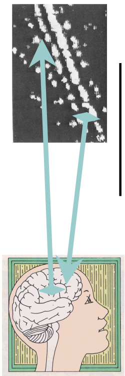
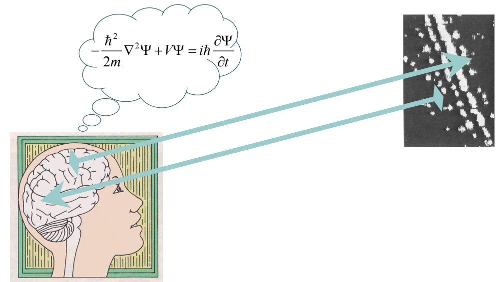


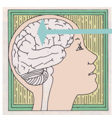
Thinking About Thinking:

Making the transition from classical to quantum physics

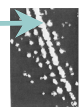
Edward F. Redish
Department of Physics
University of Maryland

What is physics, anyway?

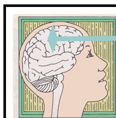




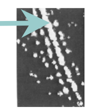
Physics is an interaction between the real world and the mind of scientists



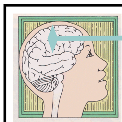
- When we only study one side of the interaction, we miss a critical part of the phenomenon of physics.



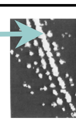
Physics is not only about the real world, it's about how to <u>think</u> about the real world.



- Some Basic Principles about Thinking
- Three Examples:
  - From the history of quantum physics
  - From contemporary research
  - From the study of how people learn quantum physics
- The Challenge: learning to think about QM

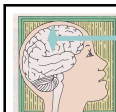


A model of thinking:

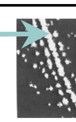


- Memory is productive and associative
  - Coherent memories are reconstructed out of smaller components.
  - Activating one element leads (with some probability) to the activation of associated elements.
- Activation and association are context dependent
  - What is activated and subsequent activations depend on the context, both external and internal (other activated elements).

**Joaquin Fuster, Memory in the Cerebral Cortex: An Empirical Approach to Neural Networks in the Human and Nonhuman Primate (MIT Press, 1999).*



Implication



- Principle #1:
  - Individuals build their knowledge by making connections to existing knowledge; they use this knowledge by productively creating a response to the information they receive.
- Corollary #1.1
  - We can only learn something we almost already know.

Example 1: The History of QM

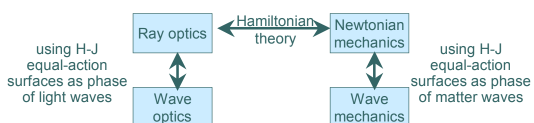
- Quantum physics is something really different from what came before.
- How did the developers of quantum physics build on what they knew?
  - *(or does this provide a counterexample to my cognitive principle #1?)*

What did they know?

- Classical electrodynamics in a variety of forms:
  - Newtonian
  - Lagrangian
  - Hamiltonian
  - Hamilton-Jacobi
- Irreducible experimental results that could not be explained classically.
  - quantization of allowed atomic orbits
  - association of radiated frequencies with the *difference* of atomic frequencies.

Schrödinger

- Schrödinger was aware of de Broglie's crude analysis of the Bohr model in terms of waves that "fit in" to an orbit.
- He generalized Hamilton's optical-mechanical analogy to matter waves by using the H-J equation as a variational function.



In his own words

- …I wish to mention that I was led to these deliberations in the first place by the suggestive papers of M. Louis de Broglie, and by reflecting over the space distribution of those "phase waves," of which he has shown that there is always a *whole number* measured along the path…The main difference is that de Broglie thinks of progressive waves, while we are led to stationary proper vibrations if we interpret our formulae as representing vibrations.

E. Schrödinger, *Ann. der Phys.* **79** (1926)
trans. by Shearer and Deans

How he did it

$$H\left(q, \frac{\partial S}{\partial q}\right) = E$$

$$\psi = e^{S/\hbar}$$

$$\frac{\hbar^2}{2m}(\nabla\psi)^2 - \left(\frac{e^2}{r} + E\right)\psi^2 = 0 \quad \text{(classical)}$$

$$J = \int d^3r \left[\frac{\hbar^2}{2m}(\nabla\psi)^2 - \left(\frac{e^2}{r} + E\right)\psi^2\right]$$

$$\delta J = 0 \quad \Rightarrow$$

$$-\frac{\hbar^2}{2m}\nabla^2\psi - \frac{e^2}{r}\psi = E\psi \qquad \text{(quantum)}$$

Heisenberg

- Heisenberg focused on the fact that the frequencies of emitted radiation were associated with frequency differences of the oscillators and not directly with their frequencies.
- He Fourier analyzed the oscillator positions and replaced the frequencies by the observed frequency combination rules.

In his own words

- …the Einstein-Bohr frequency condition…already represents such a complete departure from classical mechanics…that even for the simplest quantum theoretical problems the validity of classical mechanics simply cannot be maintained.…it seems more reasonable to try to establish a theoretical quantum mechanics…in which only observable quantities [i.e., frequencies] occur.

W. Heisenberg, *Z. für Phys.* **33** (1925)
trans. by van der Waerden et al.

How he did it

Consider an anharmonic oscillator $\frac{d^2x}{dt^2} + \omega_0^2 x + \lambda x^2 = 0$

Expand in fourier components $x = \sum a_\alpha e^{i\alpha\omega_0 t}$

For the n[th] stationary state $x = \sum a_\alpha^n e^{i\omega(n,\alpha)t}$

Classically $n \to n-\alpha$ has frequency $\omega(n,\alpha) = \alpha\omega_n$

$$\omega(n,\alpha) + \omega(n,\beta) = \alpha\omega_n + \beta\omega_n = \omega(n,\alpha+\beta)$$

Quantally $n \to n-\alpha$ has frequency $\omega(n,n-\alpha) = \frac{1}{\hbar}\left(E_n - E_{n-\alpha}\right)$

$$\omega(n,\alpha) + \omega(n,\beta) = \alpha\omega_n + \beta\omega_n = \omega(n,\alpha+\beta)$$

Rewrite $x = \sum a_{n-\alpha}^n e^{i\omega(n,n-\alpha)t}$

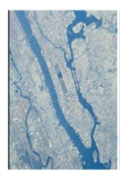
Work out: What do functions of x and p look like?
Result: matrices

What's my point?

- For both Schrödinger and Heisenberg, the new mechanics they created were small modifications of theoretical structures they were well familiar with to accommodate unexpected experimental results.
- In both cases, a familiar classical formalism was chosen in which a small and plausible change produced a dramatic new result.
- Despite the fact that QM was a major leap, both S. and H. built their new theories out of what they already knew.

Example 2: Contemporary Research

- What are we trying to build? or…
- When is an answer considered to be an answer?
  - Case: Quantum many-body theory
  - Case: Lattice gauge theory



What's my point?

- When we do much of the current work in physics we know and love, we are not simply trying to "discover new laws of the universe".
- We are trying to figure out new ways of thinking about what we already know but can't easily make sense of.
- We are "changing our minds" and figuring out how to "change the minds" of our colleagues so the community learns to think about the physics in new ways

.

Example 3: Teaching and learning QM

- QM is different from CM in many ways that have cognitive implications.
- In addition to the standard internal paradoxes and dualities QM has teaching "paradoxes/dualities".
- Thinking about the relation between QM and thinking about QM helps design appropriate courses for different populations.

Intrinsic difficulties with teaching QM: 1

- Quantum systems cannot be directly observed with our senses. Their properties must be inferred. This is different from the way we make sense of CM.
- QM is not deterministic in the way we have been taught CM is.
- QM (as usually taught)requires the interpretation of functions of many variables – wave mechanics.

Intrinsic difficulties with teaching QM: 2

- QM requires building together many classical items and going beyond them.
  - A basic understanding of probability
  - An ability to work with energies instead of forces
  - An understanding of basic wave properties
  - Lots more
    - motion of charges and magnets under EM forces
    - rigid body rotations (for spin)
- QM uses lots of math
  - Students tend to focus on math and not notice the physics underlying it.

Intrinsic difficulties with teaching QM: 3

- We learn by analogy and by extending understandings we already have. No macroscopic model exists that behaves like QM.
  - *Do not keep saying to yourself, if you can possibly avoid it, 'But how can it be like that?' because you will get 'down the drain', into a blind alley from which nobody has yet escaped. Nobody knows how it can be like that.*

R. P. Feynman

The delicate art of teaching quantum physics

- Quantum mechanics has many well-known “paradoxes / dualities”.
  - Wave / particle
  - Position / momentum
  - Quantum character / classical limit
  - Localized “free” electron / delocalized band structure
- How to teach QM effectively also poses “paradoxes / dualities.”

Paradoxes in Teaching QM

- QM builds on a classical base.
  - Strengthening that base can increase the likelihood that students will misinterpret the quantum results.
- We want our students to build a coherent and consistent picture of physics.
  - “Quantum thinking” requires the ability to use models that appear contradictory in a coherent way.
- We want our students to build powerful mathematical skills quickly
  - But we need them to be able to interpret the math in a way that makes physical sense.

What’s my point?

- Understanding how to teach quantum physics (or any physics) appropriately and effectively requires that we understand a great deal
  - about how people think (in general) and
  - about what our students (specifically) bring to the classroom.

The Challenge: Learning to think about QM

- For the teacher
  - How can we organize an introduction to quantum physics so students make sense of it more effectively?
- For the professional
  - How can we learn to think about the quantum world in a way that makes sense to us?

For the teacher

- Today, many populations of students should be learning quantum physics.
  - Physicists
  - Molecular biologists
  - Electrical engineers
  - Computer scientists
  - Materials scientists
  - Elementary school teachers
  - …

Responding to diverse populations

- Each population of students brings skills, assumptions, and deficiencies.
- Teaching them appropriately requires that we understand the resources they have with which they can build new knowledge.

An Example

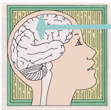
- The UMd PERG studied how much QM could be taught to upper division electrical engineering students in one semester.
- *A New Model Course in Applied Quantum Physics*
  - Tutorials
  - Research
  - Essay (JiTT) and Exam Questions
  - …
- Long term goal: To help students who would take a course in analog circuits that used a hybrid model for transistors (drift current + shift of bands) make sense of that model.
- Short term goal: To see if we could get students to understand the hybrid model of conductivity (esp. band structures).

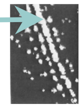
What they brought

- The population was skilled in handling diffEQs but weak in linear algebra.
- The population was surprisingly weak in the fundamental classical components. (Most students had done well in the 3 semester intro physics class.)
- The population was surprisingly weak in elementary concepts of probability.
- The population spontaneously constructed models of conductivity that incorrectly combined their chemical and physical knowledge.

How we did

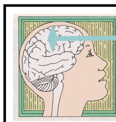
- We chose to focus on 1D QM and on building conceptual understanding of fundamental quantum issues.
  - no 3D (→ no atoms)
  - limited treatment of spin
- By designing particular instructional environments based on what we learned in our research, we were able to produce significant improvements in helping students
  - learn to think physically about the shape of a potential well and the wave functions in that well
  - build a model of conductivity that successfully used both the classical and the band-structure model.

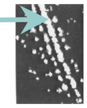




Some speculations

- Some of the difficulties we observed with engineers are likely to be found with physics majors.
- Some of the techniques we developed could help physics majors with some of the conceptually difficult issues in QM.
- Other populations could benefit from substantially different QM courses.
  - computer scientists
  - biologists





For the professional

- There are many ways of thinking about QM (“pictures”)
  - wave functions operators
  - sum over histories …
- What’s missing is a link to personal experience with the world.
- Macroscopic quantum systems already exist and more are to come. Hands-on experience with these sort of systems could fundamentally change how we think about QM.
  - lasers liquid helium II
  - superconductors Bose-Einstein condensates
  - …