Quantum State Reduction and Conditional Time Evolution of Wave-Particle Correlations in Cavity QED

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We report measurements in cavity QED of a wave-particle correlation function which records the conditional time evolution of the field of a fraction of a photon. Detection of a photon prepares a state of well-defined phase that evolves back to equilibrium via a damped vacuum Rabi oscillation. We record the regression of the field amplitude. The recorded correlation function is nonclassical and provides an efficiency independent path to the spectrum of squeezing. Nonclassicality is observed even when the intensity fluctuations are classical.

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The seminal work of Hanbury-Brown and Twiss [1] marks the beginning of the systematic study of the quantum fluctuations of light. Two lines of experiments are notable: those measuring correlations between pairs of photodetections (particle aspect of light) [2–6] and squeezing experiments which measure the variance of the electromagnetic field amplitude (wave aspect of light) [7–9]. No attempt has been made previously to draw the particle and wave aspects together by correlating a photon detection with fluctuations of the electromagnetic field amplitude. We have done this, extending the ideas of Hanbury-Brown and Twiss to record the conditional time evolution of the amplitude fluctuations of an electromagnetic wave. Measurements are made in the strong-coupling regime of cavity quantum electrodynamics (QED) [10] and exhibit the nonclassical fluctuations of light in a dramatic new way.

A photon correlator detects the emission of a photon from an optical source and correlates it with a second photon detected after a delay τ [6]. We correlate a photon emission with the photocurrent of a balanced homodyne detector [11] which simultaneously measures a quadrature amplitude of the optical field. We average over many photocurrent samples, each triggered on a photon count, and recover the conditional time evolution of the field amplitude out of the shot noise. The recorded wave-particle correlation is subject to classical bounds that are more general than those which test particle or wave aspects of light individually. In particular, there is an upper bound on the variance, in addition to the usual lower bound, whose violation shows that *both* quadrature amplitudes of squeezed light are nonclassical [12].

Cavity QED systems have been used in numerous studies in quantum optics [10], recently to detect a single microwave photon nondestructively [13], and to produce photon number states and trapping states of the electromagnetic field [14,15]. Operated at optical frequencies, they are sources of nonclassical light, as demonstrated in earlier photon correlation [3,4] and squeezing [16] measurements. The system in our laboratory consists of a beam of optically pumped Rb atoms traversing a high finesse Fabry-Perot cavity driven by laser light [4,5]. The cavity defines a TEM_{00} Gaussian standing wave mode, of waist $w_0 = 21\ \mu$m and length $l = 410\ \mu$m. We use a one-sided configuration with a 10 ppm transmission input mirror and a 285 ppm transmission output mirror. Three rates characterize the system: the optimal dipole coupling g_0, field decay rate κ, and polarization decay rate $\gamma_\perp$. Their values are $(g_0, \kappa, \gamma_\perp)/2\pi = (12, 8, 3)$ MHz, which places the system in the strong-coupling regime of cavity QED [saturation photon number $n_0 = 1.33(\gamma_\perp/g_0)^2 = 0.08$] where equilibrium is approached via the vacuum Rabi oscillation of a fraction of a photon [17–20]. Time scales are compatible with present digitizing electronics.

A schematic of the experimental apparatus is shown in Fig. 1. Light of wavelength 780 nm from a Ti:sapphire laser enters a Mach-Zehnder interferometer, driving the cavity QED system in one arm and providing a local oscillator (LO) for the balanced homodyne detector (BHD)

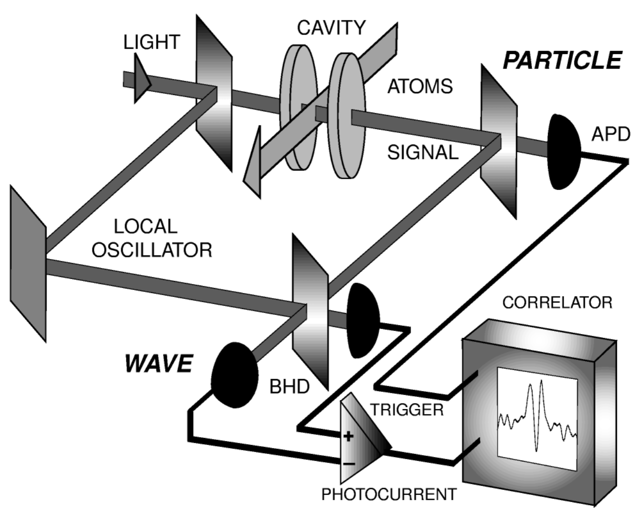

FIG. 1. Simplified diagram of the experimental setup. The fluctuations of the signal field are detected with a balanced homodyne detector (BHD) situated at the output beam splitter of a Mach-Zehnder interferometer. Sampling is triggered by counts out of the avalanche photodiode (APD).

in the other [11]. The BHD has strong common mode suppression and permits the phase sensitive measurement of the field quadrature to be made. Part of the signal escaping the cavity creates photodetections at the avalanche photodiode (APD), which trigger a digital oscilloscope to record in coincidence the BHD photocurrent. To maintain the cavity on resonance we utilize a lock cycle, chopping between a higher power auxiliary lock beam and the weak drive. A beam splitter directs 15% of the signal to the particle detector, represented in the figure by a single APD (the experiment uses a pair of APDs behind a 50/50 beam splitter). The other 85% of the signal goes to the wave detector, provided by the BHD, where it is mixed with the LO—whose relative phase we control with a piezoelectric transducer—and detected with fast photodiodes. The photocurrent is ac amplified by 65 dB, 70 MHz low pass filtered, and sampled and averaged with a fast digital oscilloscope at 2 Gs/s. The photon detections at the APDs which trigger the digital oscilloscope to record the BHD photocurrent can alternatively be used to obtain the intensity correlation function, $g^{(2)}(\tau)$.

We sketch what this apparatus measures by considering the reduction of the equilibrium state of the cavity QED system on the occasion of a triggering photon detection. Defining $\hat{Q}_\theta \equiv \frac{1}{2}(\hat{a}e^{-i\theta} + \hat{a}^\dagger e^{i\theta})$, where $\hat{a}$ is the annihilation operator for the cavity field and θ is the BHD phase, we consider the quadrature amplitude, $\hat{Q}_{0^\circ}$, in phase with $\lambda \equiv \langle \hat{a} \rangle$. For weak excitation, and assuming fixed atomic positions $\{\vec{r}_j\}$, to lowest order in λ the equilibrium state is the pure state [21,22]

$$|\psi\rangle = [|0\rangle + \lambda|1\rangle + (\lambda^2/\sqrt{2})(1+\zeta)|2\rangle + \cdots]|G\rangle + \cdots, \quad (1)$$

where $|G\rangle$ is the many-atom ground state and ζ is a complicated function of the dipole coupling strengths $\{g_j \equiv g(\vec{r}_j)\}$ [23]. After the detection, the conditional state is initially the reduced state $\hat{a}|\psi\rangle/\lambda$, which then relaxes back to equilibrium. The reduction and regression is traced by [21,22]

$$|\psi\rangle \to \{|0\rangle + \lambda[1 + \zeta f(\tau)]|1\rangle + \cdots\}|G\rangle + \cdots, \quad (2)$$

where

$$f(\tau) = e^{-(1/2)(\kappa+\gamma/2)\tau}\left\{\cos\Omega\tau + \frac{\frac{1}{2}(\kappa + \gamma/2)}{\Omega}\sin\Omega\tau\right\}, \quad (3)$$

with

$$\Omega = \left[\sum_j g_j^2 - \frac{1}{4}(\kappa - \gamma/2)^2\right]^{1/2}. \quad (4)$$

Thus, the quadrature amplitude expectation makes the transient excursion $\langle \hat{Q}_{0^\circ}\rangle \to \lambda[1 + \zeta f(\tau)]$ away from its equilibrium value $\langle \hat{Q}_{0^\circ}\rangle = \lambda$. This is to be contrasted with the excursion of the photon number, $\langle \hat{a}^\dagger\hat{a}\rangle \to \lambda^2[1 + \zeta f(\tau)]^2$, away from $\langle \hat{a}^\dagger\hat{a}\rangle = \lambda^2$.

A photon correlator measures the photon number transient $[1 + \zeta f(\tau)]^2 = g^{(2)}(\tau)$ [3–5] while the apparatus of Fig. 1 measures the transient $1 + \zeta f(\tau)$ in the field. The two measurements record the same information for $|\zeta| \ll 1$, but are different outside this limit of weak nonlinearity. If $|\zeta| > 2$, for example, the fluctuation $1 + \zeta f(0) < -1$ in the field is not allowed classically, compared with $g^{(2)}(0) > 1$ which is classically allowed. Differences can also appear when an average over ζ (atomic positions) is taken, as it is in our experiment, due to the atomic beam.

To address the issue of nonclassicality qualitatively, we must introduce appropriate inequalities. Intensity fluctuations in cavity QED have been shown to violate the classical inequalities $g^{(2)}(0) \geq 1$ [3,5] and $|g^{(2)}(\tau) - 1| \leq |g^{(2)}(0) - 1|$ [4,5]. For our field measurements, a generalized set of inequalities applies. Formally, the measured wave-particle correlation function is $\langle :(\hat{a}^\dagger\hat{a}) \times (0)\hat{Q}_\theta(\tau):\rangle/\langle \hat{a}^\dagger\hat{a}\rangle$. Normalizing by $\sqrt{\eta}\,\lambda$, where η is the coupling efficiency into the BHD, separating the fluctuation $\Delta\hat{a}$ from the mean amplitude λ, and assuming Gaussian fluctuations (third-order moments vanish), we express the measured correlation function as

$$h_\theta(\tau) = \cos\theta + 2\frac{\langle :\Delta\hat{Q}_{0^\circ}(0)\Delta\hat{Q}_\theta(\tau):\rangle}{\lambda^2 + \langle \Delta\hat{a}^\dagger\Delta\hat{a}\rangle} + \xi(\tau), \quad (5)$$

where $\xi(\tau)$ is the residual shot noise, with $\overline{\xi(0)\xi(\tau)} = (\Gamma/16\kappa\lambda^2\eta N_s)e^{-\Gamma\tau}$, where Γ is the BHD bandwidth and N_s is the number of samples. The inequalities

$$0 \leq \overline{h_{0^\circ}(0)} - 1 \leq \frac{2}{1 + \lambda^2/\langle \Delta\hat{a}^\dagger\Delta\hat{a}\rangle}, \quad (6a)$$

$$|\overline{h_{0^\circ}(\tau)} - 1| \leq |\overline{h_{0^\circ}(0)} - 1| \quad (6b)$$

follow from Eq. (5) when the quantum average is replaced by an average over a classical stochastic field [the overbar denotes the average with respect to $\xi(\tau)$]. The lower bound in (6a) addresses similar physics to the inequality $g^{(2)}(0) \geq 1$. From it [together with (6b)], $\overline{h_{0^\circ}}(0)$ is necessarily a maximum if the fluctuations are classical. This requirement follows from the conditional sampling, with its bias towards intensity maxima at $\tau = 0$; the observed field fluctuations should thus be *in phase* with the average field λ, and also *in phase* with the LO when set to $\theta = 0^\circ$. The upper bound in (6a) does not exist for $g^{(2)}(0)$. It sets a bound on the overall size of the fluctuation, $|\overline{h_{0^\circ}(\tau)} - 1|$. Quantum squeezing violates this bound [24]—sometimes when $\Delta\hat{Q}_{0^\circ}$ is the squeezed quadrature and always when it is the unsqueezed quadrature. Extremely large violations of the upper bound are possible.

From Eq. (5), our measurements give the spectrum of squeezing [9,25]

$$S(\nu, \theta = 0^\circ) = 4F\int_0^\infty d\tau \cos(2\pi\nu\tau)[\overline{h_{0^\circ}(\tau)} - 1], \quad (7)$$

where $F = 2\kappa\langle \hat{a}^\dagger\hat{a}\rangle$ is the photon flux into the correlator. This expression is independent of the BHD efficiency η.

Thus, through conditioning, we avoid the degradation of squeezing encountered by a standard measurement [8,9]. While a related conditional detection scheme has been proposed for observing photon number states in parametric down-conversion [26,27], that scheme does not display the efficiency independence observed here.

Figure 2 shows a theoretical prediction for our cavity QED system, comparing results for one optimally coupled atom with those for an atomic beam; the detection scheme has been implemented in a quantum trajectory treatment [28]. The result for one atom [Fig. 2(a)] shows a very large violation of the classical inequalities, which remains substantial, though reduced, in the presence of atomic beam fluctuations [Fig. 2(b)]. In both cases, the phase of the vacuum Rabi oscillation is anomalous, violating the lower bound of inequality (6a). For the atomic beam typical values of ζ are a little smaller than unity, so that even after an average is taken $g^{(2)}(\tau) \approx [h_{0^\circ}(\tau)]^2$, as one deduces from Eq. (2) [Fig. 2(c)]. Our measurements are made at higher photon number where spontaneous emission is significant. Under these conditions, $h_{0^\circ}(\tau)$ and $g^{(2)}(\tau)$ are not so closely related (compare Fig. 4 below).

In the experiment, we characterize the system with measurements of the intensity correlation function $g^{(2)}(\tau)$ and then measure the wave-particle correlation function. We lock the phase θ of the LO relative to the signal by actively stabilizing the length of the Mach-Zehnder interferometer with an auxiliary He-Ne laser; θ can be changed by 146° increments $[180^\circ \times (\lambda_{\text{He-Ne}}/\lambda_{\text{Ti:sapphire}})]$ in a controlled manner.

Figure 3 shows results for the conditionally averaged BHD photocurrent, and illustrates what we obtain in a raw, unnormalized measurement. In-phase and out-of-phase settings of the LO verify that the signal becomes inverted under this change of phase. The vacuum Rabi oscillation at frequency $g_0\sqrt{\bar{N}}/2\pi \approx 40$ MHz is clearly present in both data sets, and at the in-phase setting of the LO [Fig. 3(a)] its phase violates the lower bound of (6a). Thus, in spite of the preference for triggering photodetections at intensity fluctuation maxima, we observe the field fluctuating to *lower* the intensity; we observe a field fluctuation *out of phase* with the average field.

To assess whether the upper bound of (6a) is violated we must scale the raw data, and also determine the ratio, $\lambda^2/\langle\Delta\hat{a}^\dagger\Delta\hat{a}\rangle$, of coherent to incoherent intensities. The scaling may be performed to fit the residual shot noise level, under the assumption $\lambda^2/\langle\Delta\hat{a}^\dagger\Delta\hat{a}\rangle \gg 1$. We are unable, however, to determine the value of the ratio itself, and we have not therefore demonstrated the expected violation of the upper bound.

Operating at higher excitation we obtain the results of Fig. 4. Here we contrast measurements of $g^{(2)}(\tau)$ and $h_\theta(\tau)$. The former shows classical photon correlations—photon bunching and super-Poissonian counts—and no significant oscillation. There is still a strong oscillation of the conditional field, however, and it still exhibits the anomalous phase. Inequality (6b) is violated with no violation of the lower bound of (6a). We interpret Fig. 4(b) by positing a background of classical fluctuations to which the nonclassical fluctuation of Fig. 2(c) is added. The amplitude of the latter is reduced due to spontaneous emission, and the background is likely caused by atomic beam fluctuations in combination with spontaneous emission. The spectrum of squeezing [Fig. 4(c)] separates the nonclassical fluctuations from the classical fluctuations. To obtain it, we symmetrize the scaled signal in Fig. 4(b) and take the Fourier transform as specified in Eq. (7). The spectrum is negative in the vicinity of the vacuum Rabi frequency, evidencing, in frequency space, the anomalous phase. We measure ~5% reduction of the noise below the standard quantum limit. This determination is independent of the coupling efficiency into the BHD.

The notable feature of our measurement is that according to the reduction postulate, the detection of a photon (particle aspect of light) conditionally selects a field amplitude (wave aspect of light) oscillating in time with an

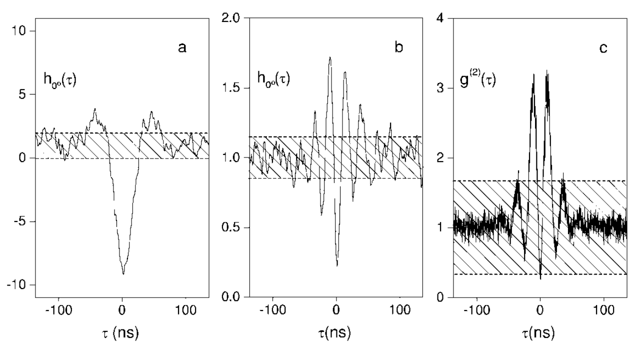

FIG. 2. Monte Carlo simulations for weak excitation ($\langle\hat{a}^\dagger\hat{a}\rangle \sim 10^{-4}$, $\lambda^2/\langle\Delta\hat{a}^\dagger\Delta\hat{a}\rangle \gg 1$): (a) $h_{0^\circ}(\tau)$ for one atom optimally coupled to the cavity mode (2×10^4 samples); (b) $h_{0^\circ}(\tau)$ and (c) $g^{(2)}(\tau)$ for an atomic beam with effective atom number $\bar{N} = 11$ (5×10^4 samples). In (a) and (b) the light parts of the traces violate the classical inequalities. The shaded region is classically allowed after taking into account the residual shot noise.

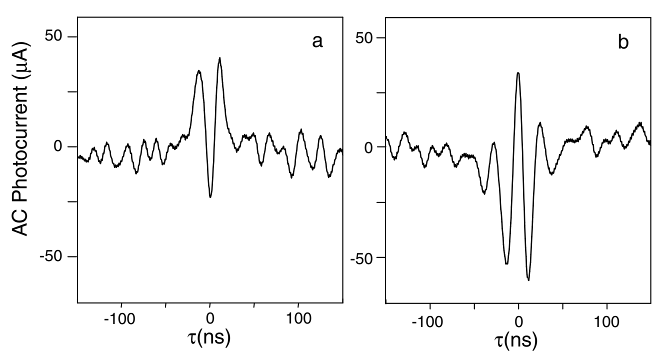

FIG. 3. Unnormalized ac photocurrent proportional to $h_\theta(\tau) - 1$; for an effective atom number $\bar{N} \approx 11$ and intracavity photon number $\langle\hat{a}^\dagger\hat{a}\rangle \approx 0.07$: (a) $h_{34^\circ}(\tau)$ (3×10^4 samples) and (b) $h_{180^\circ}(\tau)$ (5×10^4 samples). The signal decreases at intermediate phases and disappears at 90°.

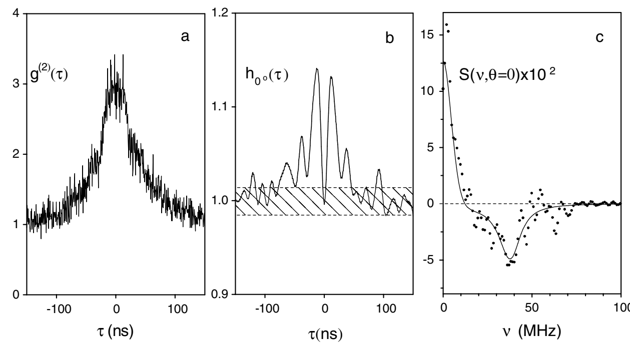

FIG. 4. Observations for an intracavity photon number $\langle \hat{a}^\dagger \hat{a} \rangle \approx 0.11$ and effective atom number $\bar{N} \approx 13$: (a) $g^{(2)}(\tau)$ shows *classical* nonoscillatory behavior, (b) $h_{0°}(\tau)$ shows *nonclassical* oscillatory behavior, and (c) $S(\nu, \theta = 0°)$ showing squeezing below the standard quantum limit (the continuous line is the optimally filtered spectrum). The shaded region in (b) is classically allowed after taking into account the residual noise. The dashed line in (c) is the boundary between classical (above) and nonclassical (below).

anomalous phase. Our experiment catches the field fluctuations as they occur, and thus we have been able to observe the anomalously phased oscillation directly. We anticipate that wave-particle correlations will find wide application not only in further studies of the fundamental nature of light, but also in practical areas of science such as imaging.

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[23] The explicit form of ζ depends on the spatial configuration of the atoms through the dimensionless coupling strengths $C_j \equiv g_j^2/\gamma\kappa$. One finds

$$\zeta = -4S(1 + C')[1 + 2C + 2(1 + \gamma/\kappa)S']^{-1},$$

where $C \equiv \sum_j C_j$, $S \equiv \sum_j C_j C_j'/(1 + C' - 2C_j')$, and $(C_j', C', S') \equiv (1 + \gamma/2\kappa)^{-1}(C_j, C, S)$. This formula is an alternative form of Eq. (1) in Ref. [3].

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