Chapter 3: Student Difficulties with Wave Physics

Introduction

From the Fall, 1995 (F95) to the present, I (together with other member of the Physics Education Research Group (PERG) at the University of Maryland (UMd)) carried out a series of investigations of student understanding of the physics and mathematical description of mechanical waves on a taut string or spring. (I will use notation "F95" or "S96" throughout the dissertation to describe Fall or Spring semesters and their years.) Student difficulties with the physics and the mathematical description of wave propagation and with the superposition of waves were investigated. From F96 onward, we also investigated student difficulties with sound waves and the propagation of waves through air. The research methods used in these investigations have been introduced in chapter 2.

We find that many students have profound and meaningful difficulties with fundamental ideas and concepts not just of wave physics but of the general ideas and approaches of physics which are often taken for granted in physics instruction yet which students must learn in our classes. For example, many students are unable to functionally describe the meaning of a disturbance to the equilibrium state of a system. Many are unable to adequately describe the concept of linear superposition, having great difficulty in considering many different points at once. The mathematics which describe wave propagation also cause trouble for students, and it seems that misinterpretations of the physics guide many students in their misinterpretation of the mathematics. We have also found evidence of the opposite, that students use misinterpretations of mathematics to guide their reasoning about the physics. These results are specific to the investigation of wave physics, but the manner in which we find students unable to build a coherent and functional understanding of the physics may cause problems for their study of physics in many other subjects.

Research setting

All data for this dissertation were collected from students in the Physics 262 class at the University of Maryland, College Park (UMd). Physics 262 is the second of a three semester, introductory, calculus-based physics course for engineers. Topics covered in the course include hydrostatics and hydrodynamics, oscillations, waves, heat and temperature, and electricity. Students are required to have taken physics 161 (or an equivalent course in Newtonian mechanics), and they are also required to be enrolled in a calculus II (or higher) course. Physics 262 has a required laboratory that meets once a week.

In the discussion section that accompanies the course, students participate in either a traditional TA-led recitation or a tutorial. In the TA-led recitation sections, a TA typically works through problems at the board. In some recitations, the TA leads a broader discussion in which some students might solve problems at the board, but the focus is still on a single person, and most of the students are not highly engaged in the discussion. Tutorials are a research-based instructional setting that replaces recitations, as has been described in chapter 2.

Students cover the topic of waves in a three or four week period (depending on the professor). Topics include wave propagation, superposition, intensity, power, wave harmonics, and usually the Doppler shift and other advanced topics. Since the more advanced ideas depend on an understanding of the basic ideas that students learn at the beginning of their study of waves, we have focused our research on the basic concepts and fundamental ideas of waves.

Chosen wave representations

To investigate student understanding, we often ask questions about the physics in an unfamiliar context that requires students to use what we hope is familiar physics. When studying waves, students often encounter only infinitely (or very) long waves which stretch (effectively) from negative to positive infinity.[1] As has been described in chapter 2, we have often chosen to investigate student understanding of wave physics by using wavepulses rather than wavetrains.[2] By a *wavepulse*, we mean a single localized disturbance that propagates along the string. By a *wavetrain*, we mean an infinitely (or very) long (e.g. sinusoidal) disturbance (see Figure 2-6).

One of the goals of PER is to see how students are able to carry over their understanding from one setting or topic to another. Our decision to investigate student understanding with wavepulses rather than wavetrains allowed us to see more clearly how students were thinking of the propagation of a wave. We could also see how students approached superposition. With a sinusoidal wavetrain, the mathematics to describe the wave is simpler than for a wavepulse, but it becomes difficult to visualize the motion of the medium due to the propagating disturbance. It also becomes difficult to interpret student responses (both sketches and descriptions) if a repeating pattern is used. Rather than simplifying the mathematics for students, we used wavepulses to find how students made sense of wave physics on a conceptual level.

Student Difficulties With Wave Propagation: Mechanical Waves

A wave is a propagating disturbance to a system. The medium of the system does not propagate with the wave and is not permanently displaced from equilibrium. Previous research (see chapter 2) has shown that students have difficulties separating the initial conditions of a system through which a wave propagates (i.e. the manner in which the wave is created) from the propagation of the wave itself. This point is often neglected when discussing wave physics, where it is possible to discuss relevant and important concepts contained in the wave equation without ever discussing the initial conditions of the system. We find that students are unable to distinguish between the manner in which a wave is created and the manner in which the wave propagates along a string.

Investigating student understanding

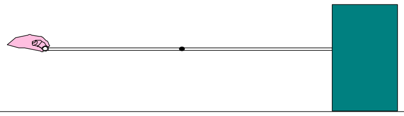
We chose to investigate how students view the relationship between how a wave is created and how the wave propagates through the system with a variety of instruments or probes. Interviews provided us with detailed descriptions of how a small number of students view the physics. With the understanding of possible student responses gained through an analysis of interviews, we can come to a better understanding of student written responses that we can give to much larger populations of students.[3]

The general question asked in all our interviews and written questions involved a taut elastic string held on one end by a hand and on the other end attached to a distant wall. A correct answer to the questions shown in Figure 3-1 and Figure 3-2 would indicate that the speed of a wave traveling along a taut string or spring depends only on the tension and mass density of the medium. The manner in which the disturbance is created does not affect the speed of the wave.

In the question shown in Figure 3-1, students were asked to describe what physical parameters could be changed to change the speed of the wave on a taut string. Even though the question asked for possible physical parameters that could affect the speed of the propagating wave (implying properties of the string on which the wave propagated, not the manner in which the wave was created), many students stated that the motion of the hand would play a role in the speed of the wave. The wording of the question may have lead more students to answer the answer the question correctly (tension and mass density are physical parameters), since some students might not consider the hand a physical parameter of the system.

Because the original wording of the question may have guided students away from their personal beliefs about the correct answer, we changed the wording of the question in later questions to the more open-ended wording shown in the free response (FR) question in Figure 3-2 (Version 1). The multiple-choice, multiple-response (MCMR) question (Version 2 of Figure 3-2) was developed in order to investigate the same student difficulties in a different fashion. In this type of question, students are asked to give all possible correct responses. They are offered a long list of possibly

Figure 3-1



A long string is attached to the wall as shown in the picture below. A red dot is painted along the string between the hand and the wall. A single pulse is created by the person holding the string and moving it up and down once. The string is firmly attached to the wall, and cannot move at that point.

When a pulse travels along a taut, elastic string, we can measure its velocity. What physical parameters could be changed to change the velocity of the pulse?

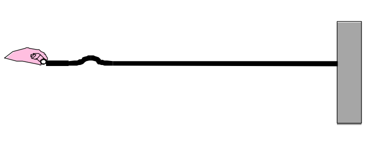
Wave propagation Question, Fall-1995, pre-instruction, N=182. Note the phrasing of the question, implying that only physical parameters can change the speed of the pulse.

correct responses. While reminding students of the correct answer, the offered responses could also remind students of many possible incorrect responses. Details of how the FR and MCMR question were asked at different times will be given when specific data are discussed.

Two sets of interviews dealt with student understanding of wave propagation concepts. In S96, nine students were asked an FR question nearly identical to the one shown in Figure 3-2 during an interview. In S97, 18 students first answered the FR question and then the MCMR question. The interviewer did not allow them to go back to change their answer on the FR question (an answer already captured on videotape). Two students interviewed during the same investigation answered only the MCMR question. The S97 interviews were part of a diagnostic test that will be described in greater detail in chapter 7.

Because of interview responses, we made two modifications to the original MCMR question. First, in the S96 interviews, we found that some students were giving an answer that we had not included in F95. They used the idea of the force needed to create the wave to explain changes in wave speed. They usually referred to the "force of the wave" when giving this explanation. This response will be described in more detail below. Second, we included the possibility of "none of the above" to give students the opportunity to describe their own model of wave propagation, even in the MCMR question. Responses *i*, *j*, and *k* were included in the MCMR question from S96 onward.

Figure 3-2



***Version 1: Free-Response (FR) format*:** A taut string is attached to a distant wall. A pulse moving on the string towards the wall reaches the wall in time t_0 (see diagram). How would you decrease the time it takes for the pulse to reach the wall? Explain your reasoning.

***Version 2: Multiple-Choice, Multiple-Response (MCMR) format*:**
A taut string is attached to a distant wall. A demonstrator moves her hand to create a pulse traveling toward the wall (see diagram). The demonstrator wants to produce a pulse that takes a <u>longer time</u> to reach the wall. Which of the actions *a-k* **taken by itself** will produce this result? More than one answer may be correct. If so, give them all. Explain your reasoning.

a. Move her hand more quickly (but still only up and down once by the same amount).
b. Move her hand more slowly (but still only up and down once by the same amount).
c. Move her hand a larger distance but up and down in the same amount of time.
d. Move her hand a smaller distance but up and down in the same amount of time.
e. Use a heavier string of the same length, under the same tension
f. Use a lighter string of the same length, under the same tension
g. Use a string of the same density, but decrease the tension.
h. Use a string of the same density, but increase the tension.
i. Put more force into the wave.
j. Put less force into the wave.
k. None of the above answers will cause the desired effect.

Free response (FR) and Multiple-choice, multiple-responses (MCMR) versions of the wave propagation question. Answers *e* and *g* are correct in the MCMR question, and we considered answers like *e* and *g* to be correct on the FR question.

Discussion of student difficulties

In this section, I will first describe student comments in interviews and then give a statistical overview of their responses to written questions.

After students have completed instruction on waves, many still use ideas of force and energy incorrectly when answering the free response (FR) question shown in Figure 3-2 Version 1. In both the S96 and S97 interviews, students had difficulties with the fundamental concepts of wave propagation. Some students used reasoning based on the force exerted by the hand to create the pulse. One student stated, “You flick [your hand] harder...you put a greater force in your hand, so it goes faster.” Other students state that creating a wave with a larger amplitude takes greater force and thus the wave will move faster. Some students state that shaking your hand harder (in interviews, this was usually accompanied by a quick jerk of the hand) will “put more force in the wave.” Another student used reasoning based on energy to describe the effect of a change in hand motion. He stated, “If we could make the initial pulse fast, if you flick [your hand], you flick it faster... It would put more energy in.” This student is failing to distinguish between the velocity of the hand, which is associated with the transverse velocity of the string, and the longitudinal velocity of the pulse along the string.

To many students, the shape of the wavepulse also determines its speed. One student stated, “Make it [the pulse] wider, so that it covers more area, which will make it go faster.” In follow-up comments, this student explained that it took more energy to create a larger pulse, and that the pulse would move faster because it had more energy. We have also found that some students state that smaller pulses will move faster. “Tinier, tighter hand movements” will allow the wave to slip more easily (thus, faster) through the medium.

Students rarely give only one kind of explanation in interviews. They can use both correct and incorrect reasoning to describe changes to wave propagation speed. One student described how to make a slower wave in the following way:

> *Well, I know that tension affects the wave speed. … [And] the amplitude would affect it {the student shows a hand motion with a larger displacement but same time length}. I think possibly, you see a slower pulse … if the force applied to the string is reduced … that is: the time through which the hand moves up and down [is reduced].*

Though the student starts with the correct response, he then describes a mixture of incorrect ideas: the “size” of the hand motion, the “force” applied to the string, and the speed of the hand motion. Of note is that there was a long pause between the correct response and the other, incorrect responses. During this time, the student was obviously thinking of the physics, so the interviewer remained quiet. Had the interviewer immediately asked a new question, the insight into the student’s understanding would not have been as deep and the student’s true understanding of the physics would not have been uncovered.

The initial conditions that determine the creation of the wave and its size play a large role in student explanations which use force and energy in their reasoning.

Though the properties of the physical system determine the wave speed and the initial conditions do not, many students believe the initial conditions play a role in propagation speed. Since a hand is used to create the wave, student explanations seem to make use of an active agent that creates the waves. This interpretation is consistent with previous findings by Maurines[4] and also with the Impetus model described in chapter 2.

Student use of multiple explanations was also observed on written questions. In F97, we asked both FR and MCMR questions on diagnostic tests at the beginning of the semester before all instruction and near the end of the semester after all instruction on waves had been completed. In the beginning of the semester, students first answered the FR question, turned it in, and were then handed the MCMR question. This ensured that they did not change their answers on the FR question as a result of seeing the list of MCMR options. During the semester, instruction consisted of lecture, textbook homework problems, and tutorials designed to address the issues discussed in this paper. (The instructional materials will be discussed in more detail in chapter 6.) The data from before and after instruction illustrate the difficulties students have even after working through specially designed research-based materials. After all instruction, students answered the FR and MCMR questions in successive weeks as a supplement to their weekly pretests given during lecture.

By comparing student responses on the FR and MCMR questions, we can probe the distribution of ideas used by students to understand the physics of wave speed. Table 3-1(a) shows how students answered both the FR and MCMR questions before instruction. Only those students who answered both FR and MCMR questions both before and after instruction are included (i.e. data are matched). Students' written explanations echo those given during interviews. By comparing student responses on the two question formats, we can see how consistently students think about wave speed.

At the beginning of the semester, very few students give <u>only</u> the correct answer, but most of them include it in the responses to one of the two questions. Almost all of the students answer that the hand motion will affect the wave speed.

Students predominantly use only one explanation when answering the FR question. The offered responses on the MCMR question appear to act as triggers that elicit additional explanations, especially from students who give the hand motion response on the FR question. Of the few students (9%) who answered the FR question using only correct reasoning, most answered the MCMR question consistently (78%). These students seem to have a robust understanding of the dependence of wave speed on medium properties. However, more than three-fourths of the students emphasize the incorrect hand motion response at the beginning of the semester (77% of the students give the hand motion response on the FR question).

Table 3-1(b) shows student responses at the end of the semester (after modified instruction, described in more detail in chapter 6). Student performance is somewhat improved, with more students giving completely correct explanations. Nearly all students (98%) recognize the correct answer on the MCMR question, but a majority of the class (58%) still believes that changes in hand motion play a role.

Table 3-1

(a)

		Student responses on free response question			
	Speed changes due to change in:	Only tension and density	both the medium and hand motion	the motion of the hand	other
Student responses On MCMR question	only tension and density	7%	1%	2%	1%
	both the medium and hand motion	1%	2%	60%	10%
	the motion of the hand	1%	1%	11%	3%

(a) Comparison of student pre-instruction responses on FR and MCMR wave propagation questions, Fall-1997 (matched data, N=92). Students answered questions before all instruction.

(b)

		Student responses on free response question			
	Student Response:	Only tension and density	both the medium and hand motion	the motion of the hand	Other
Student responses On MCMR question	Only tension and density	40%	2%	2%	2%
	Both the medium and hand motion	8%	17%	20%	2%
	the motion of the hand	2%	1%	2%	0%

(b) Comparison of student post-instruction responses on FR and MCMR wave propagation questions, Fall-1997 (matched data, N=92). Students answered questions after all instruction on waves.

In both the pre and post instruction tables, the most common off-diagonal elements of the tables show that students who answer the FR question using only hand motion explanations are triggered into additionally giving correct medium change responses on the MCMR question. Apparently, they recognize the correct answer, but do not recall it on their own in an FR question. Because fewer students are triggered in the other direction (from correct medium change explanations to additionally giving the hand motion response), we believe that the quality of understanding of those students who give the correct FR response is higher than those who are triggered to give multiple explanations. Nevertheless, it is noteworthy that so many of the students answer incorrectly even after explicit instruction on the topic. The issue of instruction will be discussed in more detail in chapter 6.

In summary, we find that students do not make a distinction between the initial conditions and the medium properties of the system. We see that most students give correct answers to describe changes to wave motion when offered the correct response, even before instruction, but they often do not think consistently about the physics, even after instruction. In a later part of the dissertation, I will discuss how individual students are able to give more than one response to describe a single physical situation, and how students use more than one model to think of waves.

Student Understanding of With Wave Propagation: Sound Waves

We have also investigated student understanding of the fundamental issues underlying a consistent physical picture of the nature of sound. Our findings show that the difficulties described in the previous section appear here as well. We find that students are unable to separate the medium from the wave, possibly because they are unable to interpret how they visualize the system in which the wave is traveling.

Investigating student understanding

To investigate how students distinguish between the motion of the wave and the medium, we posed two different types of questions about sound waves (see Figure 3-3). We asked students to describe the motion of a dust particle sitting motionlessly in front of a previously silent loudspeaker after the speaker had been turned on (Figure 3-3(a)). In addition, we asked students to describe the motion of a candle flame placed in front of a loudspeaker (Figure 3-3(b)).

The physics of these two situations merits discussion. The dust particle, we told the students in interviews, is floating motionlessly in a room with no wind (i.e. no outside air currents). This is plausible, when considering that buoyancy can support a dust particle of the right density at the desired height. The lack of air currents is not plausible when considering the candle flame because the heat from the candle causes convection currents. These currents only occur in the near vicinity of the candle, though, and add little to the effective size (i.e. width) of the candle. For both systems and at the appropriate size and time scale, we can assume that the medium through which the sound waves travel is motionless except for the motion from equilibrium caused by the sound waves themselves.

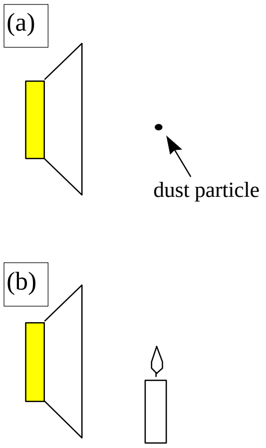
In both questions, we asked about audible frequency sound waves, between 10 and roughly 5,000 Hz. Assuming a speed of sound in air of roughly 340 m/s, this gives a range of wavelengths between 7 cm and 34 m. All of these wavelengths are much greater than the size of either the dust particle or the candle flame (roughly 1/2 to 1 cm wide). The shortest wavelengths occur at a frequency that is already outside of the common frequencies heard on a daily basis in speech or in music. (The highest of these are usually around 2000 Hz, giving a wavelength of 17 cm.) Based on our choice of dust particle size and candle flame size, we can treat them as point particles which move in response to the motion of the medium in which they are embedded.

We expected students to point out that the dust particle and the candle flame would oscillate longitudinally from side to side due to the motion of the air. We expected that the detailed physics of the differences between the dust particle (or candle flame) and the medium of air discussed in the previous paragraph were beyond the level of all the students probed . None ever raised these issues. This is consistent with our use of interviews to help determine the state space of possible responses

Figure 3-3

Consider a dust particle sitting motionlessly in front of a loudspeaker. Also, consider a candle flame where the dust particle had been.

Question: Describe the motion of the dust particle (or candle flame) after the loudspeaker is turned on and plays a note at a constant pitch and volume. How would the motion change if the frequency or volume of the sound were changed?



Two different situations in which the sound wave question was asked, (a) the dust particle sound wave question, (b) the candle flame sound wave question. In interviews, students were not given a diagram, but had a loudspeaker and a candle and were asked to imagine the dust particle or the candle flame. In pretests and examination questions, students had a variety of diagrams, all equivalent to the ones shown.

students might give in a situation. Though we were prepared for the discussion, the students had difficulties with different fundamental issues.

Two sets of individual student interviews gave us insight into how students made sense of the physics. In the first (F96), 6 students answered questions related to the motion of both the dust particle and the candle flame. These students had completed lecture instruction on sound waves and most were above average (getting either an A or B in the course) according to their descriptions of their grades.[5] They were asked to describe the motion of the object, if any, once the loudspeaker was turned on. They were also asked how that motion would change if the frequency and the volume of the loudspeaker were changed (and the dust particle or candle flame began its motion in the same location as the original object). In the second set of interviews (S97), twenty students who had completed either traditional or tutorial instruction in waves answered the dust particle question. In these interviews, a multiple-choice, multiple-response (MCMR) format version of the dust particle question was given. Because of the interview setting, it was possible to probe their responses to the question in this format to see how they arrived at their answers and how they were using the offered responses to choose their own beliefs about the movement of the dust particle. Some students seemed to focus on only the first instant of motion of the dust particle away from the speaker and did not state that there was motion due to the rarefaction of air until asked to extend their first response in time. The question responses were subsequently rephrased to account for this possibility and to suggest to students that they consider more than just one instant in time. The final version of the MCMR question is shown in Figure 3-4.

Once we had used interviews to describe student difficulties with sound waves in detail, we administered a variety of written questions to gain an understanding of how common these difficulties were in the classroom during the course of the semester. We asked the dust particle questions in three different situations: before any instruction on waves, after traditional instruction on waves, and after traditional and tutorial instruction. In one semester (F97), we asked students both the FR and MCMR versions of the dust particle question after they had completed instruction on waves. We asked and collected the FR question first to ensure that students would not change their response based on the offered MCMR answers.

Discussion of student difficulties

We found that students' difficulties did not change during the course of the semester, but the frequency with which they occurred did change depending primarily on the type of instruction that students had on waves. The state space of responses that we had developed using the interviews was therefore productive in describing student difficulties at all times of instruction.

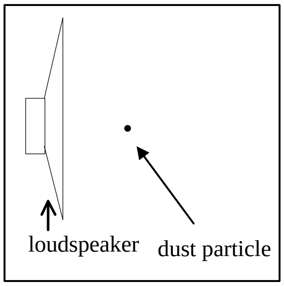
In the interviews carried out in F96, we found that most students had great difficulty separating the propagation of the sound wave from the motion of the medium through which it travels. One student's responses were representative of the reasoning used by 4 of the 6 students in the interviews.

"Alex" (names used are aliases chosen by the interviewed students), described how the dust particle would be pushed away by the sound wave. In the following quotes, the interviewer is referred to with "I" and Alex with "A."

> I: *The loudspeaker is turned on, and it plays a note at a constant pitch. Could you describe the behavior of the particle after the speaker is turned on?*

Figure 3-4

A dust particle is located in front of a silent loudspeaker (see figure). The loudspeaker is turned on and plays a note at a constant (low) pitch. **Which choice or combination of the choices** *a*-*f* (listed below) can describe the motion of the dust particle after the loudspeaker is turned on? **Circle the correct letter or letters.** Explain.



Possible responses for question 2:

a) The dust particle will move up and down.
b) The dust particle will be pushed away from the speaker.
c) The dust particle will move side to side.
d) The dust particle will not move at all.
e) The dust particle will move in a circular path.
f) None of these answers is correct.

MCMR format sound wave question, F97, N=92 students (matched) answered this pre and post-instruction on waves.

> A: *It should move away because the sound vibration, the sound wave is going away from the speaker, especially if constant pitch means you have one wave going … It’s going to move away from the center.*

Later, when asked the same question in a slightly different fashion, Alex stated:

> A: *It would move away from the speaker, pushed by the wave, pushed by the sound wave … I mean, sound waves spread through the air, which means the air is actually moving, so the dust particle should be moving with that air which is spreading away from the speaker.*
>
> I: *Okay, so the air moves away* --
>
> A: *It should carry the dust particle with it.*
>
> I: … *How does [the air] move to carry the dust particle with it?*
>
> A: *Should push it, I mean, how else is it going to move it? [Alex sketches a typical sine curve.] If you look at it, if the particle is here, and this first compression part of the wave hits it, it should move it through, and carry [the dust particle] with it.*

Here, Alex was describing the peak of the sine wave exerting a force on the dust particle, only in the direction of propagation.

Alex had a clear and complete description of the motion of the dust particle due to the sound wave. He believed that a sound wave consisted of air moving away from its source, and that the dust particle would therefore move with the air, away from the speaker. The sound wave provided the force which acted on the particle to make it move away from the speaker. Alex did not use the idea of rarefaction during the interview.

To see whether Alex used this description even when the physical situation changed, he was asked the following question:

> I: *We have the same loudspeaker, and we create the same situation as previously. We have the loudspeaker turned off, and you place a new piece of dust, exactly like the previous one, in the same location as before. Now you turn the speaker on, but rather than having the original pitch, the frequency of the note that is produced by the speaker has been doubled ... How would this change the answer that you’ve given?*
>
> A: *That would just change the rate at which the particle is moving. … The wave speed should be, it should double, too. … Yeah, speed should increase.*
>
> I: *How did you come to that answer?*

> A: *I was thinking that the frequency of the wave, a normal wave, shows us how many cycles per some period of time we have. … You can have twice as many cycles here in the same period of time. …*
>
> I: *And what effect does it have to go through one cycle versus to go through two cycles?*
>
> A: *If it goes through one cycle of the compression wave like this, then the first wave should hit it here [points to the peak of the sine curve that he had previously sketched]. And … the second wave which has frequency which is twice as big should hit it twice by then, which should make it go faster.*

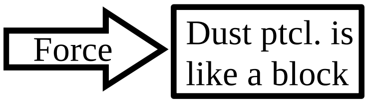
Due to a hand motion that he made repeatedly when referring to the “hit” on the particle, Alex was asked the following question:

> I: *So each compression wave has the effect of kicking the particle forward?*
>
> A: *Yeah.*
>
> I: *So when you’ve been kicked twice, you’re moving twice as fast?*
>
> A: *Basically, yeah. Right, because the force … [referring to a sketch he drew, like the one in Figure 3-5] If you have a box, and you apply a force once, the acceleration is, force equals mass times acceleration, you can find the acceleration. Then, if you apply the same force a second time to the same object, you give it more, more, well, it just moves faster.*

I have given these lengthy interview excerpts to show the robustness with which Alex could describe his conceptual understanding of sound waves and to show the general difficulties Alex had with basic and essential physics concepts.

Alex’s misinterpretation of frequency illustrates how students can use language correctly but misinterpret its meaning. He stated, “the frequency of the wave, a normal wave, shows us how many cycles per some period of time we have,” but he was unable to use his definition when describing the physical behavior of the system. At another point in the interview, he indicated that he thought the wavelength of the sound wave would stay constant when the frequency changed. At no point during the interview did he state that the speed of the sound wave depended on the medium properties of the air through which it traveled. In an equation like $v = fl$, he was free to choose one of the

Figure 3-5



Alex’s sketch of the sound wave exerting a force on the dust particle. Alex described the wave exerting a force on the dust particle (and later candle flame) only in the direction of wave propagation.

variables to remain constant. He could not clearly explain why he believed the wavelength was constant.

Another point of interest is his confusion between acceleration and velocity (possibly the confusion between acceleration and impulse). The description that the sound wave exerts a force only in the direction of wave propagation shows that Alex thinks of the leading edge of the wave pushing everything in front of it away from the sound source, much like a surfer riding on an ocean wave. To explain the surfer analogy, he described the motion of a ring on a string on which a pulse is propagating. The effect of a pulse on a small ring placed on the string was to push it along. (Alex gave a partially correct answer for a ring that is on a string; the wavepulse will make the ring move in some longitudinal fashion that depends on the angle of the string as the wavepulse passes by.) He used the term "impulse" to describe the wavepulse <u>and</u> the effect of the wavepulse on its surroundings.

> A: *This impulse will hit the ring here, and … should go and make it move forward, the same way it should be with a dust particle in the air.*

When I asked him the effect of changing the volume of the sound produced, he stated the following:

> A: *I guess I'm not thinking physics too much. … [I'm thinking of a] stereo system at home, if you turn it up, you can feel the vibration from farther away from the speaker, so basically [the dust particle] should move, once again, it should move faster.*
>
> I: *What effect did changing the volume have on the compression wave?*
>
> A: *Increased the amplitude…*
>
> I: *And that has the effect of the compression wave moving faster?*
>
> A: *Not quite, it just hits the particle with more force. … If you kick the thing, instead of kicking it faster, you're just kicking it harder. It's going to move faster.*

Again, Alex described the effect of the wave exerting a force only in the direction of propagation to make the dust particle move forward. Of the 6 students who participated in the interviews, four gave similar descriptions of the effects of the sound wave on the dust particle.[6]

Of the other two students, one student gave responses which were inconsistent, stating the correct answer (horizontal oscillation) while also stating that the dust particle would not move. Even with continued questioning, the student was unable to provide a clear response, showing that a profound confusion lay behind the student's correct responses. It is possible that this student would perform very well on examinations where the student is aware of the correct answers that the instructor is seeking, but still not have an actual understanding of the physics of sound waves.

The last student interpreted the common sinusoidal graph used to describe sound waves (either pressure or displacement from equilibrium as a function of time or position) as a picture rather than a graph and used this misinterpretation to guide his reasoning. He described transverse motion by the dust particle (and no motion by the candle flame, since it was unable to move up and down due to its attachment to the wick). This student misinterpreted the common sinusoidal graph of sound waves (where the vertical axis describes *horizontal* displacement from equilibrium as a function of position or of time), and used this misinterpretation to guide his understanding of the motion of the medium. He described that the longitudinal compression of the sound wave would squeeze the dust particle to push it up or suck it back down (due to the "vacuum" caused by a sort of rarefaction between longitudinal waves). The longitudinal wave would cause transverse motion in the dust particle. The detailed physical explanation this student gave is indicative of how seemingly simple misunderstandings (reading a graph as a picture) can have a profound effect on how students come to make sense of the physics they learn.

When asking the dust particle question of many students, we have found that lecture instruction had little effect on student understanding of the relationship between the motion of sound waves and the motion of the medium through which they travel. Table 3-2 shows (unmatched) student responses from the beginning of S96 and the end of F95. Students answered a slightly different version of the question in which the loudspeaker was enclosed in walls to form a tube. A non-trivial number of students (roughly 10% in both cases, listed within the "other oscillation" category) sketched standing wave patterns (e.g. sinusoidal standing waves with the correct nodes and antinodes at the end of the tube) to describe the motion of the dust particle. The tube walls were removed in later questions to remove this source of student confusion, but the result is an important one. Students seem to pick the familiar details or surface features of a problem to guide their reasoning in their responses. The tubes triggered a response based on common diagrams with which they were familiar, but this response showed the difficulties that students have in understanding the material.

Table 3-3 shows student explanations from F97 to the dust particle question before instruction, after traditional instruction, and after modified instruction (described in more detail in chapter 6). We see that very few of the students enter our courses with a proper understanding of the nature of sound wave propagation. Before instruction, half the students state that the sound wave pushes the dust particle away from the speaker. Some, like Alex, describe the dust particle moving in a straight-line path. Others describe the dust particle moving along a sinusoidal path away from the speaker. The latter students seem to misinterpret the sinusoidal graph of displacement

Table 3-2

Time during semester: MM used:	Before all instruction S96 (%)	Post lecture F95 (%)
CM (longitudinal oscillation)	14	24
Other oscillation	17	22
PM (pushed away linearly or sinusoidally)	45	40
Other and blank	24	14

Comparison of student responses describing the motion of a dust particle due to a loudspeaker. Data are from F95 post instruction and S96 pre-instruction and are not matched (S96, N=104. F95, N=96).

Table 3-3

Time during semester: Explanation:	Before all instruction (%)	Post lecture (%)	Post lecture, post tutorial (%)
Longitudinal oscillation	9	26	45
Other oscillation	23	22	18
Pushed away linearly or sinusoidally	50	39	11
Other	7	12	6
Blank	11	2	21

Student performance on sound wave questions before, after traditional lecture, and after additional modified tutorial instruction. Data are matched (N=137 students). The large number of blank responses in the post-all instruction category is due to the number of students who did not complete the pretest on which the question was asked.

from equilibrium as a picture of the path of the particle. After specially designed instruction, student performance has improved greatly, but lingering difficulties remained. The curriculum materials and an analysis of their effectiveness will be described in chapter 6.

With sound as well as with mechanical waves, students have great difficulty distinguishing between the medium and the propagating disturbance to the medium. The difficulties we have found include:

- the use of surface features of the problem and misinterpretations of graphs to help (mis)guide reasoning about sound wave propagation, and
- the use of descriptions of force and pushing to describe the movement of the medium only in the direction of wave propagation (like a surfer riding a wave).

Student understanding of both mechanical and sound waves indicates that their functional understanding of the physics is not as robust as we would like. Their focus on surface features of the problem indicates that they are unsure of their understanding of the material and will try to make sense of the situation using inappropriate clues in the problem. Their focus on forces that are originally exerted on the system to create the wave and make it move forward indicates that they are not thinking correctly about the relationship between the creation and the propagation of waves.

Student Understanding of the Mathematics of Waves

One of the fundamental topics of wave physics when it is first introduced is the mathematical description of propagating waves. Students are confronted with functions of two variables, often for the first time. The difficulties they have with the mathematics of waves (hereafter referred to as wave-math) can have lasting effects on their understanding of such advanced topics as the wave equation (often not covered in the introductory sequence), the propagation of electromagnetic radiation, and the mathematical description of quantum mechanics. The difficulties that we observe should therefore indicate what sort of problems students might have with mathematics at later stages in their careers. We find that many students do not have a good understanding of how an equation can be used to describe a propagating wavepulse, and we find that some students have serious difficulties interpreting the meaning of the equation which describes the wavepulse at a given instant in time.

Investigating student understanding

We investigated students using both interviews and written questions on the following issues:

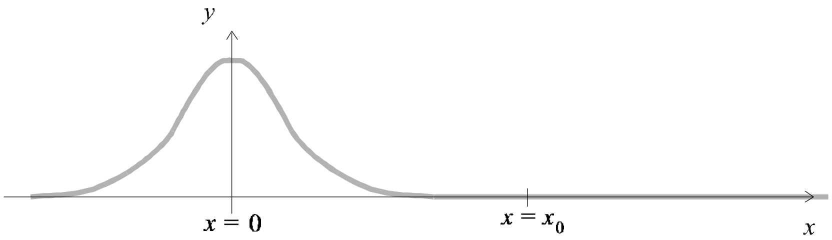
- the mathematical transformation that describes translation of a disturbance through a system, and
- the physical interpretation of the mathematics that describe propagating waves.

The question shown in Figure 3-6 presents students with an unfamiliar setting in which to describe wave motion. Students most commonly encounter sinusoidal shapes when discussing waves due to the ease of the mathematical interpretation and the usefulness of the sinusoidal description in physics. By presenting students with a Gaussian pulseshape, we are able to probe their understanding of the mathematics of wave propagation while ensuring that they are not responding by using partially recalled responses from previous questions. Part A of the question asked students to sketch the shape of a (Gaussian) wavepulse traveling to the right that had propagated a distance x_0 along a taut string. Part B asked students to write an equation to describe the shape of the string at all points once the wavepulse had traveled a distance x_0 from the origin.

Figure 3-6

Consider a pulse propagating along a long taut string in the +*x*-direction. The diagram below shows the shape of the pulse at $t = 0$ sec. Suppose the displacement of the string at this time at various values of *x* is given by

$$y(x) = Ae^{-\left(x/b\right)^2}$$



A. On the diagram above, sketch the shape of the string after it has traveled a distance x_0, where x_0 is shown in the figure. Explain why you sketched the shape as you did.

B. For the instant of time that you have sketched, find the displacement of the string as a function of *x*. Explain how you determined your answer.

Wave-math question answered by N=57 students in S96. The question has since been used in other semesters and in interviews with individual students.

We considered a response to part A to be correct when students showed the pulse displaced an amount x_0 and the amplitude essentially unchanged, as shown in Figure 3-7(a). We considered any answer to part B that replaced x with x - x_0 to be correct.

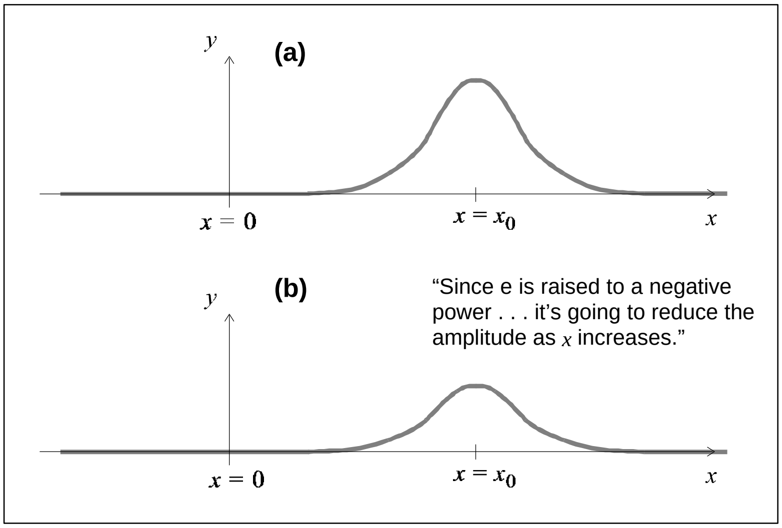
Three different student populations participated in individual interviews. In the first, students (N=9) had not seen the wave-math question before. In the second, four students (in a different semester) had seen the wave-math question in a post-lecture pretest (i.e. pre-tutorial quiz) they had taken within the previous 48 hours. We used these interviews to validate the students' pretest responses. In the third student population, ten students answered the wave-math question in a diagnostic test two months after they had traditional and tutorial instruction on waves.

In addition to the interviews, which provided the basis for our understanding of student difficulties, we asked the wave-math question in a number of written pretests. These pretests were given after students had traditional instruction but before they had tutorial instruction on the mathematics of waves. As in the other areas that have been investigated, the nature of difficulties did not change according to where the students were in their instruction, only the frequency with which a group of students had specific difficulties changed.

Discussion of student difficulties

Students often used misinterpretations of the mathematics to guide their reasoning in physics or they used misinterpretations of the physics to guide their understanding of the mathematics. Though most students describe the physical shape of the propagated wave correctly, those who do not have a consistent incorrect answer. This provides an opportunity for us to gain insight into the ways in which students

Figure 3-7



"Since e is raised to a negative power . . . it's going to reduce the amplitude as x increases."

Correct and most common incorrect response to the wave-math problem in Figure 3-6. (a) A correct sketch of the shape of the pulse at a later time, showing the amplitude unchanged, (b) An apparently correct sketch of the shape of the pulse showing the amplitude decreased - but typically accompanied by incorrect reasoning.

arrive at an incorrect understanding of physics.

In S96, the correct answer was given by 44% of the students who were interviewed and 56% of the students who took the pretest. Most of the rest of the students (56% of the interview-students and 35% of the pretest-students) drew a pulse displaced an amount x_0, but with a decreased amplitude, as shown in Figure 3-7(a). On the surface this appears to be a reasonable response in that it is consistent with what would actually happen as a result of the physical phenomena (not mentioned in the problem) of friction with the imbedding medium and internal dissipation. However the explanations given by students suggest that they are not adding to the physics of the problem but are misinterpreting the mathematics. All of the interview students and many of the pretest students cited the equation describing the shape of the string at $t = 0$ as the reason for the decrease in the amplitude.[7] As one interviewed student[8] said, "Since *e* is raised to a negative power . . . it's going to reduce the amplitude as *x* increases."

But the exponential given in this problem represents a decrease in *y* in space (at $t = 0$) and not time. These students are failing to recognize that *x* corresponds to a variable which maps a second dimension of the problem, not the location of the peak of the pulse. Also, these students are interpreting the variable *y* as the peak amplitude of the wavepulse, not the displacement of the string at many locations of *x* at different times, *t*.

One student was explicitly misled by the mathematics of the Gaussian function, though he originally stated the correct response.

> *Okay. Umm … Let's see. "Sketch the shape of the spring after the pulse has traveled* (Mumbling as he rereads the problem) *… Okay. Over a long, taut spring, the friction or the loss of energy should not be significant; so the wave should be pretty much the exact same height, distance, -- everything. So, it should be about the same wave. If I could draw it the same. So, it's got the same height, just a different X value.*
>
> *No, wait. Okay, "… the displacement of* (More mumbling, quick reading) *… is given by" – B, I guess, is a constant, so – It doesn't say that Y varies with time, but it does say it varies with X. So – that was my first intuition – but then, looking at the function of Y … Let's see, that – it's actually going to be – I guess it'll be a lot smaller than the wave I drew because the first time – X is zero, which means A must be equal to whatever that value is, because E raised to the zero's going to be 1. So, that's what A is equal to. And then as X increases, this value, E raised to the negative, is going to get bigger as we go up. So, kind of depending on what V is… Okay. So, if X keeps on getting bigger, E raised to the negative of that is going to keep on getting smaller. So the – So the actual function's going to be a lot smaller. So, it should be about the same length, just a lot shorter in length.*

This student describes the physics (including relevant approximations and idealizations) correctly, but then revises his physical understanding to fit his misinterpretation of the mathematics. Here we see a clear example of the way (first discussed when comparing student response to the FR and MCMR question) that students can have two conflicting descriptions of the same situation. In this case, the mathematics triggers in the student a form of reasoning that contradicts the simple physical description the student originally used.

Part B of this problem asked about the mathematical form of the string at a later time. We considered any answer that replaced x with $x - x_0$ to be correct. However, none of the S96 interview students and fewer than 10% of the pretest students answered this way (see Table 3-4).

The most common incorrect response was to simply substitute x_0 for x in the given equation. These students write constant functions that have no x-dependence, $y(x) = Ae^{-\left(x_0/b\right)^2}$ or $y(x_0) = Ae^{-\left(x_0/b\right)^2}$. This response was given by 67% of the interview students and 44% of the pretest students. All students drew a string with different values of y at different values of x, yet many of them wrote an equation for that shape with no x dependence. There were other students who wrote a sinusoidal dependence for y, again in conflict with what they drew for the shape of the string. Even after instruction on waves, many students seemed to be answering the mathematical part of this problem independently of the way they were answering the physical part. In another class (where students had participated in traditional instruction on this subject), a modified version of this question was asked on a post-instruction midterm examination. In this case, 45% of the students gave a sinusoidal answer to mathematically describe the shape of a pulse.

In S97, the pretest was asked of a another class (which had the same modified instruction). The percentages of correct and incorrect responses were nearly exactly the same as for S96, as shown in Table 3-4. More detailed analysis of student responses showed that 2/3 of those students who drew a smaller amplitude displaced wave explicitly mentioned the exponential in the equation when explaining how they

Table 3-4

	Example(s)	**% of interview respondents (N=9)**	**% of pretest respondents (N=57)**
correct response	$y(x) = Ae^{-\left(\frac{x-x_0}{b}\right)^2}$	0%	7%
Constant function with no dependence	$y(x) = Ae^{-\left(x_0/b\right)^2}$ $y(x_0) = Ae^{-\left(x_0/b\right)^2}$	67%	44%
Sinusoidal	$y(x) = \sin(kx - wt)$	22%	2%
Other	$x = b\ln\left(\frac{y}{A}\right)$; $\frac{dy}{dx} = -\frac{2x}{a^2}Ae^{-(x/a)^2}$	11%	47%

Student use of functions to describe a propagating Gaussian pulseshape. Students were asked to write an equation to describe the shape of the string once the pulse had moved a distance x_0 from the origin.

arrived at their answer (i.e. 25% of the class used this reasoning). The other 1/3 of the students describing a smaller amplitude wavepulse gave many different reasons, the most common being that the variable “b” described a damping constant, so the amplitude must be smaller. These students are using a surface feature of the equation (the variable “b,” used in their textbook to describe the damping constant in air resistance) to interpret the physics. Again, we see that students have difficulties interpreting the mathematics they are presented and use a variety of interpretations of the physics to guide their reasoning.

The difficulties described in this section include students failing

- to recognize the relationship between the physical situation and the associated equation,
- to understand the meaning of a function, and
- to treat a coordinate axis as a mapping of a dimension.

The interpretations that students give the mathematics focus only on the point of maximum displacement. The misinterpretation of a wavepulse as a single point of displacement rather than an extended area of displacement implies that students are thinking of waves differently from how physicists understand waves.

Student Understanding of Wave Superposition

For multiple mechanical waves traveling through a one-dimensional system, the concept of linear superposition describes the summation of the displacement due to each wave. As described in chapter 2, superposition occurs at each location in space (i.e. the sum of displacement occurs locally and due to local influences), but every location in the system must be considered (i.e. one must do the addition everywhere, or globally). The distinction between local and global phenomena is subtle in this situation, though not new to students who have used free body diagrams of extended bodies in their previous physics courses. Also, the topic is of great importance for later studies in physics. We find that students have difficulty understanding wave superposition to occur on a point-by-point basis, and some students have a “collision” model of wave superposition related more to particle mechanics than to wave physics. In wave “collisions,” waves are treated much like objects that bounce off each other, such as carts or gliders on air tracks.

Investigating student understanding

In our investigations, we focused on three different elements of the physics of wave superposition where students might have difficulties. We investigated student understanding of superposition for

- the instant when the peaks of waves overlapped,
- the instant when the wave overlapped and the peaks of the waves did not, and
- an instant some time later, when the waves were no longer overlapping at all and had passed through each other.

Our questions used wavepulses rather than wavetrains so that we could clearly separate what students thought was happening.

We chose these three topics in superposition for three reasons. First, students often are asked about wave superposition in instances where sinusoidal waves overlap either perfectly constructively or perfectly destructively. By asking for a sketch when peaks are not overlapping, we are able to investigate whether students add displacements due to each wavepulse at all points along the string or only at the peaks. Second, by asking for the sketch when the peaks overlap exactly, we see how students sketch the shape of the entire pulse, and if they change the width of the pulse in addition to its amplitude. (Student comments in office hours led to this question.) Finally, students rarely consider what happens to superposed waves after they no longer have an effect on each other; wavetrains in problem sets never end, so the issue never arises. By asking for a sketch long after the peaks have passed through each other, we can investigate what ideas the students have about possible permanent effects of the wavepulses on each other. We have used the same three time periods in our questions, time limitations permitting.

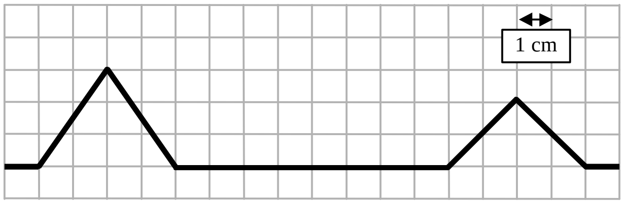
A variety of questions was used to investigate student understanding of superposition. Figure 3-8 shows two wavepulses on the same side of a string propagating toward each other. Figure 3-10 shows two wavepulses on opposite sides of a string propagating toward each other. In both cases, students were asked to sketch the shape of the string at the three times described above. Correct responses to the questions shown in Figure 3-8 and Figure 3-10 are shown in Figure 3-9 and Figure 3-11, respectively.

In each of the questions, a correct response would show point-by-point addition of the displacement due to each wavepulse at every point along the string. Furthermore, wavepulses that had superposed and then separated would look exactly as they did before interacting, without any sign of a permanent effect on each other. One of the reasons for the chosen representation of wavepulses was to facilitate the drawing of these sketches and to allow easier interpretation of student sketches.

Two sets of interviews on the topic of superposition were carried out. In S96, in a tutorial class, four volunteers answered the pretest question shown in Figure 3-12 in an interview that came after their lecture instruction on the material but before any tutorial instruction. This allowed us to validate the written responses we saw on pretests by comparing them with the more detailed verbal responses students given in interviews.

In diagnostic test interviews carried out in S97 with twenty students who had completed either traditional or tutorial instruction on waves, we asked a series of questions similar to the ones shown in Figure 3-8 and Figure 3-10. These were given in multiple-choice format, and students had a long list of possible responses from which to choose. Each response could be a possible correct answer for more than one question, and students were aware that they could use the same response more than once when answering up to five different questions. (This is a variation of a multiple-choice, multiple-response question, as described in the wave propagation section above.) Because these questions were asked during an interview, it was possible to

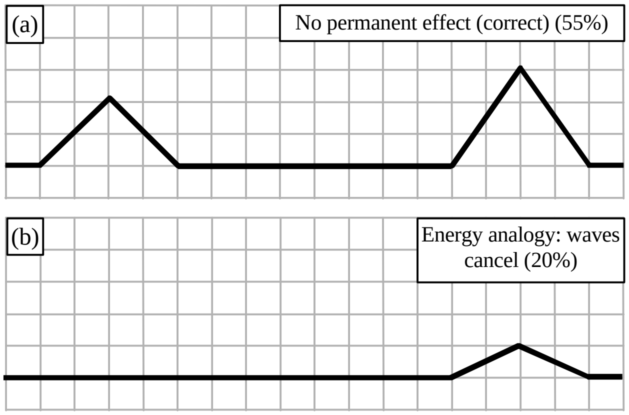
Figure 3-8



Two wavepulses are traveling toward each other at a speed of 10 cm/s on a long spring, as shown in the figure above. Sketch the shape of the spring at time t = 0.12 sec. Explain how you arrived at your answer.

Wave superposition question from a diagnostic test given, Fall-1995 semester, N = 182 students. Students had no instruction on waves when they took this diagnostic.

Figure 3-9



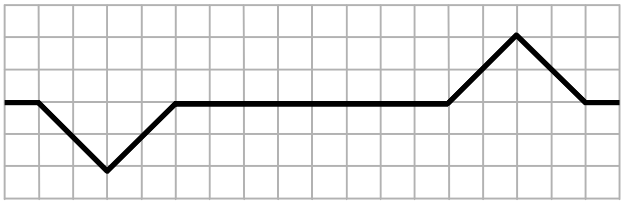
Common responses to diagnostic question from Fall-1995. (a) Correct response, (b) Most common incorrect response.

follow up on student responses and gain insight into the reasoning they used to explain their understanding of physics.

In the S96 semester, after we had developed a tutorial to address student difficulties with superposition, we asked a pretest question shown in Figure 3-12. This pretest followed lecture instruction on the basic concepts of waves (including superposition) but preceded the tutorial on wave superposition. Rather than using symmetric wavepulses of different amplitudes, we chose to use asymmetric wavepulses with the same amplitude. Though we had found interesting student ideas about the permanent effects of wavepulses meeting, we wanted to investigate in more detail how students did or did not use superposition when only parts of the pulses (but not the peaks) overlapped. The correct responses and the most common incorrect responses are shown in Figure 3-13.

During the F97 semester, we modified the pretest question from S96 and asked for an additional sketch of the string when the peaks overlapped but the bases of the pulses no longer perfectly overlapped. This question was asked in pre-instruction and post-instruction diagnostic tests.

Figure 3-10



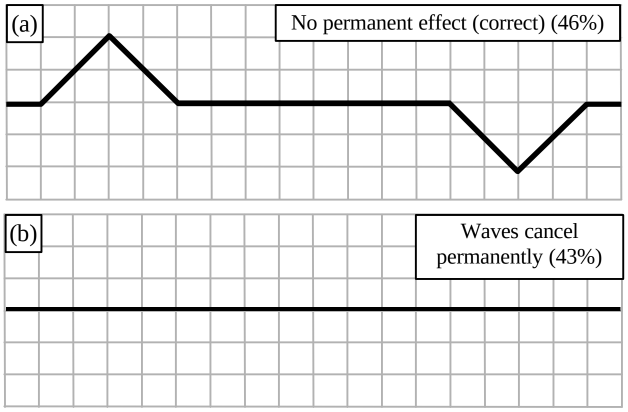
Two wavepulses are traveling toward each other on a long, taut string.

a. Sketch the shape of the string at the moment of maximum overlap. Explain.

b. Sketch the shape of the string a long time after the moment of maximum overlap. Explain.

Wave superposition question from a diagnostic test given, Fall-1995 semester, N = 182 students. Students had no instruction on waves when they took this diagnostic.

Figure 3-11



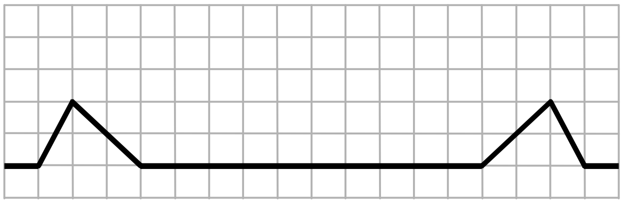
Common responses to part *b* of the diagnostic question in Figure 3-10, Fall-1995. (a) Correct response, (b) Most common incorrect response.. Note that response (b) is correct for part *a* of the question in Figure 3-10.

Discussion of student difficulties

Our results show that students have difficulties with each of the three areas of wave superposition investigated in our questions. As in the other areas of wave physics, a few student difficulties dominated the responses. These difficulties did not change during the course of instruction, but the frequency of their occurrence did. I will first discuss student descriptions of permanent effects of wavepulses on one another. Then I will describe the superposition of waves whose peaks do not overlap, and finally I will describe the superposition of waves whose peaks do overlap.

In the F95 pre-instruction diagnostic test, 182 students answered the question shown in Figure 3-8. A correct response to the question, given by 55% of the students (see Figure 3-9(a)), shows that the wavepulses pass through each other with no

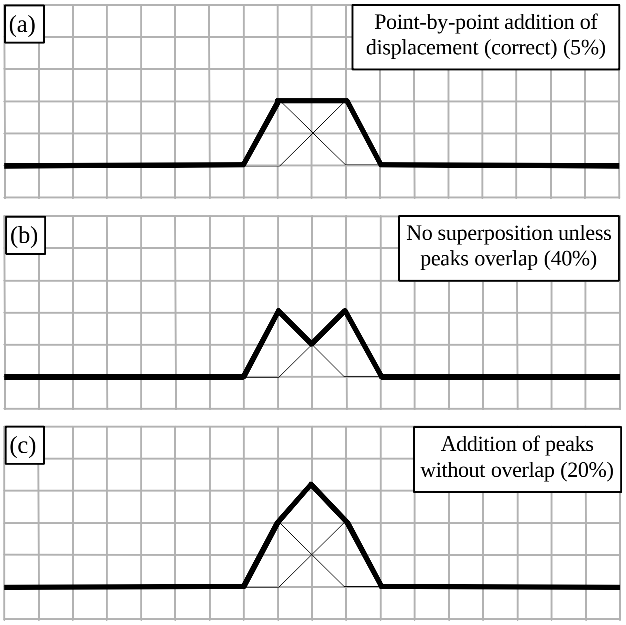
Figure 3-12



Two wavepulses are traveling toward each other at a speed of 10 cm/s on a long spring, as shown in the figure above. Sketch the shape of the spring at time t = 0.06 sec. Explain how you arrived at your

Wave superposition question from pretest given after traditional instruction, Spring-1996, N= 65. Students had completed lecture instruction on superposition.

Figure 3-13



Common responses to pretest question from Spring-1996. (a) Correct response, (b) common incorrect response, (c) common incorrect response. These responses were given on pretests and in interviews which followed lecture instruction on superposition and preceded tutorial instruction.

permanent effect on each other. One student summarized the most common incorrect response, given by 20% of the students (shown in Figure 3-9(b)), by saying “[Part of] the greater wave is canceled by the smaller one.” A further 8% of the students state that the wavepulses bounce off each other.

In explanations, students implied that they were thinking of wave interaction as a collision. If we imagine two carts of unequal size moving toward each other at the same speed and colliding in a perfectly inelastic collision (imagine Velcro holding them together), then the unit of two carts would continue to move in the direction the larger was originally moving, but at a slower speed. The size of the pulse, in this situation, seems to be analogous to the momentum or energy of the pulse. One student's comment (given when answering a similar question in a later semester) supports this interpretation: "The smaller wave would move to the right, but at a slower speed." These students appear to be thinking of wavepulses as objects that collide with each other or cancel one another out.

Of the 182 students who answered the question on destructive interference in Figure 3-10 before instruction, 43% had difficulties with the question related to the ideas of bouncing or canceling waves. Of the other students, 10% did not answer the question, and 46% correctly indicated that the wavepulses continue in their original directions with their original shapes. The correct response and the most common incorrect responses are shown in Figure 3-11. We did not further investigate student understanding of destructive interference because their difficulties were similar to (though usually more common than) the difficulties students had with constructive interference. Students described the waves canceling out or bouncing off of each other much like they did with unequal amplitude waves interfering constructively. We believe that the students who described the waves bouncing off each other interpreted the shapes of the waves such that the wavepulses had equal strength or size. Like in a perfectly elastic collision between billiard balls, the wavepulses would bounce off one another, rather than cancel each other out completely and permanently.

When investigating student understanding of superposition when waves overlap but their peaks do not, we find that many students have a different type of difficulty than thinking of the waves as colliding. Very few students were able to answer this question correctly on the pretest (only 5% sketched Figure 3-13(a)). Of the students who said that there was no superposition unless the peaks of the pulses overlapped (40% of the students sketched Figure 3-13(b)), a common explanation was that "the waves only add when the amplitudes meet."

We have found that students giving this explanation use the word "amplitude" to describe only the point of maximum displacement, and they ignore all other displaced points in their descriptions. These students view superposition as the addition of the maximum displacement point only and not as the addition of displacement at <u>all</u> locations.

Other students also had difficulty with the process of wave addition. One-fifth of them sketched Figure 3-13(c) and stated that the points of maximum displacement would add even though they weren't at the same location on the string. This question was also asked in an interview setting. One interviewed student who used the word "amplitude" incorrectly, as described above, explained, "Because the [bases of the] waves are on top of each another, the amplitudes add." This student uses the base of the wave (its longitudinal width) rather than the (transverse) displacement of a point on the wave to guide his reasoning about superposition.

In investigating student difficulties with wave propagation, we found that students were using more than one explanation to guide their reasoning. We find similar results in our investigations of student difficulties with superposition. One student who answered the question in Figure 3-12 drew a sketch like the one shown in Figure 3-13(c). He explained,

> *[the pulses] are both colliding, and as they collide … if two of the same amplitude were to collide, it would double their amplitude. And so I believe this amplitude would get higher… They would just … come together.*

This student was using the idea of a collision between waves to explain how the amplitudes (inappropriately) add up to make a larger wave. He did not use the collision analogy to describe the waves canceling each other out, though, and gave the correct response for the shape of the string after the wavepulses had passed each other. Rather than showing an explicitly incorrect prediction on his part, his comments give evidence of the analogies he used to guide his reasoning. (As previously noted, students using the collision analogy often state that waves of equal size bounce off each other and do not cancel out, so their shapes will be the same once the waves have "passed through each other," which, in the case of a bounce, they have not done.)

In summary we observe that students have the following difficulties in understanding the physics of wave superposition:

- Waves are described as if they were solid objects which can collide with each other, bounce off each other, or permanently affect each other in some way.
- A wavepulse is described only by its peak point, and no other displaced parts of the system are superposed. When peaks do not overlap, the highest point due to a wave is chosen rather than the sum of displacements due to each wave. When the peaks of wavepulses do overlap such that the waves then add, only the peaks add.

In general, we find that students show difficulty with the concept of locality and uniqueness of spatial location. Students describe wavepulses with single points rather than as extended regions which are displaced from equilibrium, much like they did when answering the wave-math problem.

Summary of Specific Student Difficulties with Waves

In this chapter, I have described student difficulties with wave physics in the context of the propagation of mechanical waves on a taut string or spring, the propagation of sound waves, the mathematics used to describe waves, and superposition. In each case, the context has been used to uncover more fundamental difficulties with wave physics.

From the research into student understanding of wave propagation speed, we see that students have difficulty differentiating between the manner in which a wave is created and the manner in which it propagates through a medium. Many do not understand the fundamental idea of a wave as a propagating disturbance. Instead, as is suggested by the results from student descriptions of sound waves, some students believe that the wave actually exerts a continuous force in the direction of motion. Many students seem to have difficulty with the idea of the equilibrium state of a system. Student difficulties with mathematics indicate that the inability to understand a wavepulse as a disturbance to the medium plays a role in how students interpret the mathematics of waves. Student descriptions of superposition indicate that students also have difficulty describing the interaction between two waves and do not think of a wave as an extended region of displacement from equilibrium. The concept of a propagating disturbance, its cause, its effects, and the manner of its interaction with its surroundings are all difficult for students.

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[1] For example, look at the textbooks by Tipler, Serway, or Halliday and Resnick, where few problems involve wave phenomena that deal with finite length disturbances from equilibrium.

[2] Arons, A. B., *A Guide to Introductory Physics Teaching* (John Wiley & Sons Inc., New York NY, 1990). 202-218.

[3] The investigation of student difficulties with the relationship between the creation of waves and their propagation through the system is similar to the research previously done by Maurines. See chapter 2 for a discussion of her findings.

[4] Maurines, L. “Spontaneous reasoning on the propagation of visible mechanical **14**:3, 279-293 (1992).

[5] We did not have access to the students’ grades, so we relied on their comments for this statement. We have found that students are usually accurate in their knowledge of their grades and are often more pessimistic than necessary about their future grade.

[6] Our results are consistent with those observed by Linder and Erickson, as described in chapter 2. While Linder and Erickson have focused on issues of what students mean by sound and how they think of the medium, our focus has been on student use of force to guide their reasoning on this topic. Many of Linder and Erickson’s interpretations apply to our observations, as our interpretations also apply to their observations.

[7] In the interview format, we had the opportunity to obtain an explanation from all of the students. In the pretests, not all of the students give explanations, but those who do cite the exponential as the reason for the decay.

[8] This student was among the best in his class, and finished the course with the highest grade of all students.