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Problem Solving and the Use of Math in Physics Courses

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Using Math

- Mathematics is commonly referred to as “the language of Science.”
- Physics is an excellent place for scientists of all stripes to learn to use mathematics in science.
- When students have trouble with math in their physics classes, we might ask them to “take more math classes.”
- But using math in science (and particularly in physics) is not just doing math.

Some differences

- In physics
  - we use many different symbols — and not just in the standard ways
  - we blur the distinction between constants and variables
  - we use symbols to stand for ideas rather than quantities.

An Example

- If

$$f(x,y) = x^2 + y^2$$

then what is

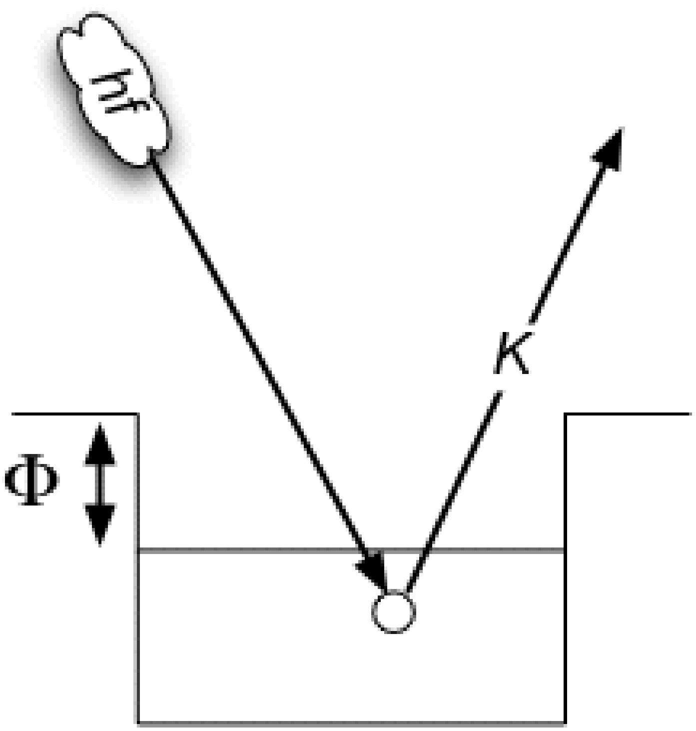
$$f(r,\theta) = ?$$

Example

- The photoelectric effect equation is

$$K = hf - \Phi$$

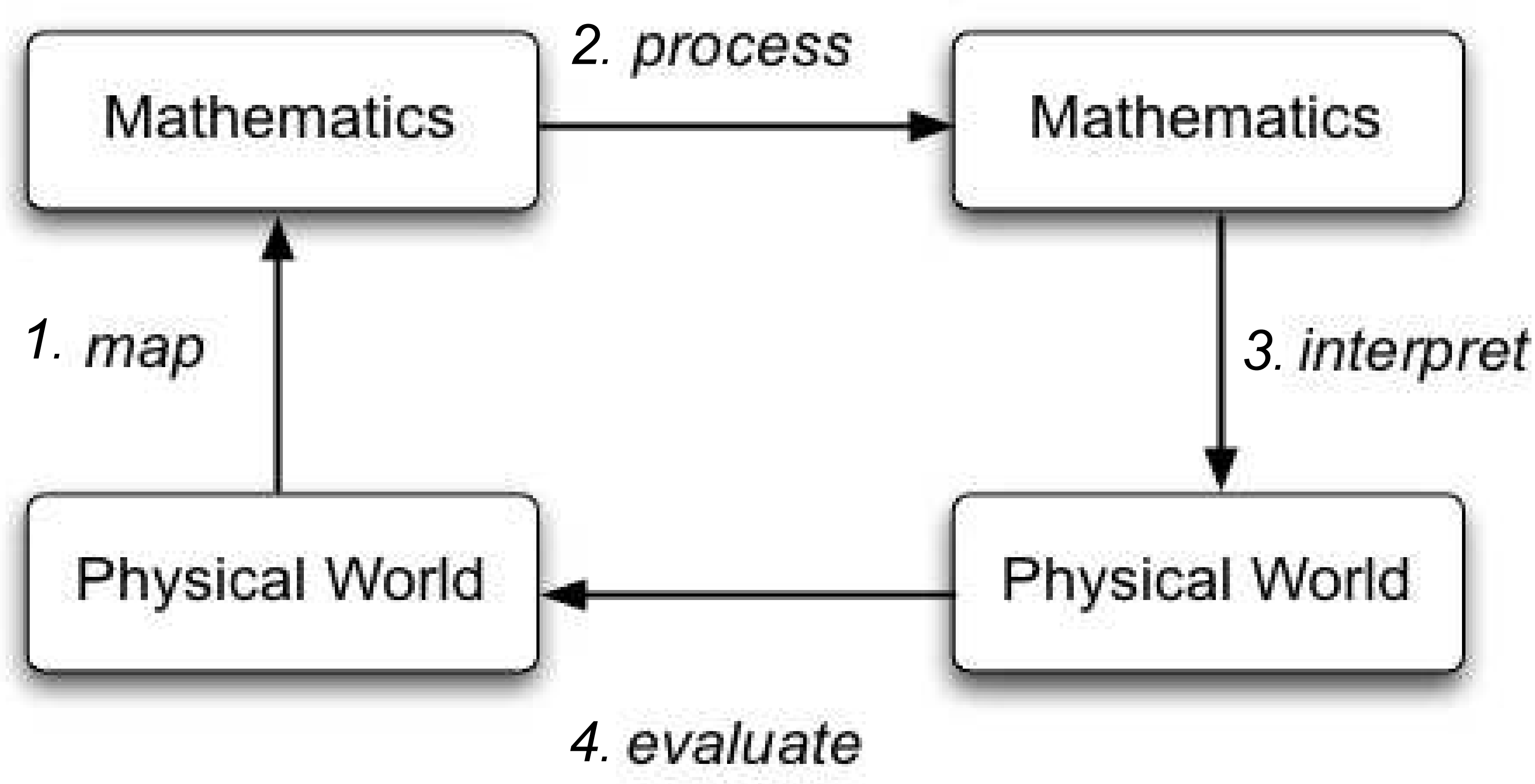
- If a frequency of *f* leads to no electrons being emitted, what will happen if we choose a <u>lower</u> frequency?



It's not just math!

- Our fundamental processing of equations is more complex than in a math class.
  - We associate our interpretation of the equation with a physical system — which lends information on how to interpret the equation
  - We use particular symbols that carry ancillary information not otherwise present in the mathematical structure of the equation
  - We use more complex quantities than in math classes and use them tacitly.
  - We use equations to organize our conceptual knowledge.

Math in Physics



Experts vs. Novices

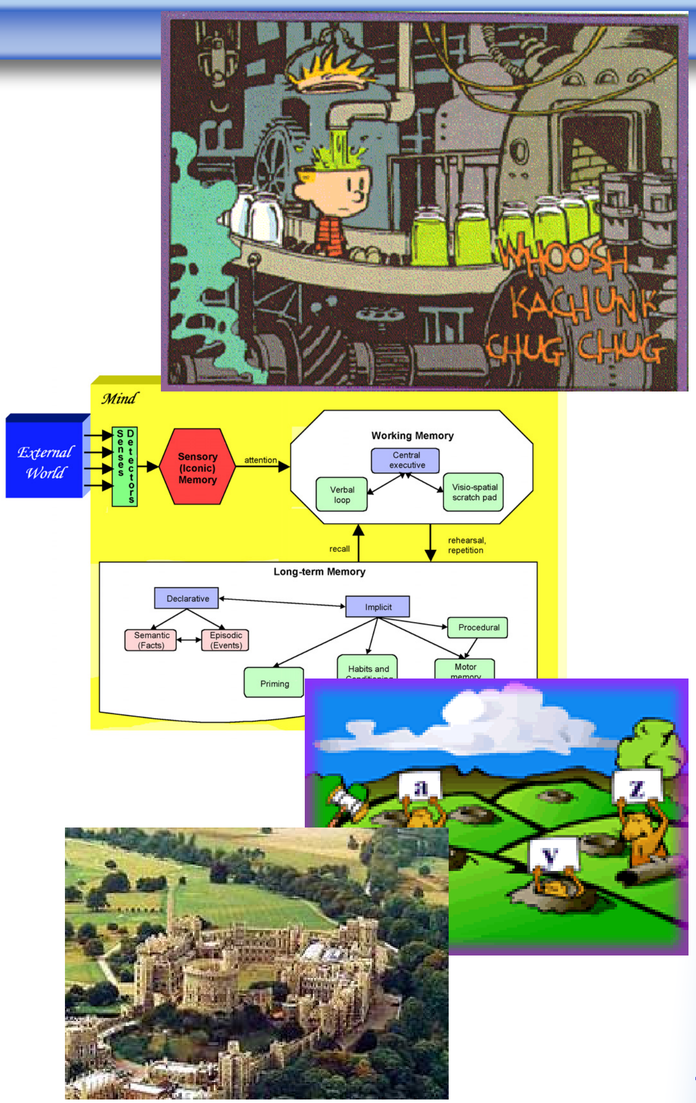
- Experts
  - Have a lot of knowledge
  - Have well structured knowledge.
  - Plan their work.
  - Choose strategies by “deep structure.”
  - Frequently evaluate their progress.
- Novices
  - Have less knowledge
  - Have weak knowledge organization
  - Just “go” without much planning.
  - Choose strategies by “surface structure”
  - Rarely evaluate their work.

Our traditional approach fails to help students focus on critical issues

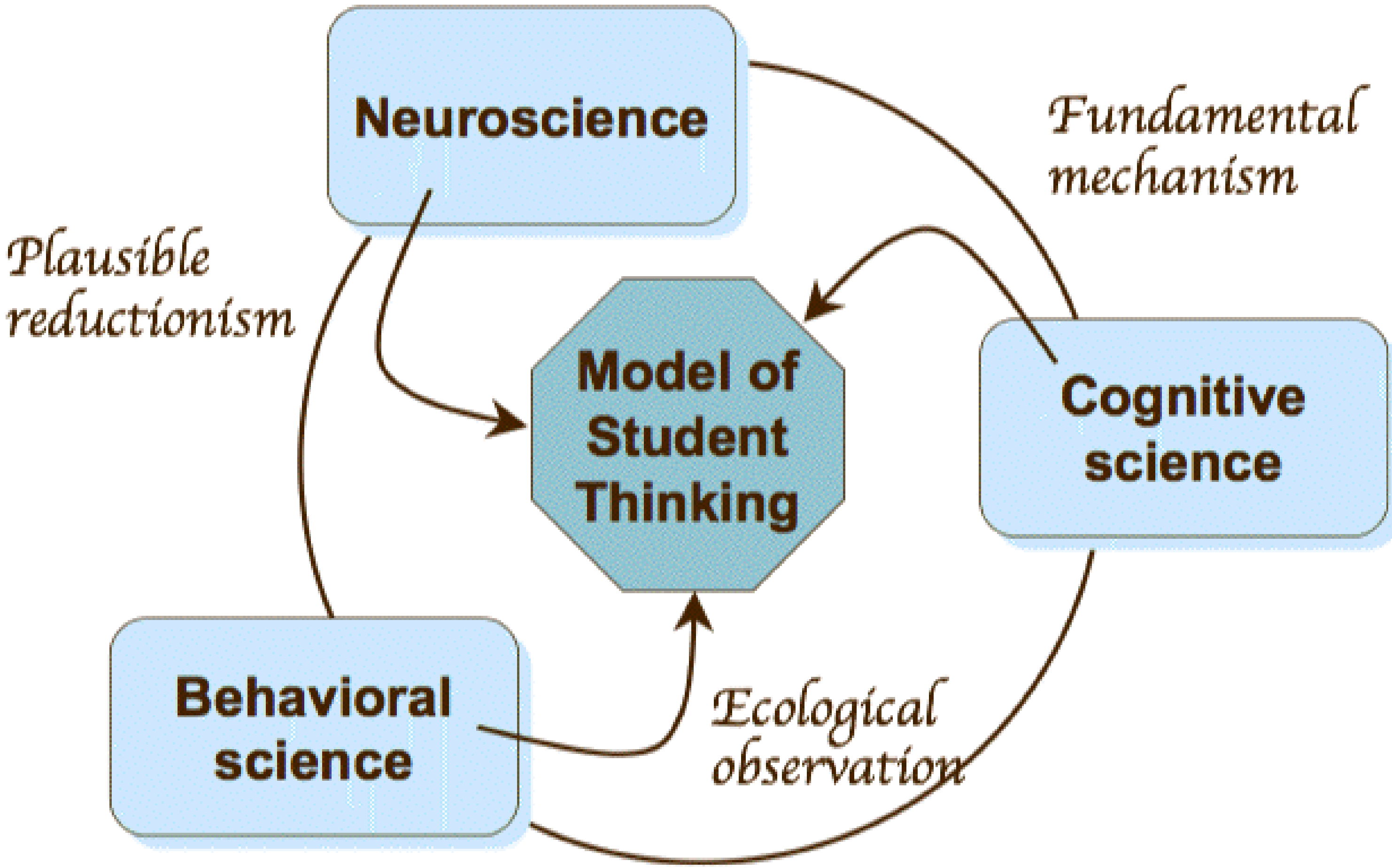
- We provide them with step 1, focus on step 2, and rarely ask them to carry out steps 3 or 4.
- Our exams focus of one-step recognition, giving “cues” so we don’t require them to recognize deep structures.
- They don’t succeed on their own with complex problem solving so we tend to “pander” by only giving simple problems.
- We often don’t recognize what’s complex in a problem, which makes it hard to design appropriate and effective problems.

Modeling the Student

- To understand what we can (and can't) do, it helps to understand how the system we are trying to modify
  (the student) "works."
- In the past 50 years,
  much has been learned
  about thinking and behavior— but there is a lot of dross.
- How can we separate
  the gold from the slag?



Triangulation



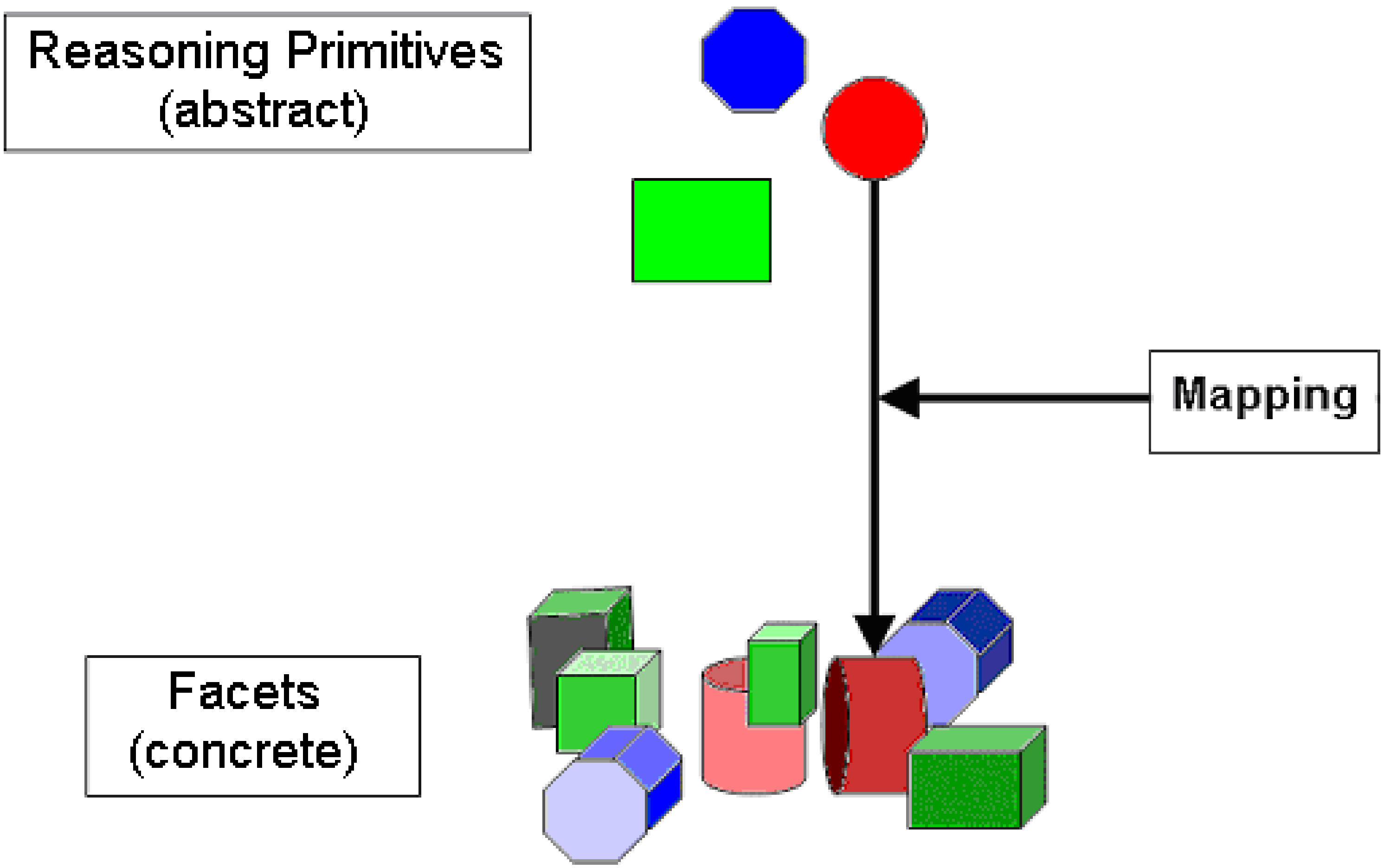
Four key ideas

- Resources
- Compilation / Binding
- Association / Activation
- Control / Selective Attention

Resources

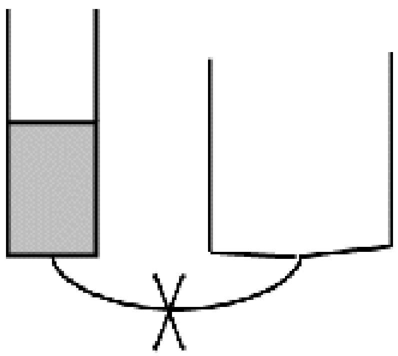
- The fundamental idea of our cognitive model is that thinking is dynamic, with different knowledge elements or processes “popping up” and activating other related elements with some (context dependent) probability.

Mapping Primitives



Example

Two containers with water are connected by a rubber tube with a pinch clamp as shown schematically in the figure below. When the clamp is opened, what will happen to the water levels?



- *It will continue to move until there's some sort of homeostasis, or whatever you call it, an equilibrium, which may be, it's either going to be that the volumes are the same, or the heights will be the same, I'm trying to think of which one it's going to be. Do you want me to tell you which? (laughs) Um, I'll go with volume for now but I'm not sure. So it'd be lower in the larger container….*

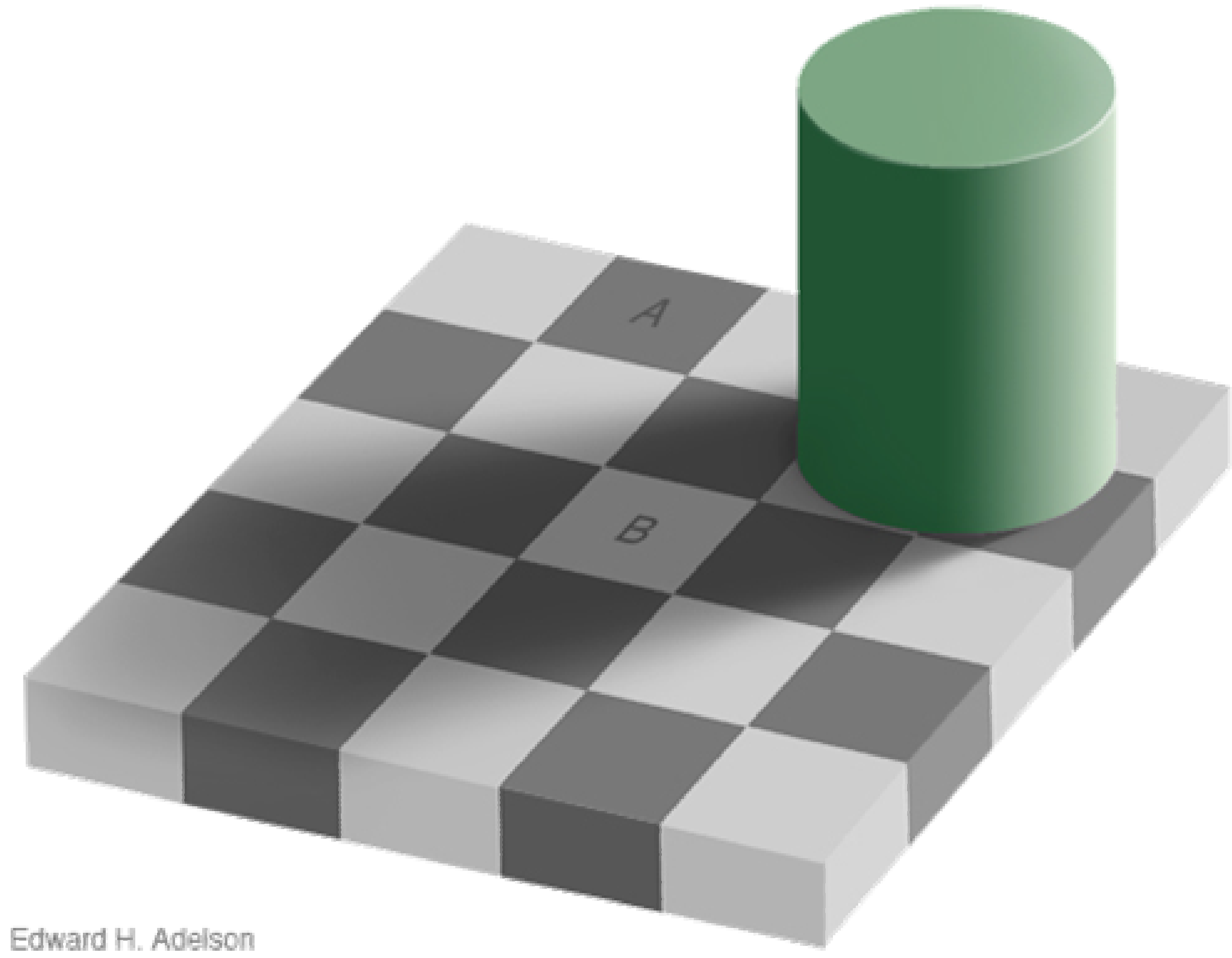
M. Loverude interview

Compilation / Binding

- As we learn, we bring together many different pieces of knowledge, binding them into a single coherent unit.
- Sometimes this process is very fast, sometimes it takes seconds, sometimes it takes years.

QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

Which square is darker?

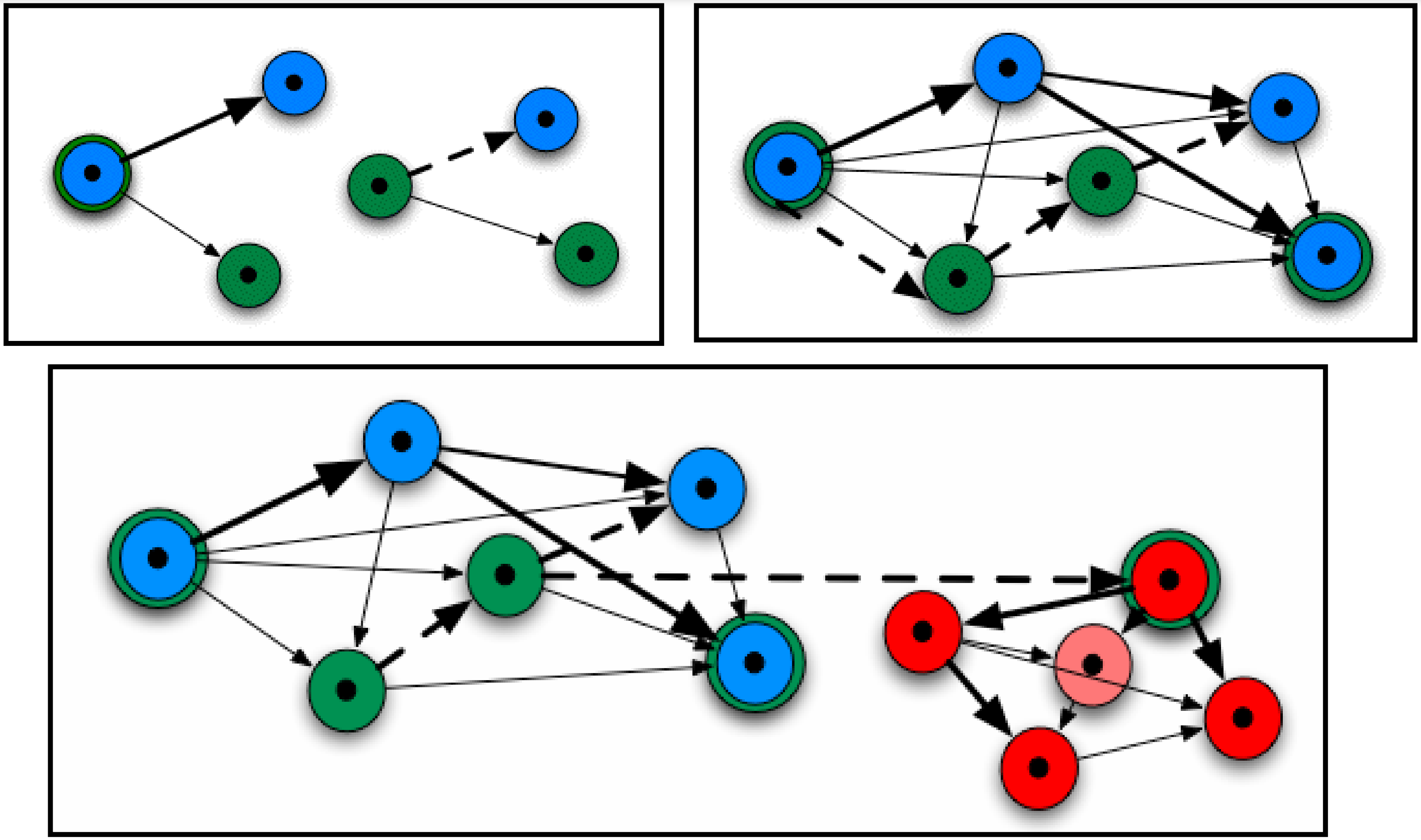


Binding is hard to undo

- Our processing of visual signals is highly relative and impossible to “unpack”.
- Things we learn can also compile — sometimes slowly, but sometimes quickly.

Finding a change

Patterns of Association / Activation



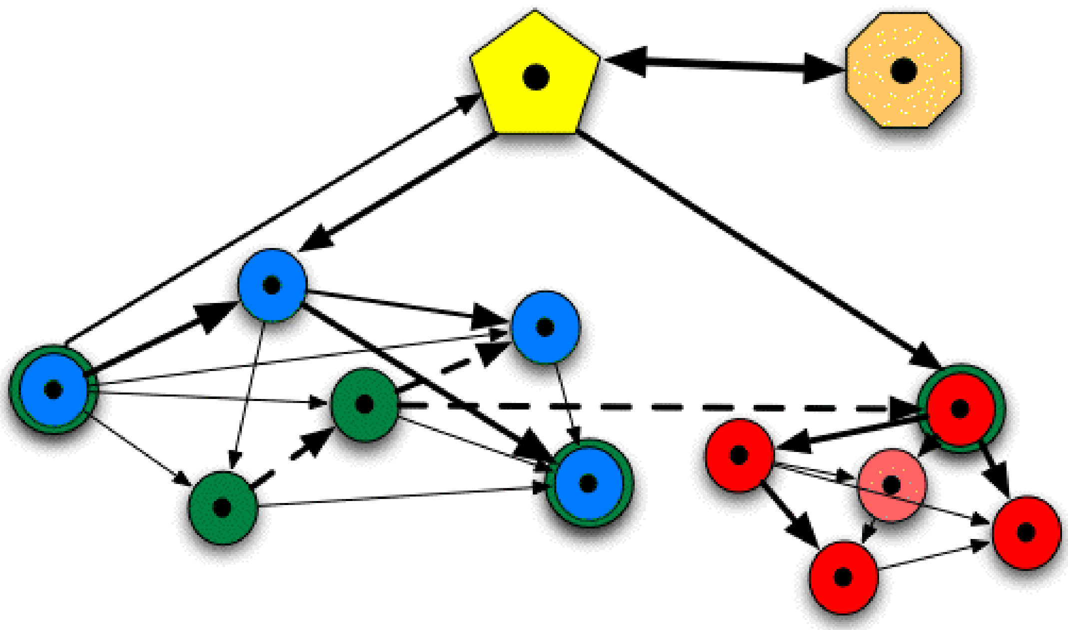
Sabella and Redish

Example

- Association ⇖ Context Dependence

Control

- Synapses can be excitatory or inhibitory.
- The brain is filled with both feedforward links (for association and activation) and feedback links (for control).



Selective Attention

- One way control plays out is through selective attention.
- There is too much in the world for our brains to process at once.
- We learn to select and ignore, *framing* our situation — deciding what matters and what doesn’t quickly and (often) unconsciously.

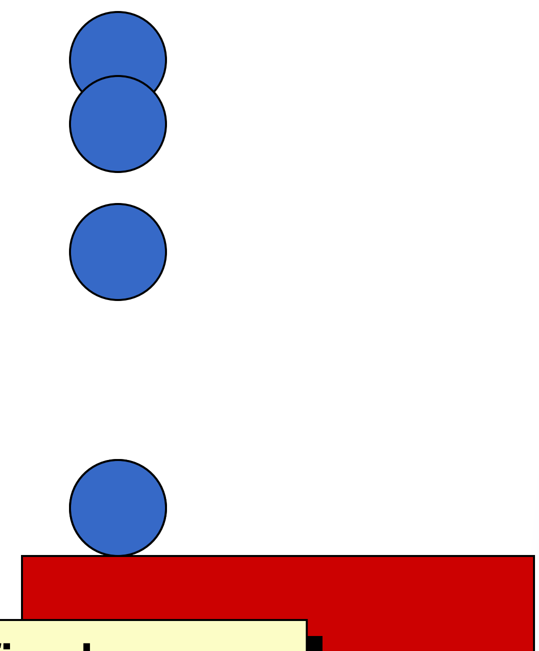
Count the passes!

Implications of the Resource Model for Problem Solving

- Activating the wrong resources
- Choosing the wrong frame
- Getting it right:
  unpacking our own compilations
  in order to better understand
  what our students need.

Activating the wrong resources

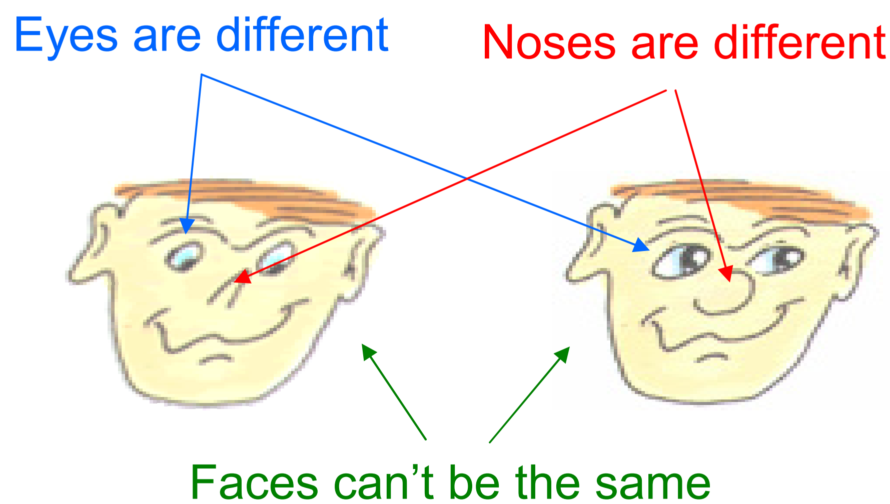
- Students often activate inappropriate resources when thinking about physics.
- In thinking about energy, some students activate *feature analysis* rather than *compensation*.



Office hours energy conservation story

Two resources

- *Feature analysis*: "Different plus different is more different."



- *Compensation*: "Robbing Peter to pay Paul."

The Data

- Learning to Learn Science project
  - 4-year $1 M NSF supported project to study algebra-based physics
  - All parts of the course were modified to
    - increase active engagement
    - focus on epistemological development
    - provide observational data (“ecological”)
  - Approximately 1000 hours of videotaped data were collected in lab, tutorial, and course center.

Organization of student problem solving

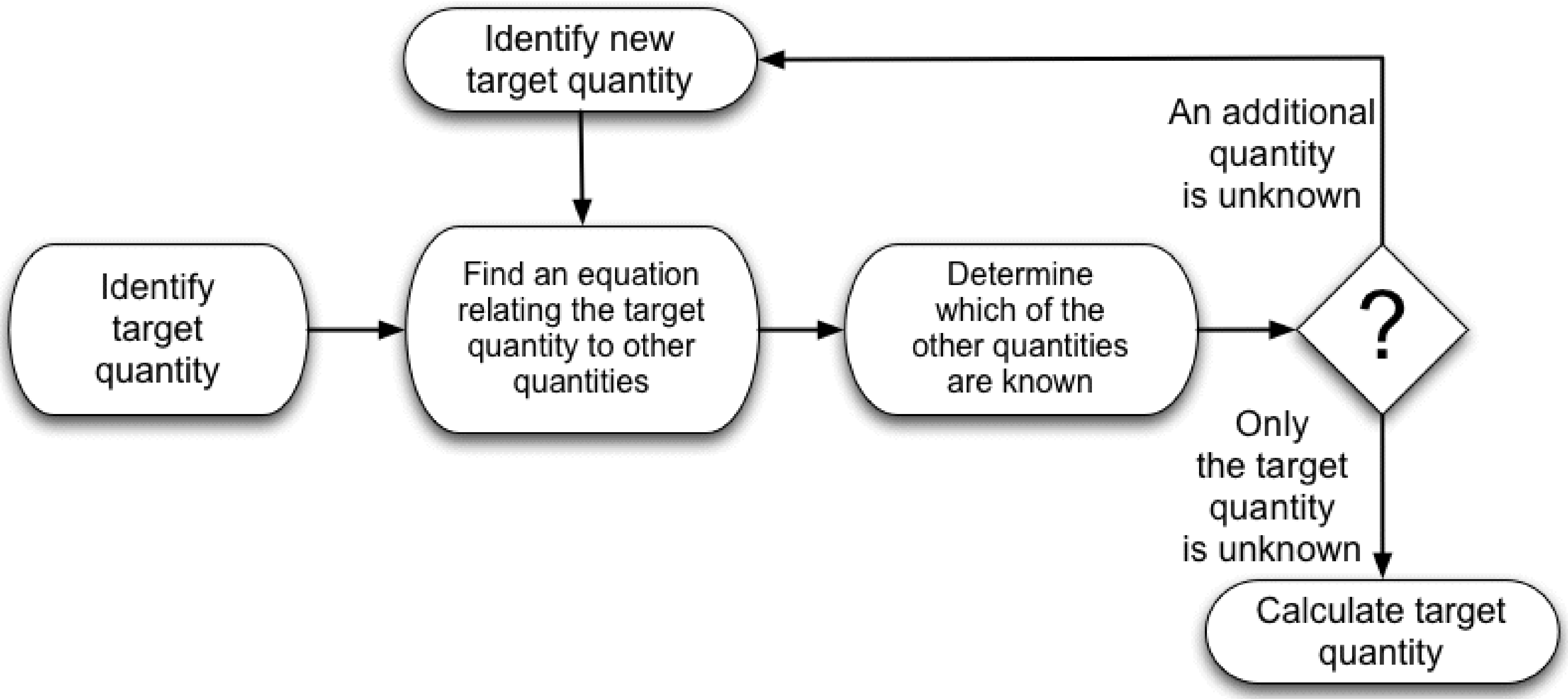
- In watching students solving physics problems we have observed:
  - They tend to work within a locally coherent organizational framework — one that only employs a fraction of their problem solving resources.
  - They may "shift gears" to a new kind of activity when one fails to prove effective.

E-games and E-frames

- *Epistemic games* — a coherent local (in time) pattern of association for building knowledge or solving a problem.
- *Epistemological frame* — a selective attention decision (often tacit) as to what are the appropriate e-games to play in a given situation.

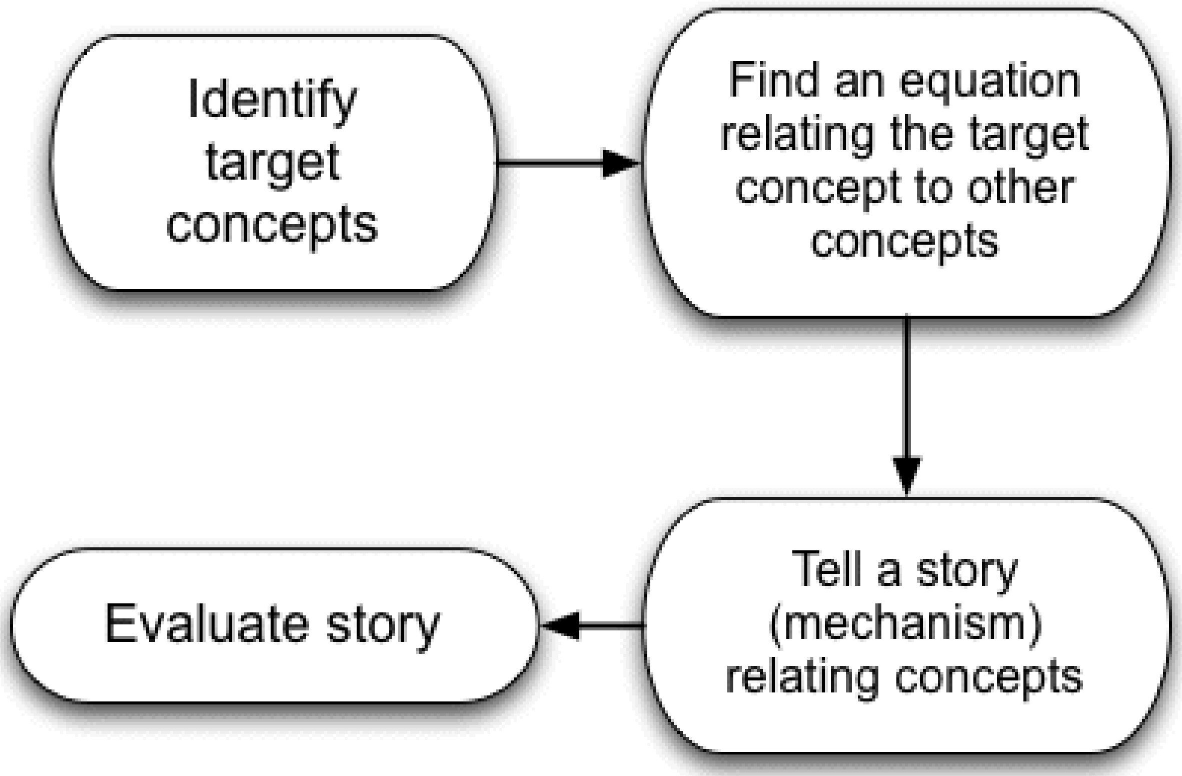
An e-game

- Recursive plug-and-chug



Another e-game

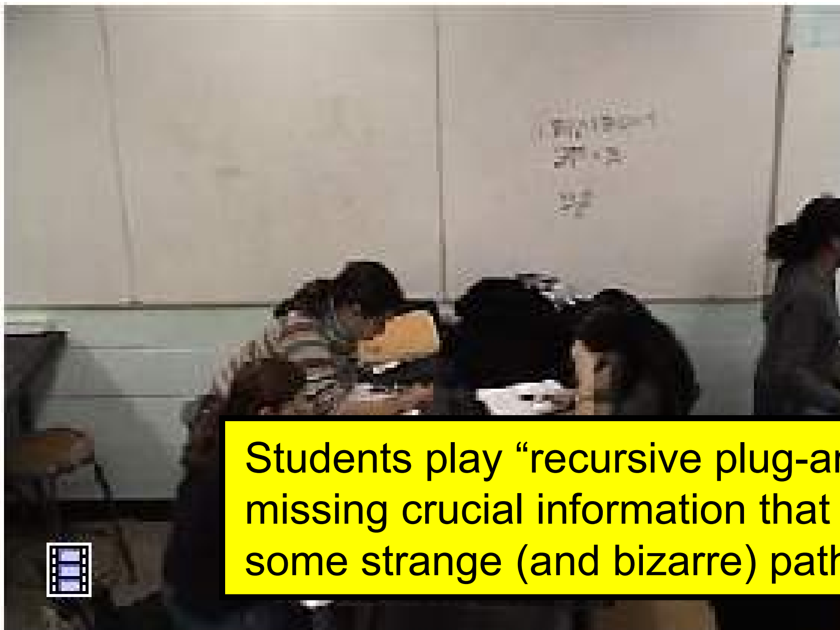
- Mapping meaning to mathematics



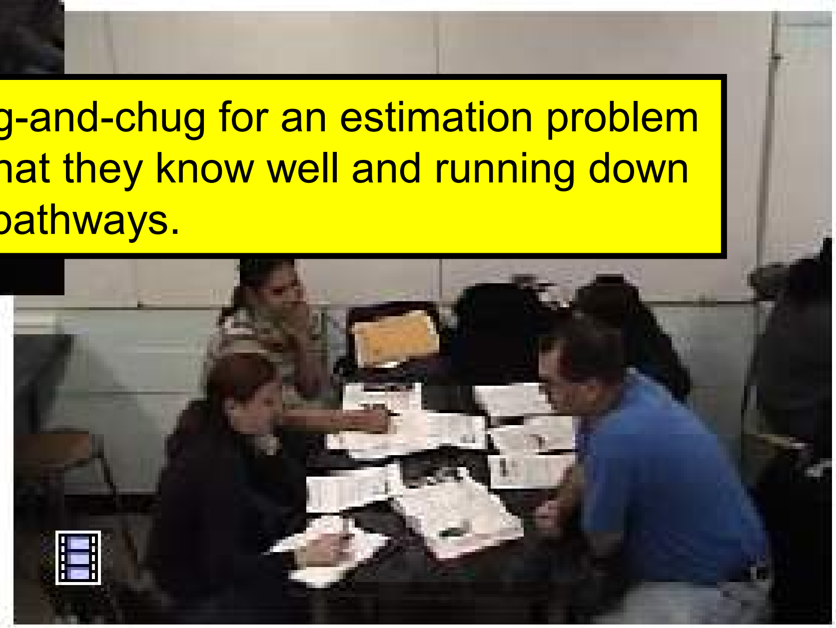
Playing the wrong game

- Problem:
  - *Estimate the difference in pressure between the floor and the ceiling in your dorm room*.
- Circumstance
  - Three students working on this problem in the course center.

What games have they chosen to play?

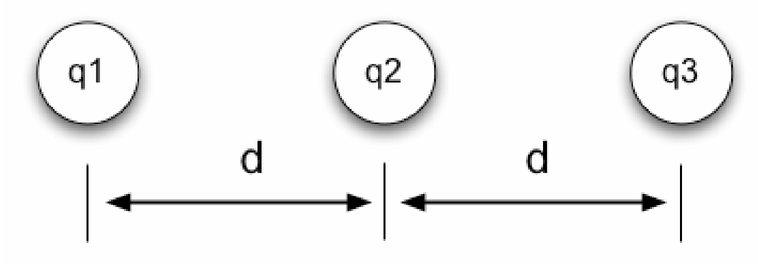


Students play "recursive plug-and-chug for an estimation problem missing crucial information that they know well and running down some strange (and bizarre) pathways.



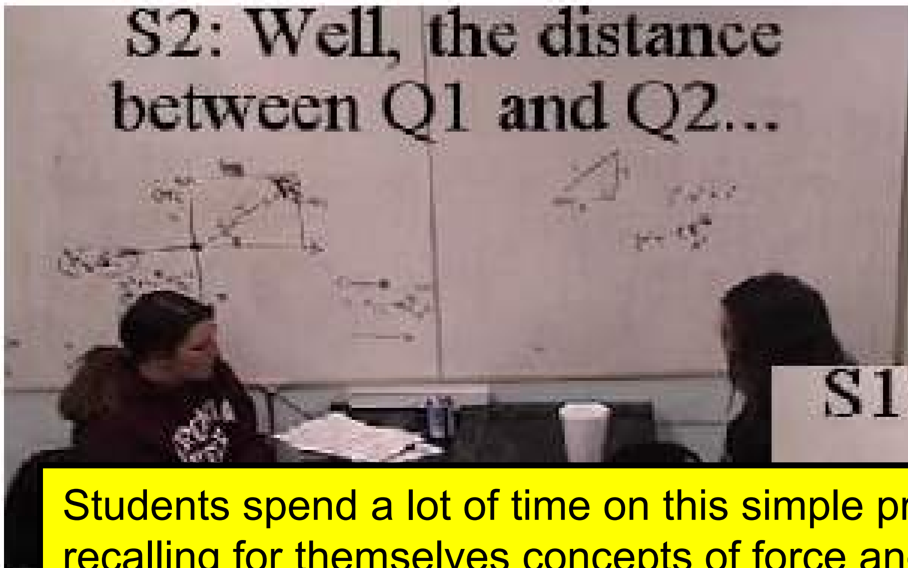
Playing the right games

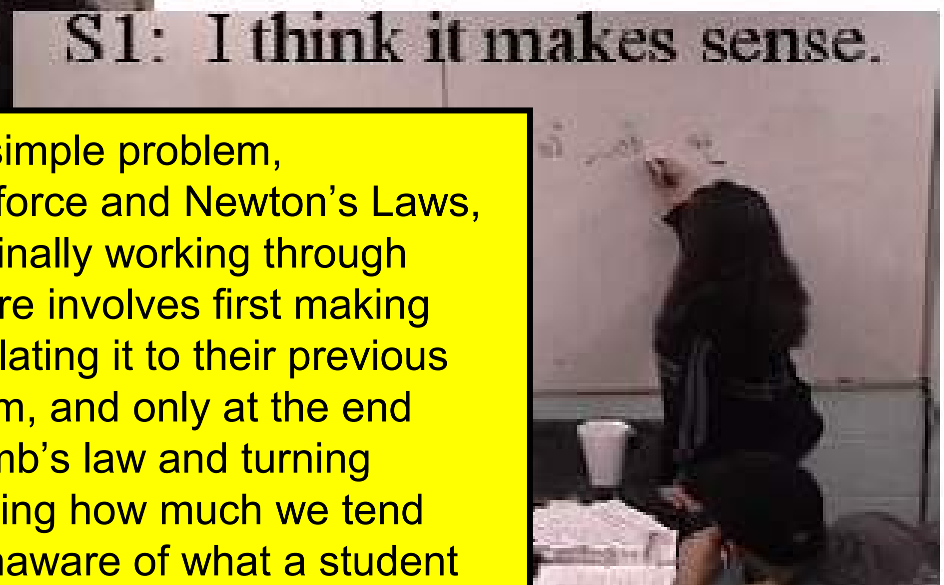
- Problem:
  - Three charged particles lie on a straight line and are separated by distances *d*. q_1 and q_2 are held fixed. q_3 is free to move but is in equilibrium (no net electrostatic force acts on it). If q_2 = Q, what value must q_1 have?



- Circumstance:
  - Four students working in the course center.

What games have they chosen to play?





Students spend a lot of time on this simple problem, recalling for themselves concepts of force and Newton's Laws, struggling with the idea of field, and finally working through to the correct answer. Their work here involves first making sense of the problem qualitatively, relating it to their previous knowledge, using a free-body diagram, and only at the end bringing in their knowledge of Coulomb's law and turning quantitative. Very good work illustrating how much we tend to "compile" and how often we are unaware of what a student really needs to learn to solve what looks like a simple problem!

Two kinds of reasoning: Logic and “getting it”

- Cognitive science suggests there are two independent cognition systems:
  - Linear processing / formal reasoning
  - Parallel processing / recognition
- In some sense, the first has been set up in part to facilitate and expand the power of the second.
- When we focus only on teaching the first, we supress our students’ intuitions and limit what they can learn.

What have we learned? 1

- There's more to problem solving than learning "the facts" and "the rules".
  - Organization and access counts.
- What experts do when they solve problems is much more complex than it looks to them.
  - We have to "unpack" what we have compiled to understand what our students need to learn.

What have we learned? 2

- Algorithmic approaches can block students from learning other important parts of problem solving if those other parts are not taught.
- Students can learn to expect to work hard and long on many components of (what might seem to us) a simple problem.
- Some of our traditional approaches may inhibit rather than facilitate the development of good physical intuition.

Some suggested guidelines

In ~35 years of teaching and ~15 doing physics education research, I have learned a few things:

- It's important to think about what it is you really want to accomplish in your teaching.
- It's essential to understand where your students are and what they can do.
- It helps to understand the structure of the tasks in light of what the students can do.
- It makes a big difference if your students understand and buy into your goals.
- It's what the students do that matters most for their learning, not what the instructors do.