Two Kinds of Student Mathematical Errors in Physics: A theoretical perspective

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Motivation

Most physics instructors have probably informally observed at least two kinds of student mathematical errors:

- Students doing the "right-kind-of-thing", but making a "minor-mistake" along the way, or
- Students doing the "entirely-wrong-kind-of-thing".

Goals for this talk

- Give explicit examples of "right-kind-of-thing-but-minor-mistake" and "entirely-wrong-kind-of-thing".

- Show how our theoretical framework (in particular, *epistemic games*) makes explicit what these errors are and how they are different.

Data for this research – *Course Center*

Student Population

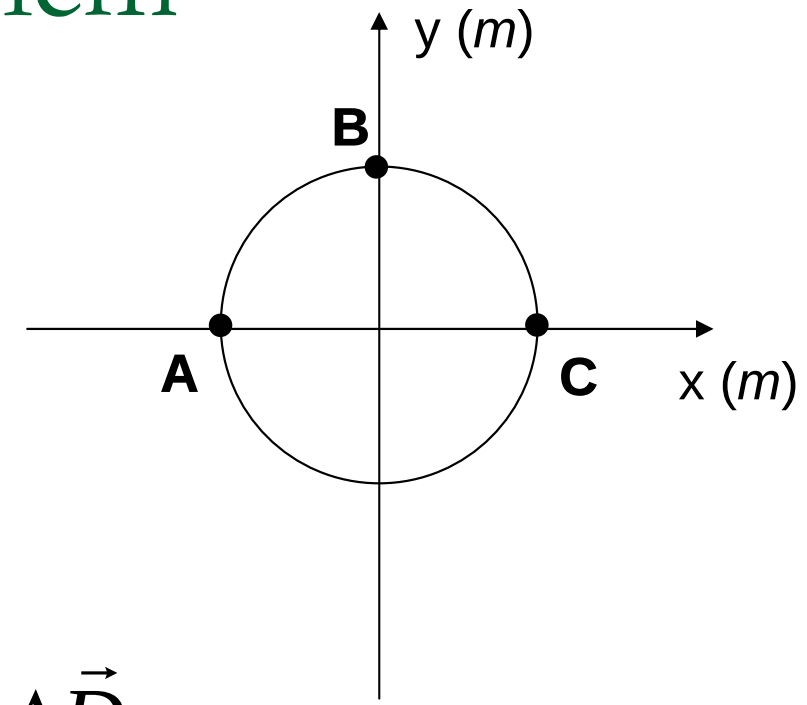
- Introductory algebra-based physics course at UMD
- >95% of the students have completed two semesters of calculus

Problem-solving Sessions

- Video-taped students solving homework problems
- TA present to assist students

Average Velocity Problem

A jogger runs around a circular track of 30 m radius shown in the figure at the right. She runs at a constant speed in a clockwise direction and completes one lap in 40 seconds.



What is her average velocity from A to C?

$$\langle \vec{v} \rangle = \frac{\Delta \vec{D}}{\Delta t}$$

$$\langle \vec{v} \rangle = \frac{60 \text{ meters } \hat{i}}{20 \text{ seconds}}$$

$$\langle \vec{v} \rangle = 3 \text{ m/s } \hat{i}$$

Analysis focuses on "Student 1"

Average Velocity Problem: Quotes from Student 1

S1: So her average velocity going from A to C is...both of these equations are going to figure out average velocity: change in distance over change in time, or velocity final plus velocity initial divided by two, right?

.....

S1: They're both - so, here... you could do it either way; but, I think if you do it this way: like, if you look at her final velocity at C, we said was down 4.7…and this one's up 4.7.

.....

S1: It's gonna be zero. So, average velocity, I think, is zero. Because the directions cancel each other out.

.....

S1: Velocity has to take into account direction. So speed, of course, is never changing.

Pattern of student activities – *Mapping Mathematics to Meaning*

Epistemic Game

Identify target "concept"

Find an equation relating target to other "concepts"

Tell a story using this relationship between "concepts"

Evaluate story

"So, her average velocity going from A to C…"

$$\langle \vec{v} \rangle = \frac{\Delta \vec{r}}{\Delta t}$$

"Both of these equations are going to figure out average velocity…"

$$\langle \vec{v} \rangle = \frac{\vec{v}_f + \vec{v}_i}{2}$$

"her final velocity at C, we said was down 4.7…and this one's up 4.7"

"So average velocity, I think, is going to be zero. "

Mapping Error

"Velocity has to take into account direction. So speed, of course, is never changing. "

"Right-kind-of-thing-but-minor-mistake"

What is an epistemic game*?

Definition (Redish):

A coherent activity that uses particular kinds of knowledge and processes associated with that knowledge to create knowledge or solve a problem.

*Collins, A. & Ferguson, W. (1993). *Educational Psychologist, 28(1),* 25-42.

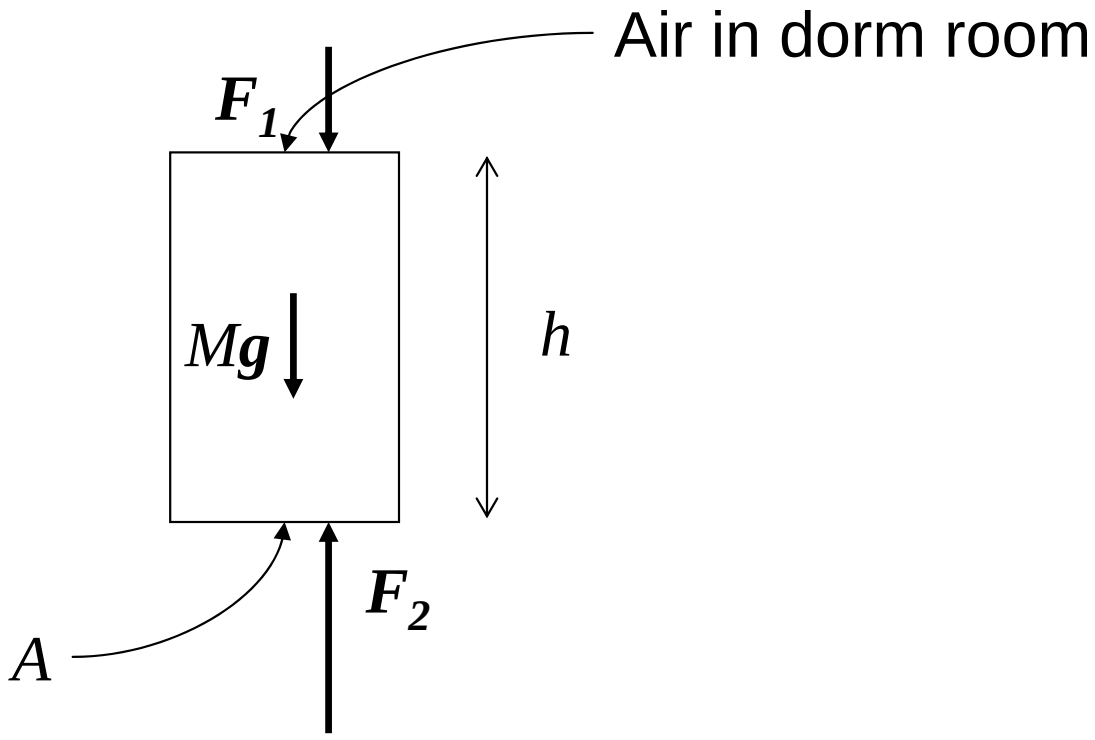
Pressure Problem

Estimate the difference in pressure between the floor and the ceiling in your dorm room. (1 atm $\approx 10^5$ Pa)

$$F_2 - F_1 = Mg$$

$$\frac{F_2}{A} - \frac{F_1}{A} = \frac{\rho V g}{A}$$

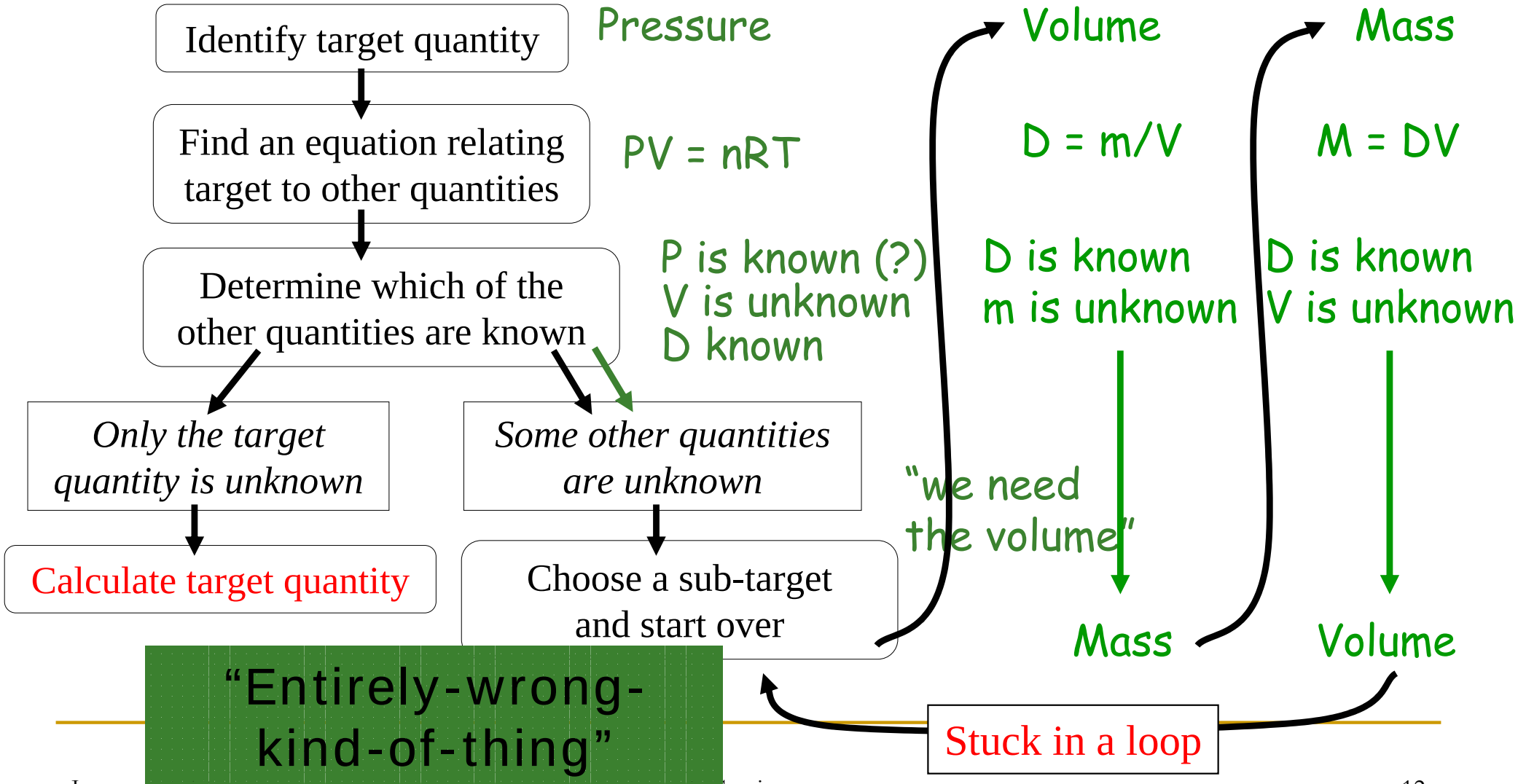
$$P_2 - P_1 = \rho g h$$



Video Clip – $PV=nRT$

A different epistemic game – *Recursive Plug-and-Chug*

Process Error



Conclusions

- The concept of *epistemic games* helps us understand students' math use in the context of physics.
- Students' conceptual errors fall into two general categories:
  - *"Right-kind-of-thing-but-minor-mistake"*
    Mapping Errors – appropriate epistemic game, inappropriate move.
  - *"Entirely-wrong-kind-of-thing"*
    Process Errors – inappropriate epistemic game

Conclusions – *cont.*

- Students play different epistemic games while solving homework problems in physics:
  - *Mapping Mathematics to Meaning*
  - *Recursive Plug-and-Chug*
  - *Mapping Meaning to Mathematics*
  - *Abstracting from Touchstone Example*
  - *Transliteration to Mathematics*
  - *Physical Sense Making*
  - *Pictorial Analysis*