Using Math in Physics: Warrants and Epistemological Frames

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Abstract: Mathematics is an essential component of university level science, but it is more complex than a straightforward application of rules and calculation. Using math in science critically involves the blending of ancillary information with the math in a way that both changes the way that equations are interpreted and provides metacognitive support for recovery from errors. We have made ethnographic observations of groups of students solving physics problems in classes ranging from introductory algebra-based physics to graduate quantum mechanics. These lead us to conjecture that expert problem solving in physics requires the development of the complex skill of mixing different classes of warrants – the ability to blend physical, mathematical, and computational reasons for constructing and believing a result. In order to analyze student behavior along this dimension, we have created analytical tools including epistemic frames and games. These should provide a useful lens on the development of problem solving skills and permit an instructor to recognize the development of sophisticated problem solving behavior even when the student makes mathematical errors.

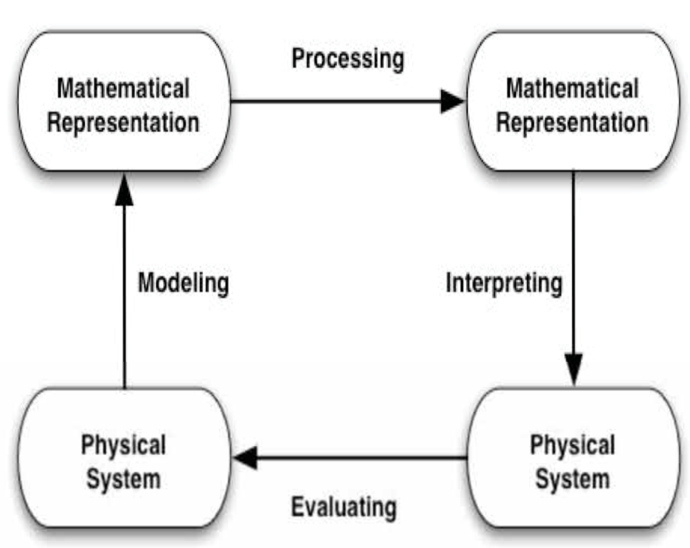
Using Math in Physics: Subtler than it looks

Physics heavily uses math in an intimate way, but very differently from the way it is typically taught in a math class. Consider two examples:

One of your colleagues is measuring the temperature of a plate of metal placed above an outlet pipe that emits cool air. The result can be well described in Cartesian coordinates by the function $T(x,y) = k(x^2 + y^2)$ where k is a constant.
If you were asked to give the function, $T(r,\theta)$, what would you write?

It is very important that you learn about traxoline. Traxoline is a new form of zionter. It is montilled in Ceristanna. The Ceristannians gristeriate large amounts of fevon and then bracter it to quasel traxoline. Traxoline may well be one of our most lukized snezlaus in the future because of our zionter lescelidge. Answer the following questions in complete sentences. Be sure to use your best handwriting.
1. *What is traxoline?*
2. *Where is traxoline montilled?*
3. *How is traxoline quaselled?*
4. *Why is it important to know about traxoline?*

- ★ Loading meaning onto symbols leads to differences in how physicists and mathematicians interpret equations.
- ★ Association with ancillary physical information is what gives equations sense and meaning.



Problem Solving and Epistemology: How do students mix different kinds of reasoning?

In order to see how students mix mathematical and physical knowledge in learning how to solve complex mathematical problems, we have observed ~100 hours of students solving problems in groups in university physics classes from Physics 1 to Quantum Mechanics.

Students do not have appear to have fixed epistemological stances, but rather they select from an array of possibilities.

Their choice of what they think is needed to decide something is right can be labile, or it can "stick" – producing a narrowed selective attention that keeps them from accessing knowledge they have and that would be useful.

The Ontology of Epistemology:

In order to talk about how students use their knowledge in solving problems we define three terms:

Epistemological framing –
the judgement (often not a conscious one) as to what knowledge is relevant to the problem at hand.

Epistemological resources –
broad general classes of warrants used to decide something is true

Warrants –
specific implementations of epistemological resources in the context of a particular problem or argument

Warrants Give Evidence of Epistemological Framing:

Much of the reasoning we have observed in novice and pre-expert physics students depends on one (or more) of four general types of proof. These classes of warrants guide our analysis of their epistemological framing:

Calculation –
algorithmically following a set of established computational steps should lead to a trustable result.

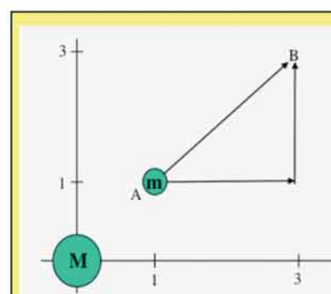
Physical mapping –
a mathematical symbolic representation faithfully characterizes some feature of the physical or geometric system it is intended to represent

Invoking authority –
information that comes from an authoritative source can be trusted

Mathematical consistency –
mathematics and mathematical manipulations have a regularity and reliability and are consist across different situations

Case Study 1:

Third-year physics majors' class in intermediate mathematical methods. Three students solving a problem together meant to demonstrate to them the mechanism by which a conservative force leads to work independent of the path and a well-defined potential energy. The



Use the definition of work: $W_{A\to B} = \int_A^B \vec{F} \cdot d\vec{r}$ *to calculate the work done by the force of gravity due to M on the small mass m as it moves along each of the two paths from A to B.*

The students have suppressed the common GmM factor and written (incorrect -- they are forgetting the cosine factors)

$$\int_{\sqrt{2}}^{3\sqrt{2}} \frac{1}{r^2} dr = \left(\int_1^3 \frac{1}{y^2+9} dy + \int_1^3 \frac{1}{x^2+1} dx \right)$$

Student 1 / ***Student 2***

S2: What's the problem? You should get a different answer from here for this

S1: No no no

S2: They should be equal?

S1: they should be equal

S2: Why should they be equal? This path is longer if you think about it.

S1: Because force, er, because work is path independent.

S2: This path is longer, so it should have, this number should be bigger then

S1: work is path independent. If you go from point A to point B, doesn't matter how you get there, it should take the same amount of work.

S2: OK. Well is this--what was the answer to this right here? What was that answer?
...
Cause path two is longer than path one, so
...
See, point six one eight, which is what I said, the work done here should be larger than the work done here cause the path

S1 is in a "by authority" frame. He knows the result (though he has written the equation incorrectly) and doesn't want to think about why.

Unable to convince S1, using physical reasoning, S2 turns to calculational warrants.

S2 is in a "physical mapping" frame. He believes the length of the paths should match with the numerical results.

Both students retain their chosen frames for a number of minutes. Eventually, S1 begins to respond to S2's demand for a different class of warrant and constructs a conceptual physical example. Their negotiation of acceptable warrants leads them to find the error and agree on a result.

Case Study 2:

Third-year physics majors' class in intermediate mathematical methods. One student explaining his solution of an exam problem in a one-on-one interview.

ρ(r,t) expresses the concentration of a chemical compound in a solvent. By conservation of matter:

$$-\frac{d}{dt}\int_\tau \rho \, d\tau = \int_{\partial\tau} (\rho\vec{v}) \cdot d\vec{A}$$

Suppose that the rate of creation (or destruction) of the compound per unit volume is given by Q(r,t). Q > 0 if the compound is being created, Q < 0 if destroyed. How would the above equation be modified?

Student 3

Yeah, if it's a, if it's a positive sign then the right hand side has to increase because something is getting sourced inside this volume. So for this to increase--

S3 had previously identified the RHS as a flux term. Physically, if there's a source (Q>0) more flows out.

Yeah, so it cannot be a positive, it has to be a negative, because then that's going to increase. For these signs to match, for the magnitude to increase, like these signs have to match, so it's probably negative.

He notes that the three signs don't make a consistent math system. The +/- on the left don't change the RHS consistently where -/- would.

Uhh, although on the other hand, when I think of a source I think of a positive sign and sink is a negative sign.

He remembers a rule "by authority" that should be relevant.

Yeah so that's where my confusion lies.

Despite having errors, this student can draw on a variety of different kinds of warrants, and expects coherence among them.

Conclusion:

Expertise in physics problem solving involves the ability to blend different epistemological framings and to flip quickly from one framing to another in response to snags and difficulties. The clustering of skills into frames can be an efficient way to proceed; if one can quickly recognize the tools needed to solve a problem, the problem can be solved without spending time hunting through a large number of possibilities. But when problems or inconsistencies arise, it can be more effective to explore a wider search space of possibilities. An instructor who is aware of the fact that students may "get stuck" in a framing that limits their access to tools and knowledge they may not only possess but be good at, will have a better understanding of the true nature of the difficulty the students may be experiencing.

References: (available at www.physics.umd.edu/perg)

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