

# LASER COOLING AND TRAPPING OF NEUTRAL ATOMS

Luis A. Orozco

Department of Physics and Astronomy  
State University of New York at Stony Brook,  
Stony Brook, NY 11794-3800, United States

## Abstract

The forces felt by atoms when illuminated with resonant radiation can reduce their velocity dispersion and confine them in a region of space for further probing and experimentation. The forces can be dissipative or conservative and allow manipulations of the external degrees of freedom of atoms and small neutral particles. Laser cooling and trapping is now an important tool for many spectroscopic studies. It enhances the density of atoms in phase space by many orders of magnitude reducing the need of large samples. These lecture notes review the fundamental principles of the field and show some of the applications to the study of the spectroscopy of radioactive atoms.

## 1 INTRODUCTION

These notes are based on the lectures I gave at the Escuela Latinoamericana de Física in México City during the summer of 1998. The purpose of the course was to familiarize the participants with the exciting new developments in atomic physics during the last decade. We have gained unprecedented abilities to control the positions and velocities of neutral atoms, that have opened new possibilities in the investigation of their spectroscopy and collective behavior.

There are excellent reviews and summer school proceedings in the literature [1, 2, 3, 4]. In these notes I treat only very general aspects of laser cooling and trapping without the careful detail given in the above reviews. The covered material follows the presentation of Ref. [5]. The aim is to develop an intuitive understanding of the principles and the basic mechanisms for laser cooling and trapping of neutral atoms.

Last century the electromagnetic theory of Maxwell gave a quantitative explanation to the pressure associated with light. This idea was not new, it

had been proposed at least in the XVII century, to explain why comet tails point away from the sun. At the beginning of this century Einstein studied the thermodynamics of emission and absorption of radiation in his paper on blackbody radiation [6]. He remarked on the transfer of momentum in spontaneous emission that ‘the smallness of the impulses transmitted by the radiation field implies that these can almost always be neglected in practice’. At that time, given the available light sources, any mechanical effects were extremely difficult to detect. Frisch observed the deflection of an atomic beam of Na by resonant light from a Na lamp in 1933 [7]. Ideas about using light to manipulate atoms and particles continued to appear in the literature and the invention of the laser helped trigger some of them. Hänsch and Schawlow [8] and Wineland and Dehmelt [9] realized that high brightness sources can exert a substantial force on atoms or ions, potentially cooling their velocity distributions. The advent of tunable lasers during the 1970s with very narrow linewidths made pioneering experiments possible. Since then a long list of people have contributed to advances in the development of laser cooling and trapping. Among the spectacular achievements facilitated by the new techniques is the Bose-Einstein condensation of a dilute gas of alkali atoms, (see the lectures of S. Rolston). Finally, the field of laser cooling and trapping received the 1997 Physics Nobel Prize in the persons of Steve Chu, Claude Cohen-Tannoudji and William Phillips [10]. It is possible to say that laser trapping and cooling is now part of the cannon of physics.

In the course of this lectures we will try to understand how to cool and trap neutral atoms using forces derived from the interaction of light with atoms. Section 2 introduces the light forces. Section 3 shows the velocity dependent force and the associated cooling mechanisms. The position dependent force is discussed in section 4. Section 5 shows how the forces combine to form an optical trap. Finally, in section 6 I have included some examples drawn from the work with radioactive neutral atoms where I have been involved.

## 2 THE LIGHT FORCES

The origin of the light force is the momentum transferred when an atom absorbs a photon from a laser beam. The momentum of the atom changes by  $\hbar\mathbf{k}$ , where  $\mathbf{k}$  is the wave vector of the incoming photon. After the emission

of a photon by an atom the atom recoils. The associated recoil velocity  $v_{\text{rec}}$  and recoil energy  $E_{\text{rec}}$  for an atom of mass  $M$  are:

$$v_{\text{rec}} = \frac{\hbar \mathbf{k}}{M}, \quad (1)$$

$$E_{\text{rec}} = \frac{\hbar^2 |\mathbf{k}|^2}{2M}. \quad (2)$$

## 2.1 *Spontaneous emission force*

If the excitation is followed by spontaneous emission, the emission can be in any direction, but because the electromagnetic interaction preserves parity, the emission will be in a symmetric pattern with respect to the incoming photon. In this case the recoil momentum summed over many absorption and emission cycles will average to zero. The atom then gains momentum in the direction of the wave vector of the incoming laser beam. The resulting force is sometimes called Doppler, radiation pressure, scattering, or the spontaneous force. The variance of the momentum transferred does not vanish, and the atom performs a random walk in momentum space as it emits spontaneously. These fluctuations limit the lowest temperature achievable when the laser beam is present.

$$\mathbf{F} = \mathbf{F}_{\text{abs}} + \mathbf{F}_{\text{em}}, \quad (3)$$

$$\mathbf{F} = \langle \mathbf{F}_{\text{abs}} \rangle + \langle \mathbf{F}_{\text{em}} \rangle + \delta \mathbf{F}_{\text{em}} \quad (4)$$

$$\langle \mathbf{F} \rangle = R_{\text{sp}} \hbar \mathbf{k}, \quad (5)$$

where  $\mathbf{F}$  is the force on the atom,  $R_{\text{sp}}$  is the rate of fluorescence scattering in cycles per second, and  $\delta \mathbf{F}$  represents the random fluctuations from the recoiling atoms. The repeated transfer of momentum from a light beam to the atom by absorption and spontaneous emission provides the spontaneous light force.

The mean number of fluorescence cycles per second from a two level atom illuminated by a laser beam near or at resonance with the transition is equal to the population in the excited state times the Einstein A coefficient  $\Gamma$ . To precisely calculate the population it is necessary to include off diagonal elements in the density matrix and solve in steady state the optical Bloch equations (see for example [11]). Here we present the result without deriving

it. The fluorescence depends on the amount of power available for the excitation (governed by the saturation parameter  $S_0$ ) and the full width at half maximum (FWHM)  $\Gamma$  of the Lorentzian lineshape. The radiative lifetime of the transition  $\tau = 1/\Gamma$  is the inverse of the Einstein A-coefficient. The fluorescent rate is:

$$R_{sp}(\Delta) = \frac{\Gamma}{2} \frac{S_0}{1 + S_0 + (2\Delta/\Gamma)^2}, \quad (6)$$

where  $\Delta$  is the laser detuning from resonance,

$$\Delta = \omega_{\text{laser}} - \omega_{\text{atom}}, \quad (7)$$

and the on-resonance saturation parameter  $S_0 = I_{exp}/I_{sat}$  is the ratio between the available intensity  $I_{exp}$  and the saturation intensity  $I_{sat}$ . At  $S_0 = 1$  and on resonance the atom scatters at half of the maximum possible rate. There are different definitions of  $S_0$  in the literature depending on particular definitions of  $I_{sat}$  and the reader has to pay attention to the particular one used. Here we follow the work of Citron *et al.* [12].

$$I_{sat} = \frac{h\pi c\Gamma}{3\lambda^3}. \quad (8)$$

With this definition, an intensity of  $I_{sat}$  corresponds to providing the energy of one photon ( $\hbar\omega$ ) every two lifetimes ( $2/\Gamma$ ) over the area of the radiative cross section of the two-level transition ( $3\lambda^2/2\pi$ ). The rate of fluorescence (see eq. 6) depends on the detuning  $\Delta$  between the atom and the laser.

At low intensities the scattering rate is proportional to the saturation parameter, but as the intensity grows it shows power broadening and the rate saturates at  $1/2\Gamma$ . The FWHM of the Lorentzian goes from  $\Gamma$  at low intensities  $S_0 \ll 1$  to  $\Gamma\sqrt{1+S_0}$  for  $S_0 > 1$ . Power broadening can be thought as arising from the absorption-stimulated emission cycles that do not contribute to the force because the emission is into the same laser beam. The on-resonance atoms are already saturated and it is only those off resonance that can contribute and broaden the width.

The force is small but a two-level atom it returns to the ground state after emission of a photon and can be re-excited by the same laser beam. When such a transition exists in real atoms it is called a cycling transition. Alkali atoms with nuclear spin  $I$  and total angular momentum  $F$  have the

transition from the  $S_{1/2}$  ground state with  $F = I + 1/2$  to the  $P_{3/2}$  state with  $F = I + 3/2$  that satisfies the cycling condition. The excited state can not decay to the other hyperfine level ( $F = I - 1/2$ ) of the ground state because of the  $\Delta F = 0, \pm 1$  selection rule. This transition in the  $D_2$  line is commonly used for trapping alkali atoms. The saturation intensities of their cycling transitions are in the range of  $1 \text{ mW/cm}^2 < I_{\text{sat}} < 10 \text{ mW/cm}^2$ .

## 2.2 *Stimulated emission force*

If the absorption of a photon is followed by stimulated emission into the same laser beam, the outgoing photon will again carry away  $\hbar\mathbf{k}$ , so there is no momentum transferred. However, if the emission is into another laser beam, there is a redistribution of laser photons causing a force proportional to the difference between the two  $\mathbf{k}$  vectors  $\Delta\mathbf{k} = \mathbf{k}_1 - \mathbf{k}_2$ . The absorption and emission are correlated events and they are coherent scattering of photons. This redistribution of momentum is what happens in an optical lens and a positive lens will be drawn towards regions of high intensity as a consequence of the third law of Newton.

To calculate the index of refraction of an atom it is necessary to add the amplitude of the incident light field with the dipole field generated by the driven atomic electrons. An optical field  $\mathbf{E}$  of the light induces a dipole moment  $\mathbf{d}$  on the atom. Considering the electron as a harmonic oscillator, the induced dipole moment can be in phase or out of phase depending on the detuning of the driving frequency with respect to resonance. When it is in phase, the interaction energy between the dipole and the field  $U = -\mathbf{d} \cdot \mathbf{E}$  is lower in high field regions. When it is out of phase  $U$  increases with  $\mathbf{E}$  and a force will eject the atom out of the field. On resonance the oscillator is orthogonal to  $\mathbf{E}$  and there is no force.

If the atom is illuminated only with a plane wave the stimulated force will be zero as all the  $\mathbf{k}$  vectors are the same. A force from stimulated emission needs a gradient in the intensity of the light such that the  $\mathbf{k}$  vectors point in different ways. This force is sometimes called the dipole or stimulated force. A force will act on an induced dipole if there is a gradient in the intensity, it can be attractive or repulsive depending on the drive detuning with respect to resonance. Any material with an index of refraction feels a force in the presence of a gradient of the intensity. The dipole force acts on cells, organelles and even DNA, providing ‘optical tweezers’ for their manipulation

. (See for example the Nobel lecture of S. Chu [10]).

### 3 VELOCITY DEPENDENT FORCE

The spontaneous force  $\mathbf{F}_{\text{spont}}$  is a velocity dependent force because the resonance condition of an atom depends on its velocity  $\mathbf{v}$  through the Doppler shift  $\mathbf{k} \cdot \mathbf{v}$ .

$$\mathbf{F}_{\text{spont}} = \hbar \mathbf{k} \frac{\Gamma}{2} \frac{S_0}{1 + S_0 + (2(\Delta - \mathbf{k} \cdot \mathbf{v})/\Gamma)^2}. \quad (9)$$

This force saturates at  $\hbar k \Gamma / 2$  and is limited by the spontaneous decay time of the atomic level. The force felt by an atom when the intensities are large ( $S_0 \approx 1$ ) are more complicated since stimulated emission is significant. We limit the discussion to the case where those processes are negligible. The velocity range of the force is significant for atoms with velocity such that their Doppler detuning keeps them within one linewidth of the Lorentzian of eq. 9. See Fig. 1. This condition states that:

$$|\Delta - \mathbf{k} \cdot \mathbf{v}| \leq \frac{\Gamma}{2} \sqrt{1 + S_0}. \quad (10)$$

#### 3.1 Deceleration of an atomic beam

The maximum acceleration of a sodium atom interacting with resonant laser light in the  $D_2$  cycling transition shows that light can decelerate an atom in a very short time.

$$\begin{aligned} a_{\text{max}} &= \frac{\hbar \mathbf{k}}{M} \frac{1}{2\tau}, \\ &= \frac{v_{\text{rec}}}{2\tau}, \\ &\approx \frac{3 \times 10^{-2} \text{m/s}^2}{2 \times 16 \times 10^{-9} \text{s}}, \\ &\approx 10^6 \text{m/s}^2, \\ &\approx 10^5 g. \end{aligned} \quad (11)$$

The thermal velocities of atomic beams are in the order of a thousand meters per second, so the stopping time is about one millisecond at  $a_{\text{rmmax}}$ ,

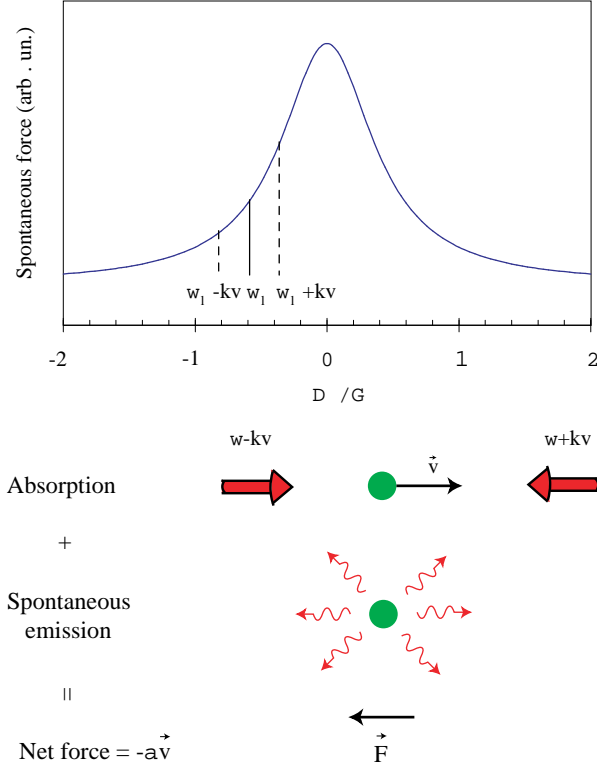


Figure 1: Velocity dependent force

stopping in about one meter. However, these estimates do not consider that the force will be different for atoms with changing velocities through the Doppler effect. The spontaneous force can act on atoms that have a velocity range where the force is significant: A Doppler shift of the order of the linewidth of the transition.

$$v_{\text{Dop}} = \frac{\Gamma}{k}, \quad (12)$$

where  $k = |\mathbf{k}|$ , and for Na  $v_{\text{Dop}} \approx 6$  m/s which is two orders of magnitude smaller than the thermal velocity and three orders of magnitude higher than the recoil velocity.

A laser beam red detuned with respect of the resonant transition and counterpropagating with a beam of atoms at velocity  $v$  can decelerate a velocity class of atoms with a width of  $v_{\text{Dop}}$  and pile them at a lower velocity.

To compensate for the resonant changing transition it is necessary to either tune the energy level of the atom in space or to change the frequency of the laser in time to keep it resonant with a group of atoms while they decelerate. Real atoms have more complications, cycling transitions are not perfect. For example, there is hyperfine structure in alkali atoms and some of the off resonant excitation can optically pump the atom into the non-cycling ground state ( $F=1$  for Na). Then the atom no longer feels the force. The methods developed for deceleration maintain the atom in a cycling transition. They use the selection rules from the polarization of the light in the presence of a magnetic field, take advantage of the Clebsh-Gordan coefficients between the levels, and sometimes require excitation at other frequencies.

### 3.2 Zeeman slowing

One approach to slowing atoms uses the Zeeman effect in a spatially varying magnetic field to tune the atomic energy levels with the changing velocity. The magnetic field is shaped to optimize the match between velocity and Zeeman detuning and keep a strong scattering of photons along the solenoid [13]. The method works if the g-factors of the levels that scale the Zeeman shifts of the ground and excited states are different so that their resonant frequency shifts. The largest ground state  $m$  sub-level in the  $D_2$  line of Na shifts 1.4 MHz/G while the excited state shifts by 2.8 MHz/G. As a result of their difference the magnetic field can shift the transition energy and can compensate for the Doppler shift along the path of a moving atom.

Assuming that the atoms decelerate with a constant acceleration  $a$  from an initial velocity  $v_0$ , the position dependent velocity  $v(z)$  is:

$$v(z) = \sqrt{v_0^2 - 2az}. \quad (13)$$

We take the changing Doppler shift  $\mathbf{k} \cdot \mathbf{v}(z)$  equal it to the Zeeman shift  $\vec{\mu} \cdot \mathbf{B}(z)$ , where  $\vec{\mu}$  is the magnetic moment of the transition, to find the shape of the compensating magnetic field.

$$B(z) = B_0 \sqrt{1 - \frac{z}{z_0}}, \quad (14)$$

$$B_0 = \frac{kv_0}{\mu}, \quad (15)$$



Table 1: Trapping and cooling parameters for alkali atoms from a source at 1000 K.

| Atom | A   | $\lambda_{D_2}$<br>[nm] | $\tau_{D_2}$<br>[nsec] | $T_{Dop.}$<br>[ $\mu$ K] | $l_{Zeeman}$<br>[cm] |
|------|-----|-------------------------|------------------------|--------------------------|----------------------|
| Na   | 23  | 589                     | 16.2                   | 235                      | 40                   |
| K    | 39  | 766                     | 26.3                   | 145                      | 84                   |
| Rb   | 87  | 780                     | 26.2                   | 145                      | 85                   |
| Cs   | 133 | 852                     | 30.4                   | 125                      | 108                  |
| Fr   | 210 | 718                     | 21.0                   | 181                      | 63                   |

$$z_0 = \frac{v_0^2}{2a}, \quad (16)$$

the field  $B_0$  induces a Zeeman shift equal to the Doppler shift of an atom having velocity  $v_0$ . A tapered solenoid produces a field of such spatial dependence. In certain applications it may be necessary to add a uniform bias field  $B_b$  to keep the field high enough to avoid optical pumping [13].

The atomic beam comes from a thermal source with a dispersion of velocities comparable to its mean velocity. It enters a tapered solenoid where the field is higher at the oven side. The laser is resonant with atoms of a given velocity  $v_0$ , usually around the mean of the thermal distribution, but this transition is modified by the Zeeman shift at the entrance and by the Doppler shift. These atoms at  $v_0$  decelerate. As their velocity changes, their Doppler shift changes but it is compensated by a different Zeeman shift. The initially fast atoms continue to be on resonance. As they decelerate and move downstream in the magnet more atoms come on resonance and start feeling the light force of the opposing laser beam. At the end of the tapered solenoid all the atoms with velocities smaller than  $v_0$  are decelerated to a final velocity that depends on the details of the solenoid and the laser detuning. The result is a significant enhancement of density in phase space; despite the fact that the diffusion process associated with the cooling increases significantly the divergence in the transverse direction. Table 1 gives lengths for Zeeman slowers, required to bring different alkali atom with velocities  $v_{\text{thermal}} = \sqrt{2k_B T/M}$  to a halt by driving it on a fully saturated transition.

### 3.3 Frequency chirping

Another method to slow atoms in a beam is to chirp the frequency of the laser maintaining the resonant interaction with a group of atoms and leaving the others without deceleration [14]. The instantaneous acceleration is negative and the varying laser detuning compensates for the changing Doppler shift.

$$\Delta'(t) = -kv(t) + \Delta, \quad (17)$$

where  $\Delta'(t)$  is the time varying laser detuning of the laser frequency. In the deceleration frame the force on an atom at velocity  $v$  is:

$$F(v) = \frac{\hbar k \Gamma}{2} \left[ \frac{-S_0}{1 + S_0 + \frac{2(\Delta + kv)^2}{\Gamma}} + \frac{S_0}{1 + S_0 + (\frac{2\Delta}{\Gamma})^2} \right], \quad (18)$$

expanding near  $v = 0$

$$F(v) = \left[ 2\hbar k^2 S_0 \frac{2\Delta/\Gamma}{[1 + S_0 + (2\Delta/\Gamma)^2]^2} \right] v. \quad (19)$$

The force is proportional to the velocity and the proportionality constant is a friction coefficient. The method is self correcting and works in batches of atoms. All velocities near  $v(0)$  damp towards  $v(t)$ . Any velocities not initially near  $v(0)$  become close to  $v(t)$  at a later time. Changes in the saturation parameter from the attenuation of the laser beam as it propagates through the beam can be compensated. The chirp rate of the laser frequency to obtain deceleration ( $\Delta < 0$ ) is

$$\frac{d\Delta'}{dt} = ka, \quad (20)$$

$$a = \frac{F(v)}{M}. \quad (21)$$

A chirp rate of 780 MHz/ms can stop an initially thermal Na atom.

### 3.4 Optical molasses

Velocity dependent forces are necessary to cool an atom and reduce its velocity. They do not confine the atom, but they provide what has been termed

‘optical molasses’. The damping felt by the atoms is substantial and the study of the cooling mechanisms has been discussed in the literature. (See for example the review paper of Metcalf and van der Straten [1]).

An atom subject to two laser beams in opposite directions will feel a force  $\mathbf{F}(\mathbf{v})$  coming from its interaction with both beams. If the counterpropagating laser beams are detuned to the red of the zero velocity atomic resonance, a moving atom will see the light of the opposing beam blue shifted in its rest frame (See Fig. 1). The beam in the same direction as the atom will be further red shifted in its rest frame. Considering only one dimension and  $S_0 \ll 1$ , the force opposing the motion will always be larger than the force in the direction of the motion, and this leads to Doppler cooling.

$$\mathbf{F}(\mathbf{v}) = \mathbf{F}_{\text{spont}}(\mathbf{k}) + \mathbf{F}_{\text{spont}}(-\mathbf{k}). \quad (22)$$

The sum of the two forces, with the semiclassical assumption that the recoil shift is negligible  $kv_{\text{rec}} \ll \Gamma$  gives in the limit where  $v^4 \ll (\Gamma/k)^4$ ,

$$\mathbf{F}(\mathbf{v}) \approx \frac{8\hbar k^2 S_0 \Delta}{\Gamma(1 + S_0 + (2\Delta/\Gamma)^2)^2} \mathbf{v}, \quad (23)$$

$$\mathbf{F}(\mathbf{v}) \approx \alpha \mathbf{v}. \quad (24)$$

The force is proportional to the velocity of the atom through the friction coefficient  $\alpha$  and depends on the sign of the laser detuning  $\Delta$ . Figure 2 shows the Doppler cooling force in one dimension as a function of velocity and detuning for the  $D_2$  line of francium. This force is limited by the spontaneous decay time of the atomic level. An estimate for the maximum velocity an atom can have and still feel the light force is when the Doppler shift is equal to the laser detuning from the transition:  $v_{\text{max}} \approx \Delta/k$ . Only a very small fraction of the thermal distribution of atoms at room temperature can be cooled in optical molasses.

### 3.5 Cooling Limits

Optical molasses provides a velocity dependent or viscous force. In the three-dimensional configuration atoms get slowed wherever they are in the region defined by the overlap of the six orthogonal beams. Large laser beams will increase the total number of cooled atoms, but the atomic density remains constant. Because of the variance of the momentum coming from the repeated

random spontaneous emission, atoms can diffuse out of the molasses region because this is not a trap. The competition between the cooling process and the diffusion of the momentum reaches an equilibrium that determines the lowest temperature of the atoms [1].

### 3.5.1 THE DOPPLER COOLING LIMIT

The atomic momentum and energy change by  $\hbar k$  and  $E_{\text{rec}}$  after each interaction with the laser beam. Following the one dimensional treatment of the force above, the change of the energy has an associated change in the frequency of the transition such that  $E_{\text{rec}} = \hbar\omega_{\text{rec}}$ . Then the average frequencies of absorption and emission are:

$$\omega_{\text{abs}} = \omega_{\text{atom}} - \omega_{\text{rec}}, \quad (25)$$

$$\omega_{\text{em}} = \omega_{\text{atom}} + \omega_{\text{rec}}. \quad (26)$$

The light field loses every cycle an average energy of:

$$\hbar(\omega_{\text{abs}} - \omega_{\text{em}}) = 2\hbar\omega_{\text{rec}}, \quad (27)$$

and the power lost by the laser field becomes atomic kinetic energy. The rate of heating should equal the rate of cooling in thermal equilibrium and

$$\mathbf{F} \cdot \mathbf{v} = \frac{\hbar\omega_{\text{rec}}}{1/R_{sc}}, \quad (28)$$

$$\alpha v^2 = \frac{\hbar\omega_{\text{rec}}}{1/R_{sc}}. \quad (29)$$

The cooling force in the optical molasses is proportional to the velocity through the friction coefficient  $\alpha$ . The temperature associated with the kinetic energy is:

$$k_B T = \frac{\hbar\Gamma}{4} \left[ \frac{1 + 2S_0 + (2\Delta/\Gamma)^2}{2\Delta/\Gamma} \right]. \quad (30)$$

This expression becomes independent of  $S_0$  in the limit of low intensity and has a minimum for  $\Delta = -\Gamma/2$ . This temperature is called the Doppler cooling limit  $T_{\text{Doppler}}$ .

$$T_{\text{Doppler}} = \frac{\hbar\Gamma}{2k_B}. \quad (31)$$

The lowest temperature in optical molasses is independent of the optical wavelength, atomic mass, and, in the limit of low intensity, also of laser intensity. The only atomic parameter that enters is the rate of spontaneous emission  $\Gamma$ . The value for Na is  $240 \mu\text{K}$  which corresponds to an average velocity of  $30 \text{ cm/s}$  four orders or magnitude smaller than the typical thermal velocities produced out of an effusive oven (See Table 1).

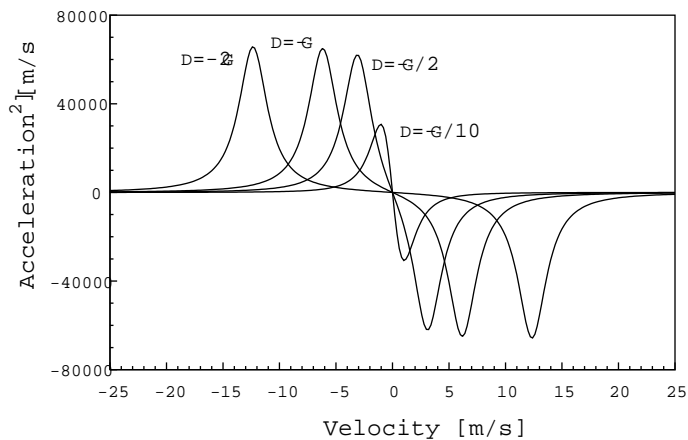


Figure 2: Doppler cooling in one-dimensional optical molasses. The numerical values are for the francium  $D_2$  line at  $S_0 = 1$ . (From Ref. [21])

### 3.5.2 BEYOND DOPPLER COOLING

In 1988 the NIST group [15] discovered that the temperature of sodium atoms in optical molasses was a factor of six lower than the Doppler cooling limit. The quantitative understanding of this result requires the inclusion of all the energy levels that are present in an atom, the effects of the polarization of the different laser beams, and the non-adiabatic response of a moving atom to the light field [3, 16, 17].

The atom has a finite response time  $\tau_{int}$  to adjust its internal state  $\sigma$  to a new environment.  $\sigma$  depends on the position  $z$  and velocity  $v$  of the atom

and in general lags behind the steady state of an atom which would be at rest in  $z$

$$\sigma(z, v) \approx \sigma_{st}(z) - v\tau_{int} \frac{d}{dz} \sigma_{st}(z). \quad (32)$$

The non-adiabaticity parameter in the problem is:

$$\epsilon = \frac{v\tau_{int}}{\lambda/2\pi}, \quad (33)$$

$$= kv\tau_{int}. \quad (34)$$

The frictional force is going to be linear in  $v$  as long as  $\epsilon < 1$ . The equilibrium temperature of the system is:

$$k_B T \approx \frac{\hbar}{\tau_{int}}. \quad (35)$$

For a two level atom there is a single internal time  $\tau_{int} = 1/\Gamma$ , the radiative lifetime of the excited state. The non-adiabaticity parameter is the ratio of the Doppler shift divided by the natural width of the transition. The temperature reachable is of the order of  $\hbar\Gamma$ . This result is in agreement with the  $T_{\text{Doppler}}$  calculated based on the change in the energy of the laser field from Eq. 31. The Doppler limit is independent of the intensity.

However; a multilevel atom, for example an alkali, has hyperfine splitting and Zeeman sublevels. There is a new internal time: The optical pumping time between ground state sublevels. Let  $\Gamma'$  be the absorption rate from  $|g\rangle$ , this number depends on the intensity and will give a different value for the lowest temperature than Doppler cooling. At low intensities  $S_0 \ll 1$  and  $\Gamma' \ll \Gamma$ . The associated  $\epsilon'$ , which is the ratio of the Doppler shift to the optical pumping rate, will be very large.

### 3.5.3 SISYPHUS COOLING

When the intensity and detuning of the laser beams are significant, a different mechanism can cool an atom. It requires an AC Stark Shift of the atomic ground state. The dressed atom formalism of the atom + photon interaction shows (see for example the contribution of Cohen-Tannoudji in Ref. [3]) that the light shift for the ground state  $\delta'$  in the presence of a field with Rabi

frequency  $\Omega$  much smaller than the absolute value of the detuning between the laser and the atomic transition  $\Delta$  is:

$$\delta' = \frac{\Omega^2}{4\Delta}. \quad (36)$$

The light shifts are proportional to the intensity ( $\Omega^2$ ), the sign depends on the detuning  $\Delta$  of the laser with respect to the atomic transition. If an atom is illuminated by two detuned laser beams counterpropagating but one with horizontal polarization and the other with vertical polarization, the atom will feel a very different force from the spontaneous force. The resulting field has polarization gradients. The field has negative circular helicity in one point in space, a distance  $\lambda/4$  away has positive helicity, and is elliptically polarized in between with linearly polarized light exactly at  $\lambda/8$  of the point with purely circular light. (See the Nobel lecture of Cohen-Tannoudji [10]). For a case where the ground state has two sublevels  $J_g = 1/2$  and the excited state four  $J_e = 3/2$  the optical pumping rates are the largest from the highest sublevel of the ground state to the lowest sublevel of the ground state. If  $v\tau_{int} \approx \lambda/2\pi$  the atom can climb a potential hill and reach the top before being pumped back to a valley. The atom is always climbing in analogy to the Greek Sisyphus. There is a decrease of the kinetic energy and the dissipation of potential energy is by spontaneous anti-Stokes Raman photons. The equilibrium temperature comes when the atoms gets trapped in one of the potential wells formed by the position dependent AC Stark Shift, then the equilibrium temperature is of the order of the well depth:

$$k_B T \approx \frac{\hbar\Omega^2}{|\Delta|}, \quad (37)$$

further cooling in the well is possible using adiabatic expansion by lowering the laser intensity at a rate slow compared to the frequency of oscillation of the trapped atom in the potential well.

Another way to understand Sisyphus cooling is the following (See Ref. [4] and the contribution of S. Chu in [2]). The induced electric dipole  $\mathbf{d}$  of an atom in the presence of an off-resonant field minimizes its energy when it aligns with the optical electric field  $\mathbf{E}$ . If an atom at a point of linearly polarized light moves a distance  $\lambda/8$  the polarization is now circular because of the way the opposite polarizations add at each point in space. The atom can

only follow a change in field alignment with a finite time delay characteristic of the damping process. This process changes kinetic energy into potential energy which is lost from damping as the dipole relaxes to the new state of polarization.

There are other configurations that produce Sisyphus cooling, for example two counterpropagating beams with  $\sigma^+$  and  $\sigma^-$  polarizations. The polarization of the field is always linear but it changes directions continuously over one wavelength. The atomic dipole sees a change in the direction it should oscillate.

All the mechanisms described before rely on absorption and spontaneous emission of photons. A natural limit to the lowest achievable temperature is given by the recoil energy  $k_b T_{\text{rec}}/2 = E_{\text{rec}}$ . Finding a way to ‘protect’ the atoms from light can bypass this limit. Two laser cooling methods are known to reduce the temperature of the atoms beyond  $T_{\text{rec}}$ : velocity selective coherent population trapping (VSCPT) and Raman cooling. VSCPT prepares the atom in a ‘dark state’ that does not absorb any light eliminating the possibility of recoil. This state is stationary and an atom that diffuses into it will be trapped (See [18] and the Nobel lecture of Cohen-Tannoudji [10]).

In Raman cooling, a series of light pulses, with well defined frequency and duration, produces an excitation profile that constitutes a ‘trap’ in velocity space for the atom. (See S. Chu in [2]).

## 4 POSITION DEPENDENT FORCE

The position dependent force is necessary to construct a trap but is more subtle than the velocity dependent force. A series of no trapping theorems constrain the distribution of electric and magnetic fields for capturing neutral atoms. (See the contribution by S. Chu in Ref. [2]).

Earnshaw theorem states that is is impossible to arrange any set of static charges to generate a point of stable equilibrium in a charge-free region. The electrostatic potential  $\phi$  satisfies  $\nabla^2\phi = 0$ , then  $\phi(x, y, z)$  at any point is the average of  $\phi$  on the surface of the sphere centered at  $(x, y, z)$ . There can not be an extremum of  $\phi$  and since the electrostatic energy is proportional to the potential, there is no minimum of the energy. Similarly:  $\nabla \cdot \mathbf{E} = 0$  and all the lines of force that go in are balanced by lines that go out of it. The optical Earnshaw theorem uses the Poynting vector of the field  $\mathbf{P}$  and it applies to



the scattering force. The light flux can not point inward everywhere, so a light trap is unstable ( $\nabla \cdot \mathbf{P} = 0$ ).

A solution is not to use static light beams, but alternate them in time to generate a trap following the ideas of the Paul trap. Another way to circumvent the optical Earnshaw theorem is to exploit the internal structure of the atoms. The effective atomic polarizability  $\mathbf{D}$  can be position dependent through the presence of an external magnetic field  $\mathbf{B}$  resulting in a negative divergence of the spontaneous light force, since the force is proportional to  $\mathbf{D}$ .

J. Dalibard proposed a solution to the neutral atom trapping using the spontaneous light force. His idea became the basis of the Magneto-Optical Trap (MOT). The solution of Dalibard was to add a spatially varying magnetic field, so that the shifts in the energy levels make the light force dependent on the position. Soon afterwards this scheme was generalized to three dimensions and it was successfully demonstrated with Na atoms by Raab *et al.* [19]. Despite many new developments the MOT remains the workhorse of laser trapping due to its robustness, large volume and capture range. The next section discusses this trap in more detail since this type has been used in the successful trapping of radioactive atoms [5].

## 5 OPTICAL TRAPS

### 5.1 *The Magneto-Optical Trap*

This section presents a simplified one-dimensional model to explain the trapping scheme in a  $J=0 \rightarrow J=1$  transition.

Figure 3 shows a configuration similar to optical molasses. Two counter-propagating, circularly polarized beams of equal helicity are detuned by  $\Delta$  to the red of the transition. In addition there is a magnetic field gradient, splitting the  $J=1$  excited state into three magnetic sublevels. If an atom is located to the left of the center, defined by the zero of the magnetic field, its  $J=0 \rightarrow J=1, m=1$  transition is closer to the laser frequency than the transitions to the other  $m$ -levels. However,  $\Delta m = +1$  transitions are driven by  $\sigma^+$  light. Atoms on the left are more in resonance with the beam coming from the left, pushing them towards the center. The same argument holds for atoms on the right side. This provides a position dependent force. The

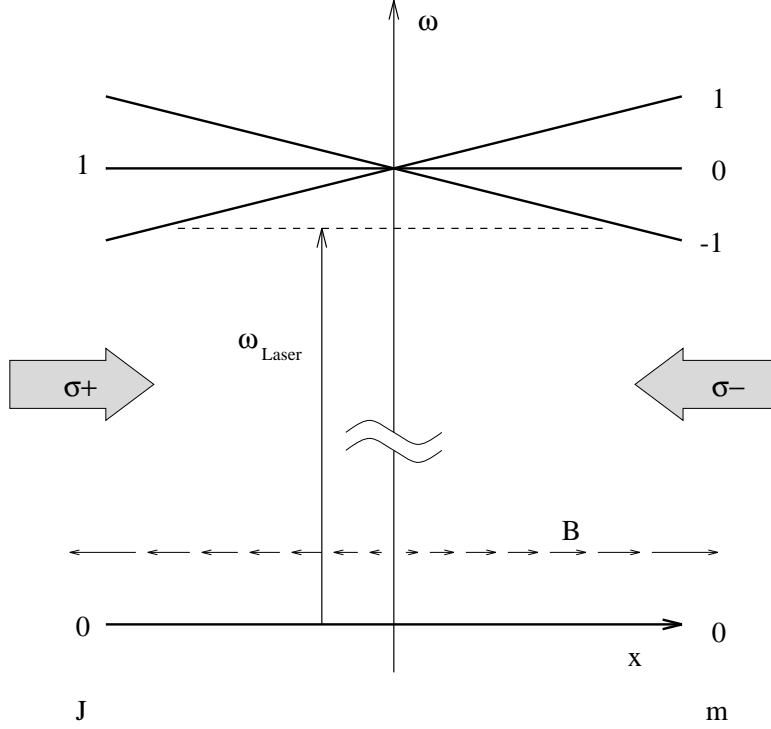


Figure 3: Simple 1-D model of the MOT. (From Ref. [21])

Doppler-cooling mechanism is also still valid, providing the velocity dependent force. Writing the Zeeman shift as  $\beta x$ , where  $x$  is the coordinate with respect to the center, the total force is:

$$F_{\text{MOT}} = \frac{\hbar k \Gamma}{2} \left[ \frac{S_0}{1 + S_0 + (2(\Delta - \xi)/\Gamma)^2} - \frac{S_0}{1 + S_0 + (2(\Delta + \xi)/\Gamma)^2} \right], \quad (38)$$

where

$$\xi = kv + \beta x. \quad (39)$$

For small detunings, expansion of the fractions in the same way as in Eq. 24, shows the force proportional to  $\xi$  (see W.D. Phillips in, [2]). In the small-field, low-velocity limit the system behaves as a damped harmonic oscillator subject to the force:

$$F(v, x) = \frac{4\hbar k S_0 (2\Delta/\Gamma)(kv + \beta x)}{[1 + (2\Delta/\Gamma)^2]^2}, \quad (40)$$

and

$$\ddot{x} + \gamma \dot{x} + \omega_{trap}^2 x = 0, \quad (41)$$

with

$$\gamma = \frac{4\hbar k^2 S_0 (2\Delta/\Gamma)}{M[1 + (2\Delta/\Gamma)^2]^2}, \quad (42)$$

$$\omega_{trap}^2 = \frac{4\hbar k \beta S_0 (2\Delta/\Gamma)}{M[1 + (2\Delta/\Gamma)^2]^2}. \quad (43)$$

The motion of the atom in the harmonic region of the trap is overdamped since  $\gamma^2/4\omega_{trap}^2 > 1$ . This same ratio in terms of the recoil energy and the Zeeman shift over one wavelength is:

$$\frac{\gamma^2}{4\omega_{trap}^2} = \frac{\pi E_{rec}}{4\lambda \hbar \beta}. \quad (44)$$

A trap with a magnetic field gradient that produces a Zeeman shift of  $\beta = 14$  MHz/cm has a trapping frequency of a few kilohertz and an Eq. 44 of the order of 10.

The real world requires three-dimensional trapping, and in alkalis a  $J = 0 \rightarrow J = 1$  transition is hard to find. For an alkali atom with non-vanishing nuclear spin the ground state ( $nS_{1/2}$ ) splits into two levels. The transition to the first  $P_{3/2}$  excited state has four levels (for  $J < I$ ), yet the trap works quite well under these conditions. Ideally, the transition from the upper ground state to the highest excited state  $F$ -level is cycling, and one can almost ignore the other states. Due to finite linewidths, off resonance excitation, and other energy levels the cycling is not perfect. An atom can get out of the cycling transition and an extra beam, a weak ‘repump’ laser, can transfer atoms from the ‘dark’ lower ground state to the upper one.

A magnetic quadrupole field, as produced by circular coils in the anti-Helmholtz configuration, provides a suitable field gradient in all three dimensions. The exact shape of the field is not very critical, and the separation between the two coils does not have to be equal to the radius. Typical gradients are 10 G/cm.

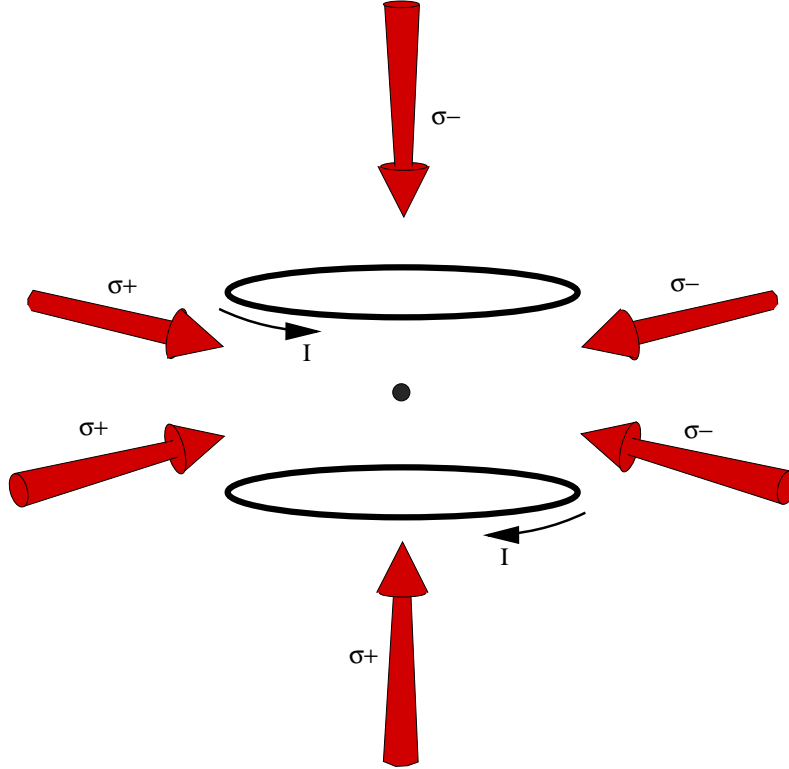


Figure 4: Laser beams and coils for a MOT. (From Ref. [22])

A large variety of optical configurations are available for the MOT. The main condition is to cover a closed volume with areas normal to the  $\mathbf{k}$  vectors of the laser beams with the appropriate polarized light. (See figure 4). The realization with three retro-reflected beams in orthogonal directions requires quarter-wave plates before entering the interaction region. In order to have the appropriate polarization on the retro-reflected beam the phase has to advance half a wavelength. The usual arrangement is to place a quarter wave plate in front of a plane mirror, but two reflections can also provide the same phase shift [20].

The intensity of the laser beams should provide a saturation parameter  $S_0 \approx 1$ . The MOT can work with significantly less intensity but it becomes more sensitive to alignment. In general the MOT is a very forgiving trap as far as polarization and intensities. The retro-reflecting technique for traps,

despite the scatter losses in the windows and the beam divergence as it propagates, works very well.

The well depth of a MOT is set by the maximum capture velocity  $v_{max}$ . For alkali atoms and  $\Delta \approx 2\Gamma$  it is close to 1 K. The background pressure around the MOT limits its lifetime and consequently the maximum number of atoms in steady state. A pressure of  $1 \times 10^{-8}$  Torr produces a trap lifetime of the order of 1 s. The characteristic size  $x_0$  of the trapping volume is set by the gradient and the detuning of the specific realization of a MOT:  $x_0 = \Delta/\beta$ .  $x_0$  is about 1 cm and to obtain larger volumes larger laser beams are required. The captured atoms concentrate in a region much smaller than the trapping volume. The size of the fluorescing ball of less than  $10^6$  captured atoms is smaller than 1 mm in diameter. It depends on the temperature and is related to the laser beams shape, magnetic environment, and polarization. The shape of the fluorescence when integrated in a charge couple device (CCD) camera is usually Gaussian (see Fig. 5).

If the alignment of the laser beams is not good there can be a torque impressed into the trap and satellites can form. Fringes in the beams can also generate satellites. As the number of atoms increases there is a limit to the size of the trap. A similar effect to space charge appears. The optical density is thick enough to create an imbalance in the two counterpropagating beams; also the atoms can absorb spontaneously emitted light that is not red-detuned from neighboring atoms. The trap is no longer optically transparent with an extra internal radiation pressure that may eject the atoms out of the trap.

To increase the density and the number of atoms beyond the point where the repelling force turns on, Ketterle *et al.* [24] developed the dark MOT. The repumping beams are blocked from the central region of the trap. The trap maintains the atoms in a non-cycling state and only repumps them to the cycling transition when they stray to the edge of the trapping volume. This approach works with alkali atoms since the ground state hyperfine splitting already requires a repumping laser.

The first experiments with a MOT by Raab *et al.* [19] reported the capture of atoms from the residual background gas in the vacuum chamber without need of deceleration. In 1990 Monroe *et al.* [25] showed trapping in a glass cell from the residual vapor pressure of a Cs metal reservoir. If the vapor pressure of an element is sufficiently high, a MOT inside a cell filled with a vapor continuously captures atoms from the low-velocity tail of the

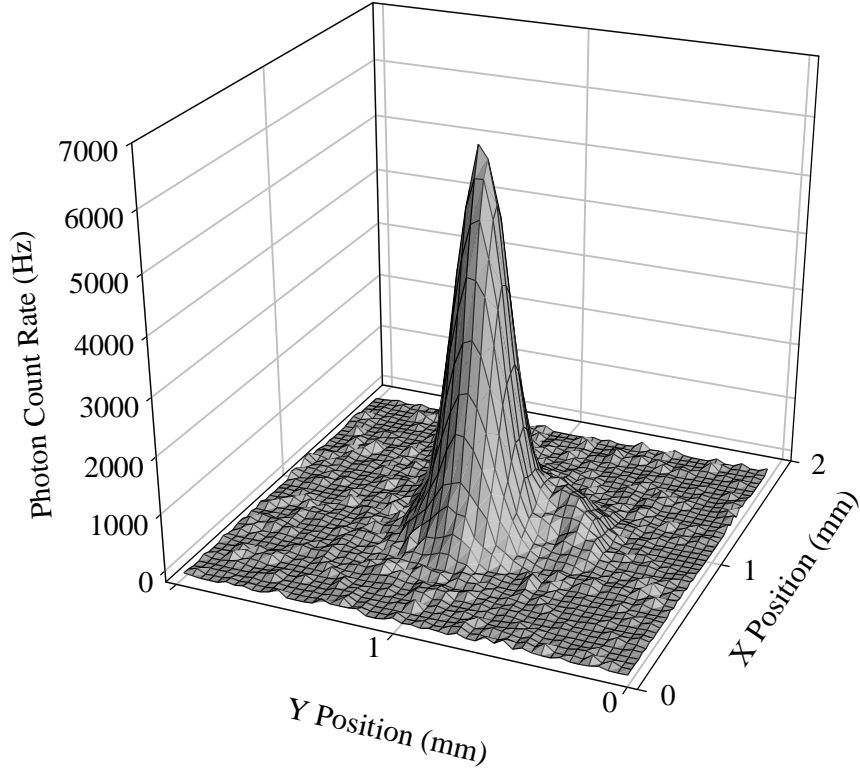


Figure 5: Two-dimensional CCD image of the fluorescence from francium atoms trapped in a MOT. (From Ref. [23]).

Maxwell-Boltzmann distribution. The remaining atoms thermalize during wall collisions and form a new Maxwell-Boltzmann distribution. From this the MOT can again capture the low velocity atoms. The trapping efficiency depends on the number of wall contacts that an atom can make before leaving the system. Since alkali atoms tend to chemisorb in the glass walls, special coatings can prevent the loss of an atom [26]. If the wall is coated, the atom physi-sorbs for a short time, thermalizes and then is free to again cross the capture region and fall into the trap.

The capture range of the MOT is enhanced with the help of large and intense laser beams. Gibble *et al.* [27] reported that for their large trap they captured atoms with initial velocities below about 18 % of the average thermal velocity at room temperature. However, the fraction of the Maxwell-

Boltzmann distribution of atom velocities below the capture velocity of the trap is too small to capture a significant fraction of scarce radioactive atoms on a single pass through the cell. Wall collisions are critical to provide multiple opportunities for capture in the vapor cell technique. On the one hand they provide the thermalization process, but they also increase the possibility of losing the atom by chemical adsorption onto the wall.

No significant vapor pressure of stable alkali atoms normally builds up unless the walls of the glass cell are coated by a mono-layer of the atom to be trapped. For most radioactive samples this is impossible, and also not desired since that will create a source of background for the study of the decay products. An alternative is to coat the cell with a special non-stick coating. The coatings are in general silanes and have been extensively studied for optical pumping applications of alkali atoms. Collisions with the bare glass walls destroy the atomic polarization and the coatings can provide a ‘soft surface’ for reflection. The Stony Brook group uses one commercially identified by the name of Dryfilm (a mixture of dichlorodimethylsilane and methyltrichlorosilane). The coating procedure follows the techniques of Swenson *et al.* [26]. The choice of a particular coating depends on many issues. For example: The difficulties in the application of the coating to the surface, how well the coating withstands high temperatures present nearby in the experimental apparatus. The coating of choice constrains the attainable background pressure in the cell and the geometry of the vacuum container. Nevertheless the vapor cell is appealingly simple. As long as a coating is known to work for a stable alkali it seems to work for the radioactive ones. The Colorado group has studied different coatings extensively [28], and have developed curing procedures to optimize the performance of the coatings.

The glass cell method relies on the non-stick coatings and works well for radioactive alkali atoms, but for other radioactive elements it may not be so easily implemented and the Zeeman slower could prove more effective to load atoms into a MOT.

The group of the University of Colorado has published a resource letter on laser trapping and cooling [29]. They also published a detailed explanation, including electronic diagrams, on how to build a glass cell MOT for Rb or Cs using laser diodes [30].

## 5.2 *The dipole force trap*

An electric or magnetic dipole in an inhomogeneous electric or magnetic field feels an attractive or repulsive force depending on the specific conditions. A strong laser field can induce an electric dipole in an atom. In 1968 Letokhov [31] proposed laser traps based on the interaction of this induced electric dipole moment with the laser field. Later, Ashkin [32] proposed a trap that combined this dipole force and the scattering force. The first laser trap for neutral atoms was of this type [33]. The trap depth is proportional to the laser intensity divided by the detuning  $\hbar\Omega^2/|\Delta|$ . In order to minimize heating from spontaneous emission, the frequency of the intense laser is tuned hundreds of thousands of linewidths away from resonance. The heating is greatly reduced since the emission rate is proportional to the laser intensity divided by the square of the laser detuning. The off-resonance nature of the trap requires very intense beams with an extremely tight focus, and is often referred to as a Far Off Resonance Trap (FORT). A single laser red-detuned tightly focused has a gradient large enough to capture atoms from a MOT. The well depth is very small, fractions of a milliKelvin, depending on precooled atoms and very good vacuum for an extended residence in the trap. The atoms reside in a conservative trap and can cool down further by other mechanisms like evaporative cooling [34]. This kind of trap has found applications in the manipulation of extended objects as a form of optical tweezers.

## 5.3 *Other traps and further manipulation*

Although the MOT is a proven trap for radioactive atoms, it may not be the ideal environment for some of the experiments now planned. The atoms are not polarized because there are all helicities present in the laser field, and the magnetic field is inhomogeneous. There have been a series of traps developed in conjunction with the pursuit of Bose Einstein condensation (BEC) [35, 36, 37, 38]. that may have application in the field of radioactive atom trapping. In this quest for even higher phase space densities, new techniques for transport and manipulation of cold atoms have also appeared.



### 5.3.1 COLD ATOM MANIPULATION

To move the accumulated atoms in a MOT to a different environment requires some care. Simply turning the trapping and cooling fields off will cause the atoms to fall ballistically. The trajectories out of the trap will map out the original velocity distribution of the captured atoms, dispersing the atoms significantly as they fall. An auxiliary laser beam can push the atoms in one direction, but it has a limited interaction range since the atoms accelerate until they are shifted out of resonance by their Doppler shift. The acceleration is in only one direction and there is still ballistic expansion of the cold atoms. Gibble et al. [39] created a moving molasses with the six beams of the MOT. By appropriate shifting of the frequencies of the beams, the atoms accelerate in the 111 direction (along the diagonal of the cube formed by the beams), but they are kept cold by the continuous interaction with the six beams.

## 6 COOLING AND TRAPPING OF FR

Francium is the heaviest of the alkali atoms and has no stable isotopes. It occurs naturally from the  $\alpha$  decay of actinium or artificially from fusion or spallation nuclear reactions in an accelerator. Its longest lived isotope has a half-life of 22 minutes. Previously, experiments to study the atomic structure of francium were possible only with the very high fluxes available at a few facilities in the world [40], or by use of natural sources [41].

Because of its large number of constituent particles, electron correlations and relativistic effects are important, but its structure is calculable with many-body perturbation theory (MBPT). Its more than two hundred nucleons and simple atomic structure make it an attractive candidate for a future atomic parity non-conservation (PNC) experiment. (See Ref. [42] for the most recent results in Cs). The PNC effect is predicted to be 18 times larger in Fr than Cs [43].

The present francium spectroscopy serves to test the theoretical calculations in a heavier alkali. This ensures that the Cs structure, calculated with the same techniques, is well understood.

Heavy-ion fusion reactions can, by proper choice of projectile, target and beam energy, provide selective production of the neutron deficient francium

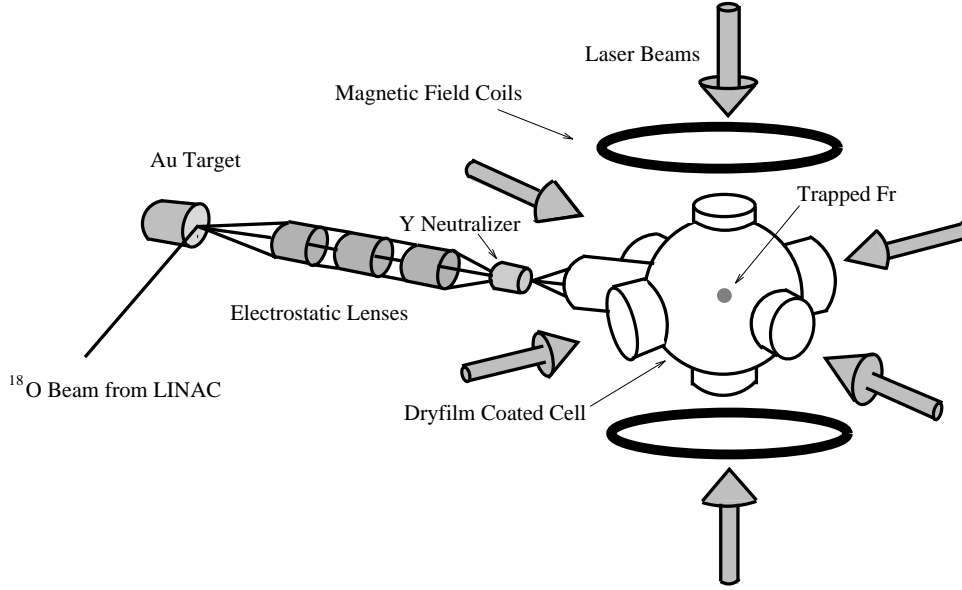


Figure 6: Schematic view of target, ion transport system, and magneto optical trap. (From Ref. [23])

isotopes. Gold is an ideal target because it is chemically inert, has clean surfaces, and a low vapor pressure. The  $^{197}\text{Au}(^{18}\text{O},xn)$  reaction at 100 MeV produces predominantly  $^{210}\text{Fr}$ , which has a 3.2 min half-life. Changing the energy and the isotope of the oxygen beam maximizes the production of isotopes 208, 209 or 211. The reaction  $^{198}\text{Pt}(^{19}\text{F},5n)$  produces  $^{212}\text{Fr}$ .

Figure 6 shows the apparatus to trap and produce Fr at Stony Brook  $10^{12}$   $^{18}\text{O}$  ions/s on Au produce  $^{210}\text{Fr}$  in the target, with less than 10% of other isotopes. The target is heated to  $\approx 1200$  K by the beam power and by an auxiliary resistance heater. The elevated temperature is necessary for the alkali elements to rapidly diffuse to the surface and be surface ionized.

Separation of the production and the trapping regions is critical in order to operate the trap in a UHV environment. Extracted at 800 V, the  $\approx 1 \times 10^6/\text{s}$   $^{210}\text{Fr}$  ions travel about one meter where they are deposited on the inner surface of a cylinder coated with yttrium which is heated to 1000 K and located 0.3 cm away from the entrance of the cell. Neutral Fr atoms evaporate from the Y surface and form an atomic beam directed towards an aperture into the vapor cell MOT.

The physical trap consists of a 10 cm diameter Pyrex bulb with six 5 cm diameter windows and two viewing windows 3 cm in diameter. The MOT is formed by six intersecting laser beams each with  $1/e^2$  (power) diameter of 4 cm and power of 150 mW, with a magnetic field gradient of 6 G/cm. The glass cell is coated with a non-stick Dry-film coating [26] to allow the atoms multiple passes through the trapping region after thermalization with the walls [25]. The trapping laser operates in the  $D_2$  line of francium, while the repumper may operate in the  $D_1$  or in the  $D_2$  lines depending on the measurement. The ground state hyperfine splitting of  $^{210}\text{Fr}$  is 46.7 GHz.

We have recently captured francium atoms [44] in a magneto optical trap (MOT), opening the possibility for extensive studies of its atomic properties. (See Fig. 4 for an image of the fluorescence of Fr atoms in a MOT).

We have been studying the spectroscopy of francium in a magneto optical trap on-line with an accelerator. The captured atoms are confined for long periods of time moving at low velocity in a small volume, an ideal environment for precision spectroscopy. Our investigations have included the location of the  $8S$  and  $9S$  energy levels [45, 48]. We have also made the first measurements of any radiative lifetime in Fr. The precision of our lifetime measurements of the  $D_1$  and  $D_2$  lines are comparable to those achieved in stable atoms [46, 47]. They test atomic theory in a heavy atom where relativistic and correlation effects are large.

## 7 Acknowledgements

During the years that I have been working in cooling and trapping of atoms I have benefited from the interaction with many people. Among them I would like to mention Jesse Simsarian, Jeff Ng, Gerald Gwinner, Jürgen Gripp, Steve Mielke, Greg Foster, Joshua Grossman, Gene Sprouse, Hal Metcalf, Tom Bergeman, Simone Kulin, and Steven Rolston. I would like to thank the organizers of the 1998 Escuela Latinoamericana de Física for their invitation and hospitality.

Support for the experiments with radioactive atoms has come from the National Science Foundation and the National Institute of Standards and Technology.

## References

- [1] H. Metcalf and P. van der Staten Phys. Rep. **244**, 203 (1994).
- [2] Laser Manipulation of Atoms and Ions Ed. E. Arimondo, W. D. Phillips, and F. Strumia (North Holland, Amsterdam 1992)
- [3] Les Houches 1990, Session LIII, Fundamental Systems in quantum optics Ed. J. Dalibard, J.-M. Raimond, and J. Zinn-Justin (North Holland, Amsterdam 1992).
- [4] C. S. Adams and E. Riis, Progress in Quantum Electronics (1996).
- [5] G. D. Sprouse and L. A. Orozco, Annu. Rev. Nucl. Part. Sci. **47**, 429 (1997).
- [6] A. Einstein Physikalische Zeit. **18**, 121 (1917); translation in, Sources of Quantum Mechanics, ed. B van der Waerden (North-Holland, Amsterdam, 1967).
- [7] O. Frisch Zeit. f. Phys. **86**, 42 (1933).
- [8] T. Hänsch and A. Schawlow Opt. Commun. **13**, 68 (1975).
- [9] D. Wineland and H. Dehmelt Bull. Am. Phys. Soc. **20**, 637 (1975).
- [10] S. Chu, Rev. Mod. Phys. **70**, 685 (1998); C. N. Cohen-Tannoudji, Rev. Mod. Phys. **70**, 707 (1998); W. D. Phillips, Rev. Mod. Phys. **70**, 721 (1998).
- [11] L. Allen and J. H. Eberly, *Optical resonance and two-level atoms*, Wiley, New York, 1975
- [12] M. L. Citron, H. R. Gray, C. W. Gabel, and C. R. Stroud Jr. Phys. Rev. A **16**, 1507 (1977).
- [13] W. D. Phillips, and H. Metcalf, Phys. Rev. Lett. **48**, 596 (1982).
- [14] W. Ertmer, R. Blatt, J. L. Hall, and M. Zhu, Phys. Rev. Lett. **54**, 996 (1985).

- [15] P. Lett, R. Watts, C. Westbrook, W. Phillips, P. Gould, and H. Metcalf, Phys. Rev. Lett. **61**, 169 (1988).
- [16] J. Dalibard and C. Cohen-Tannoudji, J. Opt. Soc. Am. B **6**, 2023 (1989).
- [17] P. Ungar, D. Weiss, E. Riis, and S. Chu, J. Opt. Soc. Am. B **6**, 2058 (1989).
- [18] J. Lawall, S. Kulin, B. Saubamea, N. Bigelow, M. Leduc, and C. Cohen-Tannoudji, Phys. Rev. Lett. **75**, 4194 (1995).
- [19] E. L. Raab, M. Prentiss, A. Cable, S. Chu, and D. E. Pritchard, Phys. Rev. Lett. **59**, 2631 (1987).
- [20] G. Gwinner, J. A. Behr, S. B. Cahn, A. Ghosh, L. A. Orozco LA, G. D. Sprouse, and F. Xu, Phys. Rev. Lett. **72**, 3795 (1994).
- [21] G. Gwinner, Ph Dr Thesis, SUNY Stony Brook, 1995, unpublished.
- [22] J. Ng, M Sc Thesis, SUNY Stony Brook, 1996, unpublished.
- [23] J. E. Simsarian, Ph Dr Thesis, SUNY Stony Brook, 1998 unpublished.
- [24] W. Ketterle, A. Martin, M. A. Joffe, and D. E. Pritchard, Phys. Rev. Lett. **69**, 2483 (1992).
- [25] C. Monroe, W. Swann, H. Robinson, and C. E. Wieman, Phys. Rev. Lett. **65**, 1571 (1990).
- [26] D. R. Swenson and L. W. Anderson, Nucl. Instr. Meth. B **29**, 627 (1988).
- [27] K. E. Gibble, S. Kasapi, and S. Chu, Opt. Lett. **17**, 526 (1992).
- [28] M. Stephens, R. Rhodes, and C. Wieman, J. Appl. Phys. **76**, 3479 (1994).
- [29] N. R. Newbury, C. E. Wieman, Am. J. Phys. **63**, 317 (1995).
- [30] C. E. Wieman, G. Flowers, S. Gilbert, Am. J. Phys. **64**, 18 (1996).
- [31] V. S. Letokhov, Pisma Zh. Eksp. Teor. Fiz **7**, 348 (1968). [ JETP Lett. **7**, 272 (1968).]

- [32] A. Ashkin, Phys. Rev. Lett. **40**, 729 (1978).
- [33] S. Chu, J. E. Bjorkholm, A. Ashkin, and A. Cable, Phys. Rev. Lett. **57**, 314 (1986).
- [34] W. Ketterle and N. J. Van Druten in Advances in Atomic Molecular and Optical Physics Vol. 37 p. 181 Academic Press (1996).
- [35] M. H. Anderson, J. R. Ensher, M. R. Matthews, C. E. Wieman, and E. A. Cornell, Science **269**, 198 (1995).
- [36] K. B. Davis, M-O Mewes, M. R. Andrews, N. J. van Druten, D. S. Durgee, D. M. Kurn, and W. Ketterle, Phys. Rev. Lett. **75**, 3969 (1995).
- [37] C. C. Bradley, C. A. Sackett, J. J. Tollett, and R. G. Hulet, Phys. Rev. Lett. **75**, 1687 (1995).
- [38] C. C. Bradley, C. A. Sackett, and R. G. Hulet, Phys. Rev. Lett. **78**, 985 (1997).
- [39] K. E. Gibble and S. Chu, Phys. Rev. Lett. **70**, 1771 (1993).
- [40] E. Arnold, W. Borchers, H. T. Duong, P. Juncar, J. Lermé, P. Lievens, W. Neu, R. Neugart, M. Pellarin, J. Pinard, J. L. Vialle, K. Wendt, and the ISOLDE Collaboration, J. Phys. B **23**, 3511 (1990).
- [41] S. V. Andreev, V. I. Mishin, and V. S. Letokhov, J. Opt. Soc. Am. B **5**, 2190 (1988).
- [42] C. S. Wood, S. C. Bennett, D. Cho, B. P. Masterson, J. L. Roberts, C. E. Tanner, C. E. Wieman, Science **275**, 1759 (1997).
- [43] V. A. Dzuba, V. V. Flambaum, and O. P. Sushkov, Phys. Rev. A **51**, 3454 (1995).
- [44] Simsarian JE, Ghosh A, Gwinner G, Orozco LA, Sprouse GD, and Voytas P. Phys. Rev. Lett. **76**, 3522 (1996).
- [45] J. E. Simsarian, W. Shi, L. A. Orozco, G. D. Sprouse, and W. Z. Zhao. Opt. Lett. **21** (1996) 1939.

- [46] W. Z. Zhao, J. E. Simsarian, L. A. Orozco, W. Shi, and G. D. Sprouse, Phys. Rev. Lett. **78**, 4169 (1997).
- [47] J. E. Simsarian, L. A. Orozco, G. D. Sprouse, and W. Z. Zhao, Phys. Rev. A **57**, 2448 (1998).
- [48] J. E. Simsarian, W. Z. Zhao, L. A. Orozco, and G. D. Sprouse, Phys. Rev. A in print (1999).