A Framework for Describing Common Mathematical Errors Students Make in Introductory Physics

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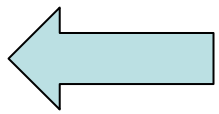
Goals of this Presentation

1. Show that Bruce Sherin's theoretical framework helps us understand how students understand equations in physics.
2. Extend this framework to understand common errors that students make while using equations in physics.

Theoretical constructs for reasoning with equations

Bruce Sherin*—[intermediate students]

1. Symbolic Forms
   i. Symbol Template
   ii. Conceptual Schema
2. Interpretive Devices 

*Sherin, B. (2001). *Cognition and Instruction*; **19**, p479-541.
*Sherin, B. (2001). *International Journal of Computers for Mathematical Learning*; **6**, p1-61.

Symbolic Forms

- **Symbol Template**: elements of knowledge that give structure to a mathematical expression; e.g. □=□ or $\left[\frac{\ldots x \ldots}{\ldots}\right]$
- **Conceptual Schema**: elements of knowledge that offer a conceptualization of the symbol template.

What's the difference?

Conceptual Schema

Balancing

e.g. "the normal force of a table is *balancing* the gravitational force of the earth."

Symbol Template

□=□

Conceptual Schema

Same amount

e.g. "the velocity of block A is the *same* as the velocity of block B"

Symbol Template

□=□

Symbolic Forms in this Presentation

PROP+

Symbol Pattern: $\left[\frac{\ldots x \ldots}{\ldots}\right]$

Description: A whole expression or term is seen as directly proportional to a quantity, x, which appears as an individual symbol in the numerator of an expression.

IDENTITY

Symbol Pattern: $x = \ldots$

Description: A quantity, associated with a single symbol that appears alone on one side of an equation, is seen as being defined by or as having the same properties as the expression on the other side.

Interpretive Devices

"[I]nterpretive stances and strategies that more broadly characterize an orientation to an equation, and thus influence the forms seen in a symbolic expression." (Sherin, 1996; p. 113)

Claim:

Formal reasoning strategies →

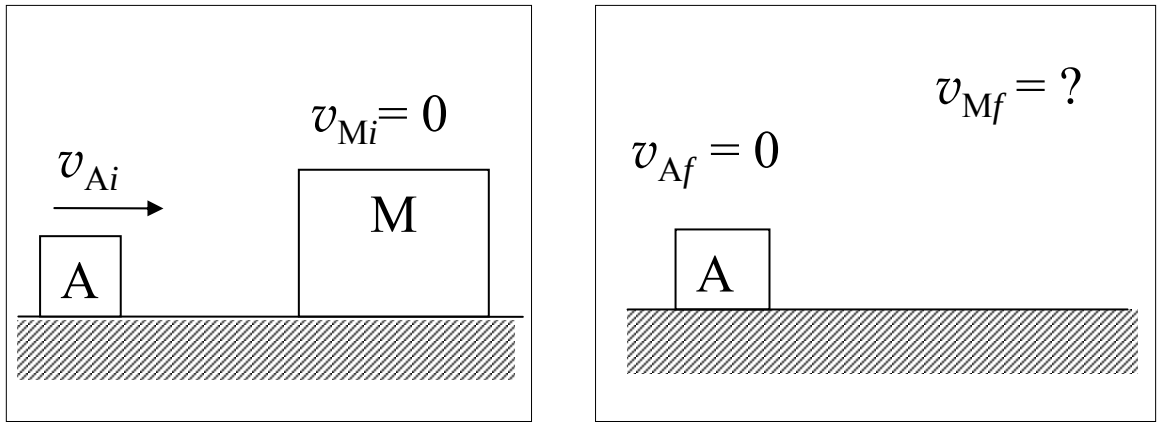
"Formal" Interpretive Devices (used for equations)

Intuitive reasoning strategies →

"Intuitive" Interpretive Devices (used in many circumstances)

Classroom Activity

- 1st Semester Algebra-based Physics Class
- Modified UW Tutorial



The mass of glider A is one-half that of glider M (i.e. $m_M = 2m_A$).

Apply Newton's second law ($F_{net} = m\Delta v/\Delta t$) to each of the colliding gliders in Experiment 1 to compare the *change in momentum* ($\Delta p = m\Delta v$) of gliders A and M during the collision. Discuss both magnitude and direction. Explain.

An Ideal Answer

$$F_{net}^{M} = m^{M} \Delta v^{M} / \Delta t^{M}$$

Newton's 2nd Law

$$F_{net}^{M} \Delta t^{M} = m^{M} \Delta v^{M}$$

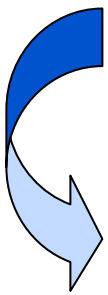
Move the change in time to the left hand side

$$F_{net}^{M} \Delta t^{M} = \Delta p^{M}$$

Definition of Change of Momentum

$$F_{net}^{A} \Delta t^{A} = \Delta p^{A}$$

Same For Glider *A*



$$F_{net}^{A} \Delta t^{A} = F_{net}^{M} \Delta t^{M}$$

Newton's 3rd Law

$$\Delta p^{A} = \Delta p^{M}$$

The change in momenta are the same.

Student Response

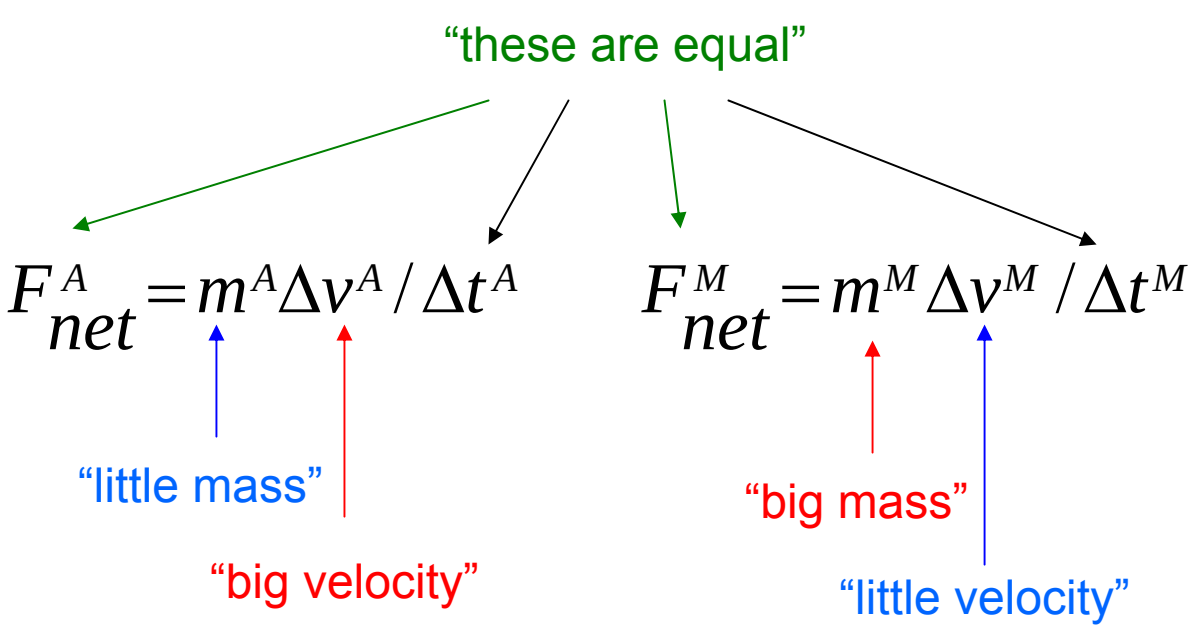
"So then the Fnet for A, the Fnet for M. This is a big mass and this is a little mass and these are equal, so this has got to be a big…velocity and this has got to be a small velocity."

$$F_{net}^{A} = m^{A}\Delta v^{A} / \Delta t^{A} \qquad F_{net}^{M} = m^{M}\Delta v^{M} / \Delta t^{M}$$

"Formal" Interpretive Device: *Changing Parameters*

Allow one parameter to change while keeping the others fixed.

Prop+



Student's Confusing Quotes

S1: "So the momentums got to be the same right?"

…

S1: "No, this is not right."

…

S1: "But the change in velocities are not the same, though…that's the problem, I was thinking [the changes in velocity] were the same."

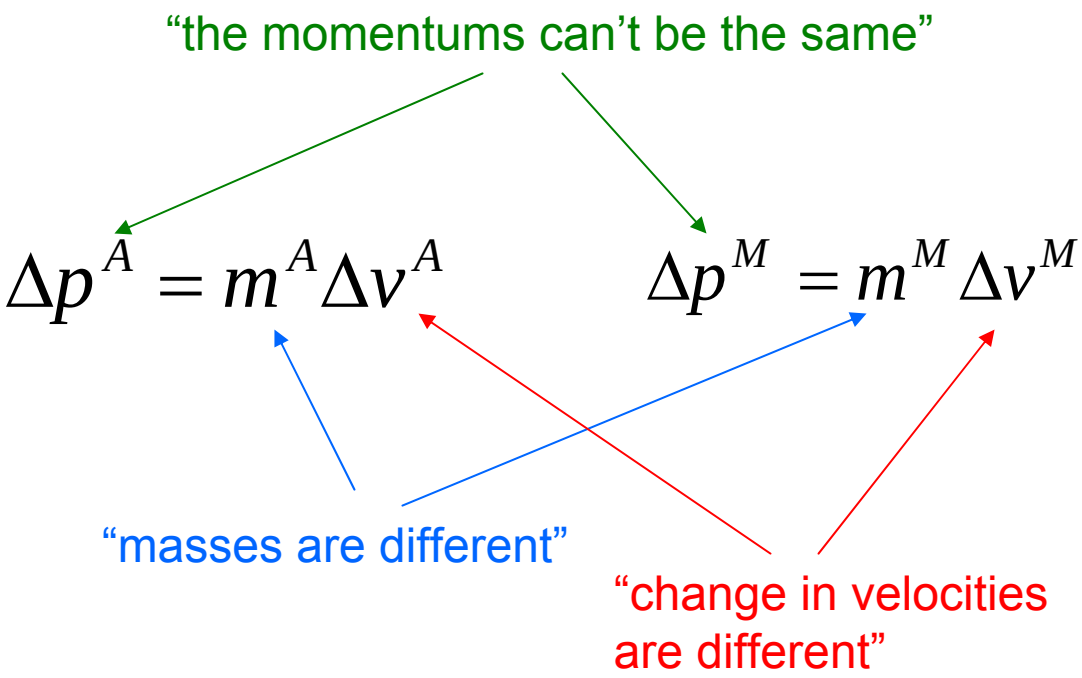
…

S1: "How could [the momenta] be the same? If the masses are different and the change in velocities are different the momentums *can't* be the same."

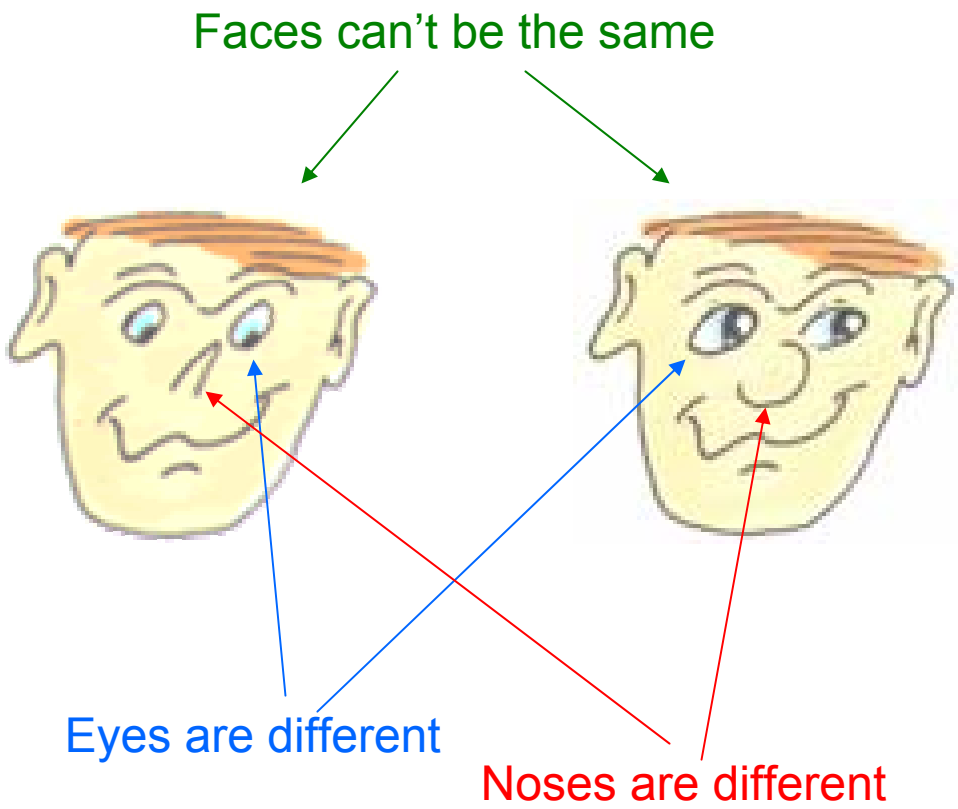
"Intuitive" Interpretive Device: *Feature Analysis*

A form of pattern recognition in which the features of a stimulus are evaluated individually.

Identity



Feature Analysis: Faces



Making Sense of Student Quotes

S1: “So the momentums got to be the same right?”

…

S1: “No, this is not right.”

…

S1: “But the change in velocities are not the same, though…that’s the problem, I was thinking [the changes in velocity] were the same.”

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S1: “How could [the momenta] be the same? If the masses are different and the change in velocities are different the momentums *can’t* be the same.”

Theoretical constructs for reasoning with equations

Bruce Sherin—[intermediate students]

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2. Interpretive Devices
   i. “Formal” Interpretive Devices (Sherin)
   ii. “Intuitive” Interpretive Devices (Tuminaro)

Conclusions

- Sherin's framework offers a lens by which to interpret students' understanding of equations in physics.
- Extending Sherin's framework to include "intuitive" interpretive devices allows us to understand some mistakes that introductory students make when reasoning with equations in physics.