Nondegenerate four-wave mixing in rubidium vapor: Transient regime

F. E. Becerra,1,2 R. T. Willis,1 S. L. Rolston,1 H. J. Carmichael,1,3 and L. A. Orozco1
1Joint Quantum Institute, Department of Physics, University of Maryland and National Institute of Standards and Technology, College Park, Maryland 20742, USA
2Departamento de Física, CINVESTAV, Apdo. Post. 14-740, 07000, México, D.F., México
3Department of Physics, University of Auckland, Private Bag 92019, Auckland, New Zealand

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We investigate the transient response of the generated light from four-wave mixing (FWM) in the diamond configuration using a step-down field excitation. The transients show fast decay times and oscillations that depend on the detunings and intensities of the fields. A simplified model taking into account the thermal motion of the atoms, propagation, absorption, and dispersion effects shows qualitative agreement with the experimental observations with the energy levels in rubidium (5S1/2, 5P1/2, 5P3/2, and 6S1/2). The atomic polarization comes from all the contributions of different velocity classes of atoms in the ensemble modifying dramatically the total transient behavior of the light from FWM.

I. INTRODUCTION

Four-wave mixing (FWM) is a nonlinear optical process that coherently combines three waves to produce a fourth one, and is used in many areas of physics. In atomic physics FWM is utilized to study spectroscopic and quantum optics processes [1–6]. It can generate specific frequencies not available with direct electromagnetic sources, and has become a widely used tool for condensed matter physics to characterize the short time correlations and coherences of semiconductors (see, e.g., Refs. [1,7,8]). Recent work with FWM for quantum information includes implementations of the atomic-ensemble-based quantum repeater of Duan et al. [9] such as [10–12], the generation of strong squeezing [13,14], and quantum imaging [15]. These implementations rely on the coherences created in the atomic systems, which are affected by dephasing mechanisms producing decoherence over time. Study of the temporal dynamics of FWM can provide insight into the decay of the atomic polarization and the process of decoherence.

We study the time evolution of the atomic coherences in FWM in rubidium vapor with transitions at telecommunication wavelengths. We use a step-down excitation of the excited states and observe the generated light from FWM in a diamond configuration, where the levels include the 5S ground state, the 5P states (D1 and D2 lines), and the 6S state (see Fig. 1). This study complements the investigation of the frequency response of the process in Refs. [16,17] and concerns effects otherwise unobservable in the frequency regime due to the nonlinearity of the system. The transient signal contains information about the temporal evolution of the induced atomic polarization in the medium and can have collective atomic effects [18,19]. Since the atomic polarization is responsible for quantum effects [9], understanding the decoherence is fundamental for applications of FWM in quantum information; our work [20] shows the limitations on nonclassicality from decoherence.

The FWM signal in our configuration in steady state is weak. We step down to zero the laser intensity that connects one of the intermediate states to the highest excited state and observe the time evolution of the FWM as it decays and propagates out of the system. The time scales of this atomic system allow us good resolution with this simple step-down process; this is in contrast to the situation encountered in FWM in semiconductors where the dynamics are very fast (picoseconds or faster) and pump-probe techniques such as time-integrated FWM and time-resolved FWM are used to study correlation dynamics in semiconductors [21–25].

The organization of this work is as follows: Sec. II describes a simplified theoretical model of the system. Section III contains the experimental setup and observations. Section IV contains the analysis incorporating other effects present in the experiment such as Doppler broadening, and Sec. V explains the results of the investigation of the transients in FWM. Section VI summarizes our conclusions.

II. THEORY

We study the evolution of the atomic coherences created by the nonlinear interaction of three fields in the atomic medium in the FWM process when one of those fields is suddenly set to zero. The density matrix treatment allows us to study the decay and possible dephasing mechanisms such as Doppler broadening.

Figure 1 shows the fields Ei at frequencies ωi coupling the four levels of the atomic system. E1 couples the transition |a⟩ → |b⟩, E2 the transition |b⟩ → |c⟩, and E3 the transition |a⟩ → |d⟩. The generated light from FWM is resonant with the transition |d⟩ → |c⟩. We assume that the generated light from the FWM process is weak in comparison with the incident fields and we ignore propagation effects at this frequency.

The density matrix equations describing this system in the rotating wave approximation (RWA) [4] are

\[ \dot{\rho}_{ll} = -\sum_{n} \Gamma_{nl} \rho_{nl} + \sum_{n} \Gamma_{nl} \rho_{nm} - \frac{i}{\hbar} \sum_{m} (V_{lm} \rho_{ml} - \rho_{lm} V_{ml}), \]

\[ \dot{\rho}_{mn} = (i \Delta_{mn} - \gamma_{mn}) \rho_{mn} - \frac{i}{\hbar} \sum_{v} (V_{mv} \rho_{vn} - \rho_{vn} V_{vm}), \]

where \( \rho_{ll} \) describes population of level \( l \), \( \rho_{mn} \) the coherence between the levels \( m \) and \( n \), \( \Gamma_{mn} \) the decay rate from level \( m \) to \( n \), \( \Gamma_{n} = 1/\tau_{n} \) the total decay rate of level \( n \) with \( \tau_{n} \) the lifetime...
of this level, and $\gamma_{mn} = \frac{1}{2}(\Gamma_m + \Gamma_n)$ the dephasing rate of the coherence $\rho_{mn}$. The term $\Delta_{mn}$ is the detuning of the external field coupling the transition $|n\rangle \rightarrow |m\rangle$ from the energy of this transition (see Fig. 1). Energy conservation in FWM, $\omega_1 + \omega_2 = \omega_3 + \omega_4$, defines the frequency and detuning for the generated light. The term $V_{nm} = -\Omega_{nm}$ describes the interaction of the atomic ensemble with the external field in the dipole approximation [4] where $\Omega_{nm}$ is the Rabi frequency for the external field coupling to the transition $|n\rangle \rightarrow |m\rangle$ given by $\Omega_{nm} = \mu_i \cdot E_i / h$, where $\mu_i$ is the electric dipole moment of the transition $|n\rangle \rightarrow |m\rangle$ and $E_i$ is the electric field coupling to this transition.

The density matrix equations are 16 coupled equations. Conservation of population, $\sum \rho_{nn} = 1$, reduces this system to a set of 15 coupled differential equations. The generated light from FWM resonant with the transition $|d\rangle \rightarrow |c\rangle$ (see Fig. 1) is proportional to the atomic polarization oscillating at the frequency $\omega_3$ [16]. The induced atomic polarization for the transition $|d\rangle \rightarrow |c\rangle$ is $P_{dc} = N |\rho_{dc}|^2 \Delta t$, where $N$ is the atomic number density [4,26]. The study of the transient behavior of the light from FWM requires solving these coupled differential equations for the atomic coherence $\rho_{dc}$.

We assume that the system starts in the steady state at time $t = 0$ when we turn off the field $E_2$ (at $t = 0$). The system evolves in the presence of only $E_1$ and $E_3$ with the steady state as the initial conditions. This allows some simplifications in the calculation. We obtain a decoupled subset of the system of 15 coupled equations that contains the term $\rho_{dc}$:

$$\frac{d}{dt} \begin{pmatrix} \rho_{ca} \\ \rho_{cb} \\ \rho_{dc} \end{pmatrix} = \begin{pmatrix} \alpha_{ca} & i\Omega_1 & i\Omega_3 \\ i\Omega_1 & \alpha_{cb} & 0 \\ i\Omega_3 & 0 & \alpha_{dc} \end{pmatrix} \begin{pmatrix} \rho_{ca} \\ \rho_{cb} \\ \rho_{dc} \end{pmatrix},$$  \tag{2}

where $\alpha_{dc} = -i\Delta_{FWM} - \gamma_{cd}$, $\alpha_{ca} = -i(\Delta_1 + \Delta_2) - \gamma_{ca}$, and $\alpha_{cb} = -i\Delta_2 - \gamma_{cb}$ with $\Delta_j$ the detuning of the laser from the corresponding atomic transition, as shown in Fig. 1, and $\Delta_{FWM} = \Delta_1 + \Delta_2 - \Delta_3$. The steady-state solution of the density matrix equations determines the initial phases and amplitudes for these coherences. The solution for $\rho_{dc}$ is a linear combination of three exponential functions of time with different characteristic energies (quasienergies),

$$\rho_{dc} = A_1 e^{-i\xi_1 t} + A_2 e^{-i\xi_2 t} + A_3 e^{-i\xi_3 t},$$  \tag{3}

where $A_j$ ($j = 1,2,3$) are complex time-independent amplitudes and the quasienergies $\lambda_j$ are complex quantities whose real parts describe the oscillatory behavior of the generated light and imaginary parts the exponential decay constants. These quantities depend on the detunings and the Rabi frequencies of the lasers, and the amplitudes $A_j$ are determined from the initial (steady-state) conditions of the system.

The real parts of the quasienergies show avoided crossings due to the interaction of the external fields with the atom. Ref. [16] describes our investigations of the FWM resonant structure where we compare those quasienergies with the measured spectra. That work, using the Schrodinger equation approach, does not take into account dissipation. With the density matrix formalism it is now possible to see the damping of the atomic polarization. The intensity of the generated light from FWM is proportional to $|\rho_{dc}|^2$. The quantity $|\rho_{dc}|^2$, using Eq. (3), contains three squared amplitudes associated with the same quasienergy,

$$F_j = |A_j|^2 e^{-2|\xi_j|^2 t}, \quad (j = 1,2,3),$$  \tag{4}

showing exponential decay with a decay constant of $2|\xi_j|$, and three cross terms plus their complex conjugates (c.c.), associated with different quasienergies,

$$G_j = A_j A_k^* e^{-i(\lambda_j - \lambda_k)^2 t} + c.c.,$$  \tag{5}

for $(j,k = 0,1,2), j \neq k$. The latter show a damped oscillatory behavior with an oscillation frequency $\beta_j = Re[\lambda_j - \lambda_k^*]$ and an exponential damping constant $\xi_{jk} = Im[\lambda_j - \lambda_k^*]$. Figures 2(a) and 2(b) show the quantities $F_j$ and $G_j$ as a function of time.

The calculations use the relevant parameters for the experiment. The relevant levels in $^{85}$Rb (Fig. 1) are $|a\rangle = |S_{1/2}\rangle$, $|b\rangle = |P_{1/2}\rangle$, $|c\rangle = |S_{1/2}\rangle$, and $|d\rangle = |P_{1/2}\rangle$, with lifetimes $\tau_{P_{1/2}} = 27.8$ ns, $\tau_{S_{1/2}} = 26.4$ ns, and $\tau_{S_{1/2}} = 46$ ns and with corresponding linewidths $\Gamma_{S_{1/2}} = 5.8$ MHz, $\Gamma_{P_{1/2}} = 6$ MHz, and $\Gamma_{S_{1/2}} = 3.2$ MHz. Figure 2(c) shows the transient FWM light using the simplified model from Eq. (3) and that numerically calculated from the complete set of density matrix equations [Eq. (1)].

For this particular set of Rabi frequencies and detunings, the transient signal of the generated field shows an oscillatory exponential decay behavior. The oscillation frequency depends on the strength of the oscillatory terms from Eq. (5). The decay of the total signal is dominated by the strongest components of the decaying terms in Eq. (4). The shape of the transient signal depends on these six terms with the phases of the oscillations determined by the initial conditions. The oscillation frequencies depend on the detunings and the Rabi frequencies of the fields.

The decoupled subset of equations describes the behavior very well, as seen in Fig. 2(c), with the difference between the two calculations of the order of one part in $10^3$, which is within the numerical error. This allows us to study the physical mechanism acting in this system using the simpler set of equations. The atomic coherences in the decoupled system

FIG. 1. (Color online) Four-level atom interacting with three external fields $E_1$, $E_2$, and $E_3$ with frequencies $\omega_1$ (780 nm), $\omega_2$ (1324 nm), and $\omega_3$ (1367 nm), respectively, and generated light (red online) via four-wave mixing at frequency $\omega_4$ (1367 nm). $E_1$ and $E_3$ are continuous wave (cw) fields and $E_2$ is time dependent. The energy levels in the diamond configuration in $^{85}$Rb are $|a\rangle = |S_{1/2}\rangle$, $|b\rangle = |P_{1/2}\rangle$, $|c\rangle = 6|S_{1/2}\rangle$, $|d\rangle = 5|P_{1/2}\rangle$, $F = 4$.
...their relative phases. The coherence of their mutual coupling. This exchange process depends on oscillations \( G \).

\[ \omega \]

their intensity profile) are \( \Omega_{795} = 45 \text{ MHz} \), \( \Omega_{1324} = 8 \text{ MHz} \), and \( \Omega_{780} = 8 \text{ MHz} \).

\[ (\text{a}) \text{ Square amplitudes } F_j \text{ from Eq. (4).} (\text{b}) \text{ Damped oscillations } G_j \text{ from Eq. (5). Dotted (red) lines correspond to the quasienergy } \lambda_1, \text{ dashed (black) lines to } \lambda_2, \text{ and solid (blue) lines to } \lambda_3. (\text{c}) \text{ Transient signal from the decoupled model using Eq. (3) (circles) and numerical simulation using the density matrix Eqs. (1) (solid line). Parameters used in the calculations are as follows: } (\lambda_1, \lambda_2, \lambda_3) = (0, 270, 30) \text{ MHz and Rabi frequencies of } (\Omega_1, \Omega_2, \Omega_3) = (50, 10, 10) \text{ MHz.} \]

can be represented as three coupled oscillators. The oscillators corresponding to the atomic coherences \( \rho_{dc} \) between \( c \) and \( d \) and \( \rho_{cb} \) between \( c \) and \( b \) can store and exchange energy because of their mutual coupling. This exchange process depends on their relative phases. The coherence \( \rho_{ca} \) between \( c \) and \( a \) acts as part of the coupling coefficient in the system of two coupled oscillators \( \rho_{dc} \) and \( \rho_{cb} \). Thus, it produces a dynamic coupling between \( \rho_{dc} \) and \( \rho_{cb} \).

III. EXPERIMENTAL OBSERVATIONS OF FWM TRANSIENTS

The specific hyperfine levels that the fields couple in the experiment are such that a field at 795 nm with frequency \( \omega_{795}(\omega_1) \) couples the transition \( 5S_{1/2}, F = 3 \rightarrow 5P_{1/2}, F = 2 \), a field at 1324 nm with frequency \( \omega_{1324}(\omega_2) \) the transition \( 5P_{1/2}, F = 2 \rightarrow 6S_{1/2}, F = 3 \), and a field at 780 nm with frequency \( \omega_{780}(\omega_3) \) the transition \( 5S_{1/2}, F = 3 \rightarrow 5P_{3/2}, F = 4 \) in \(^{85}\text{Rb} \) (see Fig. 1). The generated light at 1367 nm from FWM is resonant with the transition \( 5P_{3/2}, F = 4 \rightarrow 6S_{1/2}, F = 3 \) in \(^{85}\text{Rb} \) with a frequency \( \omega_{1367}(\omega_A) \). The estimated Rabi frequencies of the external excitations (40% uncertainty due to their intensity profile) are \( \Omega_{795} = 45 \text{ MHz} \), \( \Omega_{1324} = 8 \text{ MHz} \), and \( \Omega_{780} = 8 \text{ MHz} \).

We use a fast fiber-coupled electro-optical modulator (EOM) to extinguish the excitation at 1324 nm in less than 4 ns, which is shorter than any atomic lifetime of the levels involved in the FWM process. A triggered single-photon detector detects FWM light as a function of delay from the excitation pulse. The angles among the beams satisfy phase matching for the FWM.

![FIG. 3. (Color online) Schematic of the experimental configuration for the study of transient FWM. An electro-optic modulator (EOM) produces a pulse of light at 1324 nm with a turn-off of less than 4 ns while two lasers at 780 and 795 nm work in continuous wave (cw) mode. A triggered single-photon detector detects FWM light as a function of delay from the excitation pulse. The angles among the beams satisfy phase matching for the FWM.](image-url)

A 10 Gbits/s lithium niobate electro-optic modulator (EOM) from Alcatel-Lucent with a bandwidth of 10 GHz modulates the amplitude of the 1324-nm light. A digital delay generator SRS DG535 produces a 300-ns pulse of 4 V and provides the ac signal driving the EOM. We use a dc power supply for the ac voltage to the EOM for maximal suppression of light in the off-stage of about 98% ± 2% at 1324 nm.

The single-photon benchtop receiver from Princeton Lightwave, model PGA-600, is an In-GaAs-based single-photon avalanche photodiode (APD) running in a gated mode to detect the generated FWM light. The gate has a 1-ns width...
which defines the temporal resolution in the experiment. We use optical attenuators to attenuate the light from FWM going into the fiber-coupled detector to keep a maximal rate less than $1 \times 10^{-2}$ photons per 1-ns window. We use a FEL 1350 long-pass filter from Thorlabs before a single-mode fiber (SMF) with transmission of 70% at 1367 nm and no appreciable transmission at 1324 nm. (We do not observe any counts caused by the 1324-nm laser when we trigger this detector at a rate of $10^3$ counts per second.) We use LabView programming and GPIB and serial communication to control the SRS DG535 and the PGA-600 for the generation of 1324-nm optical pulses and data collection of the light from FWM.

We study the transients in the light from FWM as a function of powers of the cw lasers and detunings of the excitation fields from specific atomic transitions in $^{85}$Rb in a diamond configuration as shown in Fig. 1. The detunings and powers determine the exact values of the complex quasienergies that govern the atomic polarization [Eq. (3)]. We observe pure decay for certain regions of power and detunings (see Fig. 4) but also oscillatory decay for other regions (see Fig. 5). Figure 4(a) shows the excitation light at 1324 nm (in black) and the light from FWM at 1367 nm (in blue) as a function of time when the light at 1324 nm turns off at $t = 0$ for the detunings $(\Delta_{795}, \Delta_{1324}, \Delta_{780}) = (0, 100, 80)$ MHz and laser powers $(P_{795}, P_{1324}, P_{780}) = (2, 0.8, 0.02)$ mW. Note the fast decay time of the light from FWM after the extinction of light at 1324 nm. Figure 4(b) shows the decay time extracted from least-squares fits to simple exponential functions of the light from FWM as a function of the power of 780-nm light.

The errors are calculated from the change of the extracted parameter to increase the minimized $\chi^2$ of the fit by one.

We observe that the decay time decreases as we increase the power of the 780-nm light, which might point to dephasing from power broadening of the level $5P_{1/2}$ affecting the coherence $\rho_{dc}$. This decay time is independent of atomic density in a range of $7.89 \pm 2.51 \times 10^{11}$ to $1.5 \pm 0.35 \times 10^{13}$ atoms/cm$^3$ corresponding to temperatures $T = 344 \pm 5$ K to $388 \pm 5$ K, respectively, as seen in Fig. 4(b). This suggests that the short decay time is not due to any superradiant effects [28] or superfluorescence in the FWM process [19]. The short decay time of the transient FWM can be caused by dephasing mechanisms related to atomic diffusion and Doppler broadening in our sample [29]. The decay does not depend on the power of the 795-nm light in a range from 20 $\mu$W to 2 mW.

When the external fields interact with the atomic system, they generate an atomic polarization grating in the atomic ensemble which can generate light from FWM [30]. This grating can diffuse due to the thermal motion of the atoms in the ensemble and contribute to dephasing in the process, shortening the decay time [20]. The study of this dephasing process as a function of thermal motion would require larger changes in the temperature of the ensemble than the temperature range used in this experiment.

We study the transient behavior as a function of detunings of the three different lasers. We use 20 $\mu$W of power for the 780-nm light and 2 mW of power for the 795-nm light. The power of the 1324 laser in the cw mode (i.e., full transmission through the EOM) is 0.8 mW. This set of powers results in a FWM transient signal with a relatively long decay time, $\tau_{dip} \simeq 6.0$ ns, as shown in Fig. 4(b), and the maximum generation of FWM light. This allows for a more detailed study of the transients in FWM as a function of detunings.
Figure 5 shows examples of the experimental observation of the light from FWM in the transient regime as a function of the detuning of the 795 light, for $\Delta_{795} = 30$ MHz and $\Delta_{1324} = 270$ MHz. We observe different features in the generated light: a FWM spike as we turn off the 1324 nm for $\Delta_{795} = -100$ MHz and $\Delta_{795} = -60$ MHz, and oscillations at different frequencies. The FWM spike has an intensity above the steady-state level, which corresponds to negative times in the figures. The decay times of the transients shown in Fig. 5 are approximately 6 ns which is consistent with the decay times in Fig. 4(b) for $\rho_{790} = 20 \mu$W. The oscillation frequency in FWM depends on the atomic polarization [see Eq. (3) and Fig. 2]. This is different from quantum beats from the hyperfine structure [20], whose frequency remains mostly fixed to power and detuning changes.

This effect has been observed in the study of the exchange excitation of a system of N two-level atoms coupled to a cavity mode in the linear [31] and nonlinear [32] regimes, where oscillations and spikes in the transient regime are the result of exchanging energy between the electromagnetic field and induced atomic polarizations (vacuum Rabi oscillations [33,34]). The asymmetry of the FWM spike in our experiment comes from the phases in the atomic polarizations that depend on the detunings and the intensities (Rabi frequencies) of the three fields at 795, 780, and 1324 nm.

We have explored the light from FWM for different regimes of laser detunings, and we have observed similar features in the transients as those shown as a function of 795 detuning in Fig. 5.

The parameter space in detuning and Rabi frequencies for FWM is large. Our exploration tries to highlight three main features: damping faster than the lifetime of the transitions (Fig. 4) when the detuning is large, while for smaller detuning FWM is large. Our exploration tries to highlight three main features in the transients as those shown as a function of 795 detuning in Fig. 5.

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where $\Delta$ is the detuning of the laser from the atomic transition, $\Gamma$ is the linewidth of the transition, and $A$ is a constant. We assume that the total absorption of light from different hyperfine levels can be described by a total linear susceptibility equal to the sum of the linear susceptibilities of each transition,

$$\chi^{(1)}_{\text{Total}} = \sum_i A_i \frac{1}{\Delta_i - i\Gamma}, \quad (8)$$

where $A_i$ defines the relative transition strengths of the transitions and $\Delta_i$ the detuning of the laser from each hyperfine state. We account for the Doppler effect by averaging over velocities to obtain the total Doppler-averaged linear susceptibility $\bar{\chi}^{(1)}_{\text{Total}}$, where the bar denotes Doppler averaged. The field propagating in the medium is \cite{4,26}

$$E(z) = E(0) \exp \left(-\frac{1}{2}d_{\text{op}}(z)\text{Im}[\bar{\chi}^{(1)}_{\text{Total}}]\right), \quad (9)$$

where $E(0)$ is the incident field at $z = 0$, $E(z)$ is the field at a distance $z$ into the atomic medium, $\text{Im}[\bar{\chi}^{(1)}_{\text{Total}}]$ is the imaginary part of the linear susceptibility, and $d_{\text{op}}(z) = 2\pi N|\vec{\mu}_i|^2|\vec{k}_i|z/\hbar$ is the optical depth at $z$, where $\vec{k}_i$ is the wave vector of the field $E_i$ and $\vec{\mu}_i$ the dipole matrix element of the transition.

We find a $d_{\text{op}} = 20$ for light at 780 nm by comparing the observed absorption profile in the experiment with the theoretical Doppler-averaged absorption profile from Eq. (9) taking into account contributions from the other hyperfine levels. There is asymmetry in the absorption profile around the detuning for the hyperfine level considered in the FWM process $S_{3/2}, F = 4$. From the absorption data and the model we infer an experimental $d_{\text{op}}$ for light at 795 nm of 9.8.

Figure 7 shows the generated light from FWM in steady state as a function of the detunings $\Delta_{780}$ and $\Delta_{1324}$ when the 795 laser is on resonance. Note the asymmetry in the intensity of the FWM light from the atomic resonances.

This asymmetry in the resonant structure shows that the Rabi frequencies entering into the FWM process are not symmetric with respect to zero detuning of the lasers. This effect modifies the transient behavior of the light from FWM at different detunings. The asymmetric behavior of the Rabi frequencies comes from the absorptive and dispersive properties of the medium due to hyperfine levels in the atomic structure of Rb. The model [Fig. 7(b)] taking into account Doppler effects and absorption shows a similar asymmetry in the resonant structure of the light from FWM.

V. RESULTS

The transient light from FWM in the experiment for different detunings shows a shortening of the decay time as theoretically predicted in Fig. 6. This is due to induced atomic polarization interference in the atomic medium \cite{36}, in our particular case, from different velocity classes of atoms in the Doppler-broadened atomic ensemble as predicted by Zuo et al. \cite{37}. Different velocity classes experience different detunings and initial phases given by their steady-state conditions. The total generated FWM field is the sum of all the generated fields from different velocity classes. The FWM intensity, proportional to the square of the total field and thus to the square of the induced atomic polarization, contains interference terms from different velocity classes.

Figure 8 shows a calculation of the intensity of the light from FWM with detunings $(\Delta_{795}, \Delta_{1324}, \Delta_{780}) = (−60, 270, 30)$ MHz and Rabi frequencies $O(1, \Omega_2, \Omega_3) = (45, 7, 7)$ MHz for (a) the zero-velocity class, (b) the sum of Doppler-broadened intensities from different velocity classes neglecting the induced atomic polarization interference in the atomic medium using $\int |\rho_{m}(v)|^2 P(v)dv$, and (c) Doppler broadening including the induced atomic polarization interference calculated from the square of the Doppler-broadened atomic response using Eq. (6) $\int |\rho_{m}(v)P(v)dv|^2$ \cite{37}. The Rabi frequencies are consistent with those estimated from the FWM in the steady-state regime (see Fig. 7).

The transient signal ignoring induced atomic polarization interference in the atomic medium in Fig. 8(b) has a decay time, extracted from a fit of the calculation to a simple exponential, of 22.31 ± 3.8 ns. The transient signal containing induced atomic polarization interference in Fig. 8(c) shows a decay time of 5.82 ± 1.09 ns with a modulation frequency of 160 ± 5.4 MHz, both numbers extracted from a fit to a simple exponential and from the peak frequency in a fast Fourier transform (FFT). This is consistent with the experimental decay time of $\tau_{\text{exp}} = 6.3 ± 0.09$ ns for the experimental Rabi frequencies shown in Fig. 5(b). The shortening of the decay.
time of the transients with Doppler broadening corresponds to a broadening of the FWM spectrum.

The oscillations in the generated light are the result of the average of the oscillatory transients from different velocity classes of atoms. The generation of light above the steady-state level (FWM spike) is due to the total contribution to the FWM light from different velocity classes whose oscillations phase after the 1324-nm light is turned off. At later times, after the in-phase interference, the oscillations get out of phase as the different induced atomic polarizations from different velocity classes evolve in time producing fast damping of the transients. The model accounting for Doppler broadening qualitatively agrees with the experimental transients shown in Fig. 5 for specific detunings of the external excitations.

Figure 9 shows examples of the modeled FWM transients taking into account Doppler broadening using Eq. (3) and pump depletion using Eq. (9), together with the FFT showing their frequency components.

The oscillation frequency depends on the detunings of the lasers. For detunings ($\Delta_{\text{795}}, \Delta_{\text{1324}}, \Delta_{\text{780}}$) = (0, 270, 30) MHz the transient FWM contains contributions from different velocity classes of atoms resulting in an averaged oscillation frequency. This dephasing mechanism produces broadening as seen in Fig. 9(b) and predicted in Ref. [37].

Figure 10 shows measured examples of the light from FWM in the transient regime and the corresponding FFT. When the laser detunings are ($\Delta_{\text{795}}, \Delta_{\text{1324}}, \Delta_{\text{780}}$) = (0, 270, 30) MHz the transient FWM contains contributions from different velocity classes of atoms resulting in an averaged oscillation frequency. This dephasing mechanism produces broadening as shown in Fig. 10(b).

Figure 11 shows the oscillation frequencies $\beta_j$ from the decoupled model of Eq. (3) as a function of different detunings and Rabi frequencies. The continuous lines are the results of the model and the points with error bars are measurements from FFTs under appropriate experimental conditions. The horizontal error bars come from uncertainty in the frequencies of the lasers. The lasers at 780 and 795 nm are locked to atomic absorption lines in Rb with linewidths of 6 MHz and frequency shifted using optical modulators (see Sec. III).
estimate the uncertainty of the frequency of the 1324-nm laser from the 30-MHz resolution of the wave meter.

This simplified model shows some characteristics of the generated light. The transients should oscillate at the natural frequencies of the induced atomic polarization in the medium, however, we observe some discrepancy between the experimental observations and the expected frequencies from the model. These discrepancies could come from the fact that this simplified model does not contain information about pump absorption or transverse effects. Pump depletion could produce abrupt changes in the Rabi frequencies entering into the FWM process as the lasers are scanned across atomic resonances because of the presence of other hyperfine levels. Transverse effects in the atomic medium could modify the wave vectors of the pumps as a function of frequency and change the phase-matching condition required in FWM [4].

The asymmetry with respect to the 795 and 780 detunings in the experimental oscillation frequencies might be related to the asymmetric behavior of the observed FWM light in the steady state in Fig. 7(a). The asymmetric absorption of the pump beams around the particular hyperfine levels predicts an asymmetry in the FWM in the steady-state regime [see Fig. 7(b)]. However, a model fully describing the asymmetric behavior of the transient oscillations in Fig. 11 might require solving the coupled density matrix and field propagation equations to account for possible coupling between the pump fields in the medium producing focusing and defocusing as well as nonlinear refractive index effects modifying the phase matching for the fields [4].

The location of the oscillations (data points) shows that the system changes among the different oscillation frequencies available. The data comes from the FFT of the time response and one oscillation dominates in every case. Further investigation will be necessary to understand this effect in the FWM process.

VI. CONCLUSIONS

Coherences, such as those created in the FWM process, are delicate. This paper shows the atomic polarization coherences that contribute to the FWM and are visible through the time response of the system. They are responsible for transients, but their fragility is evident in the qualitative and quantitative behaviors that strongly depend on detunings and intensities. Our work also shows that the presence of absorption and dispersion with the Doppler-broadened medium significantly changes the coherences and their dynamics: oscillations and decay time.
The observations of pure decay agree with the model for resonant excitation and low power; changes in the detuning bring oscillations and jumps above the steady-state value of the FWM signal which come from induced atomic polarization interference in the atomic medium.

This work provides a better understanding of coherent effects and energy exchange processes in multilevel atomic systems and how they are affected due to broadening as a source of decoherence. We find that, in this Doppler-broadened system, it is possible to obtain coherent processes with decay times of the order of the radiative lifetimes for transitions in the diamond configuration.

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