Affecting Student Reasoning in the Context of Quantum Tunneling

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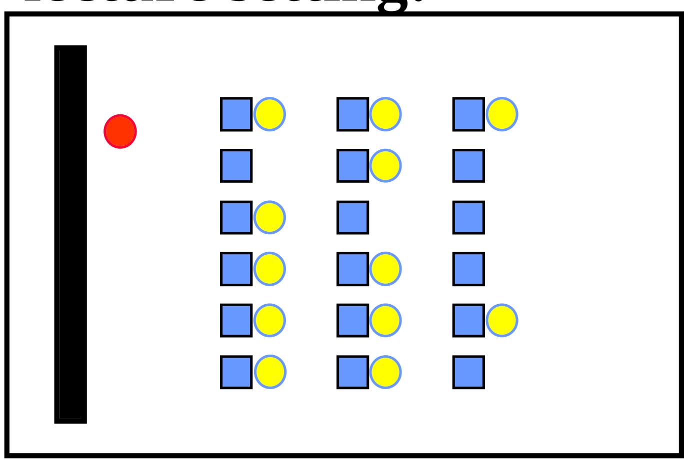
Course Context

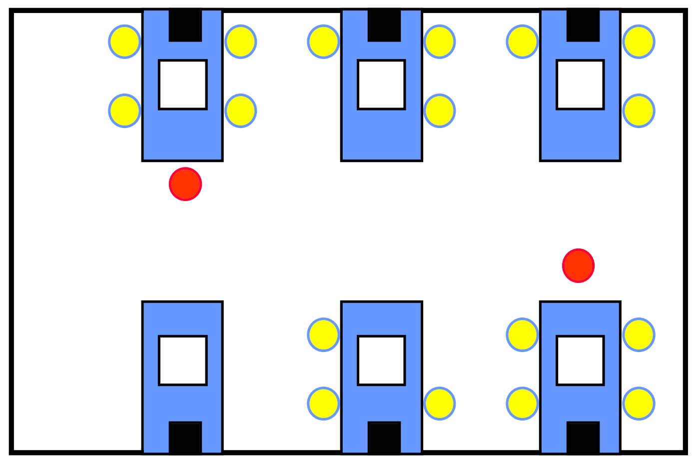
Junior/senior level quantum physics course (mostly EEs):

- Lecture: 2 hours / week
- Tutorial: 1 hour / week
  UMd-PERG-developed tutorials which often
  make use of appropriate software packages*
- Class assignments: JITT essays due each morning before class

lecture setting:



tutorial:



* *Photoelectric Tutor*, *Visual Quantum Mechanics*, *Physlets*, *MUPPET*, *MBL (Vernier*, *Pasco)*, *CUPS*

Tunneling:
A touchstone or a peripheral example?

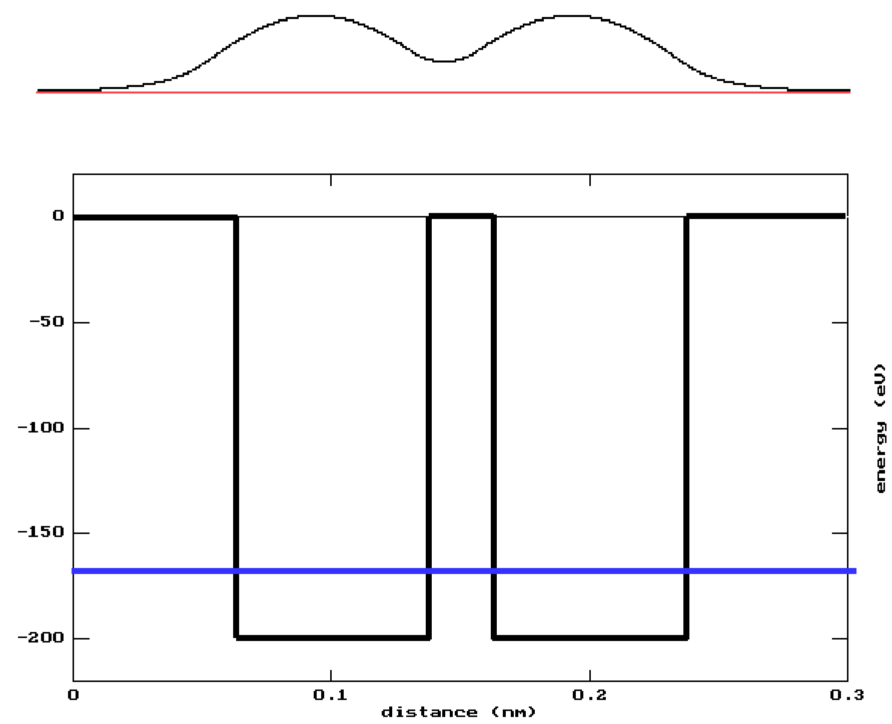
- Tunneling is usually treated as one of a large collection of QM results
  - a “solvable” mathematical problem
    (a bit messy unless in δ-function approx.)
  - a “transmitted” result
    (many books cite the formula for the transmission probability with no derivation)
  - a “qualitative” result
    (the basis of devices such as the STM or of phenomena such as nuclear decay)
- Conceptually, tunneling plays a much more significant role.

Tunneling as a Conceptual Touchstone: 1

- Barrier penetration (tunneling) underlies two central physical phenomena usually not discussed in this way
  - molecular bonding
  - formation of band structures

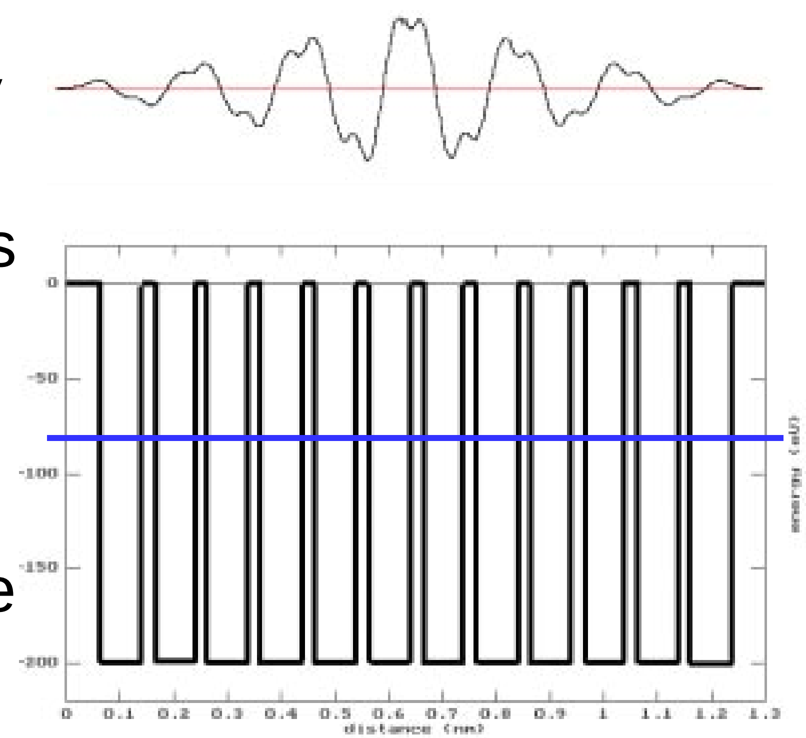
Molecular Bonding as Tunneling

- Molecular bonding arises when an electron see 2 effective binding potentials separated by a barrier.
- The ability of the electron to penetrate the barrier is critical for producing bonding.



Band Structure as Tunneling

- Tunneling in a crystal allows individual electrons to be shared among many atoms.
- Tunneling is what students fail to consider when they say electrons must be “plucked out” of an atom (at a significant energy cost) in order to participate in conduction.



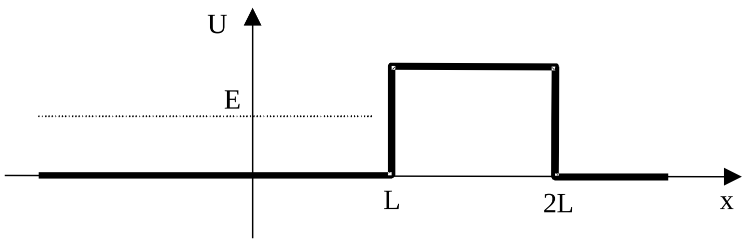
Tunneling as a Conceptual Touchstone: 2

- Understanding tunneling requires understanding a number of fundamental but difficult conceptual issues.
  - Students must separately track total, potential, and local kinetic energies.
  - Students must understand the difference between energy, wave function, and probability.
  - Students must understand the difference between stationary (no time dependence) and static (no flow).

Pretest Question and responses

Consider the following potential:

A *quantum mechanical* particle with energy *E* (indicated on the diagram) in the region where U = 0 is incident from the left on the potential well shown in the figure.



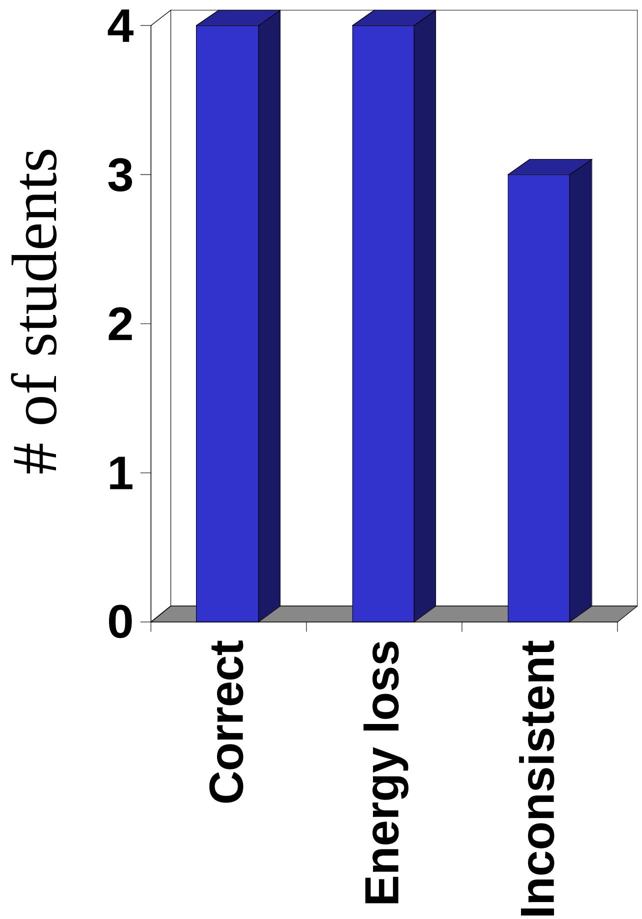
a. Does the particle energy
1) *increase*, 2) *decrease*, or ☑ 3) *stay the same*

Explain, briefly.

b. Compare the energy of the particle in the region I (x < L) and III (x > 2L). The energy in region I is
1) *greater than*, 2) *less than*, or ☑ 3) *the same as*
the energy in region III.

Explain, briefly.

Common student pretest responses

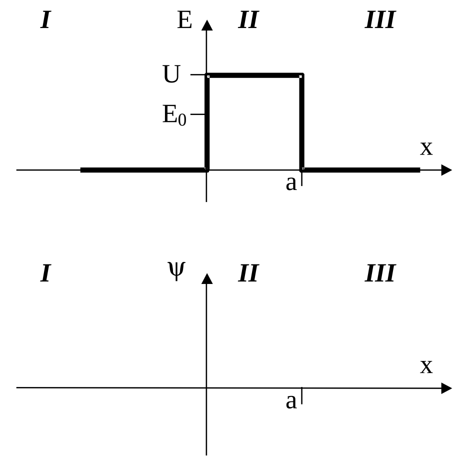


- Energy loss:
  - Particle loses energy when tunneling through the barrier.
  - *"collides and penetrates and loses energy in barrier."*
- Inconsistent:
  - Students state the particle loses energy in the barrier, but has the same energy once it passes through.
  - *"The particle regains its lost energy after passing through the barrier."*

Exam Question: Traditional (n=11) and Modified Instruction classes (n=13)

Consider a beam of electrons with energy E_0 incident from the left (x<0) on a potential barrier of height U and width a (see energy diagram to the right). Three regions are indicated on the energy diagram as *I*, *II*, and *III*.

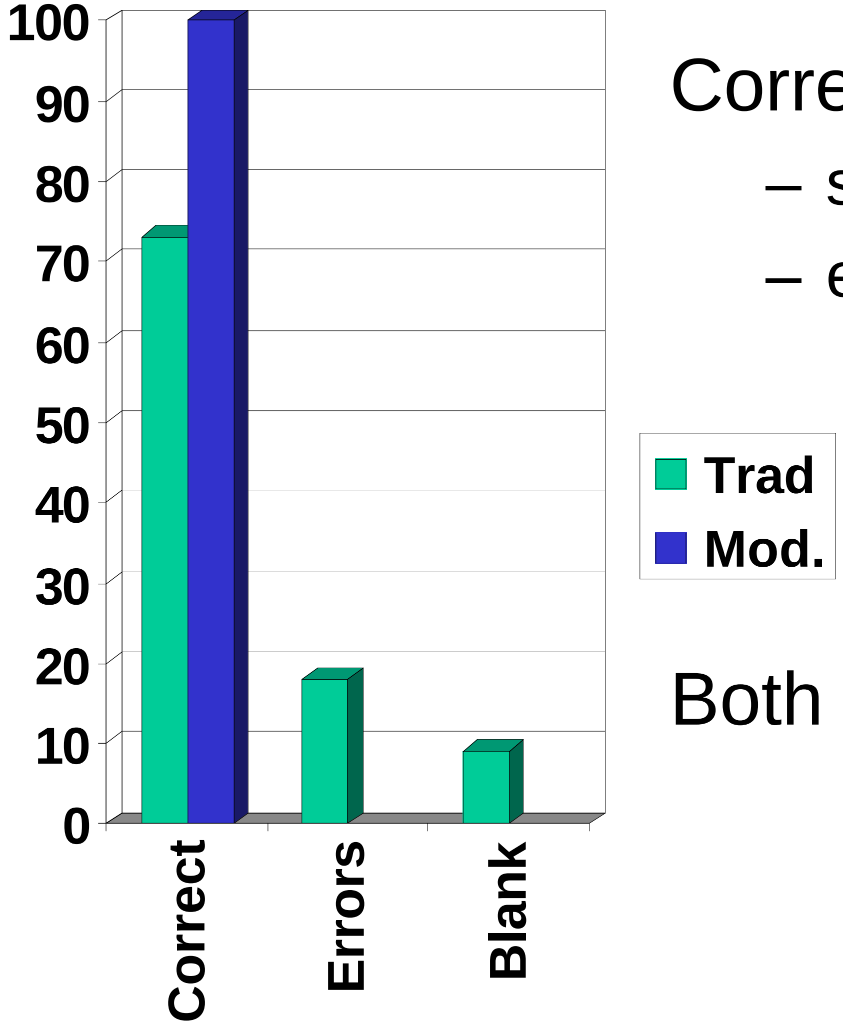
a. Sketch the shape of the total wave function of an electron in regions *I*, *II*, and *III* in the diagram to the right. Explain how you arrived at your answer.



b. Write equations for the wave function in each of the regions in the diagram. Leave normalization constants unspecified.

c. Compare the energy of electrons found in regions *I* and *III*. Explain how you arrived at your answer.

a. Student use of mathematics

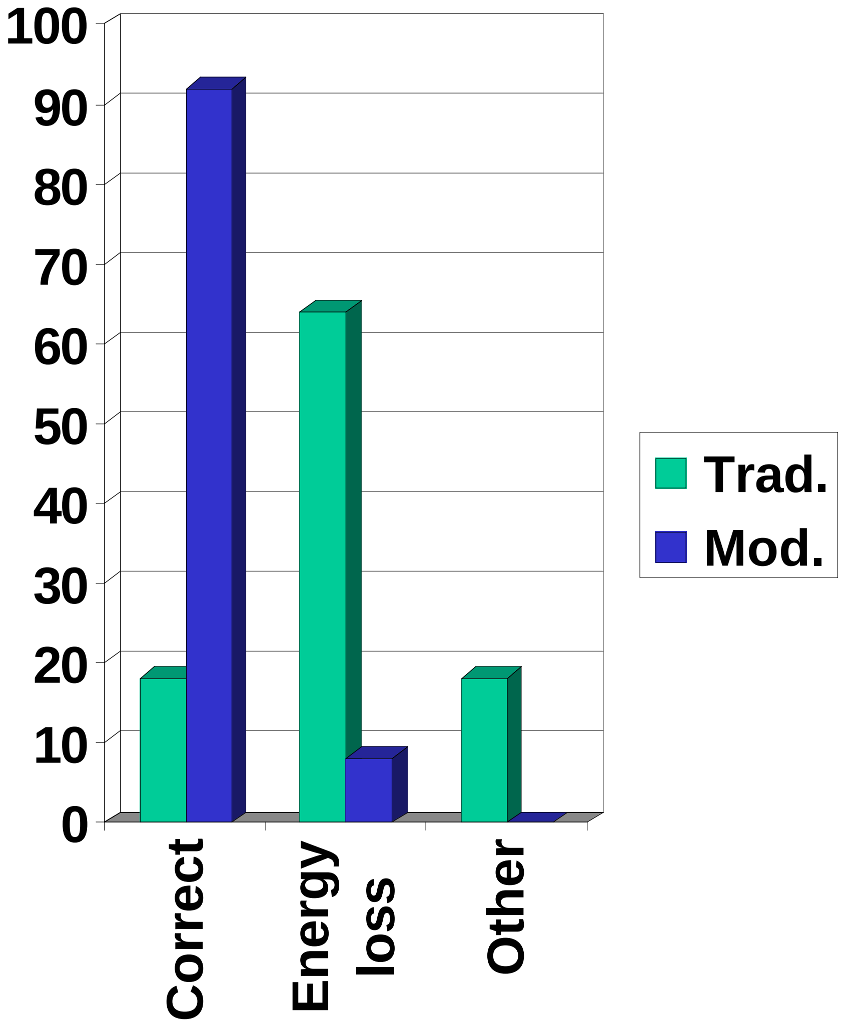


Correct response:

- sinusoidal in regions I, III
- exponential in region II

Both classes perform well.

c. Description of energies (as in pretest)



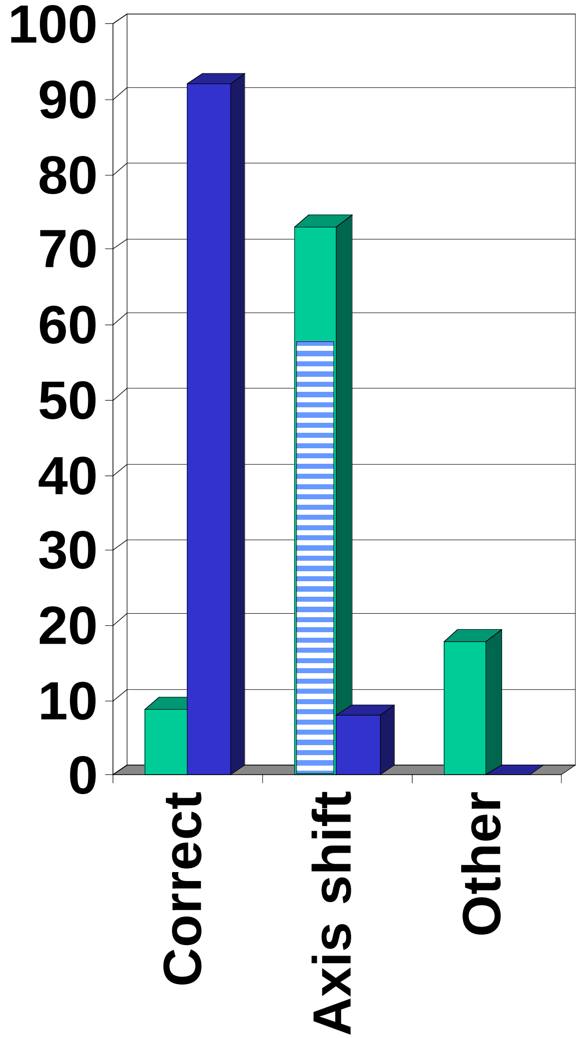
- ★ Quotes from traditional instruction class:

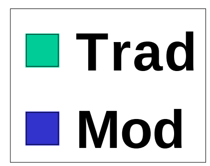
  "[the particles] in region 3 'lost' energy while tunneling through the barrier"

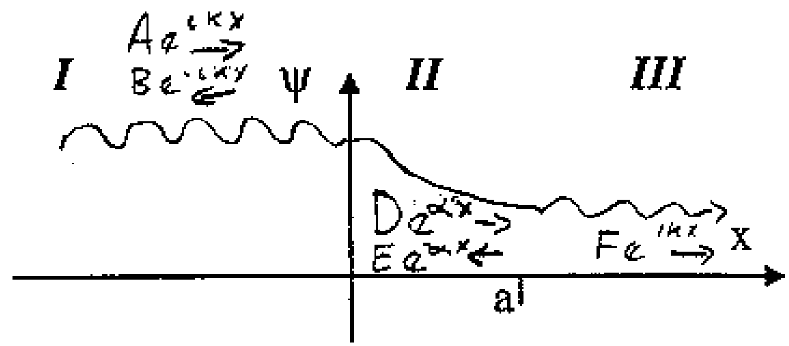
- ★ Modified instruction:
  - – tutorials
  - – online essays
  - – inclass concept test

  Can't be disentangled…

b. Student sketches of the wave function







Axis shift response

Energy loss and axis shift consistent:

- Traditional instruction students who gave
  BOTH axis shift
  AND energy loss responses
  (this student quoted earlier saying energy lost in barrier)

Additional issues

- Where is the reflection occurring?
  - Particle interpretation
    of the steady state wavefunction
  - If the probability is higher at the beginning of the barrier than the end - then what?!?
- What about additional representational issues
  - comparing wavelength and amplitude: smaller is smaller