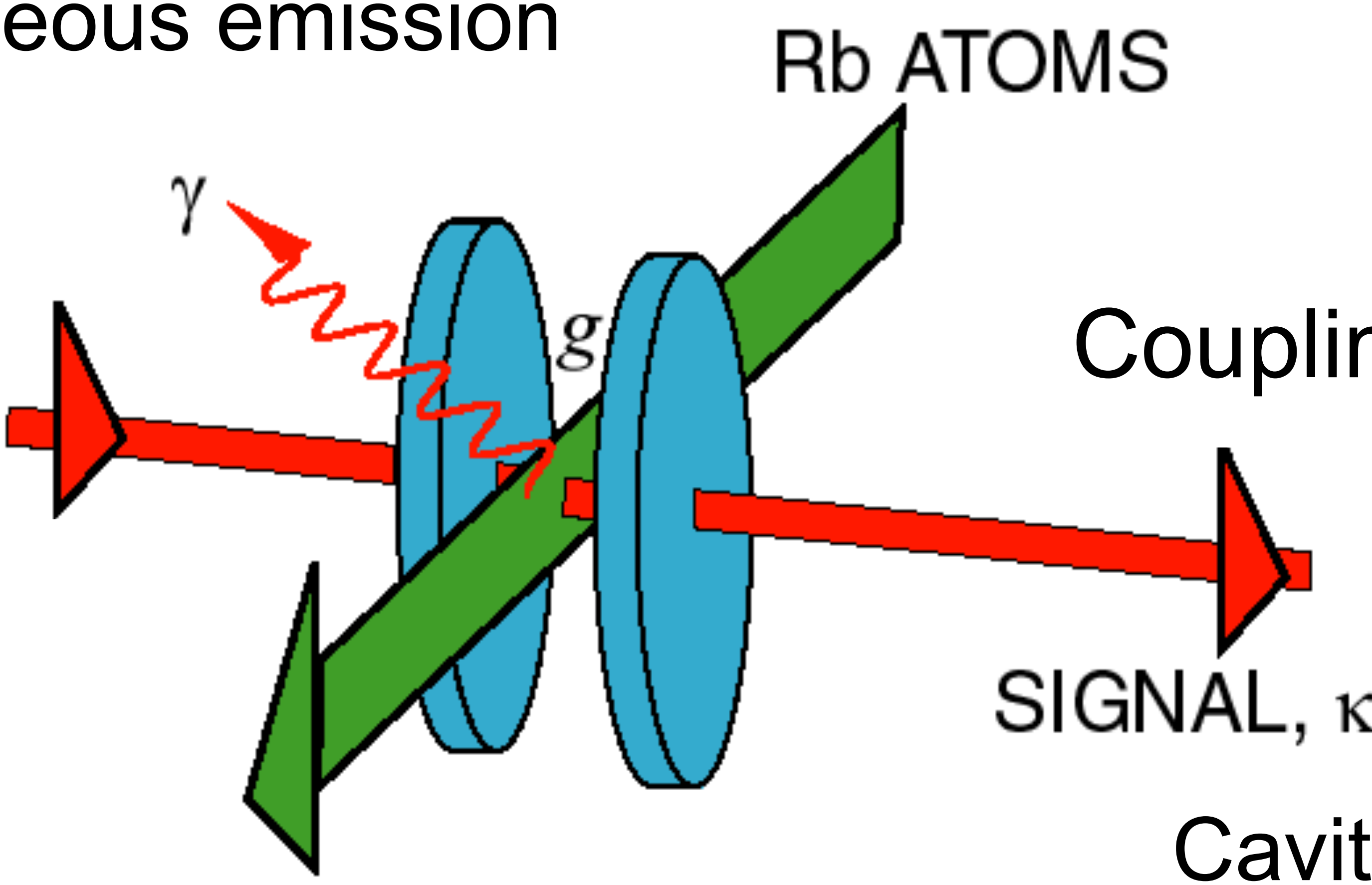
Dispersive limit when Θ=0 and Δ >> 1 :

$$y = -ix\frac{2C\Delta}{1+\Delta^2+|x|^2}$$

The system shows rich dynamical instabilities in the high intensity


EMPTY CAVITY
LIGHT
PD
SIGNAL
Transmission [Arb. U.]
Frequency offset [MHz]
0.0
0.2
0.4
0.6
0.8
1.0
0.0
0.5
1.0
1.5
2.0
2.5

Spontaneous emission



Cavity decay

Cooperativity for one atom: C_1

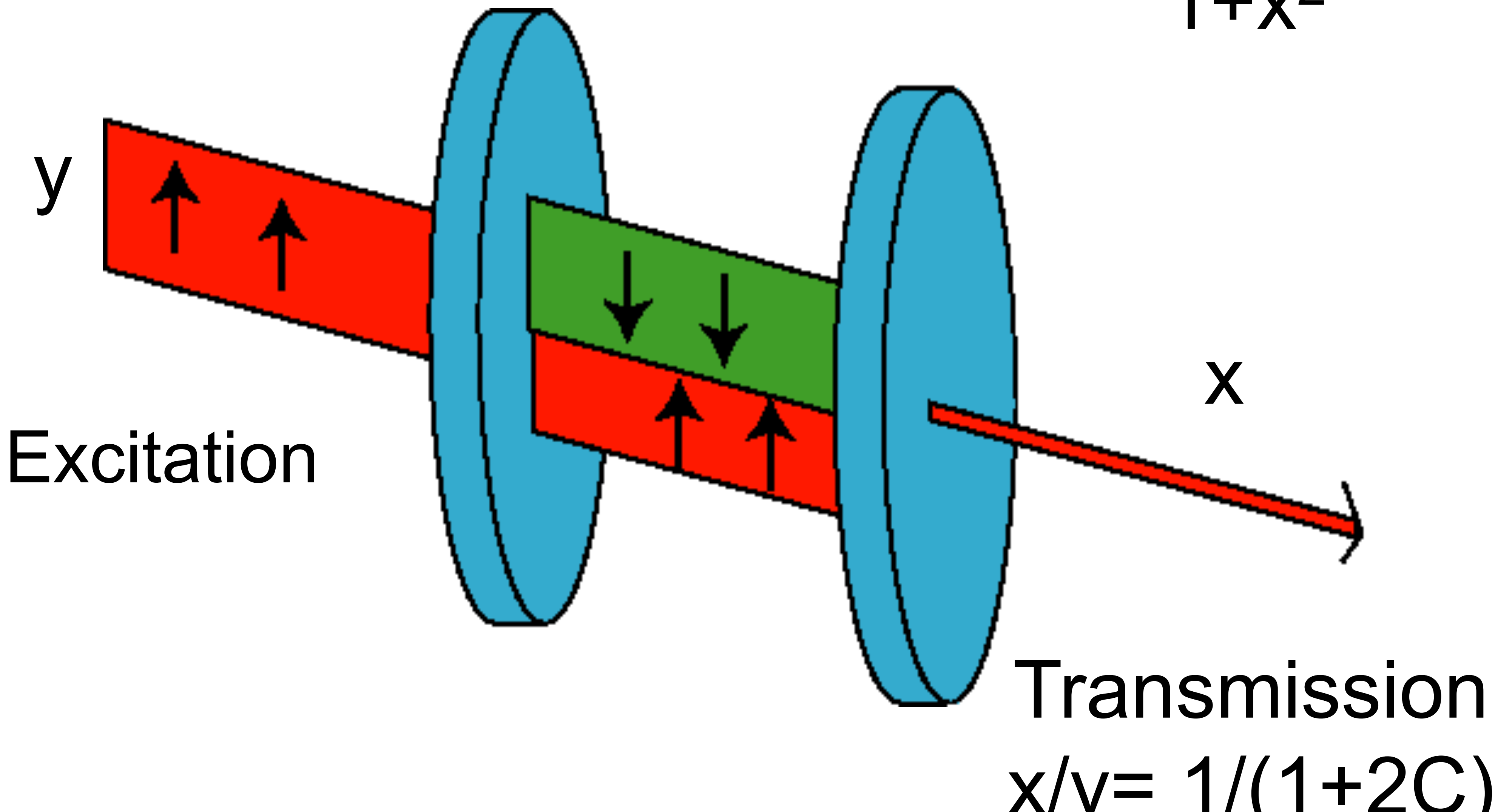
$$C_1 = \frac{g^2}{\kappa\gamma} \qquad C = C_1 N$$

Cooperativity for N atoms: C

$$g \approx \kappa \approx \gamma$$

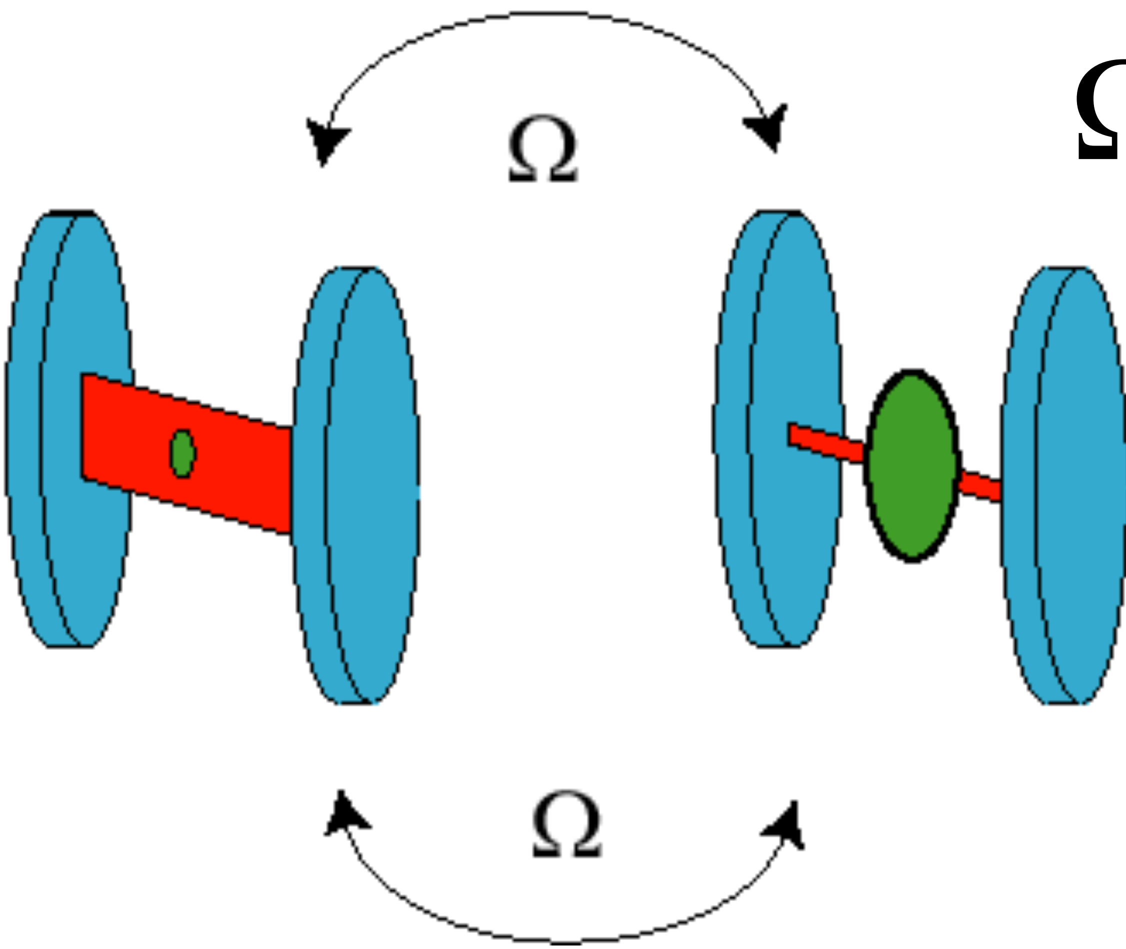
Steady State

Atomic polarization: $\dfrac{-2Cx}{1+x^2}$



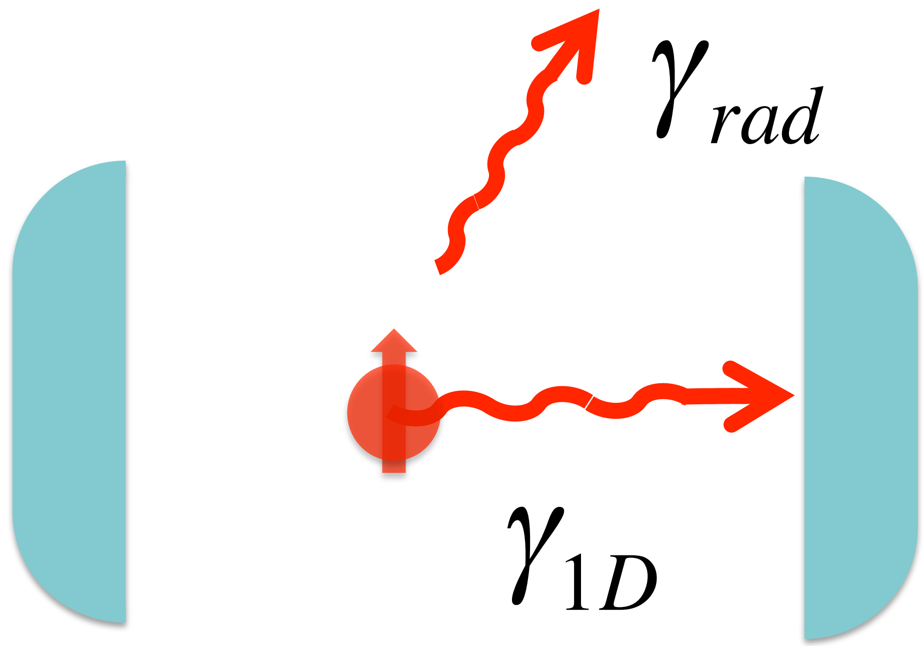
Jaynes Cummings Dynamics
Rabi Oscillations

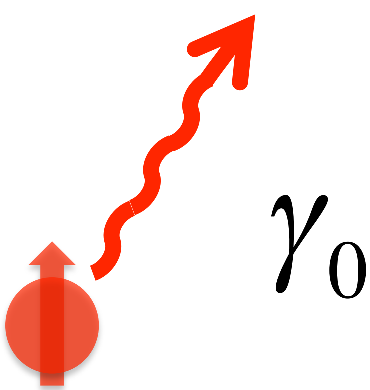
Exchange of excitation for *N* atoms:



$$\Omega \approx g\sqrt{N}$$

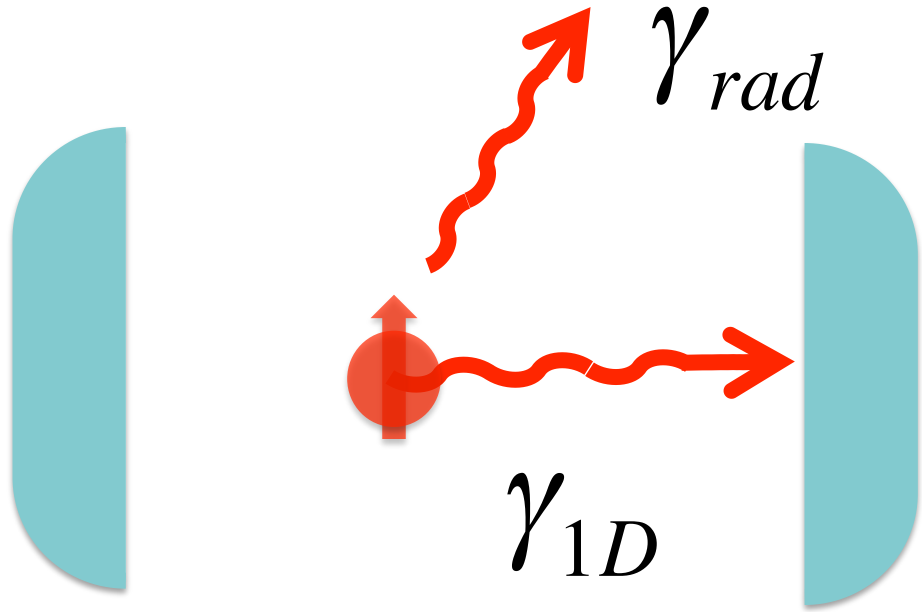
Coupling Enhancement

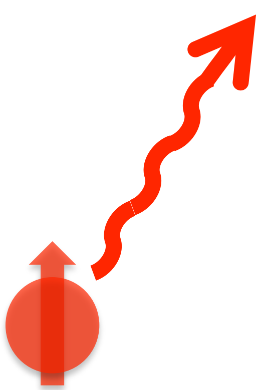




$$\alpha = \frac{\gamma_{1D}}{\gamma_0}$$

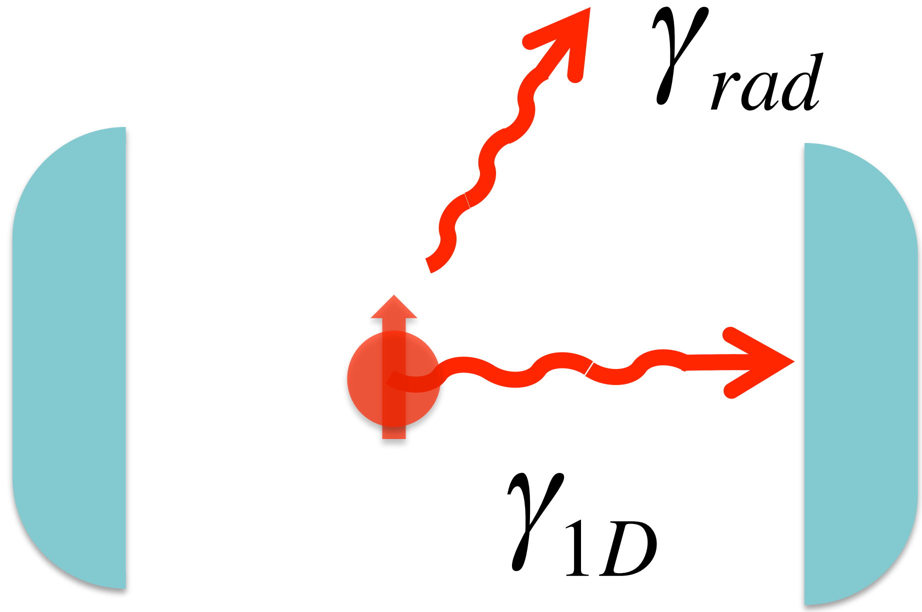
Coupling Efficiency

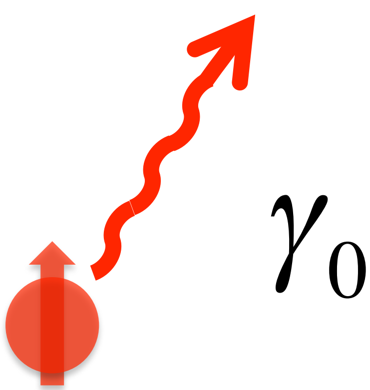




$$\beta = \frac{\gamma_{1D}}{\gamma_{Tot}} \quad ; \quad \gamma_{Tot} = \gamma_{1D} + \gamma_{rad}$$

Purcell Factor

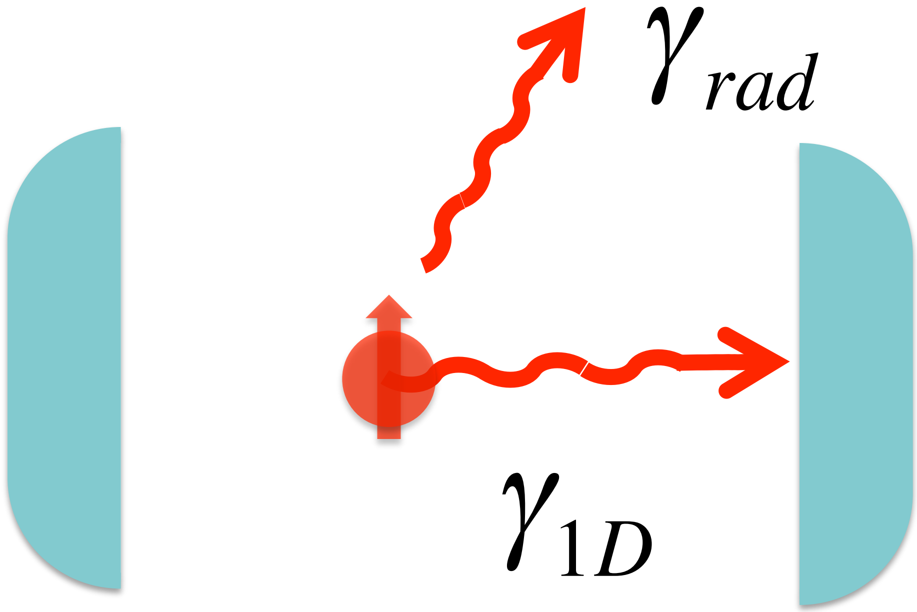


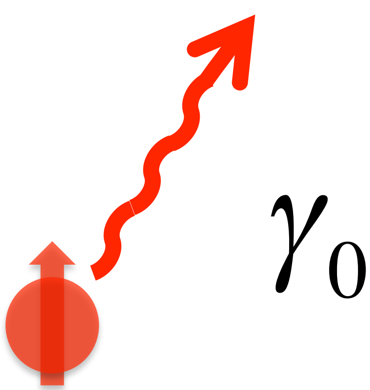


$$F_P = \frac{\gamma_{tot}}{\gamma_0} = \frac{\alpha}{\beta}$$

$$\gamma_{Tot} = \gamma_{1D} + \gamma_{rad}$$

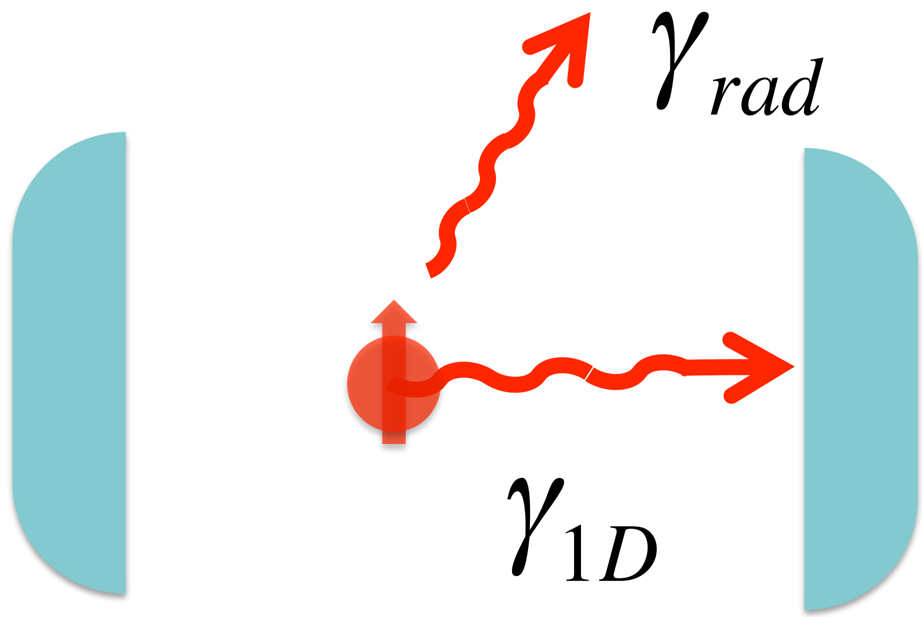
Cooperativity

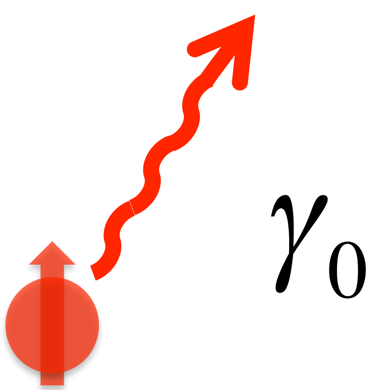




$$C_1 = \frac{\beta}{(1-\beta)} = \frac{\gamma_{1D}}{\gamma_{rad}}$$

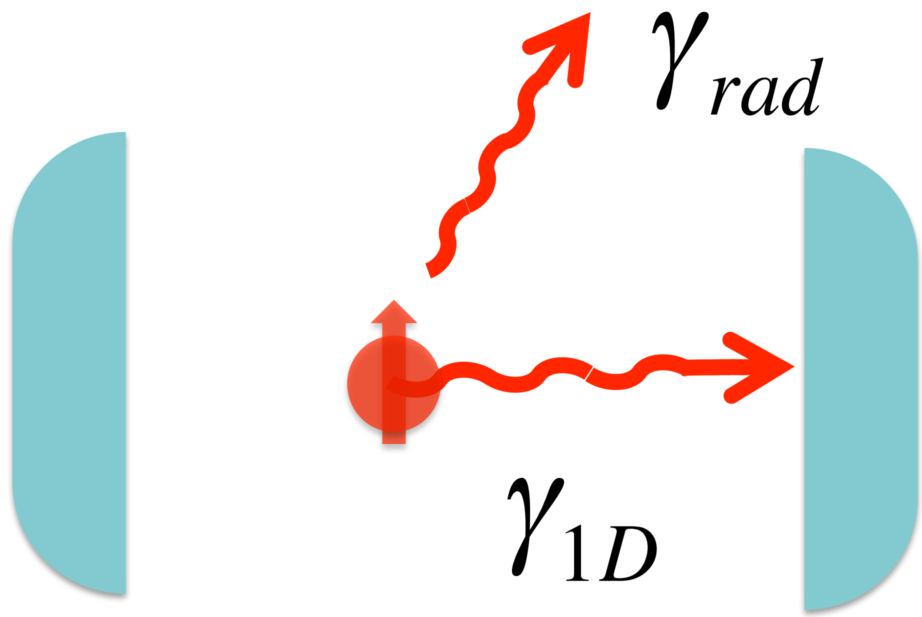
Cooperativity

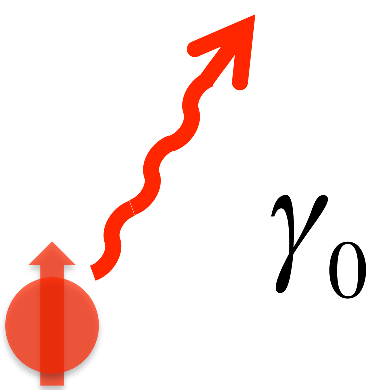




$$C_1 = \frac{g^2}{\kappa \gamma_0}$$

Cooperativity





$$C_1 = \frac{\sigma_0}{Area_{\text{mode}}} \frac{1}{T}$$

The coupling enhancement α is proportional to the total number of photons emitted into the cavity mode,

The coupling efficiency β is the percentage of photons emitted into the mode relative to the total number of emitted photons.

The cooperativity is the ratio between the photons going into the mode and those emitted out to other modes. It is the inverse of the number of atoms that are necessary to observe non-linear effects in the cavity.

Some Implementations

- Rydbergs on Superconducting cavities (Microwaves)
- Alkali atoms on Optical Cavities (Optical)
- Quantum dots on microcavities (Optical)
- Trapped ions and vibrational mode (phonons)
- Circuit QED Superconducting qbits on microwave resonators (Microwaves)
- Polaritons on optical microcavities (photons)

The cooperativity has become the figure of merit for many quantum optics experiments, it is not limited to cavity QED.

How to choose a platform?

$$C = \frac{\sigma_0}{Area_{\mathrm{mode}}} \frac{1}{T} N \qquad\qquad C = \frac{g^2}{\kappa\gamma_0} N$$

Take the area of the mode to be $\pi(\lambda/2)^2$, and σ as $3\lambda^2/2\pi$ then C does not depend on the "atom"

Another approach is to maximize *g* through a large $E_{0,}$ then minimize the cavity volume *V*

The solutions are guided by your resources and where you can approach the ideals

Microwaves can be confined to cavities with mode areas close to the atomic cross section of the Rydberg Atoms. (Experiments led by S. Haroche)

This is more difficult in the visible for free space with atoms, but recent developments at ENS on making micrometric mirrors are helping.

Quantum Hamiltonian for *N* atoms

$$\hat{H} = \hat{H}_1 + \hat{H}_1 + \hat{H}_2 + \hat{H}_3 + \hat{H}_4 + \hat{H}_5 \ ,$$

$$\hat{H}_1 = \hbar\omega_c \hat{a}^\dagger \hat{a} + \frac{1}{2}\hbar\omega_a \sum_{j=1}^{N} \hat{\sigma}_j^z \ ,$$

Free atoms free field

J.C

$$\hat{H}_2 = i\hbar \sum_{j=1}^{N} g_j \left(\hat{a}^\dagger \hat{\sigma}_j^- e^{-i\vec{k}\cdot\vec{r}_j} - \hat{a}\hat{\sigma}_j^+ e^{i\vec{k}\cdot\vec{r}_j} \right)$$

Interaction

$$\hat{H}_3 = \sum_{j=1}^{N} \left(\hat{\Gamma}_A \hat{\sigma}_j^+ + \hat{\Gamma}_A^\dagger \hat{\sigma}_j^- \right) \ ,$$

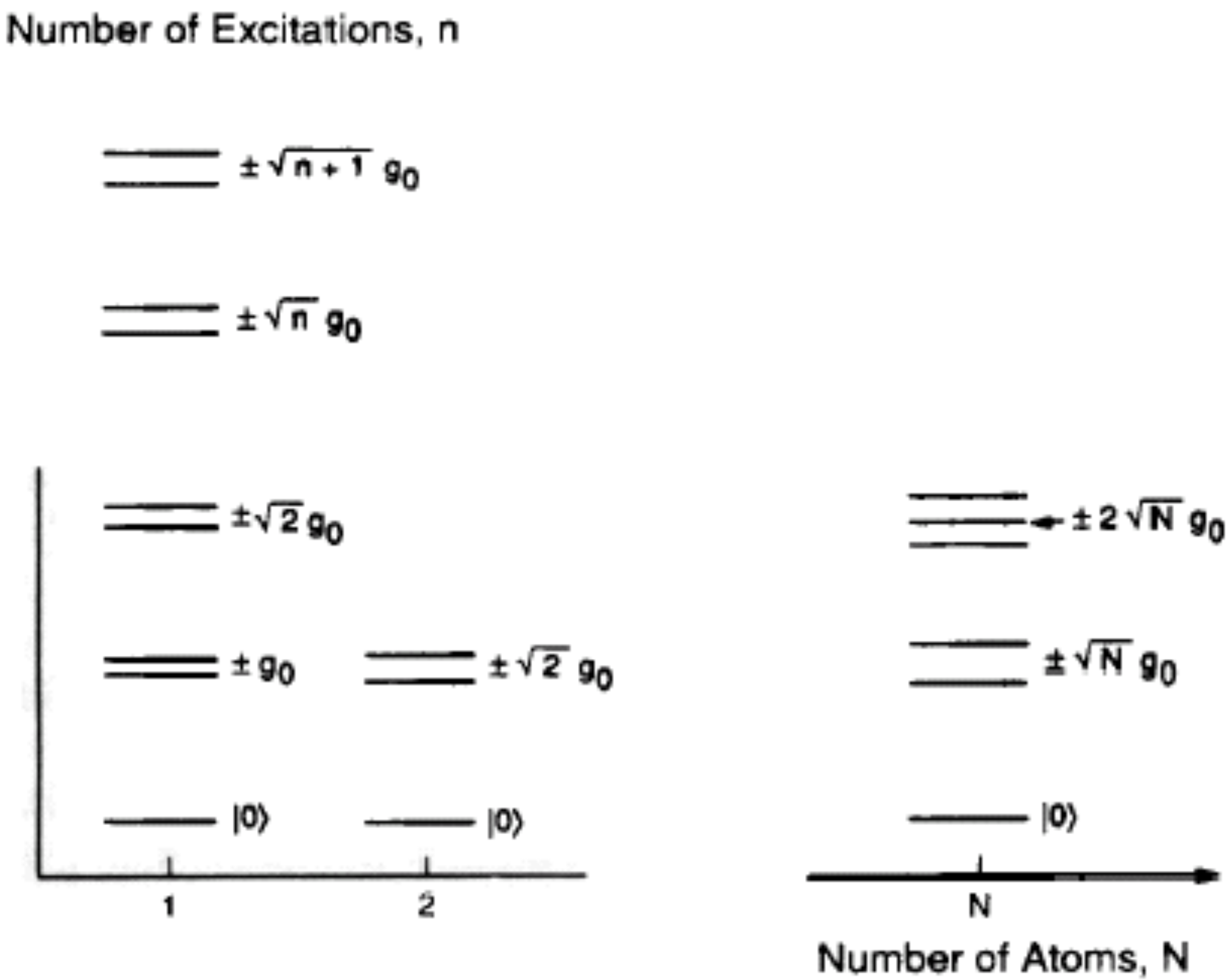
Atomic decay

$$\hat{H}_4 = \hat{\Gamma}_F \hat{a}^\dagger + \hat{\Gamma}_F^\dagger \hat{a} \ ,$$

Cavity decay

$$\hat{H}_5 = i\hbar \left(\hat{a}^\dagger \mathcal{E} e^{-i\omega_l t} - \hat{a}\mathcal{E}^* e^{i\omega_l t} \right) \ .$$

Drive



Number of Excitations, n
$\pm\sqrt{n+1}\,g_0$
$\pm\sqrt{n}\,g_0$
$\pm\sqrt{2}\,g_0$
$\pm g_0$
$\pm\sqrt{2}\,g_0$
$|0\rangle$
$|0\rangle$
1
2
$\pm 2\sqrt{N}\,g_0$
$\pm\sqrt{N}\,g_0$
$|0\rangle$
N
Number of Atoms, N

Quantum Rabi Oscillation: A Direct Test of Field Quantization in a Cavity

M. Brune, F. Schmidt-Kaler, A. Maali, J. Dreyer, E. Hagley, J.M. Raimond, and S. Haroche
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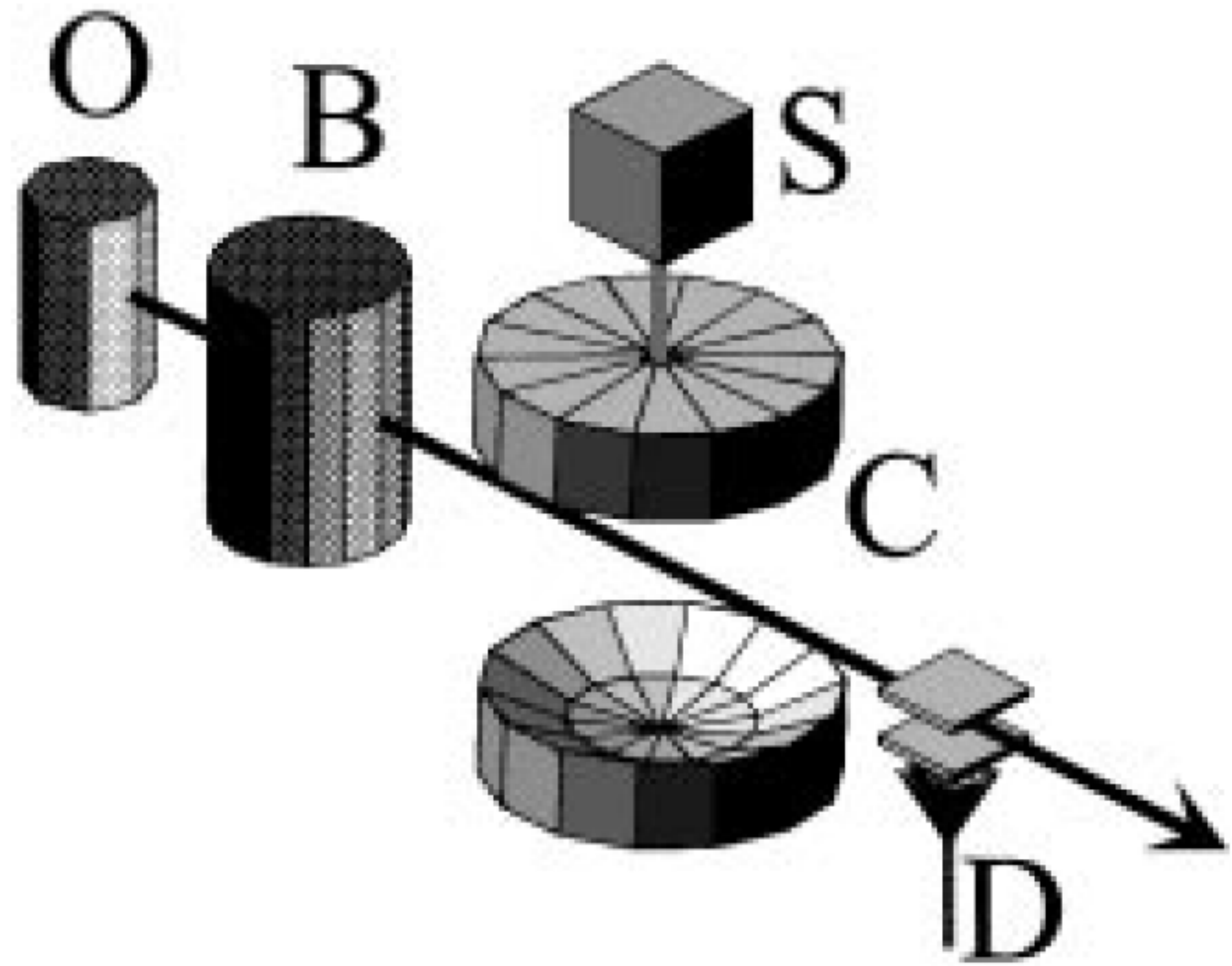
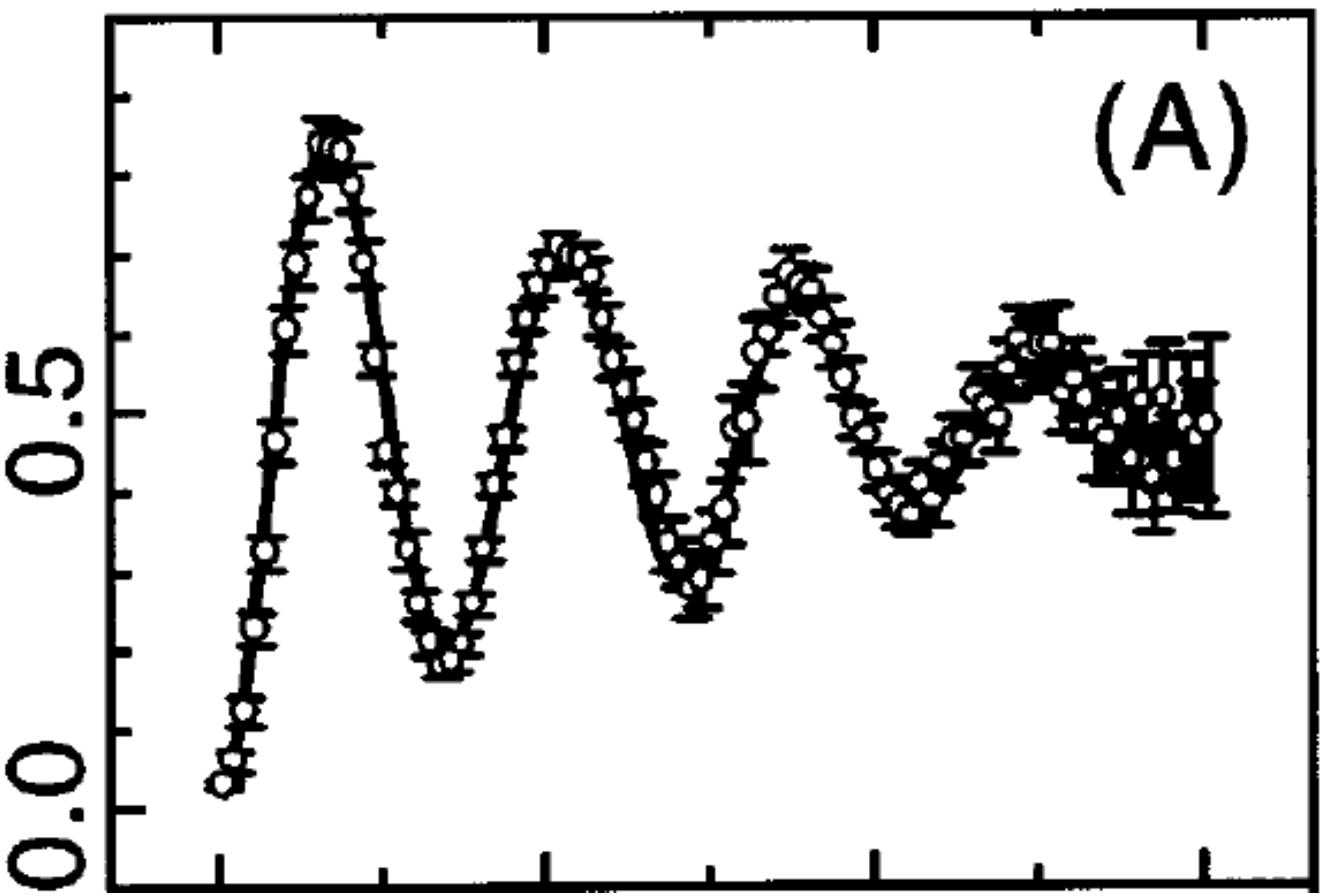
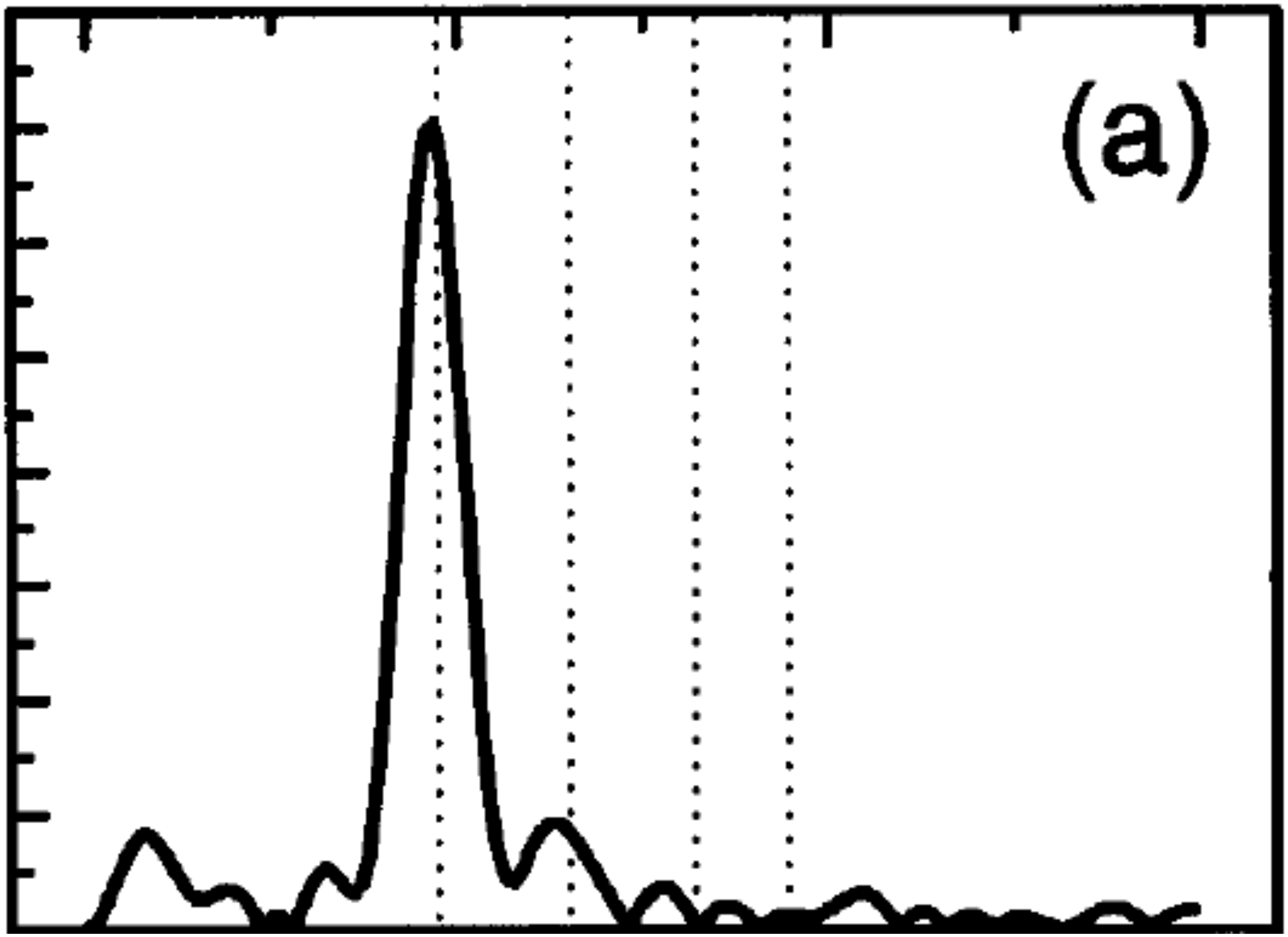
(Received 9 November 1995)

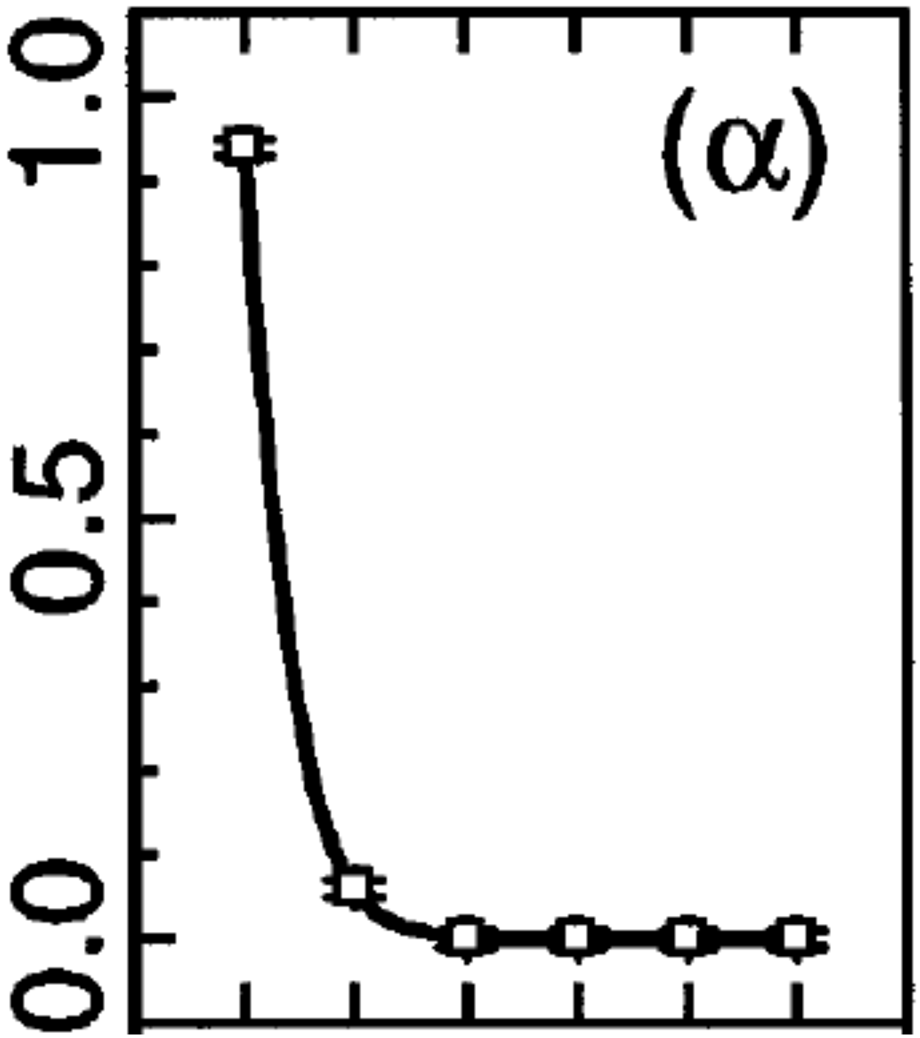
We have observed the Rabi oscillation of circular Rydberg atoms in the vacuum and in small coherent fields stored in a high Q cavity. The signal exhibits discrete Fourier components at frequencies proportional to the square root of successive integers. This provides direct evidence of field quantization in the cavity. The weights of the Fourier components yield the photon number distribution in the field. This investigation of the excited levels of the atom-cavity system reveals nonlinear quantum features at extremely low field strengths.


O
B
S
C
D



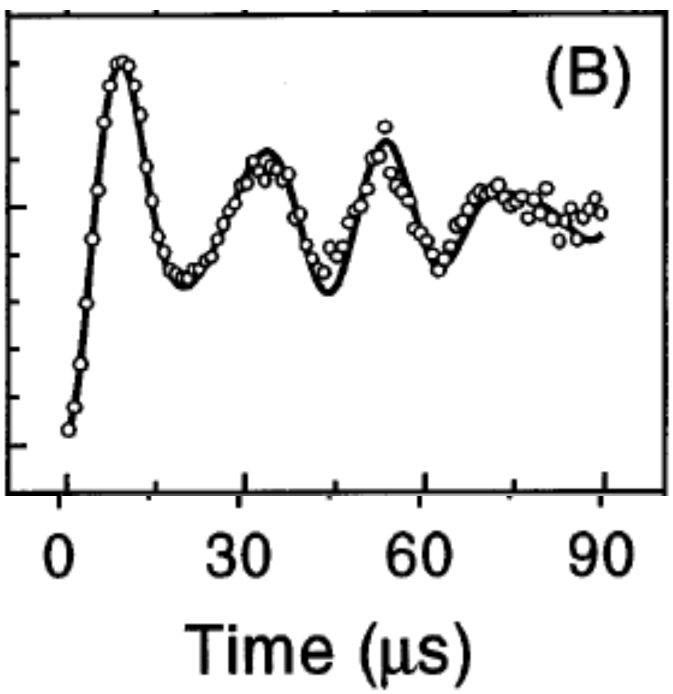
(A)
0.5
0.0


(a)

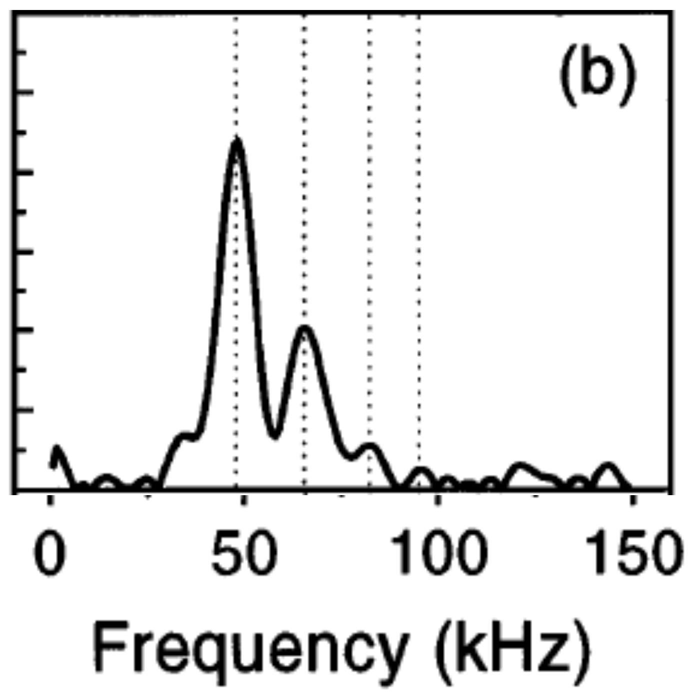

(α)
1.0
0.5
0.0

For two photons

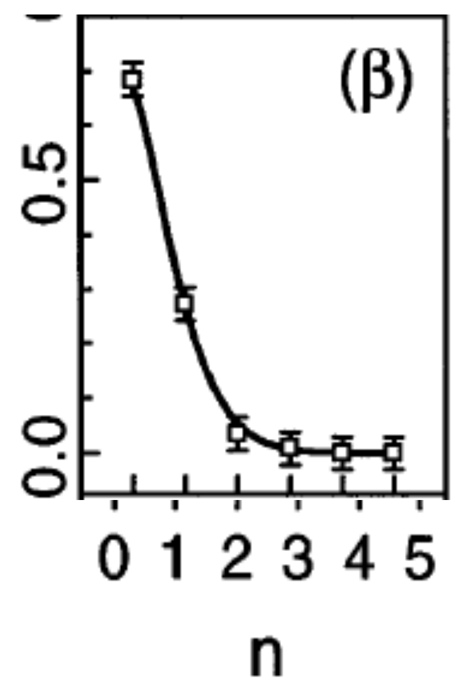
Oscillations



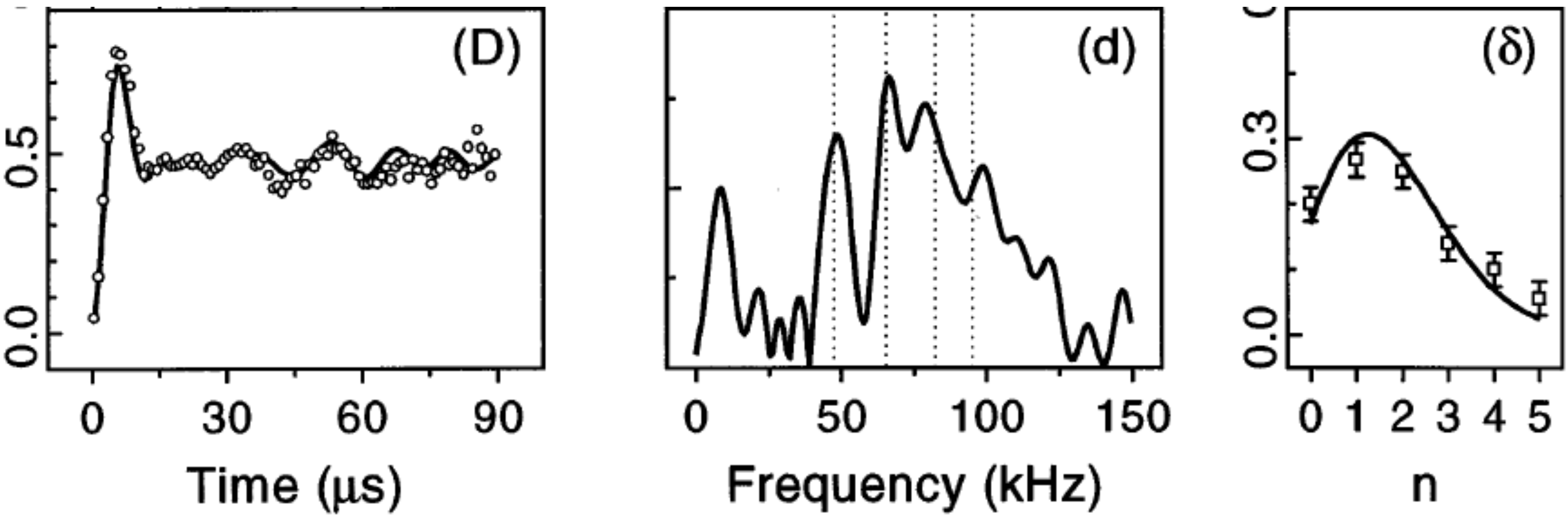
Fourier Transform



P(n)



For four photons



LETTERS

Nonlinear spectroscopy of photons bound to one atom

I. SCHUSTER, A. KUBANEK, A. FUHRMANEK, T. PUPPE, P. W. H. PINKSE, K. MURR AND G. REMPE*
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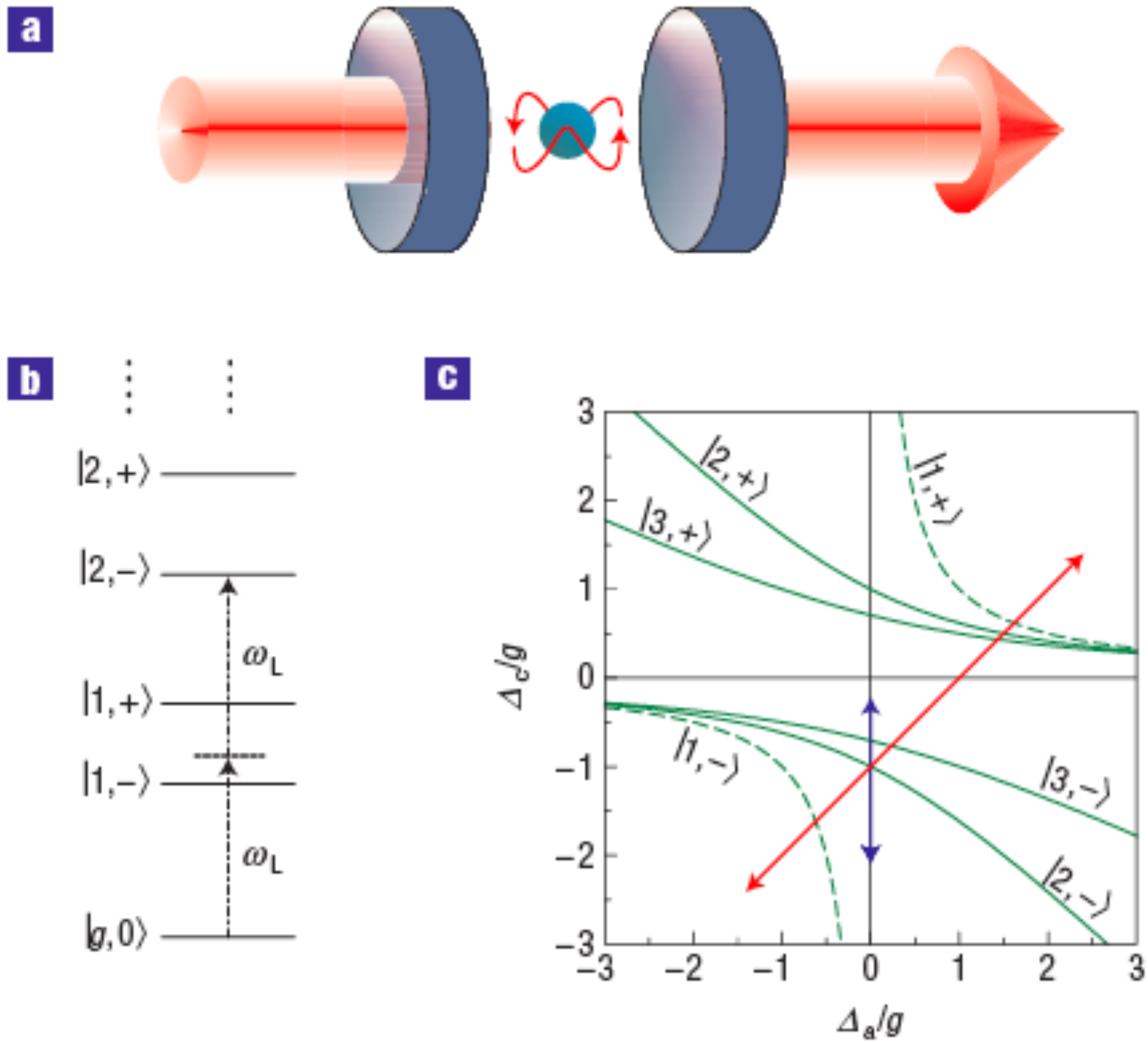
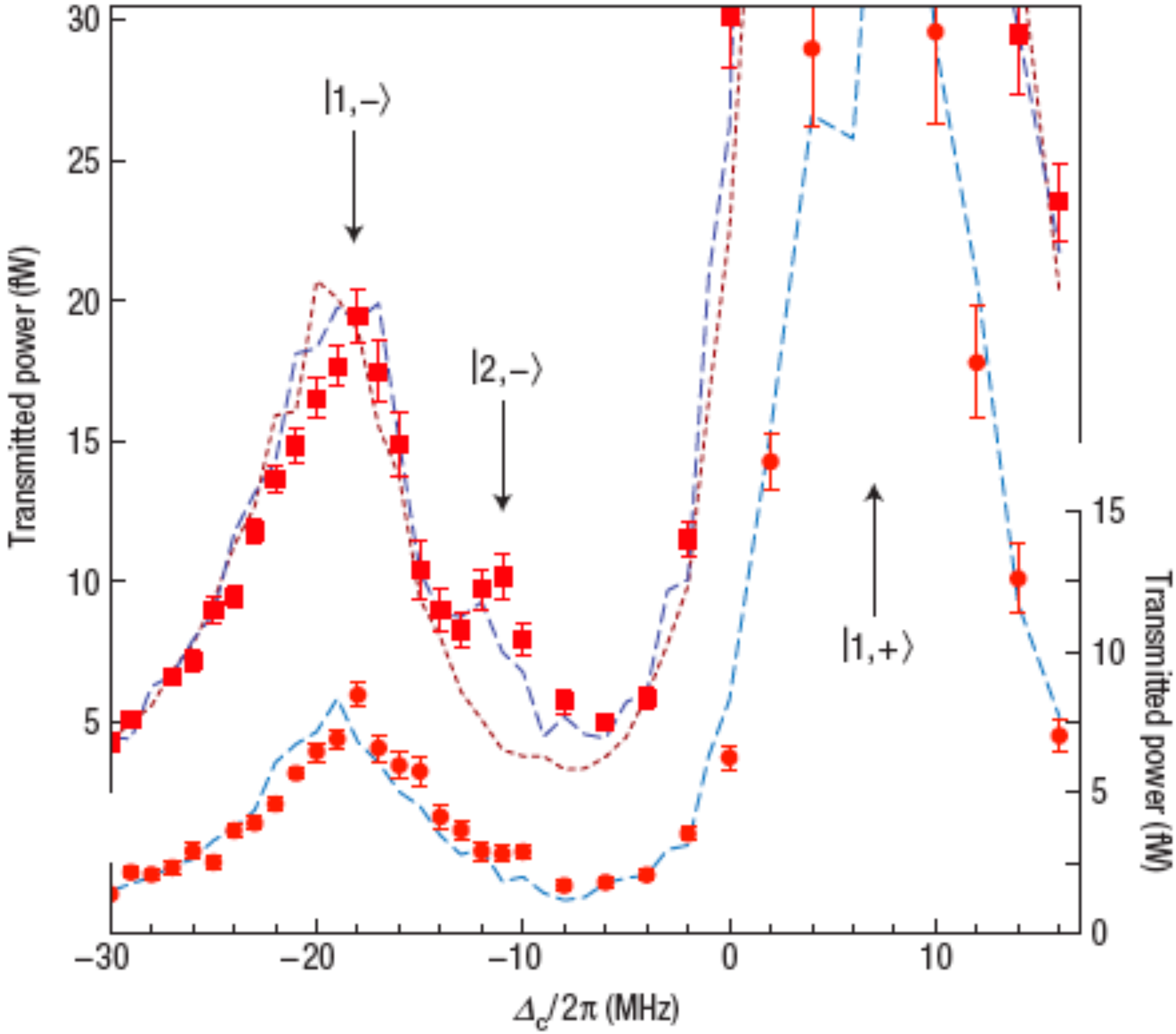

a
b
|2,+⟩
|2,−⟩
|1,+⟩
|1,−⟩
|g,0⟩
ω_L
ω_L
c
|2,+⟩
|3,+⟩
|1,+⟩
|1,−⟩
|3,−⟩
|2,−⟩
Δ_c/g
Δ_a/g
3
2
1
0
−1
−2
−3
−3
−2
−1
0
1
2
3



|1,−⟩
|2,−⟩
|1,+⟩
Transmitted power (fW)
Transmitted power (fW)
Δc/2π (MHz)
30
25
20
15
10
5
15
10
5
0
−30
−20
−10
0
10

nature physics

LETTERS

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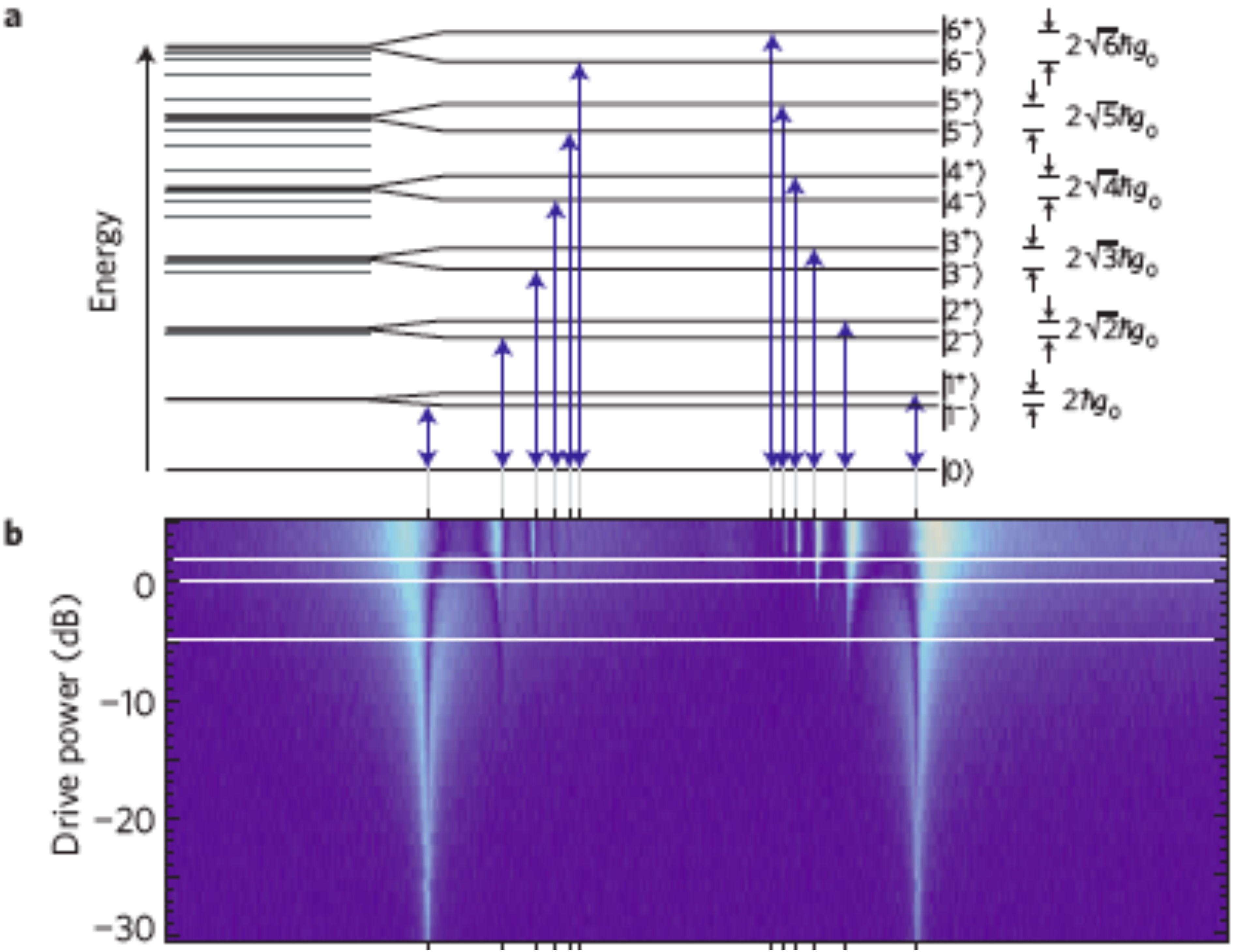
Nonlinear response of the vacuum Rabi resonance

Lev S. Bishop[1], J. M. Chow[1], Jens Koch[1], A. A. Houck[1], M. H. Devoret[1], E. Thuneberg[2], S. M. Girvin[1] and R. J. Schoelkopf[1]*


a
b
Energy
|6⁺⟩
|6⁻⟩
|5⁺⟩
|5⁻⟩
|4⁺⟩
|4⁻⟩
|3⁺⟩
|3⁻⟩
|2⁺⟩
|2⁻⟩
|1⁺⟩
|1⁻⟩
|0⟩
$2\sqrt{6}\hbar g_0$
$2\sqrt{5}\hbar g_0$
$2\sqrt{4}\hbar g_0$
$2\sqrt{3}\hbar g_0$
$2\sqrt{2}\hbar g_0$
$2\hbar g_0$
Drive power (dB)
0
−10
−20
−30

Photon Antibunching and Squeezing for a Single Atom in a Resonant Cavity

H. J. Carmichael

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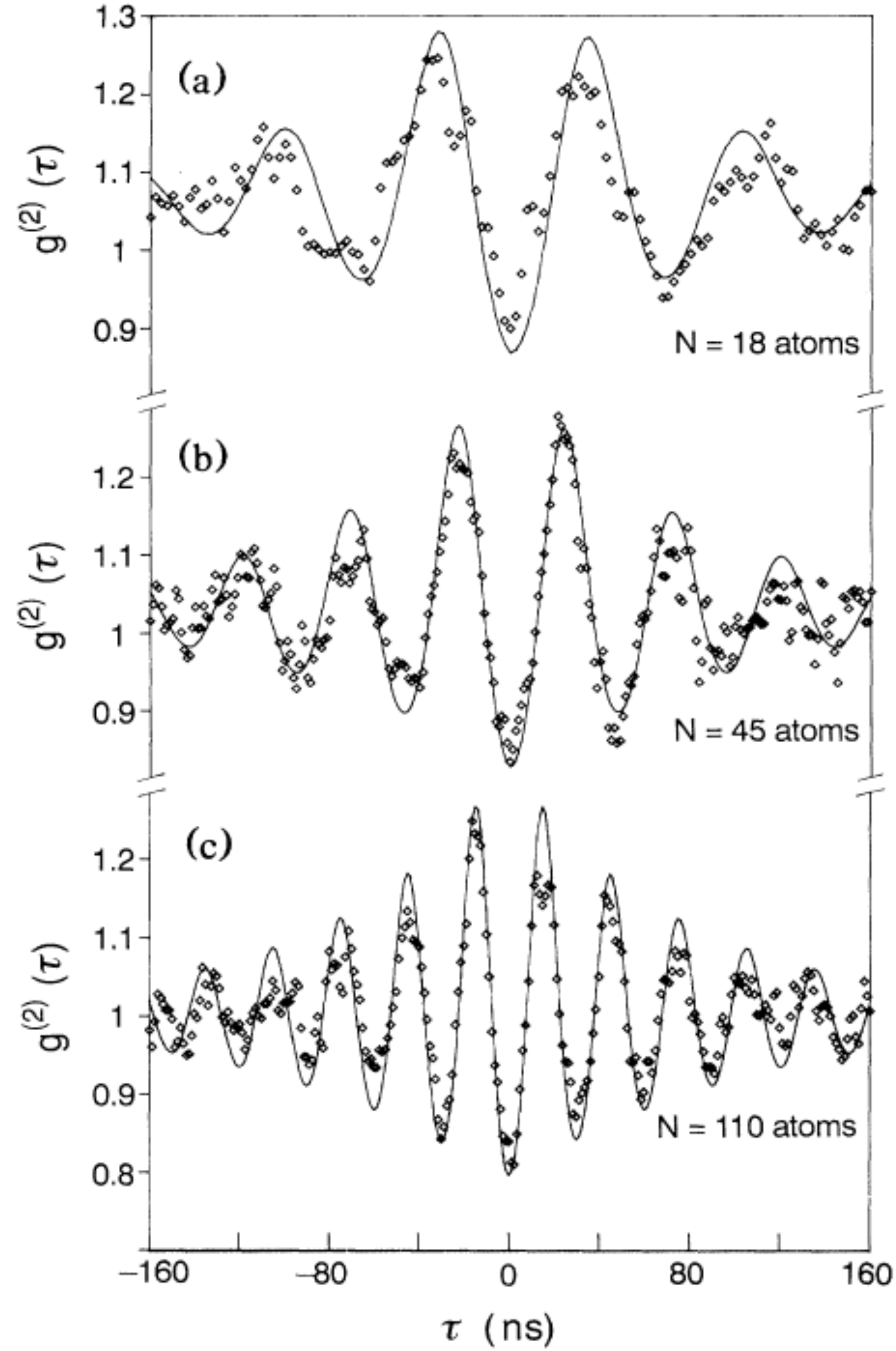
(Received 12 August 1985)

The transmitted light from an optical cavity containing a single two-level atom may show photon antibunching and squeezing. The two effects are closely related and simply understood in terms of the theory of single-atom resonance fluorescence. It follows that corresponding nonclassical effects in optical bistability do not originate in atomic collectivity.

Optical Bistability and Photon Statistics in Cavity Quantum Electrodynamics

G. Rempe, R. J. Thompson, R. J. Brecha,[a] W. D. Lee,[b] and H. J. Kimble
Norman Bridge Laboratory of Physics 12-33, California Institute of Technology, Pasadena, California 91125
(Received 28 June 1991)

The quantum statistical behavior of a small collection of N two-state atoms strongly coupled to the field of a high-finesse optical cavity is investigated. Input-output characteristics are recorded over the range $3 \lesssim N \lesssim 65$, with bistability observed for $N \gtrsim 15$ intracavity atoms and for a saturation photon number $n_0 \simeq 0.8$. For weak excitation the transmitted field exhibits photon antibunching as a nonclassical manifestation of state reduction and quantum interference with the magnitude of the nonclassical effects largely independent of N.



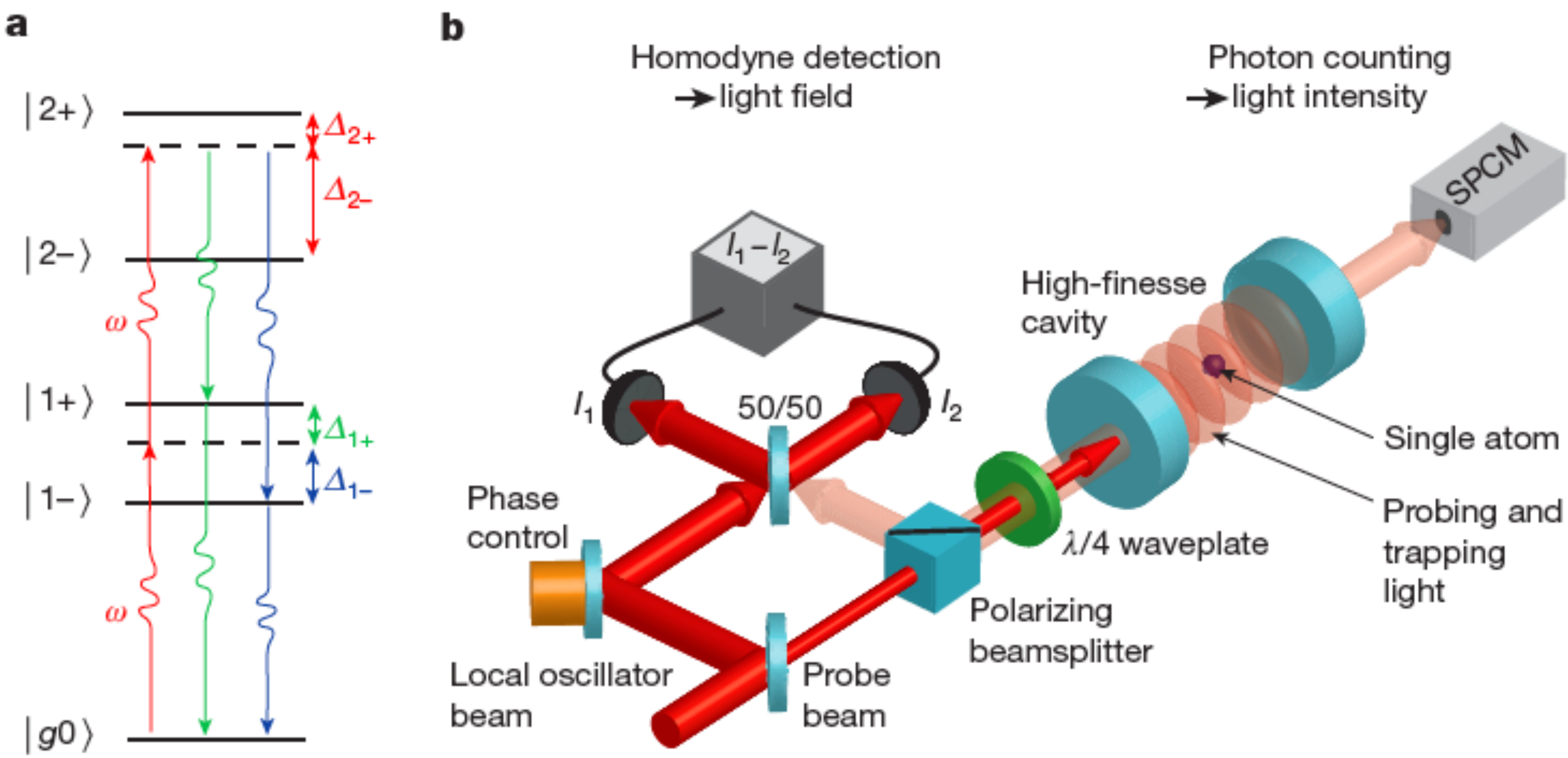
1.3
1.2
1.1
1
0.9
(a)
N = 18 atoms
(b)
N = 45 atoms
(c)
N = 110 atoms
0.8
$g^{(2)}(\tau)$
–160
–80
0
80
160
τ (ns)

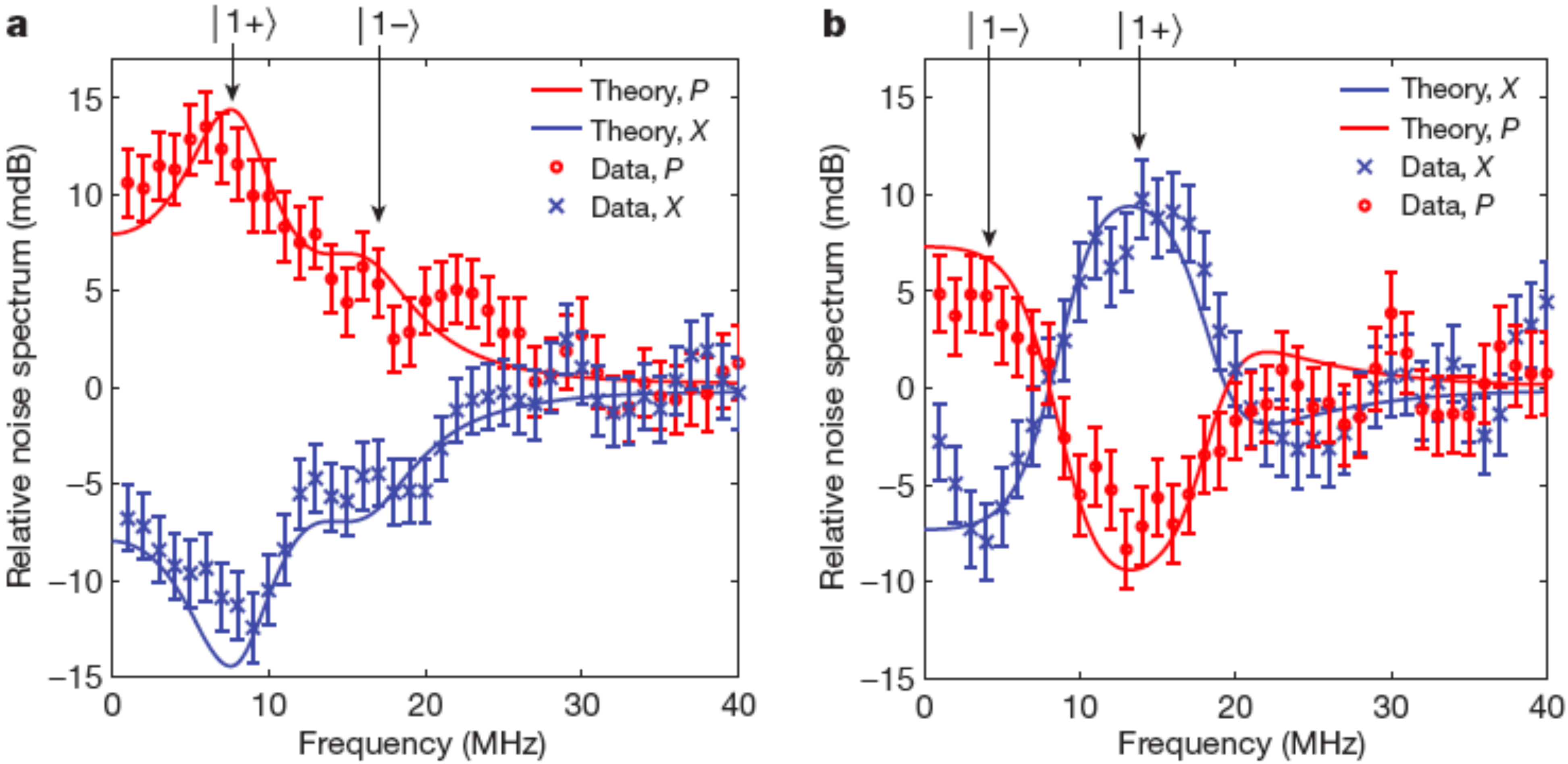
LETTER

doi:10.1038/nature10170

Observation of squeezed light from one atom excited with two photons

A. Ourjoumtsev[1,2], A. Kubanek[1], M. Koch[1], C. Sames[1], P. W. H. Pinkse[1]†, G. Rempe[1] & K. Murr[1]


a
|2+⟩
|2−⟩
|1+⟩
|1−⟩
|g0⟩
Δ2+
Δ2−
Δ1+
Δ1−
ω
ω
b
Homodyne detection
→light field
Photon counting
→light intensity
SPCM
I1 − I2
High-finesse
cavity
I1
50/50
I2
Single atom
Phase
control
λ/4 waveplate
Probing and
trapping
light
Polarizing
beamsplitter
Local oscillator
beam
Probe
beam



a
|1+⟩
|1−⟩
Theory, P
Theory, X
Data, P
Data, X
15
10
5
0
−5
−10
−15
0
10
20
30
40
Relative noise spectrum (mdB)
Frequency (MHz)
b
|1−⟩
|1+⟩
Theory, X
Theory, P
Data, X
Data, P
Relative noise spectrum (mdB)
Frequency (MHz)

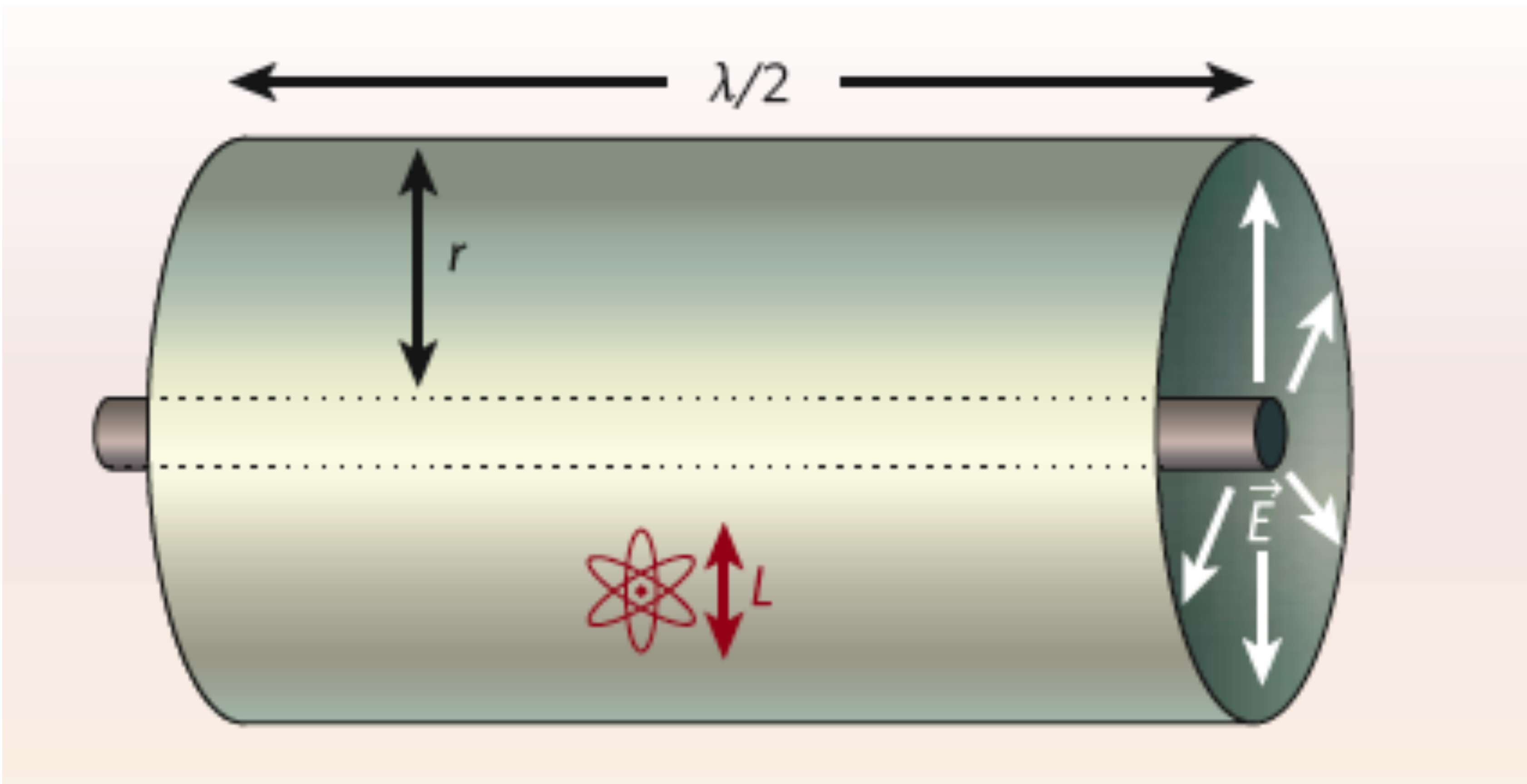
If the interest is just in *g*, the dipole-mode coupling constant then let us look at circuit QED

nature Vol 451|7 February 2008

Wiring up quantum systems

R. J. Schoelkopf and S. M. Girvin

The emerging field of circuit quantum electrodynamics could pave the way for the design of practical quantum computers.



The dipole *d* with characteristic length *L* is in a coaxial cavity of lengh $\lambda/2$ and radius *r*

The coaxial mode volume is much more confined than λ^3

$$g = \frac{dE_\upsilon}{\hbar}; \quad d = eL$$

$$V_{eff} = \pi r^2 \lambda / 2;$$

$$E_\upsilon = \frac{1}{r}\sqrt{\frac{\hbar\omega^2}{2\pi^2\varepsilon_0 c}}$$

$$\frac{g}{\omega} = \left(\frac{L}{r}\right)\sqrt{\frac{e^2}{2\pi^2\varepsilon_0\hbar c}} = \left(\frac{L}{r}\right)\sqrt{\frac{2\alpha}{\pi}}$$

Now the coupling constant can be a percentage of the frequency!

$$\frac{g}{\omega}=\left(\frac{L}{r}\right)\sqrt{\frac{2\alpha}{\pi}}=0.068\left(\frac{L}{r}\right)$$

Be careful as the Jaynes Cummings model may no longer be adequate

Some bibliography:

1. Hyatt M. Gibbs, *Optical Bistability: Controlling Light with Light* (Academic Press, Orlando, 1985).

2. L. A. Lugiato, *Optical Bistability*, in Progress in Optics, edited by E. Wolf (North-Holland, Amsterdam, 1984), Vol. XXI, pp. 69-216.

3. *Cavity Quantum Electrodynamics*, edited by Paul R. Berman, Advances in Atomic Molecular and Optical Physics Supplement 2, Academic Press, Boston 1994.

4. S. Haroche and J. M. Raimond, *Exploring the Quantum: Atoms, Cavities, and Photons* (Oxford Univ. Press, 2006).

5. J. Gripp, S. L. Mielke, L. A. Orozco, and H. J. Carmichael, "Anharmonicity of the Vacuum Rabi Peaks in a Many-Atom System," Phys. Rev. A **54**, R3746 (1996).

6. S. L. Mielke, G. T. Foster, J. Gripp, and L. A. Orozco, "Time Response of a coupled atoms-cavity system," Opt. Lett. **22**, 325 (1997).

7. Gripp, S. L. Mielke, L. A. Orozco, "Evolution of the Vacuum Rabi Peaks in a detuned Atoms-Cavity System," Phys. Rev. A **56**, 3262 (1997).

Merci