Giant Violations of Classical Inequalities through Conditional Homodyne Detection of the Quadrature Amplitudes of Light

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(Received 25 February 2000)

Conditional homodyne detection is proposed as an extension of the intensity correlation technique introduced by Hanbury-Brown and Twiss [Nature (London) **177**, 27 (1956)]. It detects giant quadrature amplitude fluctuations for weakly squeezed light, violating a classical bound by orders of magnitude. Fluctuations of *both* quadrature amplitudes are anomalously large. The squeezed quadrature also exhibits an anomalous phase.

PACS numbers: 42.50.Dv, 42.50.Lc

The development by Hanbury-Brown and Twiss (HBT) of the technique of intensity cross-correlating light [1] has had important ramifications in quantum optics. It provided the stimulus for a systematic treatment of optical coherence within the framework of quantum mechanics [2] and led to digital, photon counting implementations, applied in measurements of the photon statistics of laser light [3,4], and, ultimately, of nonclassical light sources [5–7].

A notable feature of the HBT technique is that it makes a *conditional* measurement, detecting the intensity fluctuations by collecting data on the cue of a conditioning photon count that identifies times when a fluctuation is in progress. The average fluctuation is recovered as a conditional evolution over time and a sensitive probe of its nonclassical features is provided. Thus, photon antibunching is detected with relative ease [5]; on the other hand, direct observation of the related sub-Poissonian effect is very difficult, because it relies on a low-efficiency nonconditional measurement [8,9].

Conventionally, quadrature squeezing is detected nonconditionally [10]; effectively, the sub-Poissonian variance of a photon counting distribution is measured with the photon counts integrated over many correlation times. The detection is insensitive for light of low photon flux, and the squeezing is degraded by inefficiencies. The time evolution of the fluctuations is not observed since the measurement is resolved in the frequency domain.

In this Letter we propose a method for the conditional homodyne detection (CHD) of the quadrature amplitude fluctuations of light. The measurement cross-correlates a photon count with the current of a balanced homodyne detector in a natural extension of the HBT technique. It is in principle very sensitive to the fluctuations of weakly squeezed light and shows only a signal-to-noise dependence on detection efficiency. Given sufficient bandwidth, it resolves the fluctuations in time.

The proposed correlator is illustrated in Fig. 1. The design is based on the "start"/"stop" scheme used in modern implementations of the HBT technique [5–7]. The main difference is the homodyne detector which replaces the photon multiplier, or avalanche photon diode, in the stop channel. Within a few correlation times before and after each start, the homodyne current $I(t)$ is digitized, recorded, and used to update a cumulative average. Averaging N_s such samples reduces the shot noise; the surviving signal is a conditional average of the quadrature amplitude fluctuations. Since the signal is nonzero only if there is a bias towards fluctuations of a definite sign, a coherent offset of the source field is generally required. By adjusting the offset phase in parallel with the phase of the local oscillator, any quadrature amplitude can, in principle, be measured; a similar offset is used in schemes for quantum state reconstruction [11–13]. In Fig. 1, the source is quasimonochromatic with emitted field $\sqrt{2}\,\hat{a}$ and photon flux $2\langle\hat{a}^\dagger\hat{a}\rangle$ per inverse half-width (the units of time are inverse half-widths throughout). The input field to the correlator is

$$\sqrt{2}\,\hat{b} = \sqrt{2}\,(\hat{a} + Ae^{i\phi}), \quad (1)$$

where it is assumed that the offset, with amplitude $Ae^{i\phi}$, is added without loss from the source field at the beam

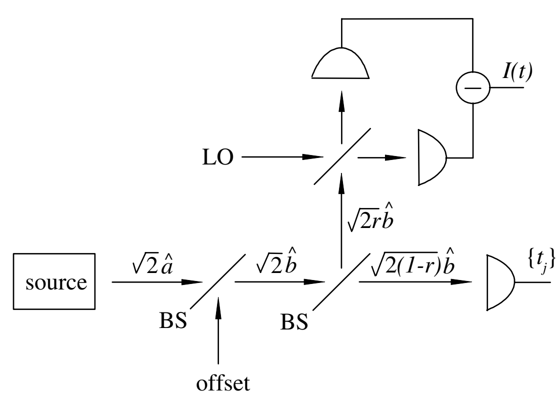

FIG. 1. In conditional homodyne detection a coherent offset is added to the source field $\sqrt{2}\,\hat{a}$ to produce the input field $\sqrt{2}\,\hat{b}$. A fraction r of the input photon flux is sent to a balanced homodyne detector, and the rest goes to a photon counter which triggers the sampling of the homodyne current. An ensemble of samples, $\{I(t_j + \tau), j = 1, \ldots, N_s\}$, $-\tau_{\max} \le \tau \le \tau_{\max}$, is averaged to recover a signal out of the shot noise.

splitter. Vacuum terms are omitted in the field operators $\sqrt{2}\,\hat{a}$ and $\sqrt{2}\,\hat{b}$.

In general, the scheme in Fig. 1 measures a third-order correlation function of the field $\sqrt{2}\,\hat{b}$. It correlates the photon number $\hat{n}_b \equiv \hat{b}^\dagger \hat{b}$ with the quadrature amplitude $\hat{b}_\theta \equiv (\hat{b}e^{-i\theta} + \text{H.c.})/2$, where θ is the local oscillator phase. Specifically, assuming the homodyne detector has the impulse response $e^{-\Gamma t}$, the measured signal is

$$H(\tau) = \sqrt{8r}\,\Gamma \int_{-\infty}^{\tau} dy\, \frac{\langle:\hat{n}_b(0)\hat{b}_\theta(y):\rangle}{\langle \hat{n}_b\rangle} e^{\Gamma(y-\tau)} + \Xi(\tau), \tag{2}$$

where r is the reflection coefficient at the correlator input, :: denotes time and normal ordering, and $\Xi(\tau)$ is the residual shot noise, with correlation function $\overline{\Xi(\tau)\Xi(\tau')} = (\Gamma/2N_s)e^{-\Gamma|\tau-\tau'|}$. The residual noise increases with detection bandwidth Γ in the standard way, and $\Gamma \gg 1$ is required to resolve the source fluctuations in time. N_s must be large enough to keep the noise below the signal level. Squeezing is revealed by a *second-order* correlation function of the field. The connection with $H(\tau)$ is made by writing $\hat{b} = \langle \hat{b}\rangle + \Delta\hat{a}$, separating the mean field $\langle \hat{b}\rangle$ from the source-field fluctuation. We then assume that third-order moments in $\Delta\hat{a}^\dagger$ and $\Delta\hat{a}$ vanish. This is true exactly if the fluctuations are Gaussian, or it may be approximately valid because the source field is weak. The offset is adjusted to match the phase of $\langle \hat{b}\rangle$ to the local oscillator phase and its amplitude to the size of the rms fluctuation. With

$$\langle \hat{b}\rangle = \sqrt{\langle \Delta\hat{a}^\dagger \Delta\hat{a}\rangle}\, e^{i\theta}, \tag{3}$$

and in the limit $\Gamma \gg 1$, we thus obtain

$$h_\theta(\tau) \equiv \frac{H(\tau)}{\sqrt{8r}\,|\langle \hat{b}\rangle|} = 1 + \frac{\langle:\Delta\hat{a}_\theta(0)\Delta\hat{a}_\theta(\tau):\rangle}{\langle \Delta\hat{a}^\dagger \Delta\hat{a}\rangle} + \xi(\tau), \tag{4}$$

where $\hat{a}_\theta \equiv (\hat{a}e^{-i\theta} + \text{H.c.})/2$, and the scaled shot noise variance is $\overline{\xi(\tau)^2} = \Gamma/16rN_s|\langle \hat{b}\rangle|^2$. In Eq. (4) we see that the proposed correlator achieves a squeezing measurement, since the spectrum of squeezing is [14]

$$S_\theta(\omega) = 8\langle \hat{n}_b\rangle \int_0^\infty d\tau \cos(\omega\tau)[\overline{h_\theta(\tau)} - 1]. \tag{5}$$

(The overbar denotes the average over the residual shot noise.)

It is notable that the measurement is independent of the fraction of light sent to the homodyne detector; r appears only in the signal-to-noise ratio [15]. Other schemes are known for autocorrelating the quadrature amplitudes of light [16,17], of which one is also efficiency independent [17]. In that case, though, a standard HBT arrangement is used, from which the field correlation is extracted in an indirect manner. Conditional sampling in homodyne detection has also been proposed before [18], however, in a limited application which (i) requires a source of photon pairs, (ii) does not correlate quadrature amplitudes, and (iii) shows the usual dependence on detection efficiency.

The efficiency independence is less remarkable than the giant fluctuations revealed by Eq. (4). The standard signature of squeezing is a reduced shot noise level, interpreted as a deamplification of the vacuum fluctuations. CHD provides a different view. The shot noise is irrelevant; ideally it is eliminated through the conditional average. The focus, then, is on the fluctuations of the light emitted by the source, which violate inequalities that a classical field must satisfy. Equation (4) uncovers the nonclassicality. It does this for the squeezed quadrature, as one would expect, but more surprisingly, it also shows the fluctuations of the unsqueezed quadrature to be nonclassical. There is a single underlying anomaly: the individual quadrature variances are incompatible with the fluctuation intensity

$$\langle \Delta\hat{a}^\dagger \Delta\hat{a}\rangle = \langle:(\Delta\hat{a}_\theta)^2:\rangle + \langle:(\Delta\hat{a}_{\theta+\pi/2})^2:\rangle. \tag{6}$$

Classically, the quadrature variances are positive. It follows then, from Eqs. (4) and (6), that

$$0 \le \overline{h_\theta(0)} - 1 \le 1. \tag{7a}$$

More generally, one proves the Schwarz inequality

$$|\overline{h_\theta(\tau)} - 1| \le |\overline{h_\theta(0)} - 1| \le 1. \tag{7b}$$

In actual fact, one of the variances is negative whenever there is squeezing below the quantum limit. This allows the squeezed (unsqueezed) fluctuations to violate the lower (upper) bound of (7a) and fluctuations in either quadrature to violate (7b). Giant violations occur if the photon flux is low. For the degenerate parametric oscillator [19], for example, with pump parameter $\lambda \ll 1$

$$\langle:(\Delta\hat{a}_X)^2:\rangle \approx \lambda(1+\lambda)/4, \tag{8a}$$

$$\langle:(\Delta\hat{a}_Y)^2:\rangle \approx -\lambda(1-\lambda)/4, \tag{8b}$$

$$\langle \Delta\hat{a}^\dagger \Delta\hat{a}\rangle \approx \lambda^2/2. \tag{8c}$$

The ratio $\langle:(\Delta\hat{a}_{X,Y})^2:\rangle/\langle \Delta\hat{a}^\dagger \Delta\hat{a}\rangle$ is of the order of $1/\lambda$. The classical bounds might be exceeded by a factor of 10 or even 100.

To illustrate this prediction, we have implemented CHD within quantum trajectory theory [20] and simulated it for various light sources. Quantum trajectory theory is formulated around the experimental data, viewed as a stochastic measurement record. In the present case, the record comprises a continuous homodyne current, $I(t)$, and a set of start times $\{t_j\}$. The source quasimode is in a quantum state $|\psi_{\text{REC}}(t)\rangle$, conditioned on the accumulated measurement record. Realizations of $I(t)$, $\{t_j\}$, and $|\psi_{\text{REC}}(t)\rangle$ obey a set of stochastic differential equations which we simulate on a computer. We sample an ongoing realization of $I(t)$ extending over many starts and calculate

$$H(\tau) = \frac{1}{N_s}\sum_{j=1}^{N_s} I(t_j + \tau). \tag{9}$$

We do not assume Gaussian noise or make explicit use of Eq. (2) or Eq. (4). [Equation (2) is derived from Eq. (9)

by writing $I(t) = \Gamma \int_{-\infty}^{t} dQ_y\, e^{\Gamma(y-t)}$ and evaluating the sum over j as an average over past and future (before and after t_j) measurement records [21].]

The stochastic process is formulated following the principles outlined in Secs. 8.4, 9.2, and 9.4 of Ref. [20], generalized to include the coherent offset and to combine a continuous evolution under homodyne detection with the quantum jump conditioning, $|\bar{\psi}_{\rm REC}\rangle \to \hat{b}|\bar{\psi}_{\rm REC}\rangle$, at the start times t_j. (An overbar indicates that the quantum state is not normalized.) The probability for a start count in time step dt is $2(1-r)\langle \hat{n}_b(t)\rangle_{\rm REC} dt$. Between starts, $|\bar{\psi}_{\rm REC}\rangle$ evolves continuously under a stochastic Schrödinger equation. The evolution is conditioned on the ongoing realization of the charge,

$$dQ_t = \sqrt{8r}\,\langle \hat{b}_\theta\rangle_{\rm REC} dt + dW_t\,, \qquad (10)$$

deposited in the homodyne detector output circuit between t and $t + dt$; the Wiener increment dW_t incorporates the shot noise. For the arrangement of Fig. 1, we obtain the conditional Schrödinger equation

$$d|\bar{\psi}_{\rm REC}\rangle = [(\hat{H}_S/i\hbar - 2Ae^{-i\phi}\hat{a})dt + \sqrt{2r}\,\hat{b}e^{-i\theta}dQ_t]|\bar{\psi}_{\rm REC}\rangle\,, \qquad (11)$$

where $\hat{H}_S$ is the non-Hermitian source Hamiltonian. The stochastic process dQ_t has bandwidth $1/dt$. Construction of the homodyne current with realistic detection bandwidth is modeled on the filtering of an RC circuit ($\Gamma = 1/RC$)—i.e.,

$$dI = -\Gamma(Idt - dQ_t)\,. \qquad (12)$$

(In Figs. 3 and 4 spontaneous emission is also present. This is included in the standard way through additional quantum jumps.)

Figures 2(a) and 2(b) present results for the degenerate parametric oscillator. They demonstrate that well below threshold, where the squeezing is small (8% at line center), the classical bounds are dramatically violated. Note in particular the anomalous phase of the fluctuation in Fig. 2(b). Here, although the sampling of $I(t)$ is triggered on photon counts, the average records a fluctuation that is out of phase with the offset—triggering should be more probable for *in phase* fluctuations. Equation (4) holds for broadband detection and matches curve (i) very well. Curves (ii) and (iii) illustrate how the time resolution is lost as the detection bandwidth is reduced.

We emphasize with Fig. 2(c) that we do not simply report an alternate method for the detection of squeezing. Conditional homodyne detection provides a wholly new view of the nonclassicality of squeezed light. As Fig. 2(c) shows, the violation of inequality (7a) *increases* as squeezing, i.e., λ, decreases. This is explained by noting that for small λ CHD detects fluctuations which are *isolated in time*—fluctuations associated with the *infrequent* emission of two-photon pulses. The pulses produce highly bunched HBT correlations—$g^{(2)}(0) \sim 1/\lambda^2$. Conditional homodyne detection resolves the intensity correlation into

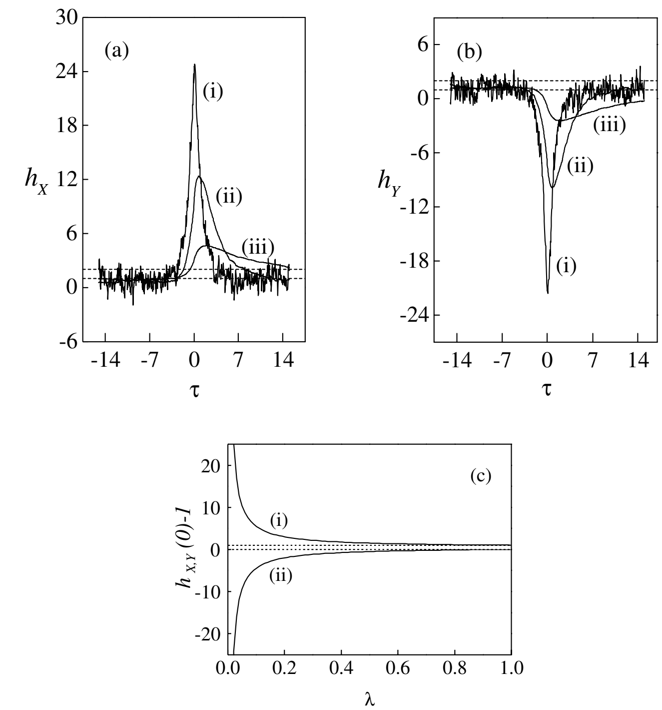

FIG. 2. Quantum trajectory simulation of CHD for the degenerate parametric oscillator: (a) X-quadrature amplitude (unsqueezed), (b) Y-quadrature amplitude (squeezed); with intracavity photon number $\langle \hat{a}^\dagger\hat{a}\rangle = 2.0 \times 10^{-4}$ ($\lambda = 0.02$), $r = 0.5$, $N_s = 10\,000$, and (i) $\Gamma = 10$, (ii) $\Gamma = 0.5$, and (iii) $\Gamma = 0.1$. The dashed lines are the classical bounds. (c) $h_X(0) - 1$ (i) and $h_Y(0) - 1$ (ii) as a function of λ.

quadrature amplitude components, uncovering field fluctuations that are similarly large, while in addition revealing the anomalous phase.

Figures 3 and 4 illustrate CHD applied to cavity QED, where nonclassical HBT correlations have been observed [6,7] and interpreted as an indirect manifestation of quadrature squeezing [17,22]. The spectrum of squeezing is

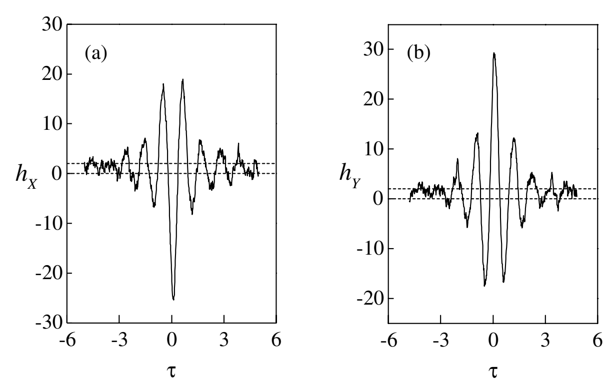

FIG. 3. Quantum trajectory simulation of CHD for single-atom cavity QED: (a) X-quadrature amplitude (in phase with the mean field), (b) Y-quadrature amplitude; with dipole coupling constant $g = 6$, atomic decay rate $\gamma = 2$, intracavity photon number $\langle \hat{a}^\dagger\hat{a}\rangle = 2.8 \times 10^{-4}$, $r = 0.5$, $N_s = 10\,000$, and $\Gamma = 10$. The dashed lines are the classical bounds.

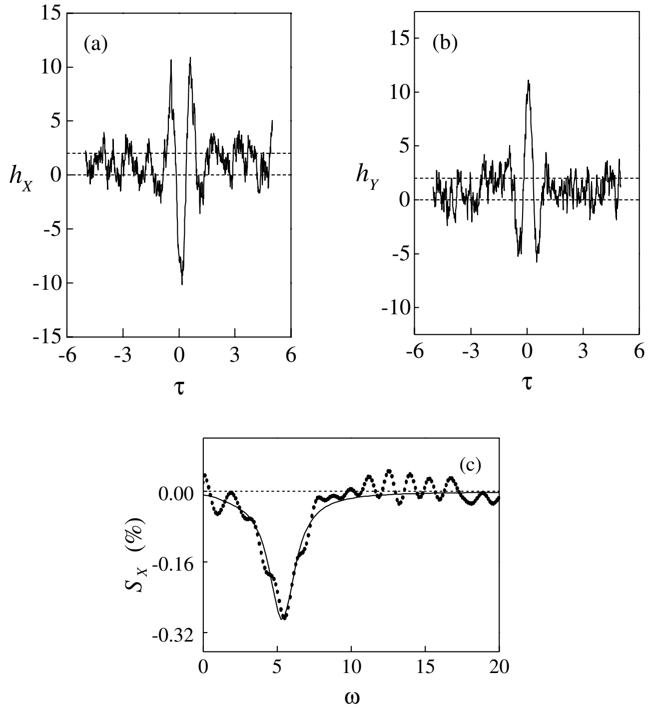

FIG. 4. Quantum trajectory simulation of CHD for many-atom cavity QED: (a) X-quadrature amplitude (in phase with the mean field), (b) Y-quadrature amplitude; with atomic beam density $\bar{N}_{\rm eff} = 3$, transit time $w_0/\bar{v} = 2.9$, dipole coupling constant $g_{\max} = 3.77$, atomic decay rate $\gamma = 1.25$, intracavity photon number $\langle \hat{a}^\dagger \hat{a} \rangle = 1.5 \times 10^{-4}$, $r = 0.5$, $N_s = 20\,000$, and $\Gamma = 10$. The dashed lines are the classical bounds. Curve (c) is the spectrum of squeezing obtained from the X-quadrature simulation.

the so-called "vacuum Rabi" doublet [23]. The fluctuations therefore develop as spontaneous Rabi oscillations. Squeezing is evident from the anomalous phase of the oscillation [Figs. 3(a) and 4(a)], where again, giant violations of the inequalities may be observed. Results for one atom, shown in Fig. 3, continue to hold in an atomic beam with a reduced size of the violation of the classical inequalities (Fig. 4). An experiment for the latter case will be reported elsewhere [24].

We have proposed a new technique for studying the quantum fluctuations of light, building on the seminal work of Hanbury-Brown and Twiss. The technique records the time evolution of quadrature amplitude fluctuations and can detect giant violations of classical inequalities.

The work was supported by the NSF under Grants No. PHY-9531218 and No. PHY-9721409 and by ONR under Grant No. N000149810419. H. J. C. acknowledges support from the Alexander von Humboldt-Stiftung, and H. M. C.-B. from CONACYT, Mexico.

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