Chapter 4: A Proposed Model of Student Learning

Introduction

One goal of physics education research is to go beyond the discovery and recitation of difficulties that students have with a specific topic in physics. By trying to organize how we see students approaching the material, we have the opportunity to gain deeper insight into how students come to make sense of the physics they are taught in our classrooms. We can then use our organization of student difficulties (and strengths) to help develop curriculum materials that more effectively address sometimes subtle and counter-intuitive student needs. This chapter presents a brief discussion of a possible organization of student difficulties according to a model of learning that will be used in chapter 5 to analyze the data presented in chapter 3.

To describe how students learn in our classroom, we need to develop a meaningful language that lets us describe, organize, and systematically discuss our observations of student reasoning.[1] Other fields, generally organized under the name cognitive studies, can provide a source of understanding and suggest models that help us make sense of student learning in physics. Their validity in the physics education research often lies in the suggestions these models make rather than in their exact details, but these suggestions can play a profound role in the manner in which we approach our classrooms.[2] Those readers less interested in the details of this learning theory are asked to read the conclusions of this chapter and Table 4-1 for a summary of the ideas contained in it.

Reasoning Primitives

Consider a simple action that is common and repeated often enough that it is not even consciously considered, e.g. pushing an object previously at rest across a surface. An effort must be exerted to get it moving. Similarly, when delegating work to another person, it is often necessary to motivate this person so that the work is begun. Though the two situations have little to do with each other, both are examples of the need for an "actuating agency" to set events (or objects or people) in motion.[2] The actuating agency can be thought of as a reasoning primitive common to many different settings.

In this sense, a primitive is a common and small logical building block that lets us describe basic elements of common events in many different situations. A suitable analogy can be made to the way physicists and chemists think of the atom. In many settings, the atom is the smallest relevant description of nature. One atom (the primitive) can be part of many different types of molecules (the situation). Of course, the substructure of the atom is of great interest, but not always relevant to the specific model one is considering. In the same way, one can discuss elements of primitives and how they develop, but the primitive itself is a relevant grain size (as discussed in chapter 2) for discussion. We can think of primitives as the building blocks with which people build their thinking.[3] Primitives can help simplify both everyday and physics reasoning situations.

For example, the common use of *actuating agency* can help explain some of the results described in chapter 2. In Clement's coin toss problem, students describe the effort needed to throw the coin in the air and speak of this "force" remaining with the coin as it rises. In this description, students use the actuating agency primitive when talking about the force exerted to set the coin in motion, but additionally assume that the force stays with the coin after it is released from the hand. Thus, students make sense of the physics of the coin toss problem by incorrectly over-applying an otherwise useful abstract idea that helps simplify our predictions about what happens when an object should be set in motion.

The most productive and relevant discussion of the use of primitives in physics has been carried out by diSessa[4,5] and by Minstrell.[6] diSessa's work has focused on very general reasoning elements used in a variety of situations including physics, such as the actuating agency described above,[7] while Minstrell's work has focused on how students apply primitives specifically in their reasoning in physics.

Table 4-1

Primitive	Definition	Example (mechanics related)
Force as mover	"A directed impetus acts in a burst on an object. Result is displacement and/or speed in the same direction."	Clement's coin toss problem as describe in chapter 2.
Working harder	"More effort or cues to more effort may be interpreted as if in an effort to compensate for more resistance."	To make a box begin to move across the floor, a larger force needs to be exerted than to keep it moving.
Smaller objects naturally go faster	Larger objects take more effort to create, see Intrinsic Resistance (to which it is related). Also related to "Bigger is Slower."	The same impulse delivered to a small object (coin) as to a large object (brick) will make the smaller one travel faster than the large one.
Intrinsic Resistance	"Especially heavy or large things resist motion."	Heavier boxes are harder to start moving across a floor (or lift up) than are lighter boxes.
Ohm's p-prim	"An agent or causal impetus acts through a resistance or interference to produce a result. It cues and justifies a set of proportionalities, such as 'increased effort or intensity of impetus leads to more result'; 'increased resistance leads to less result.' These effects can compensate each other; for example, increased effort and increased resistance may leave the result unchanged."	The speed of a coin tossed in the air depends on its mass and the force exerted on it to throw it in the air (see Force as Mover example).

Table 4-1 (continued)

Primitive	Definition	Example (mechanics related)
Dying away	"All motion, especially impulsively or violently caused, gradually dies away."	A coin tossed in the air slowly loses speed and stops (related to an impetus theory, that it has "used up" the ability to move, see chapter 2).
Guiding	"A determined path directly causes an object to move along it."	A ball traveling a circular path (guided by a wall, for example) will continue on a curved path even after the wall is no longer there (see FCI question…)
Canceling	"An influence may be undone by an opposite influence."	An object will move after one kick (see Force as Mover) and stop after another in the opposite direction.
Bouncing	"An object comes into impingement with a big or otherwise immobile other object, and the impinger recoils." (see Overcoming below.)	An small object will bounce off a large one, or two equal sized objects will bounce off each other.
Overcoming	"One force or influence overpowers another"	To get a box moving along a rough floor, the exerted effort must be larger than the resistance of the object (related to Ohm's in terms of competing proportionalities).

Primitives as defined by diSessa in his monograph (see reference 4). For each primitive, a general definition is given, and an example (if possible, taken from the discussion in the chapter) is included.

General Reasoning Primitives

diSessa has developed a description of student use of primitives through observations of students' interpretations and generalizations of the everyday phenomena around them and their use of these interpretations to guide their reasoning in physics. Even though he draws his conclusions mainly from extensive investigations of student difficulties in the field of mechanics, he emphasizes the general nature of student primitives.

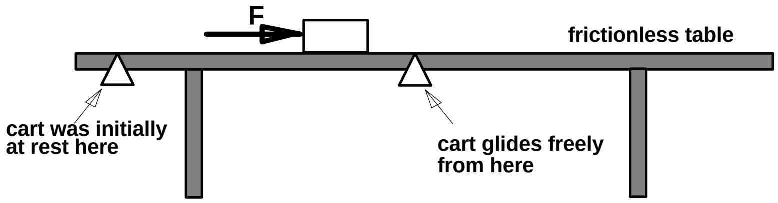
To illustrate how diSessa discusses student use of primitives, let us consider one example in detail (for a complete list of the primitives discussed in this chapter, see Table 4-1). The actuating agency primitive has already been introduced. A refinement of this primitive comes when one considers how different objects with different properties (such as different masses) are to be brought into motion. Consider two boxes with different masses resting on the same rough surface. The goal is to set them in motion. More effort will be needed to move a larger box. The physics of the situation is complicated, requiring an understanding of normal forces, friction (both the threshold nature of the friction force and difference between static and kinetic friction), and Newton's Second Law. A simpler way to think of the situation is to use the reasoning that "more requires more" (mass and effort, respectively) or "less requires less." In the simple linear reasoning that we often use, it is possible to say that the larger effort is then proportional to the resistance afforded by the larger mass such that the two boxes are set in motion in the same fashion.

diSessa refers to the compensatory reasoning based on resistance as the Ohm's primitive. The name comes from the correct physics reasoning found in Ohm's law, $V = IR$. If voltage changes, the current depends on the resistance of the circuit. We often see students use the reasoning "bigger mass requires bigger force" in our classroom interactions. This is not necessarily incorrect, but it is often overly simplistic. A more refined use of the Ohm's primitive than the example of setting a box in motion is the analysis of the acceleration of an object due to a force exerted on it, as described by Newton's Second Law, $F = ma$. In this case, the net force on the box and the acceleration of the box after the exerted force is larger than the maximum possible friction force can be compared. The effect of the force is not simply motion, as is implied by the simplistic application of the Ohm's primitive, but acceleration of the box. As illustrated by this situation, the use of the Ohm's primitive may be correct and appropriate, correct but overly simplistic, or even incorrect.

Student use of the Ohm's primitive can be seen in other, more difficult settings that are discussed at the introductory physics level. In research done at the University of Washington, students were asked to compare the change in kinetic energy and the change in momentum of two objects with unequal mass which start from rest and are moved a fixed distance by a constant force (see Figure 4-1).[8] A correct answer would

say that the change in kinetic energy was equal for the two but the change in momentum was unequal. By the work-energy theorem (Net work equals the change in kinetic energy, $\int \vec{F} \bullet d\vec{r} = \Delta KE$), both objects are moved the same distance by the same force, so their change in kinetic energy is the same. But the same force exerted on the two objects leads to a different acceleration for the two and the lighter object will have the force exerted on it for a shorter time. By the impulse-momentum theorem (i.e. the definition of force, rewritten as Impulse equals the change in momentum, $\vec{F}\Delta t = \Delta\vec{p}$), the object in motion for less time has a smaller change in momentum. We often encounter students who state that *both* the change in kinetic energy and the change in momentum should be equal. In the first case, they state that the mass is higher but the velocity is less and therefore the kinetic energy, $KE = 1/2\ mv^2$, is equal for the two objects. These students are getting the correct answer while using inexact reasoning that does not sufficiently analyze the physics. In the second case, these students again state that the higher mass and lower velocity compensate each other such that the change in momentum ($\vec{p} = m\vec{v}$) for the two objects is equal. Obviously, both cannot be true since the exponent on the velocity differs in the two equations. But we see that students are applying the Ohm's primitive incorrectly to both questions. In one case,

Figure 4-1



Two carts, A and B, are initially at rest on a frictionless, horizontal table. They move along parallel tracks (only one cart is shown in the figure above). The same constant force, F, is exerted on each cart, in turn, as it travels between the two marks on the table. The carts are then allowed to glide freely. The carts are *not* identical. Cart A appears larger than cart B and reaches the second mark before cart B.

Compare the *momentum* of cart A to the *momentum* of cart B after the carts have passed the second mark. Explain your reasoning.

Compare the *kinetic energy* of cart A to the *kinetic energy* of cart B after the carts have passed the second mark. Explain your reasoning.

Question asked to compare student understanding of momentum and kinetic energy. A correct answer to the first question would state that cart B spent more time being accelerated by the force, so its change in momentum (from rest) was larger. A correct answer to the second question would state that both carts had equal forces exerted over equal distances, so the change in kinetic energy (from rest) was equal for the two carts. Student responses to the question can be interpreted by means of common discrete reasoning elements, called primitives that students apply inappropriately to the situation.

though it is not linear, they get the right answer, while in the linear case, they give an incorrect response.

The Ohm's primitive involves proportional, compensatory reasoning and involves the recognition of different elements of the system. This makes it one of the more complicated primitives that diSessa describes. Rather than show how each of the primitives described by diSessa was developed and how it is used, I will describe those which will play a role in this dissertation and give examples of student reasoning which can be interpreted as using these primitives.[9] The primitives relevant to this dissertation fall into two categories, those related to force and motion and those related to collisions between objects.

Force and Motion Primitives

Three primitives effectively describe how students approach reasoning about force and motion in a way that will be important in later parts of this dissertation. These are the working harder, smaller is faster, and dying away primitives.

The working harder primitive describes the "more is more" or "less is less" element of the Ohm's primitive. This primitive describes reasoning where there is a simple linear relation between different objects and the idea of resistance is not included. Examples of the common reasoning using the working harder primitive include people who work more and get better grades or objects that have larger forces exerted on them move faster. This primitive seems very reasonable in some settings but can be easily misapplied. Force is proportional to acceleration, not velocity, for example.

The smaller is faster primitive describes how a small object is more easily made to go fast than a larger object. This is closely connected to the bigger is slower primitive. (Elephants seem slower than mice, though they usually aren't.[10]) This primitive makes sense, as long as one assumes that the same force is exerted on the light and the heavy objects (while again assuming that force is proportional to velocity and not acceleration). In terms of common sense reasoning, it is harder to move a large object than a small object (See chapter 2 for a discussion of common sense physics related to force and motion.)

Finally, the dying away primitive can be related to our existence in a frictional world. Every motion we experience eventually comes to an end. Many students generalize this inappropriately to situations such as Clement's coin toss example, given in chapter 2, where the dying away primitive plays a role in the impetus theory explanations given by students. The force that is "used up" as the coin is thrown into the air can be thought of as having "died away" in the process. In this example, we see how multiple primitives can play a role in the reasoning about a single physical situation.

Primitives Describing Collision

The collision primitives will also play a role in our descriptions of student difficulties with wave physics. These primitives include canceling, bouncing, and overcoming.

The canceling primitive is directly related to collisions and describes that motion stops when two objects collide with each other (thus, their motions have been canceled). Another example of reasoning using this primitive is the description that a box that is brought into motion by a force will be stopped by an equivalent force in the opposite direction. These forces can then be said to cancel out (even though the actual physics of the situation is more complex than such a simple description). This example illustrates how students applying primitives may ignore various elements of the problem to come up with a (in this case correct) answer through the use of overly simple reasoning.

The bouncing primitive describes the common sense reasoning used to describe a ball hitting a wall, for example. While ignoring the detailed physics of collisions, one can use the idea that objects simply bounce off of other objects that are in the way and immovable. This same reasoning (the object is in the way) plays a role in some student's descriptions of normal forces for objects lying on a surface, though the element of collisions is missing in the case of normal forces.

Finally, the overcoming primitive gives a less phenomenological and more analytical description for the same bouncing phenomena. For example, the force of the wall overpowers the force of the ball and sends the ball back from whence it came. This reasoning is very similar to the impetus theory described in chapter 2 in the sense that the moving ball has an intrinsic force that is overcome by the larger force of the wall. The confusion lies in describing force as an object or quantity specific to an object rather than the interaction between objects.[11] (This same confusion seems to play a role when students use the dying away primitive in Clement's coin toss problem.) Incorrect use of the overcoming primitive may be caused by students trying to make sense of their experiences in the language of the physics classroom rather than the real world description of the bouncing primitive (where balls just bounce off walls because that's what they do).

Facets of Knowledge: Context-Specific Interpretation of Primitives

diSessa is not the only physics education researcher to investigate the usefulness of using common elements to describe student difficulties with physics. Minstrell developed the idea of "facets" to describe the common elements of student reasoning that he found in his work as a high school teacher in Washington state.[12] Minstrell's facets are similar to diSessa's primitives in that they describe small observable relevant pieces of student reasoning. Minstrell chooses to look at specific observable elements of student reasoning, which, he states, is only possible by choosing a "grain size" of reasoning that is small enough to contain general ideas which can be applied in a great variety of situations. In the process, he focuses on the student's reasoning and not the correct physics (Compare this to the description of Halloun and Hestenes's work in chapter 2.)

As an example of the use of facets when describing student reasoning about force and motion in the classroom, Minstrell describes a set of facets commonly found in classroom discussions of the physics of motion (see Table 4-2). The Goal Facet is the desired explanation that an instructor would like to see. The others are examples of explanations that students give. The Mental Model Facet gives a broad description that links together many facets that can be applied incorrectly to a given physical situation. Note that none of the facets are always incorrect. Instead, all but the Goal Facet are often inapplicable in certain situations and are not general enough to be used in all situations.

Minstrell describes an example of the application of facets in student reasoning that comes in response to a question describing two students leaning (motionlessly) against each other, where one student (Sam) is "stronger and heavier" than the other (Shirley). Students are asked to compare the forces Sam and Shirley exert on each other. Students are offered a series of choices: Sam exerts a greater force, they exert equal forces on each other, Shirley exerts a greater force, or neither exerts a force on the other. The correct answer would be to say that they are exerting equal forces on each other (by Newton's third law). Some students state that Sam is bigger and must therefore exert a larger force (facets 475 and/or 478), but others state that they are motionless because Sam is hard to move and Shirley must be pushing, so she exerts a larger force. In a similar question, some students use the facet that "Passive objects don't exert forces." Thus, since Sam and Shirley are not moving, neither exerts a force on the other. Minstrell shows that these types of reasoning are consistently used to describe forces relating to motionless objects, moving objects, and forces caused by many different objects such as magnetic, gravitational, or pushing forces.

Student facets can be discussed as applications of diSessa's primitives to a specific setting. The answer stating that Sam is bigger and exerts a larger force is consistent with the overcoming and the Ohm's primitives (he has less resistance and therefore exerts a larger force). But the idea that Shirley must be pushing harder is also consistent with the Ohm's primitive. Thus, the same primitive can lead to contradictory facets and answers. We see that the Ohm's primitive can be considered the source primitive for facets 475 through 478 in Table 4-2.

Another example of facets as applications of primitives in a specific setting comes from the description that Sam and Shirley are exerting no forces because they are not moving. This is consistent with the actuating agency primitive, because (in this primitive) forces only occur when there is motion.

Neither diSessa nor Minstrell discuss how students come to apply specific primitives in their reasoning, nor do they discuss how students choose and use specific facets in a given setting. A variety of questions remain. How do students choose to use one or another primitive when answering specific questions about specific physical situations? How do their choices manifest themselves in the facets that we observe? And are students consistent in their use of facets? These questions play a large role in the dissertation. In later chapters, I will discuss how students come to choose specific

Table 4-2

470	Goal facet: All interactions involve equal magnitude and oppositely directed action and reaction forces that are acting on separate, interacting bodies.
472	Action and reaction forces are equal and opposite forces on the same object
475	The stronger/firmer/harder object will exert the larger force
476	The object moving the fastest will exert the greater force
477	The more active/energetic object will exert the greater force
478	The bigger/heavier object will exert the larger force
479	Mental Model facet: in an interaction between objects the one with more of a particular perceptually salient characteristic will exert the larger force.

Common facets described by Minstrell that relate to collisions between objects. Note that the xx0 facet is the "goal facet" that we would like students to have in our classrooms, while the xx9 facet is the "mental model facet" that is the organizing theme for incorrect student facets.

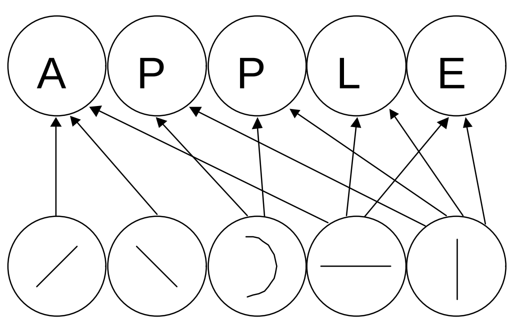
facets in a specific setting. We find that students can be described as using guiding analogies in their reasoning as they approach a specific physics setting. These analogies help determine which of the many (possibly contradictory) facets which could be applied to a situation actually are. This idea will be discussed in more detail in the section describing mental models, below.

Parallel Data Processing

In some of the examples described above, students could be described as using more than a single primitive (or facet) in their reasoning. For example, in Clement's coin toss problem, it was possible to describe some students as using both the actuating agency and dying away primitives. In order to describe the manner in which multiple primitives are used by students, we can ask how students connect primitives in their reasoning.

Consider reading the word APPLE. To perceive the individual letters in the word, one can break each letter into its simplest shapes. This creates a set of vertical, horizontal, and diagonal lines along with half circles (see Figure 4-2). Experienced readers do not read each letter based on its parts and then piece together the word from its constituent letters. Instead, the entire word is perceived at the same time. Researchers have effectively described the process of visual perception of entire words by focusing on how the individual elements of the words are perceived and interpreted in connection to each other.[13] For example, the combination of diagonal lines and a horizontal line in the right configuration creates an "A." The combination of vertical and three horizontal lines when connected correctly creates an "E." By assuming that the lineshapes are all interpreted and connected to each other at the same time (i.e. in parallel), one can describe how a finite set of symbols can form a single word. Because of the way in which many small elements are connected simultaneously to present one word to the reader, the theory of perception described in this example is called parallel data processing, or connectionism.[14] The latter term is used to emphasize the connections between different "nodes" of information. In this section, I will describe how children's learning of torque was modeled by using a connectionist model.

Figure 4-2



The word APPLE and the simple line shapes that can be combined to form all the letters in the word. According to connectionist theory, as the entire word APPLE is perceived, each letter is interpreted as the conjunction of different line shapes; all lineshapes are interpreted at the same time.

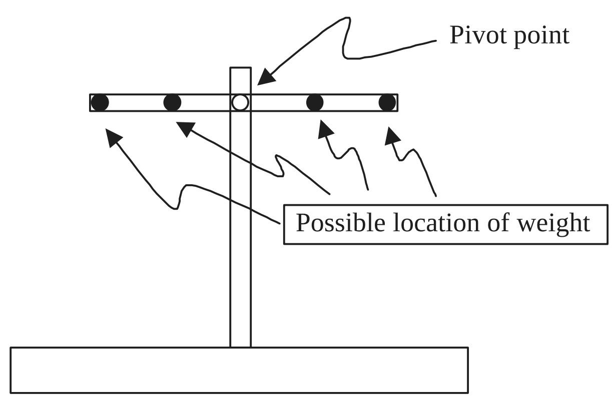
The APPLE example shows how a description in terms of parallel data processing involves taking individual, basic building blocks of perception and combining them into much more complicated structures like words. Most research into the use of parallel data processing has taken place in perception or linguistics, where the basic building blocks of perception (or grammar) are possibly quite different from those in physics. The purpose of this section is to show that the structure of parallel data processing can be helpful for understanding how students apply primitives to their reasoning.

Children investigated for their understanding of balance were asked to describe whether a set of weights placed a certain distance from a pivot point would balance the beam on which they hung (see Figure 4-3).[14] Pegs were placed at equal distances on equal-length arms of a balance beam. Small weights all of equal mass were placed at different locations on the beam while the beam was held in place. Subjects were asked to predict how, if at all, the balance beam would rotate if released. A correct answer would explain that the number of weights (proportional to the mass and therefore the force of gravity at that point) times the distance from the pivot point was the relevant measure (i.e. the torque is proportional to force and distance by $t = Fd$ in this simple situation). The beam will rotate in the direction of the side of the beam with the largest torque.

Observations show that children slowly come to realize that the relevant variables are weight of the object and distance from the pivot point.[15] Furthermore, observations show that, over time, children develop four different levels or patterns of reasoning with which they answer the question of how to balance the beam on which weights are already hanging.

The first and simplest pattern involves counting the number of weights hanging from each side of the balance beam. In the second pattern, children still look for the number of weights first, but if these are equal, then distance from the pivot is included in children's reasoning. In the third pattern, distance and weight are both always

Figure 4-3



Sketch of the torque balance task. Pegs are located at equal distances along equal-length arms of a balance beam. Small equal-sized weights were placed at different locations on the beam while the beam was held in place. Subjects were required to predict how, if at all, the balance beam would rotate if released.

considered, but with a special emphasis on equality. If one is equal, the other determines imbalance. If both weight and distance are greater for one side, the child states that side will drop. If one side has greater weight and the other has greater distance, the child using this model is unable to resolve the inconsistency. Finally, in the fourth pattern of reasoning, children learn to make a full explanation based on the sum of the products of weight and distance. Here, students are using both the weight and the distance from the pivot point in their reasoning. Evidence shows that children progress through these four patterns of reasoning as they gain experience, and that even college students are unable to consistently use the fourth pattern at all times in their reasoning.[16]

In describing the four patterns of reasoning that students use, the working harder primitive was applied in two different fashions to lead to two facets that the subjects appear to use in their reasoning. The weight facet seems to involve counting how many weights are being hung from each end of the balance beam. In the first pattern, if there is more weight, the balance beam will tilt in that direction. The distance facet involves the simple operational measurement of distance from the pivot point. In the second pattern, if the weights are equal but the distances from the pivot are unequal, the balance beam will tilt in that direction. In the second pattern, the distance facet is less important and its use dependent on an inability to apply the weight facet. In the third pattern, students use a refined version of the second model. Now, the distance facet is isomorphic in its reasoning utility with the weight facet. Balance is determined by a combination of the two, but without a refined description of what happens if they vary covariationally (i.e. one variable goes up while the other goes down). In the fourth pattern, the two facets are linked together to create a quantity (torque) which determines balance. One can describe the students using the fourth pattern as applying the Ohm's primitive, since they are now able to reason with three variables, two of which compensate for each other covariationally. Only when the two facets are correctly linked together is the concept of torque fully operationally understood.

Further research into student understanding of the physics of this situation has shown that students more easily answer the question (i.e. use a better model) when the weights or distances are very distinct, rather than nearly equal to each other.[17] When the weights or distances are distinct, it is possible to use only one facet to guide one's reasoning to the correct answer. This suggests that it is more difficult to use two facets at the same time than one.

Patterns of Association, Guiding Analogies, and Mental Models

In the previous sections, specific student difficulties were described as inappropriate applications of sometimes useful facets of knowledge or reasoning primitives. Primitives are too general, though, to be of much use by themselves. They are too general and can lead to contradictory responses. The organizational structure of primitives seems critical when we discuss how students make sense of the physics through the use of primitives. We use the idea of a "guiding executive" that guides students to use and interpret particular primitives in particular ways to particular situations. In general, we refer to this guiding executive as a pattern of association, or a mental model when it is highly structured, complex, and coherent.

When students consistently use a set of primitives inappropriately in a given setting, we can say that they have a pattern of association with which they approach the physics. The term is used to describe the semi-structured manner in which students bring a large body of knowledge to a situation. Some of this knowledge is applicable, while other pieces of what the student believes may be problematic.[18] Where primitives are single, individual, prototypical units of reasoning, a pattern of association can be thought of as a linked web of primitives and facets associated with a topic. Note, though, that analyzing student responses in terms of patterns of association can be helpful in trying to make sense of what we observe but does not imply that students have a specific fixed model in mind when they approach a situation. Patterns of association are more fluid and less precise than a physical model.

The term "model" has very specific meaning in physics. Patterns of association and even mental models are not physical models. They have certain traits that possibly make them problematic when used by students. Student patterns of association are often incomplete, self-contradictory, and inconsistent with experimental data. Based on the description of patterns of association as linked sets of primitives which students often use incorrectly, this should be no surprise. Note that incompleteness, self-contradiction, and inconsistency are possible traits of physical models, too. We may refer to an accepted physical model, determined through theoretical and experimental work and the agreement of the research community to be valid in certain physical realms with certain limitations, as a Community Consensus Model (CM). For example, the model of waves that we present to students in the introductory level is only the linear model, which is technically incomplete and sometimes inconsistent with the experimental data. Furthermore, the simple linear model of waves is sometimes not self-consistent. For example, as described in chapter 2, two superposing waves may create a situation that violates the small angle approximation in some part of the medium. But, a trained physicist is able to know the limits of the given CM, while students usually do not know the limits of validity of a given pattern of association.

Due to the accepted and understood limitations of the CM of waves, we can describe it as a mental model. Physicists agree on certain common elements to the model and are aware of shortcomings of the model, but use it to guide their general reasoning about a large number of wave phenomena. The terminology represents the distinction between the accepted and understood limitations of a mental model (as a reasonably complex, coherent, but partially contradictory model) and the looser form of a pattern of association.

Analyzing student reasoning in terms of patterns of association can be highly productive in trying to make sense of student reasoning about advanced topics in physics. In a paper which organizes research into student difficulties with light and optics, Igal Galili uses patterns of association to describe how students develop their understanding.[19] The paper builds on previous investigations of student understanding of light and optics, many of which have been used to develop curriculum designed to help students overcome their difficulties.[20]

Galili goes beyond a description of student difficulties and tries to explain the cognitive structure of student thinking in order to better develop curriculum that can address student needs. A comparison can be made to the way in which Halloun and Hestenes go beyond Clement's investigations, as described in chapter 2. A difference, as will be pointed out, is that Galili focuses on students' responses and does not categorize students according to the correct model. As Galili says, "Students' views are certainly organized. However, their organization is different from that employed in scientific knowledge." He cites Minstrell's facets as basic building blocks of knowledge, and writes, "clusters of facets, connected by causal links, are ... appropriate to describe mental images and represent operational models." As an example, Galili discusses three conceptual topics: understanding of light sources, image formation by a converging lens, and image formation by a plane mirror. In each case, he distinguishes between the

- naïve (pre-instructional),
- novice (post-instructional), and
- appropriate formal (or community consensus)

facets of knowledge. The novice facet of knowledge in each case is a hybrid between the naïve and the formal facet.

In the case of conceptual understanding of light sources, the naïve facet of knowledge is the "static light model." Some students, previous research has shown, believe that light fills space, i.e. like a gas filling a room. Researchers often find that after instruction students state that light emanates only in radial directions from the light source, with a preferred direction being toward the observer. (Galili calls this the "flashlight model.") This novice facet seems to be a hybrid between the naïve view and the formal facet, which states that light emanates in all directions from all sources. As Galili points out, the "flashlight model" can be the source of many reported student difficulties in unique settings (such as pinholes, lenses, mirrors, etc.) and more advanced settings.[20]

In the case of conceptual understanding of real image formation by a converging lens, Galili describes the difference between what he calls the holistic (naïve), the image projection (novice), and the point-to-point mapping (formal) facets of knowledge. In the naïve conceptualization of image formation, the full image moves to the lens, is inverted by the lens, and moves to the screen, where it can be seen.[21] The novice facet of knowledge is a modified version of the naïve facet, containing the idea of a light ray but with the idea of unique rays which are more important than others. Furthermore, Galili states, in this facet "each ray carries structural information about the point of origin," meaning that physical significance is attached to each ray in a way that is inconsistent with the formal, point-to-point mapping of object to image. In the formal facet of knowledge, light flux emerges in all directions from all points of the object. Some light rays interact with the lens and converge to an image point of each individual point. The role of the screen is not to create the image but to scatter light in all directions for observers who are not in the region where light diverging from the image source would reach them.

In the case of image formation from a plane mirror, Galili again describes holistic (naïve), image projection (novice), and point-to-point mapping (formal) facets of knowledge. Students using the naïve view state that the image of the object is "on the mirror," where it can then be observed. Galili describes two versions of the novice image projection conceptualization. In the first, light rays move first to the mirror in the shortest possible path, and then reflect to the observer. This reasoning violates the law of reflection (angle of incidence equals angle of reflection for a light ray). In the second novice conceptualization, the law of reflection is used correctly but students still use only single, specific, individual rays to show where the image is. They do not think of light emanating from all points of the source in all directions, a concept which is part of the formal conceptualization. Again, the novice facet of knowledge seems to be a mixture of the naïve and the formal conceptualization.

Table 4-3 summarizes Galili's description of the different patterns of association held by students. Using the language we have introduced, the formal pattern of association can also be referred to as the community consensus model. Research has shown that the novice mental model is often the one with which our students leave our courses.[20] Students use light rays, but rarely consider a full set of them.[22] Students describe the laws of reflection and refraction correctly, but only use special rays in their reasoning. This leads to difficulties where students believe that blocking one of the special rays leads to an incomplete image being formed.[19] Also, a screen is necessary for images to be observed (even in an area where the light which forms the image can be observed), since image formation and image observation are two distinct things in the novice model.

Galili also discusses how the hybrid model might come into being due to classroom instruction. He describes possible conceptual change where students move from a naïve, holistic mental model to the image projection mental model by "the transformation of certain naïve facets of knowledge into other facets which often implement the [image projection mental model]." The idea of conceptual change will be discussed in more detail below.

Certain issues and questions remain. Galili describes three different primitives (facets) that students use in the three patterns of association, but seems to assume that students can be described by a single pattern of association at any given time in their learning. Galili does not discuss the possibility that students might use facets

Table 4-3

Pattern of Association Physical Topic:	**Naïve**	**Novice**	**Formal**
Understanding of Light Sources	Static light fills space	Special Flashlight rays	Light emanates in all directions
What a Lens Acts on to Create an Image	Full images that travel through space	Special rays with physical significance	All rays (some then form an image)
What a Mirror Acts on to Create an Image	Full image (located on the mirror)	Special rays, not necessarily with law of reflection	All rays (some then form an image)

Galili's description of the facets students use in three different patterns of association.

inconsistently in different physical situations. Students might have more than one association pattern for a situation and they might use different patterns of association depending on which pattern the question brought up in the student. In such a situation, each association pattern might act as a guideline for student reasoning but not lead to firm rules of use. The pattern of association would act as a guiding executive in helping students choose which primitives to apply to a situation. I will discuss this idea of patterns of association as guiding executives of student reasoning later in the dissertation.

Models of Conceptual Change

A fundamental goal of education is to change the way that students look at the world around them. Previous research has shown that students do not enter our classrooms as blank slates, but that they bring a body of knowledge to the lecture halls and classrooms in which we teach them. In the previous section, students after instruction were described as possibly having a hybrid novice pattern of association containing aspects of both the naïve and the community consensus (or formal) model. For those students who are using the novice pattern of association, both the naïve pattern of association and the formal mental model seem to contain reasonable and useful elements. In a teaching situation, one can create a situation where students might apply the naïve pattern of association while also being aware of the formal and correct response. Thus, a situation of "cognitive conflict" may arise in the student though an awareness of the inconsistency of one's own beliefs. This provides an opportunity to help the student determine whether the elements of the naïve reasoning are valid in a given situation.

To describe the process by which students change their ideas about the world around them, we need a description that accounts for the development of student understanding. Such a model, referred to as the Conceptual Change Model (CCM) has been proposed and developed by Hewson and others.[23–26] As stated by Demastes et al.,[26] the process by which a student's conceptual model changes can be described in two different fashions. In the first type of conceptual change, a gradual change can occur, where "competing conceptions remain but eventually only one is consistently applied by the learner." Also possible are wholesale changes, which are not evolutionary in nature but instead can be described as complete, relatively sudden changes. The distinctions between gradual and wholesale change of knowledge play a fundamental role in this dissertation.[27]

Hewson and Hennessey have used the CCM to investigate student understanding of force and motion. The task involved a book placed on a table. Students in sixth grade were asked to choose which free-body diagram from a set of offered responses best represented the book. They were then asked to justify their response with both written and verbal explanations. The paper details how the understanding of a single student, Alma, changed during instruction.

Alma began the semester by stating that only a downward force was needed to keep the book on the table. She spoke of how her response was consistent with other responses she had given, and how the response was useful in her reasoning. Thus, her original conception was satisfactory to her needs. But, the authors point out, she was not very committed to it. In other words, though she gave an incorrect response, she did not explain in detail how she arrived at the response.

At the midpoint of the semester, Alma says, "My theory has definitely changed... I think that there are equal forces... because the book isn't moving… The two forces are equal." She has obviously changed her conception of forces from one in which a single force is required to hold the book down to one in which equal forces keep the book from accelerating from its present resting state (though she does not use these terms). She adds "I can now see why I picked [the previous answer], and I don't really believe this reason anymore." Alma has left a previous conception behind and has shifted into a new understanding of force and motion.

By the end of the semester, Alma has not only correctly described the forces on a book at rest, but she has been able to describe the need for these forces. Hewson and Hennessey refer to the process by which her ability to justify and explain the need for her response as "conceptual capture." To her conception of force, she added the idea that the table must be exerting an upward force. In her own words, she now believes that the table can exert a force (something she did not believe at the beginning of the semester).

We can describe Alma's learning during the semester in terms of facets and patterns of association. Alma's description matches difficulties that Minstrell has described.[28] In terms of the primitives that Alma uses, only one is needed at first. Gravity pulls down. She seems to be using the primitive (actuating agency) that only moving objects exert forces (i.e. the table is not exerting a force on the book). As the semester progresses, she learns to think not in terms of motion alone, but in terms of sums of forces. While dropping the actuating agency primitive, she must now account for the book not moving. To do so, she seems to add another facet to her reasoning: the table can exert a force on the book. Thus, the at-rest condition of the book can now be described by the link between two facets, and her association pattern of motion has changed from a simple to a more complex one. In terms of the use of multiple facets that must be linked together for a complete understanding of the physics, Alma's learning is similar to the development of children's learning about torque and the balance beam, as described above in the section on parallel data processing.

Demastes et al.[26] have pointed out that students in biology do not necessarily switch conceptions (or patterns of association, or mental models) in a wholesale fashion. Instead, Demastes et al. expand Hewson's description to say that students can go through different patterns of conceptual change which they describe as, "(a) cascade, (b) wholesale, (c) incremental, and (d) dual constructions." Since their paper does not deal with physics, I will not emphasize details here, but I will summarize their most interesting findings. They point out that "students are often not as logical or exclusive in their cognitive restructuring as researchers assume." Demastes et al. state that students do not necessarily rebuild or exchange their conceptual understanding when confronted with evidence that shows that their previous understanding is incorrect or insufficient. Instead, students may build a completely new and separate conceptual model that accounts for the new observations. The authors give an example where students have dual, conflicting conceptions, are aware of the conflict, and still say, “I have no problem with that.”

The CCM, as described by Hewson and others and expanded by Demastes et al., describes how students come to develop an understanding of class content. The model provides insight into events that happen within our students in our classrooms, and it provides predictions about student performance in our research. As illustrated in the research by Hewson and Hennessey, the CCM model is consistent with the idea that a shift in student understanding involves a change in the patterns of association used by students to describe a physical situation. Furthermore, the shift seems to function at the level of new primitives being introduced to the association pattern. But, as pointed out by Demastes et al., we should not expect our students to completely change their conception of a physical situation. They may be learning the material while still holding on to their previous beliefs about the applicability of specific facets of knowledge to settings outside of our classrooms.

Summary

In this chapter, I have described a description of student understanding and a model that may be used to describe student learning of physics. This model has been developed to serve as a productive simplification of the different elements of student reasoning that occur in the classroom.

We have chosen to describe student reasoning in terms of basic logical elements that are common to many areas of reasoning, not just physics. These reasoning elements are helpful in making sense of the world around us and are applicable in many different situations. For example, the notion that it takes effort to bring an object into motion is similar to the idea that it takes effort to motivate a lazy person. For both phenomena, an actuating agency is needed to cause a movement from rest. We refer to logical building blocks like the actuating agency as primitives. Primitives can be applied to a specific context in a variety of ways, so that the same primitive may lead to different interpretations of the situation. We refer to each such interpretation of a primitive in a context as a facet of knowledge. It is possible to have a single primitive lead to different and contradictory facets.

Students seem to use a variety of primitives (and facets) in connection with each other to describe certain sets of phenomena. We call these systems of primitives (or facets) patterns of association, or , when they are coherent and consistent, mental models. Often, students are guided in their choice of facets by the association patterns that they already have of what are deemed similar situations. Patterns of association can effectively describe analogies that students use to guide their reasoning. Thus, a researcher can use the idea of a pattern of association in two ways. In the first, a pattern of association describes the incomplete and possibly inconsistent knowledge that students bring to a physics problem in terms of the facets applied in their reasoning. In the second, it describes the knowledge that they believe should apply to the situation, and this knowledge they use as a guiding analogy to help guide their choice of facets in their solution of the problem.

In the context of student use of patterns of association and mental models, it is possible to describe student learning in terms of the facets that students use to guide their reasoning at different points of instruction. Students may re-interpret old primitives, learn new facets, or stop using certain primitives when they no longer apply to the physical situation. Also, depending on the domain size of analysis with which one approaches student difficulties with the physics, one can say that an individual student may use multiple patterns of association or mental models simultaneously. This can be interpreted at the level of facets, where students have different, non-overlapping sets of facets, and at the level of mental models, where students use different guiding analogies to develop their understanding of a given situation.

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[1] See, for example, Redish, E. F. "Implications of Cognitive Studies for Teaching Physics" Am. J. Phys. **62**, 796-803 (1994) and Hestenes, D., "Wherefore a science of teaching?" Phys. Teach. **17**, 235-242 (1979).

[2] An excellent discussion of this approach can be found in: Hammer, David "More than misconceptions: Multiple perspectives on student knowledge and reasoning and an appropriate role for education research," Am. J. Phys. **64**, 1316-1325 (1996).

[3] The discussion of primitives is closely related to many ideas of schema theory. For the most concise definition of schema theory, see Alba, Joseph W. And Lynn Hasher, "Is Memory Schematic," Psych. Bull., **93**, 203 (1983). The authors critique a large amount of the schema theory literature while not denying the existence of schemas (or primitives) in everyday, common reasoning patterns. Since we are concerned with the use of everyday reasoning patterns in the classroom, a schema theory is still applicable to this analysis.

[4] diSessa, A. A., "Towards an epistemology of physics," Cognit. and Instruct. **10**, 105-225 (1993).

[5] In reference 4, diSessa refers to his units of basic reasoning as "phenomenological primitives" (or "p-prims" for short), but we have found that "p-prims" are essentially the same as the schemas referred to as prototype theories in the cognitive studies literature. In order to keep the number of terms introduced in this chapter to a minimum, we will refer to an individual p-prim as a specific primitive while still using the classifications given by diSessa.

[6] Minstrell, J. "Facets of students' knowledge and relevant instruction," In: *Research in Physics Learning: Theoretical Issues and Empirical Studies, Proceedings of an International Workshop*, Bremen, Germany, March 4-8, 1991, edited by R. Duit, F. Goldberg, and H. Niedderer (IPN, Kiel Germany, 1992) 110-128.

[7] The term "actuating agency" has been proposed by David Hammer to more accurately describe diSessa's phrase "Force as Mover." See Hammer, D., "Misconceptions or p-prims, How might alternative perspectives of cognitive structures influence instructional perceptions and intentions?" J. Learn. Sci. **5**:2, 97-127 (1996) for more details.

[8] For more detailed description of the research which included the described experiment, see Pride, T. E. O'Brien, S. Vokos, and L. C. McDermott, "The challenge of matching learning assessments to teaching goals: An example from the work-energy and impulse-momentum theorems," Am. J. Phys. **68**, 147-157 (1998) and references cited therein.

[9] For a more complete description, see reference 4.

[10] The hardware store, Hechinger's, recently ran television advertisements for a "Big and Fast" sale, stating that things in nature were never big AND fast. To illustrate this, they showed a variety of small but fast objects such as a mouse and large and slow objects such as an elephant. They then stated that sometimes objects could be large and fast. To illustrate, they showed a Saturn V rocket at the beginning of the take-off sequence (when it is actually moving very, very slowly).

[11] We can also interpret this confusion as an example of the failure to distinguish between a quantity ($\vec{p}$) and its rate of change ($\vec{F} = d\vec{p}/dt$).

[12] See reference 6, p. 92.

[13] For example, people can still read words where parts of certain letters have been covered up; the parsing process seems to include the ability to fill in a partially complete pattern using the context in which it appears (i.e. the other letters).

[14] See Klahr, D. and B. MacWhinney, "Information Processing." In *The Handbook of Child Psychology*, *Vol.2, Cognition, perceprtion, and action*, edited by W. Damon (Wiley, New York, 1998) 631-678.

[15] Klahr, D. and R. S. Siegler, "The representation of children's knowledge," in *Developmental psychology: An advanced textbook* (3rd ed.), edited by M. H. Bornstein and M. E. Lamb (Erlbaum, Hillsdale, NJ, 1992).

[16] See, for example, Ortiz, L. G., P. R. L. Heron, P. S. Shaffer, and L. C. McDermott, "Identifying and Addressing Student Difficulties with the Static Equilibrium of Rigd Bodies," *The Announcer* **28**:2 114 (1998).

[17] See reference 14 for more details.

[18] Norman, D. A. "Some Observations on Mental Models" In *Mental Models*, D. Gentner and A. L. Stevens (Eds.) (Lawrence Erlbaum Associates, Hillsdale NJ, 1983) 7-14.

[19] Galili uses the term mental model for what we have called a pattern of association. For more details, see Galili, I., "Students' conceptual change in geometrical optics," Int. J. Sci. Educ. **18**:7, 847-868 (1996).

[20] For a summary of research into student understanding of geometrical optics, see reference 19 and references cited therein. For an example of how research into student difficulties with light and optics leads to curriculum development, see Wosilait, K., P.

R. L. Heron, P. S. Shaffer, and L. C. McDermott, "Development and assessment of a research-based tutorial on light and shadow," Am. J. Phys. **66**:10, 906-913 (1998) and references cited therein.

[21] For a detailed discussion of student descriptions of this conceptualization, see Galili, I., S. Bendall, and F. M. Goldberg, "The effects of prior knowledge and instruction on understanding image formation," J. Res. Sci. Teach. **30**:3, 271-301 (1993) and references cited therein.

[22] Bruce Sherwood, of Carnegie-Mellon University, has proposed that all textbooks follow a certain theorem: light attracts glass. In other words, only those rays of light which leave the source and pass through a lens or are reflected by a mirror are shown, and all other rays are left off the diagram. The result is that students who use the novice, hybrid mental model may be reaffirmed in their belief that only some specific and special rays are important to the physics. This will often lead them to the correct answer while using incomplete reasoning.

[23] Hewson, P. W. And M. G. A'B. Hewson. "The role of conceptual conflict in conceptual change and the design of science instruction," Instr. Sci. **13**, 1-13 (1984); "The status of students' conceptions," In: *Research in Physics Learning: Theoretical Issues and Empirical Studies, Proceedings of an International Workshop*, Bremen, Germany, March 4-8, 1991, edited by R. Duit, F. Goldberg, and H. Niedderer (IPN, Kiel Germany, 1992) 59-73.

[24] Hewson, P. W. And M. G. Hennesy. "Making status explicit: A case study of conceptual change," In: *Research in Physics Learning: Theoretical Issues and Empirical Studies, Proceedings of an International Workshop*, Bremen, Germany, March 4-8, 1991, edited by R. Duit, F. Goldberg, and H. Niedderer (IPN, Kiel Germany, 1992) 176-187.

[25] Posner, G. J., K. A. Strike, P. W. Hewson, and W. A. Gertzog, "Accommodation of a scientific conception: Toward a theory of conceptual change," Sci. Educ. **66**:2, 211-227 (1982).

[26] Demastes, Sherry S., Ronald G. Good, and Patsye Peebles, "Patterns of Conceptual Change in Evolution." J. Res. Sci. Teach. **33**, 407-431 (1996).

[27] The different processes are usually referred to as "assimilation" and "accommodation." See reference 25 for a brief review and references therein for more detailed descriptions.

[28] Minstrell, J. "Explaining the 'at rest' condition of an object," Phys. Teach. **20** 10-14 (1982).