Time response of a coupled atoms–cavity system

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Received September 13, 1996

We study the time response of a quantum optical system to a step excitation. The system is composed of a collection of N two-level atoms coupled to a single mode of the electromagnetic field of an optical cavity. The size of the step excitation is not limited to the low-intensity regime. Before the system reaches steady state there is an oscillatory exchange of energy between the atoms and the cavity. We compare the experimental results quantitatively with theoretical calculations and with previous transmission spectroscopy measurements. © 1997 Optical Society of America

The report by Kaluzny *et al.*[1] of the exchange of energy between a collection of Rydberg atoms and a microwave cavity demonstrated one of the key characteristics of the interaction of N two-level atoms with a single mode of the electromagnetic field. This model system has been extensively studied in the optical bistability literature[2] and in cavity quantum electrodynamics[3] (QED). Most of the research on the exchange of energy has been in the low-intensity regime,[4–8] where the number of energy quanta in the system is much less than one. This permits the treatment of the problem as two coupled harmonic oscillators, one for the field and the other for the atoms. Under these conditions the quantal and semiclassical predictions are the same: The coupling lifts the degeneracy of the normal modes, producing a spectral doublet separated by twice the vacuum Rabi frequency. We have studied this system in the optical regime, analyzing the transmitted spectrum and making quantitative measurements not restricted to weak excitation.[9,10] There have also been important developments in this area through the use of semiconductor microcavities,[11–14] in which the time evolution and spectroscopy show properties of low-intensity cavity QED. Recently Brune *et al.*[15] presented a study with Rydberg atoms in driven microwave cavities, in which they probed beyond the low-intensity regime.

There are two general ways to explore the dynamical response of the system in the optical regime. One is to study its spectrum, and the other is to study its time evolution. When the system is linear, the results obtained with these methods are equivalent. If the system is nonlinear, they are not. We showed recently[9] that our apparatus can operate in the nonlinear domain. The result is that in the transmitted spectrum the vacuum Rabi doublet observed for very low excitation[4–8] becomes anharmonic and evolves into a single peak for higher intensities.[9]

In this Letter we show the time response to step excitation of our system. We drive the system with enough excitation to explore the nonlinear regime, where the model of two coupled harmonic oscillators is no longer valid. The information gathered is different from that obtained out of the transmitted spectrum.

The theoretical description of the system begins with a generalization of the Jaynes–Cummings Hamiltonian for the reversible coupling of N two-level atoms to the field of a single cavity mode in the dipole coupling and rotating wave approximations: $\hat{H} = i\hbar g(\hat{J}_- \hat{a}^\dagger + \hat{a}\hat{J}_+) + (\hbar\omega/2)\hat{J}_z + \hbar\omega\hat{a}^\dagger\hat{a}$, where $\hat{J}_{\pm,z}$ are collective atomic operators and $\hat{a}^\dagger$, $\hat{a}$ are creation and annihilation field operators, respectively. The dipole coupling between N two-level atoms and the cavity field is $g\sqrt{N}$, where $g = (\mu^2\omega/2\hbar\epsilon_0 V)^{1/2}$; μ is the transition-dipole moment of the atom, ω is the resonance frequency of both atoms and cavity, and V is the cavity-mode volume. We add a driving field, include reservoirs both for the decay of the atoms through spontaneous emission and for escape of the field from the cavity mode to the outside world, and use a semiclassical decorrelation from the resulting master equation to obtain the Maxwell–Bloch equations for the atoms–cavity system. The result is the model of optical bistability describing a collection of N purely radiatively broadened two-level atoms (decay rate of $\gamma_\perp$) interacting with a single plane-wave traveling mode of the electromagnetic field of a cavity (decay rate of κ) in the uniform field limit.[2] We normalize the intracavity field in the presence of atoms (x) and in an empty cavity (y) by the square root of the saturation photon number, $n_0 = \gamma_\perp{}^2/2g^2$. Note that n_0 expresses the ratio of dissipative coupling into the vacuum modes to reversible coupling into the cavity mode. x is proportional to the output field, and y is proportional to the driving input field that excites the system as a function of time. The normalized intensities associated with the driving and transmitted fields are $Y = |y|^2$ and $X = |x|^2$. The vacuum Rabi frequency for this system is $\Omega_{\mathrm{VR}} = \{g^2N - [(\kappa - \gamma_\perp)/2]^2\}^{1/2}$.

The heart of the apparatus[16] is a high-finesse optical cavity formed by two 1.3-cm-diameter mirrors, each with radius of curvature of 7.5 cm and transmission coefficient of 2.4×10^{-4}, separated by 4.1 mm (see Fig. 1). An oven heated to ≈ 430 K produces an effusive beam of Rb atoms that is optically pumped before intersecting the cavity mode at 90°. This provides a continuous stream of two-level atoms that on average spend 11 lifetimes crossing the waist of the TEM_{00} Gaussian mode of the cavity. The parameters of the experiment are $(g, \kappa, \gamma_\perp) = 2\pi(1.5, 1.4 \pm 0.1, 3.1) \times 10^6$ rad/s, falling within the intermediate regime of cavity QED. The excitation source is a cw Ti:sapphire laser locked on resonance to the $5S_{1/2}$, $F = 3 \rightarrow 5P_{3/2}$, $F = 4$ transition of ^{85}Rb. Part of the laser beam is split into an intense auxiliary beam used to lock the cavity on

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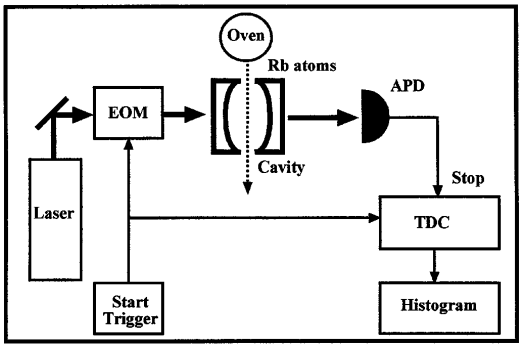

Fig. 1. Simplified diagram of the experimental setup.

resonance with FM sidebands. The other beam, much weaker in intensity, serves as the signal beam. It passes through two electro-optic modulators (EOM's; Gsänger LM0202), where the light is turned on and off. The $1/e$ (power) fall time is 5 ns, with an extinction ratio of greater than 300:1. The light is mode matched into the cavity. We detect the emerging photons with an avalanche photodiode (APD; EG&G SPCM-AQ-151).

A chopper wheel alternately passes the locking beam or the signal beam at a rate of ≈ 1 kHz. While the locking beam is blocked, the EOM's turn the signal beam on and off with 1-μs high transmission and 3-μs low transmission. A LeCroy 3377 time-to-digital converter (TDC) starts with the EOM trigger, measuring as many as 16 photon arrival times during the 4-μs data collection.

During a data run, we let the oven temperature stabilize and then performed a measurement of the on-resonance intensity bistability by sweeping the signal beam intensity and recording the output characteristics.[16] This scan identifies I_{crit}, the lowest input intensity at which there is on-resonance bistability.[10] We then fixed the signal beam input intensity and collected data, which we stored as a histogram. This was repeated for different values of the input intensity, both above and below I_{crit}. We also explored other coupling strengths by changing the number of atoms with the oven temperature (the smallest value was $N \approx 25$). The results are qualitatively similar to those presented here.

When the effective number of atoms[6] in the cavity mode is $N \approx 300$, Fig. 2(a) shows the response of the system to a step excitation of $1.2I_{\mathrm{crit}}$. The transmitted intensity grows from zero and then decreases once the atoms start building up a polarization that then exchanges energy with the cavity mode. The measured intensity oscillates at two frequencies, because the output field is composed of a part oscillating at approximately Ω_{VR} and the slow buildup of a steady-state field ($x < 1$). For the first part of the turn-on, the oscillations in the field are near zero and large enough that they cause an intensity oscillation at $2\Omega_{\mathrm{VR}}$. At the end, field and intensity both oscillate at Ω_{VR} since the oscillations are smaller than the steady state. Figure 2(b) shows the freely evolving system after the sudden turn-off of the excitation. The frequency of oscillation in this case is twice the frequency of oscillation of the field. Note that the intensity first decreases and then grows to a value much larger than the steady state. There is a large amount of energy stored in the atomic polarization opposing the incoming field. When the incoming field is turned off, the energy is free to go into the cavity and then escape, but part of it returns to the atoms. We can observe more than 12 exchanges before the signal decays into the background noise [see the inset in Fig. 2(b)].

For the purpose of quantitative comparisons some refinements in the model are necessary. We include the Gaussian transverse profile and the standing-wave structure of the cavity mode. We calculate the time evolution of the Maxwell–Bloch equations for a random distribution of atoms in the cavity mode volume. Our model does not take into account the remaining inhomogeneous and transit broadening of the atoms, the finite turn-off time of the EOM's, or any fluctuations in the model parameters.

Figure 3(a) shows the frequency of oscillation of the output field as a function of the input intensity, obtained from the fast Fourier transform of the time response and from the transmission spectroscopy.[9] The dashed and the dotted lines come from the time-response calculations, when the driving field is suddenly turned on and off, respectively. The solid curve is the prediction for the evolution of the anharmonic vacuum Rabi peaks as the intensity of the spectroscopic probe changes, with parameters appropriate for the experimental data. It describes the condition that there be no phase shift between input and output fields.[10] The experimental conditions for the time and frequency observations are different, but scaling frequencies with Ω_{VR} and intensities with I_{crit} permit a direct comparison. The model predictions with this scaling differ at most by 8% at the highest intensity. The oscillation frequency of the turn-off response is fairly constant at Ω_{VR} since the system oscillates at

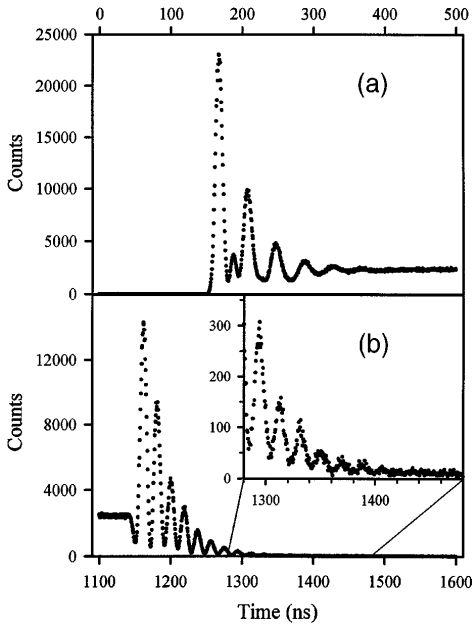

Fig. 2. Time response of the coupled atoms–cavity system to (a) turn-on and (b) turn-off step excitation of $I/I_{\mathrm{crit}} = 1.2$. The inset in (b) is an amplification by a factor of 30 of the vertical scale.

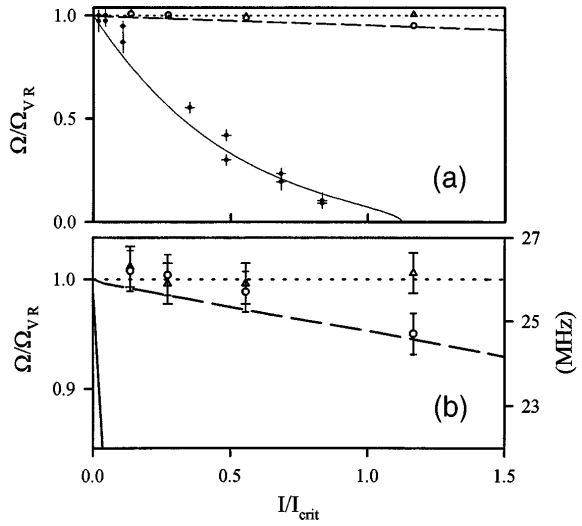

Fig. 3. (a) Frequency of the exchange of excitation as a function of input intensity measured by transmission spectroscopy of the atoms–cavity system (solid curve) with experimental data (filled dots) from Ref. 9 and by fast Fourier transform of the time response from turn-on (dashed line) and turn-off (dotted line). The open circles (turn-on) and triangles (turn-off) come from data similar to those in Fig. 2 but for different driving intensities I/I_{crit}. (b) Expanded vertical scale of (a).

low intensity (no drive) most of the time. [See the expanded vertical scale in Fig. 3(b).] The turn-on shows a frequency shift toward smaller values for larger input intensities since the system oscillates in the nonlinear regime (strong drive) for most of the time. The data obtained spectroscopically follow closely the zero-phase condition, eventually merging the two peaks into a single one for $I \approx I_{\mathrm{crit}}$. The values obtained in the small-intensity region, $I/I_{\mathrm{crit}} \ll 1$, show two distinct features. First, the spectroscopy and the time domain measurements reach the same value of the oscillating frequency $\Omega/\Omega_{\mathrm{VR}}$. In this regime the system is linear and its response to step excitation gives the same information as the transmission spectroscopy. Second, the slopes of the curves are very different at $I/I_{\mathrm{crit}} = 0$. The zero-phase condition, established in the driven system between the input and output fields, creates a relationship between the oscillating frequency Ω and the intensity I. Its derivative with respect to the intensity is nonzero even for zero intensity, as shown by the theoretical curve.

The spectrum obtained for the coupled atoms–cavity system by analysis of the time response to step excitation shows the oscillatory exchange of energy with an excitation-dependent period. Despite a high excitation for the turn-off time response, the system oscillates at the low-intensity frequency, providing no information of the possible nonlinear regime. The turn-on time response frequency shows a small-intensity dependence. This behavior contrasts with the transmission spectroscopy, in which the system is driven and can reach nonlinear steady states that drastically modify the spectrum.

In this Letter we have shown our investigation of the time response of N two-level atoms coupled to a single mode of the electromagnetic field. It is a step in our research toward studying the quantal behavior of the electromagnetic field in such an optical system. The effective number of atoms present in the cavity mode can be small ($N \approx 25$), and the observations in the region explored are quantitatively explained by the Maxwell–Bloch equations. The experiment has a thermal beam of atoms traversing a Gaussian mode with standing waves that produce a distribution of coupling constants, as has recently been observed with slow single atoms.[17] This causes inhomogeneous broadening that averages out the correlations between atoms and field. A combination of time and frequency measurements with slow atoms seems a promising avenue for overcoming those difficulties in the future, as recently suggested by Carmichael *et al.*[18]

We thank G. Ramos for help with some of the experiments. This research was supported in part by the National Science Foundation.

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