

Chapter 2. Background:

An Overview of Relevant Education Research

In the last twenty years physics education research (PER) has shown that the traditional lecture course is not working for many of the students in introductory physics courses.^{1, 2, & 3} PER has succeeded in defining some of the fundamental problems with the introductory course and how to solve them. PER and other areas of educational research including math education and cognitive science have contributed greatly to our understanding of student learning in the introductory course. This chapter is an overview of those education research results relevant to this investigation.

I. WARNING SIGNS

I will discuss three examples that illustrate the need for assessment that takes into account conceptual understanding and student expectations. The first example is from Eric Mazur,⁴ a physics professor at Harvard, the second from David Hammer⁵ while he was a graduate student at Berkeley, and the third comes from Sheila Tobias,⁶ a non-traditional researcher of student difficulties with math and science. These three studies represent a small fraction of the available literature; they were selected primarily for their value as specific illustrations of what happens in traditional introductory physics lecture courses.

A. Mazur's Example: Student Difficulties With Conceptual Understanding

Eric Mazur is the Gordon McKay Professor of Applied Physics and a professor of physics at Harvard University. He currently leads a research program in applied physics and maintains an active interest in innovative instruction. He has developed his

own strategy for teaching physics to lecture classes which is described in his book, *Peer Instruction: A User's Manual*. Mazur taught the introductory course for science and engineering majors for six years in the traditional lecture format before he became aware of a series of PER articles by Halloun and Hestenes concerning the persistence of students' common sense beliefs about mechanics.^{7, 8} He was at the time quite satisfied with his teaching. In Mazur's own words,⁹

...I taught a fairly conventional course consisting of lectures enlivened by classroom demonstrations. I was generally satisfied with my teaching - my students did well on what I considered difficult problems and the evaluations I received from them were very positive. As far as I knew there were not many problems in my class.

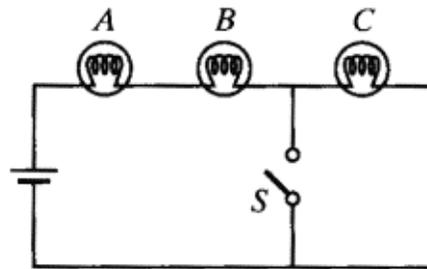
Like most instructors when they first become aware of the poor performance of students on multiple choice concept tests like the Force Concept Inventory (FCI) following instruction, Mazur thought, "Not my students."¹⁰ When he gave his students the test, his first warning that there was a problem was when one of his students asked, "Professor Mazur, how should I answer these questions? According to what you taught us, or by the way I think about these things?" Despite the concept tests' simplicity, the students scored little better than they had on their last midterm examination which Mazur thought was of far greater difficulty.

To better understand these results, Mazur gave his students the two problems shown in Figure 2-1 as the first and last problems on a midterm examination in a traditional lecture class. Note that the first problem is purely conceptual and requires only a basic understanding of simple DC circuits. The second problem is a traditional circuit problem that might be found in any introductory physics text. Problem 2 deals

Figure 2-1: Conceptual (top) and Traditional exam problems on the subject of DC circuits. Eric Mazur of Harvard gave these problems as the first and last problems on a midterm examination in the spring of 1991. (Figures from E. Mazur, *Peer Instruction: A Users Manual*, Prentice Hall 1997.)

1. A series circuit consists of three identical light bulbs connected to a battery as shown here. When the switch S is closed, do the following increase, decrease, or stay the same?

- The intensities of bulbs A and B
- The intensity of bulb C
- The current drawn from the battery
- The voltage drop across each bulb
- The power dissipated in the circuit



2. For the circuit shown, calculate (a) the current in the $2\text{-}\Omega$ resistor and (b) the potential difference between points P and Q .

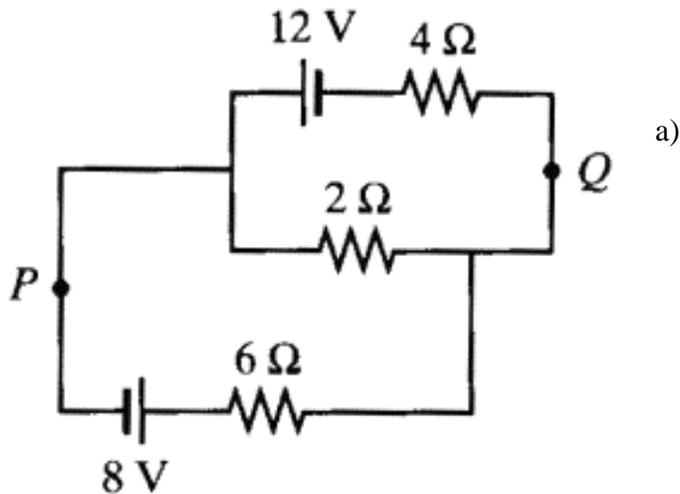


Figure 2-2: Test scores for the problems shown in Figure 2-1. For the conceptual problem, each part was worth a maximum of 2 points. (a) Test score distribution for the conceptual problem. (b) Test score distribution for the traditional problem. (c) Correlation between the scores on each problem. The radius of each data point is a measure of the number of students represented by that point. (Figures from E. Mazur, *Peer Instruction: A User's Manual*, Prentice Hall 1997.)

Figure 2-2a: Problem 1 Histogram

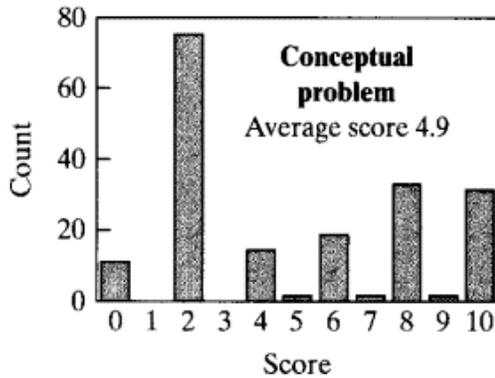


Figure 2-2b: Problem 2 Histogram

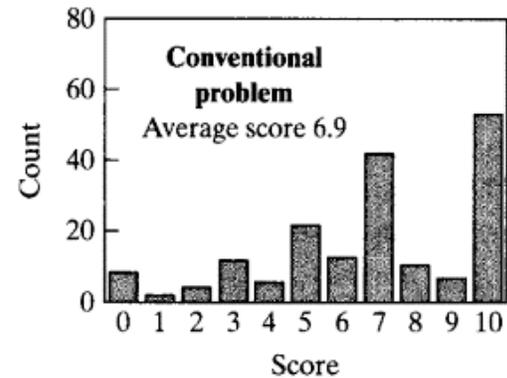
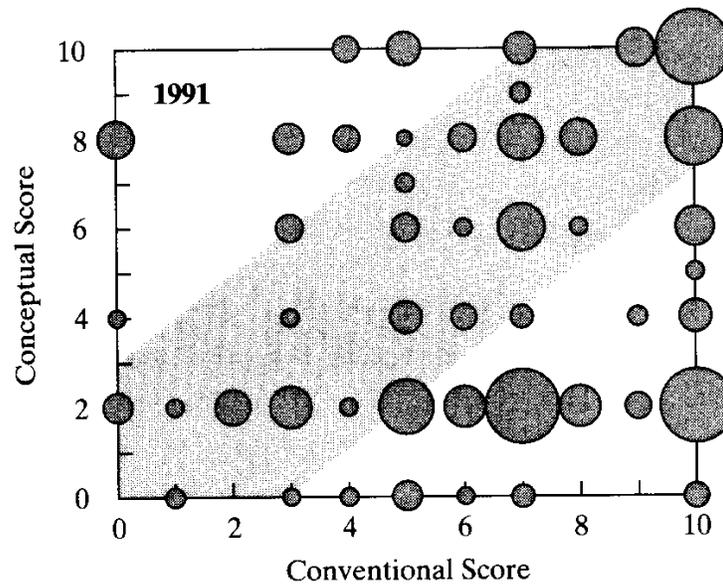


Figure 2-2c: Students Conceptual Problem Score vs. Traditional Problem Score



with the same concepts as problem 1 but also requires setting up and solving equations using Kirchhoff's laws. Most physics instructors consider the first problem to be easier than the second. The results shown in Figure 2-2 show that the students disagree. Both problems were graded out of ten points. The average on the conventional problem is significantly higher than on the conceptual problem (6.9 vs. 4.9). Figures 2-2 a and b are histograms of the student scores for the two problems. Note that two-thirds of the students scored at least 7 out of 10 on the conventional problem while only one third of the students did as well on the conceptual problem. In fact, almost one third of the students were unable to answer more than one of the five parts correctly.

The reason for the large peak at two for the conceptual problem was that 40% of the students only answered 1b correctly. Mazur suggests that these students believe that closing the switch doesn't change the current through the battery but that the current splits into two at the top junction and rejoins at the bottom. Despite this many of them still managed to correctly solve the traditional problem. Also note that the result on problem 1b is an overestimate since the responses of students who believe that bulb C will dim rather than go out will be scored correct.

In Figure 2-2c the student scores for each problem are plotted against each other. Notice the apparent lack of correlation between student scores on the two problems. The broad diagonal band represents the boundary for roughly equal scores (± 3 points) on the two problems. Although 52% of the students lie on the band, 39% of the students did substantially worse on the conceptual problem. A large number of these students managed to score 2 points or less on the conceptual problem while getting 10 points on the traditional problem.

According to Mazur, this result was repeated many times on pairs of problems during the remainder of the semester. The students performed significantly better on standard end-of-chapter textbook problems than on simple conceptual problems covering the same material. Mazur suggests that this example has three implications for physics instructors:

- 1) It is possible for students to do well on conventional problems by memorizing algorithms (recipes) without understanding the underlying physics.
- 2) It is possible even for an experienced instructor to be misled by traditional assessments into thinking that students have been taught effectively.
- 3) Students who use this algorithmic approach may believe they have mastered the material and then are severely frustrated when they learn that their plug-and-chug strategies don't work on all problems.

B. Hammer's Example, Two Approaches to Learning Physics

One of the few studies of student epistemological beliefs on learning and physics in introductory undergraduate physics courses was done by David Hammer while he was a graduate student at University of California, Berkeley.¹¹ In his early work on expectations, Hammer reported results from his case studies on students in the introductory "pre-med" physics course at UC Berkeley. In this study, Hammer investigated three questions:

1. What general conceptions of physics do students have, if any?
2. How do these conceptions affect their understanding and performance?
3. How are these conceptions affected by the instruction in the class?

Students were selected at random from the roster and asked to participate in the study until five students had volunteered. Each student was interviewed individually five times during the semester. Each interview took about an hour and involved a variety of tasks including open-ended discussions of the student's impressions of the course and of

physics, semi-directed tasks such as going through a midterm exam, and specific discussions of physics concepts as well as quantitative and quantitative problem solving.

Hammer chose to report initially on two of the five students, Liza and Ellen (pseudonyms) who on paper appeared very similar.¹² They were both planning to go to medical school: both had math SAT scores around 700 and had A's in mathematics courses through calculus. Liza's record was stronger, including a 5 on the BC Calculus Advanced Placement Exam, an A- in her first semester chemistry course, and an A in her high school physics class. Ellen received a C in the same chemistry course and had not taken a physics course beforehand.

1. Two approaches to learning physics

Hammer chose to report on these two students because, unlike the other subjects in the study, Liza and Ellen both put a great deal of time and effort into the course and, while their grades were similar (Liza B+, Ellen B), initially their approaches to the course were not. Liza relied heavily on memorization and pattern matching without trying to understand the material. From Hammer's paper,¹³

Throughout her transcripts there are both explicit and implicit indications that Liza's approach to the course was to learn formulas and facts based on the authority of the instructor and the text. To Liza, the formulas were the physics and that the professor said it in lecture or that it was in the book constituted sufficient justification.

She almost always said she understood the lecture or the reading; it was only when I pressed for explanations beyond citations that she 'didn't think about it' or was 'not sure'." [Excerpt from Hammer's transcripts follows]

Liza: ...*he* [the instructor] *defines the acceleration of circular motion in this formula, and then later on he proves it.*

Hammer: OK, how does he prove it?

Liza: *Um, (flips pages) this acceleration is referred to, um, inward acceleration.*

Hammer: Why is the acceleration inward?

Liza: *He said there is an inward acceleration for circular motion.*

Hammer: Does that make sense?

Liza: (pause) *I didn't think about it (laughs).*

She solved problems by 'figuring out which formula to use:'

Liza: *I look at all those formulas, say I have velocity, time, and acceleration, and I need to find distance, so maybe I would use a formula that would have those four things.*

But having an explanation for the formula itself did not seem relevant.

Liza: *This if v_f equals v_0 plus at ($v_f = v_0 + at$).*

Hammer: OK, and where did that come from?

Liza: *It came from here (laughs and indicates book).*

Hammer: Why is it true?

Liza: *Well, there is an initial velocity, and there is a final velocity, we know what the a is which is the acceleration, and I just use this to find what time it takes.*

Liza felt no need to check presented facts for consistency

On the other hand, Ellen tried to build her own understanding of the course material. She worked very hard to reconcile what she was learning in class with her experiences and her intuition. In Hammer's words,

Ellen's attitude was more independent, and, to her, the formalism was only one way of looking at the physics. She did want to make sense of the material, to integrate it with her own intuitions ... Ellen criticized lectures for failing to connect 'theory' with 'reality,' for emphasizing the formalism over simple explanations. She felt, for example, that the professor's treatment of finding the range of a projectile was needlessly complicated: [excerpt from transcript follows]

Ellen: *...obviously when it stops being in the air it stops going horizontally. ... it seems like we spent a couple of lectures just*

trying to get that through people's heads, and I'm sure if you just say that to someone they'll (say) well obviously. ... I guess that's what it is, we get theory with theory, and then we get application with application.

Hammer: *What do you mean by theory?*

Ellen: *It means formulas, ... let's use this formula because it has the right variable, ... instead of saying OK, we want to know how fast the ball goes in this directions, because if we know that, all we have to do is find out how long it goes in that direction before it hits the ground and we can find out how far. [Note that this quote seems to indicate she believes that the theory is the mathematical formalism.]*

After the first few weeks she became frustrated, finding it difficult to reconcile different parts of the formalism with each other and with her intuition ... Eventually she compromised her standards:

Ellen: *I'd rather know why for real.*

Hammer: *What do you mean by 'know why for real?'*

Ellen: *Well, sometimes I rationalize things, and I know that it's probably not the soundest rationalization, or something, it's just that I can kind of make sense for the time being, if I don't have time to really figure everything about it...even though I don't really know how it was derived.*

Like Liza, Ellen was able to apply prescribed methods to solve problems, but for her this was not sufficient:

Hammer: *so part b, can you do it?*

Ellen: *No. (laughs) I can pretend to. I can do it the way I know it's supposed to be done, but I pretty much already did that.*

Both students put in a great deal of time and effort to learn physics in this course.

Liza worked extra problems and attended every lecture, recitation, and lab. While Liza felt she was succeeding in learning physics, Ellen did not feel she was getting much out of the course. While at least initially she became comfortable with the material, even then she was aware of large gaps in her understanding.

2. Two approaches to problem solving

The two approaches to learning physics had a significant effect on how the two students approached physics problems. As Hammer relates,

An assigned problem asked: “One ball is thrown horizontally with a velocity v_0 from a height h , and another is thrown straight down with the same initial speed. Which ball will land first?”¹⁴ Liza wrote out $x = x_0 + v_0 t + (\frac{1}{2})at^2$ for the horizontal and vertical components of each ball, substituting appropriate initial positions, speeds, and accelerations. [Author’s note: this problem is very difficult for introductory students to solve symbolically using only the equations for position and velocity as a function of time.] She then used the vertical components to show, not without some difficulty, that the second ball travels a greater vertical distance after time t than the first, so it hits first. Ellen, in contrast, said immediately that the answer was ‘obvious,’ and went on to explain qualitatively that the second ball hits first because it has a greater speed downward to cover the same vertical distance.

... When Ellen was able to make sense of the material, her intuition helped her understand the formalism and guided her solutions. After the first few weeks, however, Ellen’s attitude interfered with her ability to answer the course questions. For a while she continued to try to reason based on her own sense of things, rather than accepting what she was told or answering by rote, but her intuitions were not generally in accord with Newtonian physics.

Liza, following the algorithms provided, often failed to apply her own knowledge from everyday life. She almost certainly had the common sense to know that the ball thrown down would hit first, but she did not think to make use of it in this context. At no time did she acknowledge that this was the answer one would expect. However, with her methodical adherence to the procedures, she was more reliable than Ellen in answering correctly.

3. Implications

For Liza, understanding physics meant knowing the facts and formulas and being able to apply them to problems. She did not deliberately or consciously use her intuition or her experience. According to Hammer, the effect of Liza’s approach was

that she was satisfied with learning physics as a set of relatively unrelated pieces, isolated both from each other and from her own everyday

experience. Her understanding remained incoherent and fragmented ..., but it served her comparatively well on problem sets and examinations. Ellen, when her approach was successful, was able to bring fragments of her understanding together, to integrate different pieces from the course with each other and with her intuitive knowledge. Unfortunately, it was not successful after the first few weeks.

Most physics instructors would prefer their students to approach an introductory physics more like Ellen than Liza, to think and reflect on the material while trying to reconcile intuition based on experience and the course material. Yet, as the course proceeded, Ellen had more and more trouble trying to build a comprehensive understanding of the course material while Liza consistently did well in the course. By the second midterm exam, Ellen was struggling and scored below the mean. Eventually she was only able to get a B in the course by abandoning her approach and doing “what everyone else does” by adopting an approach more like Liza’s.

It is also worth noting Liza’s response in her last interview when she was asked if she had liked the course. She answered, “Not really...it was kind of boring...all those formulas and derivations.” When asked if she was describing the course or physics in general, she replied, “Both.”

The course did not support Ellen’s thinking, reflective approach but seemed to reward and encourage Liza’s memorization approach. While Liza’s approach allowed her to be ‘successful in the class’ as measured by grades, the course left her with a very naive and boring view of what physics is and what it means to do physics. Is this course an aberration or a typical course? Again from Hammer’s paper,

This was a standard introductory physics course. The method of instruction was to present material, demonstrate its validity, provide examples of its application, and assign further problems for students to do on their own. Lectures paralleled the text at a pace of about a chapter a

week. ... Similarly, two of the ten labs involved determination of an empirical rule ... All others concerned verification of known results or the use of known results to measure some quantity. In every case, the procedures were specified.

This description would fit many traditional lecture format classes. Hammer indicates that while he only reported on the results for two students, the other students in the study made similar comments implying that these two students are not atypical. What is happening in the typical traditional lecture course to cause results like these?

The previous two examples illustrate some of the difficulties with the introductory physics course taught by the traditional lecture method uncovered by PER. Note that in both cases, traditional assessment offered no indications of problems with the course. Both Mazur and Hammer offered explanations of what was wrong with traditional lectures. Mazur uses an analogy to explain.

If I were lecturing not on physics but, say, on Shakespeare, I would certainly not spend the lectures reading plays to the student. Instead, I would ask the students to read the plays before coming to the lecture and I would use the lecture periods to discuss the plays and deepen the students' understanding of and appreciation of Shakespeare.

Yet in physics (and many other introductory courses) lecturing is often little more than reading the book to the students with demonstrations as illustrations. But the problem with traditional lectures is more complicated than that. To explain his observations described in the previous section, Hammer offers a more analytical view.

The flow of reasoning was always from the theory to the phenomena, the flow of information always from the professor and text to the students. The students applied the theoretical concepts of the course, but they were not involved in or even witness to the formation of those concepts. The laws were simply provided, and students were to become familiar with them through practice in solving problems. ...

The emphasis on formalism and the goal of problem solving facility seem consonant with conceptions of physics as a collection of facts and

formulas. The style of instruction, with knowledge passing from instructor to student, procedures and results to experimentation specified in advance, seems consistent with a reliance on authority. Furthermore, the pace of the course, with a chapter of reading and ten problems due every week, would not allow much independent exploration, except for those students who find the material easy.

Here, Hammer is making some serious accusations concerning why the course seemed to encourage student views of learning and physics that are contrary to what most instructors want. He obviously believes that the problem is caused by the nature and structure of the traditional lecture course, specifically the emphasis on problem solving and the over-reliance on authority. However, he also states that these results must be considered tentative both for the small sample size and the inexactness of the process of deriving information from student comments. But he is not alone in his finding that the nature and structure of the traditional lecture course is interfering with student learning. Similar results were found in a study conducted by Sheila Tobias.

C. Tobias' Example: Observations on the Traditional Lecture Teaching Method

In her study, Tobias was trying to learn why many qualified undergraduate students with good math and science backgrounds decide to turn away from majoring in science after exposure to introductory courses.¹⁵ Of college freshman who switch out of science and engineering majors, only about a third switch because they found the course work too difficult. For the rest, 40% find other fields more interesting and 25% believe they would have better job prospects elsewhere.¹⁶ She refers to these students who have the ability to pursue science but choose not to as “the second tier.” These second tier students are some of the same students who are not being reached by traditional lecture

instruction. In this study, she wanted to understand the second tier students' view of the process and problems of learning science in these introductory classes.

Because she wanted her observers to record their thoughts on the learning process as it was happening, she could not use students who had already taken introductory science classes. Instead she chose to use as her observers mature postgraduate or senior undergraduate students with good high school math and science background who chose not to take science in college. All the observers had demonstrated ability in their own fields. Tobias believed (correctly as we see below) that the responses made by the observers would be typical of students in the introductory science courses.

Three humanities graduate students (Eric, Jackie, & Michel) and one fifth-year social science undergraduate (Vicki) acted as student observers in three different calculus-based introductory physics classes at the University of Arizona and the University of Nebraska, Lincoln. To make sure they had the proper background, the study candidates were required to have taken at least one semester of college calculus in addition to four years each of high school mathematics and science. Two of the student observers with rusty but solid calculus skills,¹⁷ Eric & Jackie, did very well in their respective courses. Another student observer, Michel, whose math skills were weaker, struggled in the course but was able to learn the concepts and solve the assigned problems. The other student observer, Vicki, found that what had worked for her in the social sciences did not work for her in physics. She had to struggle and would have dropped the course if she were not involved in the project. Tobias considers Vicki's viewpoint to be closest to that of an average "C" student.

Each student observer was paid to seriously audit one semester of different calculus-based introductory physics courses as observers. In return for the stipend, they were expected to perform as well as they could in the class including attending all classes, submitting all work, and taking all examinations up to the final. In addition, they were expected to keep a journal where they would monitor the instructor's style of presentation, the material in the book, and the assignments, as well as to record their personal experiences with the course and where possible those of their classmates. At the end of the semester, they were asked to address the following two questions in a final essay:

1. How and in what ways was this course different from other college courses you have taken in other fields?
2. What were the specific knowledge, skills, and experience deficits you noticed in yourself and in your fellow students that got in the way of your mastery of the material?

Since her study also uses a very small sample set, Tobias engaged Abigail Lipson, a psychologist and senior member of the Harvard University Bureau of Study Counsel, to compare the student observers' results with an analysis of interviews of eighty science and non-science Harvard-Radcliffe students at the beginning of each of the students' four years of study.¹⁸ These interviews were part of a much larger study of Harvard-Radcliffe students who were traced, tested, and interviewed throughout their undergraduate careers to look at predictors of success in science and non-science degrees as well as gender differences.¹⁹ Although the Tobias' student observers were not typical of the students who participated in Lipson's study, the student observers' reactions to the courses and the subject matter were similar to those of Lipson's interviewed Harvard-Radcliffe students who started as science majors and then switched out.

While Tobias' student observers' comments on their own progress and difficulties are very illuminating, it is their observations on how the class was taught that help us understand what is happening in traditional lecture classes. For example:

Eric: To some extent science is hard because it simply is hard. That is to say, the material to be learned involves a great many concepts, some of which are counterintuitive. The process of mastering these concepts and being able to demonstrate a computational understanding of actual or theoretical situations required a great deal of time and devotion. In my experience, this fact is well understood by the students, the professor, and the general public. What is not as well understood are the various ways in which this already hard subject matter is made even harder and more frustrating by the pedagogy itself.

All four student observers commented on three particular aspects of the class that they felt made learning physics more difficult:

1. The lack of a narrative or story line,
2. An overemphasis on quantitative problem solving, and
3. A classroom culture that discourages discussion and cooperation.

To understand the nature of these difficulties, it is instructive to review some of their comments on the standard lecture course. These comments include excerpts from their journals and the final essay as well as comments from Tobias. Despite being in different classes, the comments are very similar.

1. Lack of a narrative or story line

In Hammer's study, Liza, the student who focused on developing recipes for problem solving from the start, indicated that to her physics was a collection of equations and formulas that applied to many different situations. She did not see physics as the process of learning and applying a few fundamental ideas to understand many different situations. While she could solve the assigned problems, she missed the big picture.

This may have contributed significantly to why she found both the course and the subject boring.

In Tobias's study, the professors' lectures apparently failed to help the student observers see common themes and links in the material. In fact, Eric, Jackie, and Michel all found it patronizing not to be told in advance where they were headed or what they needed to understand as indicated by their comments below.

Eric: We had marched through the chapters, doing the required work but never digging deeper ... I was able to keep myself on track by concentrating on one chapter at a time. But I never got the idea that the professor had any understanding of how the concepts were related, as he rarely tied together information from more than one chapter. His lectures did not seem to build upon each other, and he gave no indication of a linear movement through a group of concepts ... The final then asked the most primary basic questions about only the most important laws of physics. We were not required, at any time, to interrelate concepts or to try to understand the "bigger picture."

Jackie: Why, I wanted to know, did we begin by studying only the idealized motion of particles in straight lines? What about other kinds of motion? If he [the professor] could tell us what's coming next, why we moved from projectile motion to circular motion, for example, I would find it easier to concentrate; I'd know what to focus on. In college, I always wanted to know how to connect the small parts of a large subject. In humanities classes, I searched for themes in novels, connections in history, and organizing principles in poetry.

Each of the four student observers were accomplished undergraduates and had at least four years of experience in undergraduate courses in other fields. Yet, not one of them was able to see the connections or the context in what they were learning in the introductory physics course.

2. Overemphasis on quantitative problem solving

The exams, homework assignments, and lecture emphasized algorithmic problem solving, especially 'how much' problems. All four student observers were more

interested in building an understanding of the physics concepts, in particular, the ‘why’ questions. As Michele noted above and again below, she felt her curiosity was not supported by the course.

Michele: My curiosity simply did not extend to the quantitative solution. I just didn't care how much. I was more interested in the why and the how. I wanted verbal explanations with formulae and computations only as a secondary aid. Becoming capable at problem solving was not a major goal of mine. But it was a major goal of the course.

Eric made a similar observation.

[Tobias] “Eric was learning that, for the most part, ‘why’ questions are neither asked nor answered. The preference is for ‘how’ questions.”

Initially this caused Eric to have a very negative view of the course. Later he found,

Eric: As I am able to ask more knowledgeable questions, class becomes more interesting. I am finding that while the professor is happy to do example problems for the entire period, he will discuss the real world ramifications of a theory if asked.

Note that this professor was only willing to discuss the ramifications in response to Eric’s questions. What would the professor do if no questions were asked? In addition, his fellow classmates did not appreciate Eric’s questions. They lost patience with his silly ‘why’ questions when they felt that they got in the way of finding the right solution to their assigned problems. According to Eric, this was what physics was all about for them. The other student observers made similar observations about their classmates.

Vicki: When [John, a classmate she was studying with,] brought up an equation, I would try to relate it to another equation to help me learn them both all the better. This confused the effort. When John works a problem he uses only what is necessary and brings nothing else into the process. If I am going to learn a subject I need to know what is similar and dissimilar about an item. Why, when you are pushing down on a moving block, is there no “work” done? Isn't there a force downward on the block? I know that force and displacement have to be in the same direction; I needed to relate this to the concepts in the problem.... John

seemed disturbed by this and thought I had not yet mastered the fundamentals.

Jackie: [In her study group, the other students'] concerns focused on the kinds of problems they would encounter on the exams, not at all on a general understanding of the concepts ... They ignored all the fun parts, seeing the whole picture, laying out the equations and solving these. Instead, they wanted to know what equaled what and solve for an answer. The elegance of problem solving was lost ...

Jackie later speculated that this might be less a difference of mind and more a matter of efficiency.

Jackie: I think the students around me are having the same sort of thought-provoking questions about the material that I put into my journal, but under time pressure they don't pursue them, eventually they learn to disregard "extraneous" thoughts and to stick only to the details of what they'll need to know for the exam. Since the only feedback we get is on the homework assignments, the students cannot help but conclude that their ability to solve problems is the only important goal of this class.

Part of the reason for this attitude in the students may have been caused by the exams. The student observers noted that the exam problems were usually more like the easier homework problems rather than (in their opinion) the more interesting harder problems.

Michele: Too easy exams in contrast to too hard homework. On philosophy exams, instructors expect their students to do more than what they've done before, not less.

Jackie: It [the exam] was nothing like the homework problems. It was simple. It didn't really test understanding. There were so many things I thought I was going to have to know that weren't on the exam: that normal forces are what a scale reads, the direction and nature of frictional force, ..., the difference between kinetic and static friction, what an inclined plane does to free fall, ..., etc. I don't completely get all of this. But these are the questions I was thinking about when I prepared for the test.

Eric: The problems [on exams] seldom required the use of more than one concept or physical principle. Only once were we asked to explain or comment on something rather than complete a calculation.

Eric found the four class exams in his class biased towards computation and away from conceptual understanding. While he understood that some level of conceptual understanding was required to complete the computations, he found that the level was not particularly high.

One consequence of this emphasis on quantitative problems is that physics becomes less of a creative process and more of a craft. This, also, is reflected in the student observer comments below:

Eric: I do not feel that what this professor is doing can be considered teaching in any complex or complete sense. My understanding is that we are to learn primarily by reading the text, secondarily by doing problems on our own and comparing our solutions to those on sale in the physics office, and thirdly by mimicking the professor's problem solving examples. Simply by intuition I know physics, and more generally science to involve creativity and finesse; but this man makes it more into a craft, like cooking, where if someone follows the recipe, he or she will do well.

Jackie: Learning to solve physics problems is a process, not a matter of insight... Understanding the free body diagram means knowing how to do them.

[Tobias comments:] The problems were of limited interest because they had all been solved before. Only occasionally did these exercises provide intellectual satisfaction; rarely were they a source of new insight. [The student observers] looked upon the effort to be training at the expense of education in science, too many scales, not enough music.

As Tobias notes, in addition to their view of creativity in introductory physics, the emphasis on quantitative problem solving also influenced their opinion of the design and goal of the course. In essence, to the student observers the course values mastery over understanding.

Michele: A course design that assumes that everyone in the class has already decided to be a physicist and wants to be trained, not educated, in the subject.

Vicki: ... so the physics instructor could not imagine mastery being demonstrated by anything but increasing skill at problem solving.

3. A classroom culture that discourages discussion and cooperation

The way material is presented in the traditional lecture format makes it appear that the book and the instructor are the acknowledged authorities and they are passing what they know to students who learn it and repeat it back in homework and exams. The student is trying to learn what the instructor knows. In this transmissionist view (knowledge is transmitted from the authority to the student) of learning, the emphasis is on learning facts, not understanding. It is interesting to note that the student observers perceived their courses to be 'low' on concepts & theory and mired in facts (*dry formulas and dull reality*).

As Eric's comment below indicates, this model of instruction can cause students to approach the course in ways that hinder student learning.

Eric: *I still get the feeling that unlike a humanities course, here the professor is the keeper of the information, the one who knows all the answers. This does little to propagate discussion or dissent. The professor does examples the 'right way' and we are to mimic this as accurately as possible. Our opinions are not valued, especially since there is only one right answer, and at this level, usually only one right way to get it.*

Part of this perception may be due to the course emphasis on quantitative problems discussed above, but part of it is due to a lack of student discussion in and out of class on the ideas and concepts being learned. The students do not feel they have ownership of the material; they have not made it an intrinsic part of what they know and understand. All of the observers commented and speculated on the passive nature of their classmates in lecture. For example, Vicki and Eric noted,

Vicki: *No one ever asks questions in class except for that rare “Will you clarify?” question. I feel like it is grab and run.*

[And in a later journal entry] *Some people seemed to go to class only to hand in their homework. ... Others would attend the lectures purposefully to get information in order to digest it later, and in private. I wanted to digest it there, in class, through questions and discussion. I learn verbally. I like being put on the spot. I am not as passive as they seemed, to me, to be.*

Eric: [after learning of the low class average on quizzes and exams] *What this means is that there are a good many people sitting quietly and not asking questions. This is always the case to some extent in college, but physics seems harder on these people than the humanities.*

Even outside of class, when Eric asked his classmates about what they were studying, they weren't able to articulate an answer.

Eric: *I wonder if this is because they lack communication skills or because they haven't had the time to reflect on what they have learned, or because they don't really know much about their subject — if knowledge is defined to mean a deep thoughtful understanding rather than a superficial ability to regurgitate formulas.*

Some of their observations speculate as to why students are so passive in lecture

Jackie: *When he goes through these problems, the work seems so obvious, the equations so inevitable, that I tend not to question what he's doing...Lectures in physics can be incredibly passive experiences for students, particularly dangerous for those who believe that if they can follow the professor, they've mastered the material.*

This makes an interesting contrast to situations which the student observers thought were helpful to them in learning the course material.

[Tobias comments:] Vicky eventually found that her best studying came while working with at least one other student, *teaching the material to one another*. Here she was able to do what she could not do in class: question and try out what she thought she understood, and then question again. ... The best class in Eric's view was one where the professor brought in five or six demonstrations, the results of which were counter-intuitive, and then asked the class to speculate as to why the particular results occurred. In this class, there was substantial interchange.

The comments shown below suggest that part of the problem lies in the course format and culture.

Eric: The lack of community, together with the lack of interchange between the professor and the students combines to produce a totally passive classroom experience...The best classes I had were classes in which I was constantly engaged, constantly questioning and pushing the limits of myself and the subject and myself. The way this course is organized accounts for the lack of student involvement...The students are given pre-masticated information simply to mimic and apply to problems. Let them rather be exposed to conceptual problems, try to find solutions to them on their own, and then help them to understand the mistakes they make along the way.

Both Eric and Vicki believed that part of the problem was due to classroom competition.

Vicki noted,

Vicki: [In my other classes] learning is done through discussion with other students and with the professor. [She found it demoralizing to be working alone, in isolation, in a culture she characterized as destructively competitive.] I have the answer. Do you? If you don't, I'm not going to share it with you."

In addition to hindering their learning of physics, this competition could also have unhealthy results later in the students' careers.

Eric: [The sense of competition is in no way beneficial.] It automatically precludes any desire to work with or to help other people. Suddenly your classmates are your enemies. ... My class is full of intellectual warriors who will some day hold jobs in technologically-based companies where they will be assigned to teams or groups in order to collectively work on projects. [But] these people will have had no training in working collectively. In fact, their experience will have taught them to fear cooperation, and that another person's intellectual achievement will be detrimental to their own.

Another factor causing the lack of community in the physics classroom may be due to the models of doing physics as portrayed in class and in the textbook. In both cases, physics is often portrayed as being done by the scientist working alone. This can

be compounded if, as was true in Vicki's case, the students have no experiences of group work in recitation or lecture to contradict the model of the lone scientist.

4. Implications

The four student observers, like Hammer's student who tried to build her own understanding, exhibited beliefs and attitudes toward learning and physics that most instructors would greatly desire to see in their students. Particularly, their drive for a deep, useful understanding of physics beyond traditional problem solving, their need for debate and discussion, and their need for a coherent structure in what they were learning. Yet, as is the case of Hammer's student, the class did not support these attitudes. In fact the classes seemed to encourage students to memorize algorithmic solutions to physics problems rather than using an understanding of the physics concepts and discourage discussion of the course material in or out of class. Even when the student observers met with study groups outside of class, the focus of the discussion was on the mechanics of the solution, not a discussion of the physics.

In both examples expounded by Hammer and Mazur, non-traditional research-based assessment methods showed indications of inconsistencies in what the students were learning that were not apparent using traditional assessment. Here, each of the four student observers felt that the traditional exams were narrowly focused on quantitative problem solving and were not reflective of what they were learning. If we wish to improve student learning in the introductory course, both the traditional lecture format and the assessment methods must change.

D. Summary

In the three examples discussed in this section, we have seen research results that suggest that even when students do well in lecture-format classes with traditional textbook-style problems, some of them have a very superficial understanding of physics. We have also seen that this superficial understanding seems to be due to a lack of coherent themes, an overemphasis on quantitative problems, and a lack of meaningful student discussion on the principles, ideas, and concepts of introductory physics in and out of class. Traditional assessment and student evaluations showed no indication of any of these problems.

Another interesting point is that the studies by Mazur, Hammer, and Tobias make use of many of the research methods currently used in physics education research. Mazur used the FCI, a multiple choice concept evaluation, and specially designed exam problems. Hammer used extensive interviews or case studies. Tobias' study used student interviews, classroom observations, and reflective journals. These are all forms of alternative assessment.

Before we can discuss how to use these methods to evaluate traditional and innovative instruction, we need a better understanding of the student difficulties that will need to be addressed. The remainder of this chapter is an overview of what is known regarding the issues of complex problem solving, conceptual understanding, multiple representations, and cognitive development for students in introductory physics classes.

II. PROBLEM SOLVING: EXPERTS VS. NOVICES

Over the last twenty-five years, cognitive scientists and artificial intelligence researchers as well as math and physics education researchers have studied how people solve physics problems. The key point relevant to this investigation is that there are significant differences between novice and expert problem solvers. This section will summarize the nature and results of some of the more pertinent studies. For further information, the reader is referred to David Maloney's excellent review article on problem solving in physics, which is paraphrased extensively in this section.²⁰

A. Characterizing Expert and Novice Problem Solvers

A number of problem-solving studies have focused on identifying differences between expert and novice problem solvers as well as good and bad problem solvers. According to Maloney, interview studies of this type found differences in knowledge structure, approach to problem solving, and the use of examples and readings. These studies are generally done by interviewing individuals as they perform a problem-solving task or by studying computer models designed to simulate human problem-solving behavior. Most of the interview studies classify graduate students and professors as expert problem solvers while students from introductory physics sequences are considered novices. Novice students are further classified as either good problem solvers or poor problem solvers on the basis of how well they do on traditional textbook-style physics problems on class exams.

1. Knowledge structure

Although physics content knowledge is an integral part of physics problem solving,^{21, 22, 23, & 24} de Jong and Ferguson-Hessler argue “that just having the knowledge is not sufficient, it must be organized in a useful manner.”²⁵ The following results tend to support this view.

- Experts problem solvers have extensive domain knowledge that is organized hierarchically, with general physics principles at the top. Novices have significantly less domain knowledge that, when organized, tends to be organized around the surface features of physical situations.^{26, 27, 28, 29, & 30}
- Experts tend to classify problems by the physical principles involved in the solution of the problem. Novices tend to classify problems based on physical objects, configurations, properties, and concepts explicitly described in the problem statement.^{31, 32, 33, & 34}
- Good problem solvers had better knowledge organization than poor problem solvers.^{35 & 36}

2. Problem solving approaches

One major emphasis of the studies on problem solving from the late 1970s through the early 1980s was to explore the differences between the problem solving approaches of novices and experts. The results can be summarized as follows:

- Expert problem solvers make use of and often require a detailed qualitative representation to plan their solution before working forward to a solution. Novice problem solvers tend to start with an equation that contains the unknown quantity asked for by the problem and work backward to the given information (means-end analysis).^{37 & 38}
- Expert problem solvers plan their solutions more carefully and in greater detail before carrying them out.^{39 & 40}
- Expert problem solvers make greater use of physical reasoning.⁴¹
- Expert problem solvers also conduct an exploratory analysis, evaluate their thinking, and check their solutions to make sure they are reasonable when solving problems.⁴²

Two of the studies found “strong evidence” that one of the main causes of poor problem solvers’ difficulties is their failure and/or inability to construct an appropriate qualitative

representation.^{43 & 44} They suggest that the inability to use qualitative representations is due to poor understanding of the physics concepts.

3. Use of example problems and reading

In 1989, Chi *et al.* reported on a study of how students used worked examples when they are trying to learn how to solve problems.⁴⁵ This study used “think aloud” interviews (described in more detail in chapter 7). They observed what students did when studying the examples and how they used the examples when they were solving problems. Chi *et al.* found the following differences between good and poor students:

- The good students were significantly better at working out the missing steps and identifying the points they did not understand in an example. They would continue to study an example until they did understand it. The poor students tended to walk through the examples without checking to see if they understood them.
- Good students worked out the procedural aspects of the problems that are usually left implicit in textbook presentations, but poor students needed assistance in identifying and understanding these aspects.
- Good students tended to refer to a short segment of an example when they needed a specific relation or when they wanted to compare what they had done on a problem. Poor students tend to go back and read major segments of examples in search of procedures to use on the problem that they were trying to solve.

Fergusson-Hessler and de Jong conducted a similar study investigating how students read segments of text.⁴⁶ At selected points in the text, the students would be asked to think aloud about what they were doing. The students were judged as “good” or “poor” performers on the basis of test scores from three exams. The think alouds from five students who did well on all the exams and from five students who did poorly on all three exams were analyzed. Ferguson-Hessler and de Jong found the following result:

- Good performers used more deep processing than the poor performers when reading the text. Deep processing here means imposing structure not given in the text and making procedures and assumptions explicit.

- Poor performers tended to take more for granted and focused more on declarative knowledge (principles, formulas, and concepts) when reading the text. Good performers focused more on procedural (when a particular relation is used) and situational (characteristics of problem situations) knowledge.

B. How to Help Novices Become Experts

The research summarized in the previous section helps us describe the characteristics of expert problem solvers. But the real question from an instructional standpoint is, “How do we help students learn to become good problem solvers?” Although this is a current area of research, few studies have been done so far and not much is known. However, the results that do exist are encouraging. In this section, I will describe several studies of methods of instruction designed to encourage students to become better problem solvers.

1. Instruction to improve problem solving

As part of their research identifying the factors that contribute to good problem solving, Larkin and Reif tested methods of improving problem solving ability.⁴⁷ They found that students who were taught with either an explicit problem solving strategy or taught in a way that emphasized a hierarchical physics knowledge structure with the main principles at the top performed significantly better at problem solving than control groups. Although these studies were successful, they were conducted on a small scale over a relatively short time period and not incorporated into a regular introductory physics course.

Three classroom-based studies on improving physics problem solving have been reported since the characteristics of good and poor problem solvers discussed above appeared in the PER literature. All three studies were conducted in modified

undergraduate introductory physics classes with typical class-size samples. Two additional studies that report on changes in the student's knowledge use are also described.

a. WISE

In the first study, conducted by Wright and Williams at a community college in 1986, the students were taught and encouraged to use an explicit problem solving strategy as part of the Explicitly Structured Physics Instruction (ESPI) system.⁴⁸ The WISE problem solving strategy consists of four steps described below:

1. What's happening
 - Identify givens and unknowns
 - Draw a diagram
 - Identify the relevant physical principle
2. Isolate the unknown
 - Select an equation
 - Solve algebraically
 - Look for additional equations if necessary
3. Substitute
 - Plug in both numbers and units
4. Evaluate
 - Check the reasonableness of the answer

Wright and Williams found that students who used the WISE strategy on homeworks and exams performed significantly better (as determined by course grade) than students who did not use the strategy. In addition, student comments were strongly favorable to the WISE strategy and class retention was better for the experimental class. However, the authors found that the students were reluctant, even actively opposed in some cases to adopting the WISE problem strategy because of the extra work.

b. Overview, Case Study

In 1991, Van Heuvelen reported on the Overview, Case Study (OCS) approach to physics instruction.⁴⁹ Although this approach restructures the entire format of the class, it does specifically address problem solving. An integral feature of the OCS approach is the explicit development of multiple representations, especially qualitative physics representations, in problem solving. The approach also includes explicit discussion of knowledge hierarchy, an emphasis on active reasoning, and the use of cooperative group activities. Students who were taught with the OCS approach scored significantly better on problems from the College Board Advanced Placement physics test than students who were taught with traditional instruction at the same school.

c. Group Problem Solving with context-rich problems

In 1992, Heller and the Physics Education Group at University of Minnesota reported on a research-based lecture-recitation-laboratory curriculum they developed that emphasizes group problem solving.⁵⁰ In their curriculum, the lecture component stresses underlying themes, i.e. the main principles of physics, and an explicit problem solving strategy that is presented and modeled for the students. In recitation, the students do group problem solving using the strategy, and in lab they work in groups on laboratory problems where they must decide on what measurements to make and how to analyze the data they collect to answer the lab problem. This is one of the three research-based curricula being evaluated in Part III of this dissertation. A more detailed description of the curriculum and their own evaluation can be found in chapter 8.

In lecture the students were taught a five-step problem-solving strategy.⁵¹ A detailed outline of the strategy is shown in Table 2-1. The five steps are as follows:

1. Visualize the problem (make a physical representation of the situation)
2. Describe the problem in physics terms
3. Plan a solution
4. Execute the plan
5. Check and evaluate

Notice that this five-step plan is very similar to the WISE plan discussed above but with less of an emphasis on the mathematical calculation. To get the students to use this strategy productively, they found it necessary to use specially constructed problems called “context-rich problems” for the groups to use in recitation and lab. These

Table 2-1: Detailed five-step problem solving strategy used in the University of Minnesota's group problem solving curriculum.⁵² [Reference Heller Part 1]

1. Visualize the problem

Translate the words of the problem statement into a visual representation:

- draw a sketch (or series of sketches) of the situation;
- identify the known and unknown quantities and constraints;
- restate the question;
- identify a general approach to the problem—what physics concepts and principles are appropriate to the situation.

2. Describe the problem in physics terms (physics description)

Translate the sketch into a physical representation of the problem:

- use identified principles to construct idealized diagram(s) with a coordinate system (e.g., vector component diagrams) for each object at each time of interest;
- symbolically specify the relevant known and unknown variables;
- symbolically specify the target variable (e.g., find v_0 such that $h_{\max} \geq 10$ m).

3. Plan a solution

Translate the physics description into a mathematical representation of the problem:

- start with the identified physics concepts and principles in equation form (e.g., $\bar{a}_x = \Delta v_x$, $\Sigma F_x = ma$);
- apply the principles systematically to each object and type of interaction in the physics description (e.g., $N_1 - W_1 \cos \theta = m_1 a_{1x}$ and $W_1 = m_1 g$);
- add equations of constraint that specify the special conditions that restrict some aspect of the problem (e.g., two objects have the same acceleration, $a_1 = a_2$);
- work backward (from target variable) until you have determined that there is enough information to solve the problem (the same number of independent equations as unknowns);
- specify the mathematical steps to solve the problem (e.g., solve equation #2 for N_1 , then substitute into equation # 1, etc.).

4. Execute the plan

Translate the plan into a series of appropriate mathematical actions:

- use the rules of algebra to obtain an expression with the desired unknown variable on one side of the equation and all the known variables on the other side;
- substitute specific values into the expression to obtain an arithmetic solution.

5. Check and evaluate

Determine if the answer makes sense:

- check—is the solution complete?
- check—is the sign of the answer correct, and does it have the correct units?
- evaluate—is the magnitude of the answer reasonable?

context-rich problems are more complex than typical textbook problems. They are described in more detail in the next section.

Heller *et al.* designed this curriculum so that three out of six hours of class time each week are spent on group problem solving. They hypothesized that,

...in well functioning groups, students share conceptual and procedural knowledge as they solve a problem together. [That is, the students discuss their ideas, their strategies, and their understanding of the physics of the problem.] During this joint construction of a solution, individual group members can request explanations and justifications from one another. This mutual critique would clarify all the members' thinking about the concepts and principles to be used, and how those concepts and principles should be applied to the particular problem. Moreover, each member can observe others perform the varied thinking strategies that he or she must perform independently and silently on individual problem assignments.

To evaluate this curriculum, Heller *et al.* made extensive observations, studied the copied solutions, surveyed the students, and interviewed them. Although they did not report on how this type of activity affects the students' thinking and reasoning processes, they did show that groups that have more elaborate discussions produce better qualitative descriptions⁵³ and that students who were taught with this curriculum produced better solutions.

Class exams included both an individual and a group component. By comparing group and individual solutions to exam problems rated equally difficult by a rating scheme of their own design, Heller *et al.* showed that the group solutions were consistently better than the solutions of the best individuals. The greatest difference between the individual and group solutions was in the qualitative analysis in steps two and three.

The students also did better on traditional textbook problems. One exam composed entirely of typical textbook problems was given to one class taught with the modified curriculum and one class taught with the traditional lecture, recitation, and laboratory curriculum. The solutions were judged on the expertness of the approach to the solution, not on the final answer.

The results from these three studies indicate that research-based instruction methods can help introductory physics students develop expert problem solving skills. Students can be taught an expert-like problem solving strategy and they can learn to use qualitative representations. In each case, learning these skills had a positive effect on the students' problem solving abilities. But is there evidence of improving students' depth and organization of knowledge?

d. Studies on improving students' depth and organization of knowledge

In their studies of the use of computer aids to help students solve problems, Dufresne *et al.* found that students who used a computer tool they created, the Hierarchical Analysis Tool (HAT), showed significant improvement in problem solving in terms of reaching the correct solution and applying the correct principle.⁵⁴ The HAT program asks the student questions about the primary principle and the ancillary concepts that apply to the problem in question. It helps the student to work from the primary principle to the specific case of the problem. Students who initially based their reasoning on surface features or a mixture of physical principles and surface features showed the most improvement. This result is even more remarkable than it seems at first because the students were not given any feedback about their performance. The change is a direct result of using the HAT.

A related study by Volet on modified instruction in computer science found that students taught a programming strategy that involved reflection and evaluation of a computer program did not know significantly more content than the control group, but they were better able to apply their computing knowledge to an unfamiliar complex problem on the final exam.⁵⁵ This study is particularly interesting for three reasons.

First, the typical introductory computer science class in this study is very much like a traditional introductory physics course, taught with a lecture and a recitation but no laboratory. New programming concepts and ideas are presented to the students in lecture. The students are assigned programming problems each week to apply what they learn in lecture. In the typical recitation, the students work on their homework problems while waiting for the recitation instructor to come around and answer their questions individually. Except for the individual instruction in recitation this sounds very much like most physics classes.

Second, the curriculum modifications are similar to the group problem solving approach used by Heller *et al.* (described above) with some interesting exceptions. There is no laboratory component and the only part of the course that was modified was the recitation section. The students were taught an explicit 5-step programming strategy.

The five steps are as follows:

1. problem definition
2. algorithm development (list the procedure step by step in plain English)
3. conversion of the algorithms into a flowchart or pseudocode representation
4. coding from the flowchart or pseudocode into a specific programming language
5. execution of the code, debugging of errors, and improvement of the program

Note that this strategy is similar to the WISE strategy and the one used by Heller *et al.* The strategy was modeled extensively for the students by the recitation instructor, but the majority of the time in recitation the students were working in groups modeling and coaching each other on exercises similar to the homework problems. The students were required to verbalize not only their results but to go through the programming process as a think aloud with opportunity for group discussion at every decision point. Very little, if any, time was spent by the instructor working with the students individually. Special efforts were made to create a cooperative atmosphere that would be more conducive to learning. A partners system was set up at the start of the semester to demonstrate that collaboration was a normal expectation of the instructor. The students were encouraged to work with their partners and/or their groups in and out of recitation.

Third, the results of the modified instruction were extremely positive. Volet compared students in the two experimental sections to an equal number of students in the same class but in different recitations with 4 different instructors. The experimental and control students were paired based on the students' background in computing, overall program of study, gender, interest in computing, and initial study goals for the computing course. The final exam was graded by a professor who was blind as to the type of instruction experienced by the students. Parts one and two of the final exam asked students questions on their factual and procedural knowledge, for example, asking them to describe programming concepts, functions, and procedures and asking them why certain techniques are used. Part three required the students to solve an unfamiliar, fairly complex programming problem. While the experimental group did not do significantly better than the control group on parts one and two, they did do significantly better on

part three. Volet claims this result indicates “that experimental and control students did not differ in the amount of computer programming knowledge they had acquired (as assessed in parts one and two [on the final]),” but the result of part three “indicates that the experimental students’ computing knowledge was more accessible and more usable than control students’ knowledge.” In addition, the experimental group did significantly better in more advanced computer science classes.

We see from these studies that it is possible for introductory physics instruction to help students acquire at least some expert problem-solving skills. But there are additional issues to consider.

2. Appropriate problems

As we saw in the examples of Mazur, Hammer, and Tobias earlier in this chapter, the fact that students can solve traditional textbook problems is not always a good indication that students have a good understanding of the physics of a situation. Are traditional textbook problems useful for learning physics? Are they useful for teaching problem solving?

For the purposes of teaching an explicit problem solving strategy, there is another aspect to consider. While developing the group problem solving curriculum described above, Heller *et al.* studied what type of problems would be useful for teaching students to use the five-step strategy. They found that this is a difficult proposition for individual problem solvers because in their words,⁵⁶

if the problems are simple enough to be solved moderately well using their novice strategy, then students see no reason to abandon this strategy — even if the prescribed strategy works as well or better. If the problems are complex enough so the novice approach clearly fails, then students are

initially unsuccessful at using the prescribed strategy, so they revert back to their novice strategy.

In researching what types of problems are effective for promoting the use of the prescribed “expert” problem solving strategy in group problem solving, Heller and Hollabaugh began by studying student problem solutions and group interactions for standard textbook end of chapter problems to characterize the typical novice strategy for solving this type of problem.⁵⁷ An example problem for motion on an inclined plane is shown in Table 2-2 Problem A. When solving problems of this type, the group discussions tended to revolve around “what formulas should we use?” rather than “what physics concepts and principles should be applied to this problem?” One group, whose solution was typical, used the following solution process:

1. Rather than begin with a discussion and analysis of the forces acting on the block, this group began by attempting to recall the force diagram and formulas from their text. The example from the text was for a book sliding down the ramp. As a result, their frictional force was in the wrong direction and there was a sign error in their force equation.
2. At no point did the students plan a solution. They plugged numbers into formulas and manipulated equations until they had a numeric solution.
3. The group discussion was concerned with finding additional formulas that contained the same symbols as the unknown variables. They did not discuss the meaning of either the symbols or the formulas. They incorrectly tried to substitute the formula for instantaneous velocity into the formula for average velocity.

Table 2-2: Comparison of (A) a typical textbook problem with (B) a context rich problem for an object on an inclined plane

Problem A. A Typical textbook style problem

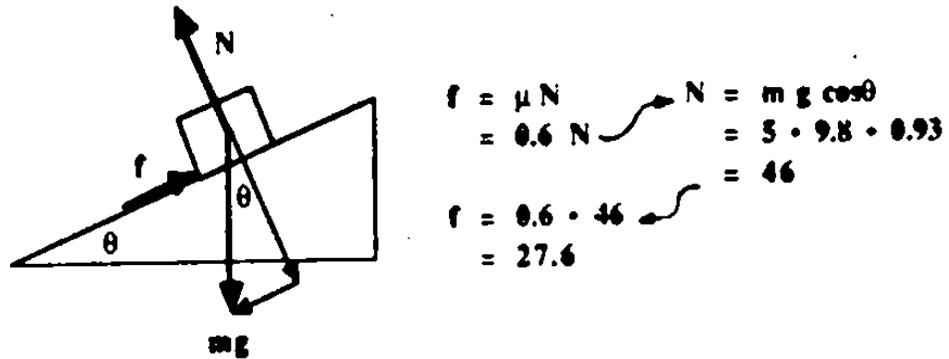
A 5.0 kg block slides 0.5 m up an inclined plane to a stop. The plane is inclined at an angle of 20° to the horizontal, and the coefficient of kinetic friction between the block and the plane is 0.60. What is the initial velocity of the block?

Problem B. Context-rich problem

While visiting a friend in San Francisco, you decide to drive around the city. You turn a corner and find yourself going up a steep hill. Suddenly a small boy runs out on the street chasing a ball. You slam on the brakes and skid to a stop, leaving a skid mark 50 ft long on the street. The boy calmly walks away, but a policeman watching from the sidewalk comes over and gives you a ticket for speeding. You are still shaking from the experience when he points out that the speed limit on this street is 25 MPH.

After you recover your wits, you examine the situation more closely. You determine that the street makes an angle of 20° and that the coefficient of static friction between your tires and the street is 0.80. Your car's information book tells you that the mass of your car is 1570 kg. You weigh 130 lb and a witness tells you that the boy had a weight of about 60 lbs and took 3.0 s to cross the 15-ft wide street. Will you fight the ticket in court?

Figure 2-3: Student group solution to the traditional textbook problem in Table 2-2A.⁵⁸



$$\begin{aligned}
 f &= \mu N & N &= m g \cos\theta \\
 &= 0.6 N & &= 5 \cdot 9.8 \cdot 0.93 \\
 & & &= 46 \\
 f &= 0.6 \cdot 46 & & \\
 &= 27.6 & &
 \end{aligned}$$

$$\begin{aligned}
 f - m g \sin\theta &= m a \\
 27.6 - (5 \cdot 9.8 \cdot 0.34) &= 5 a \\
 \frac{27.6 - 16.8}{5} &= a \\
 2.17 &= a & v &= a t \\
 & & &= 2.17 t & x &= v t \\
 & & & & &0.5 = v t \\
 & & & & &\frac{0.5}{t} = v \\
 & & & & &\frac{0.5}{t} = 2.17 t \\
 & & & & &\frac{0.5}{2.17} = t^2 \\
 & & & & &0.48 = t \\
 v &= a t \\
 &= 2.17 \cdot 0.48 \\
 &= 1.04 \text{ m/sec}
 \end{aligned}$$

A typical incorrect solution of a group for a standard textbook problem. The arrows show the progression of the mathematical solution.

Their solution is shown in Figure 2-3. Heller *et al.* estimate that two-thirds of the groups used this formula-based approach. They concluded that standard textbook problems were not effective in promoting group discussions that would help the student become better problem solvers.

Next, Heller and Hollabaugh compared textbook problems to real world problems to determine which characteristics of textbook problems encourage the use of novice strategies and which characteristics of real world problems require the use of an expert strategy. They found several characteristics of textbook problems that encourage students to use the formulaic approach described above “despite the instructor’s effort to teach a more effective strategy.” Textbook problems typically use idealized objects and events that have little or no connection to the student’s real world. They suggest that this reinforces the student’s tendency to memorize algorithms to deal with specific objects or situations. The unknown quantity is specified in the problem (usually in the last sentence) and all the other quantities needed to solve the problem are given (usually with the correct units). This encourages students to solve the problem by searching for formulas that have the right quantities and then plugging in numbers until a numeric answer is obtained. This numeric answer can then be checked in the back of the book.

On the other hand, solutions to real world problems are motivated by the solver wanting to know something about actual objects or events with which the solver is familiar. Before any calculations can be done, the solver must decide which quantities are useful for solving the question, which physics concepts and principles are relevant, what additional information is needed, and determine which information can be determined and which must be estimated. In other words, students solving a real world

problem have to think about the problem, try to understand what is going on, and make a number of decisions before reducing the problem to plug and chug mathematics. Since most textbook problems have removed the need for this type of analysis, they make algorithmic problem solving appear to be the correct way to solve problems.

To encourage students to use the prescribed problem solving strategy, Heller and her group created what they call “context-rich problems.” The context rich problems are designed to utilize many of the characteristics of real world problems. They are short stories that include a reason (although sometimes humorous or farfetched) for calculating specific quantities about real objects or events. They are more complex than typical textbook problems. In addition, they typically have one or more of the following characteristics:

1. The problem statement may not specify the unknown variable.
2. More information may be provided than is necessary to solve the problem.
3. Some information may need to be estimated or recalled.
4. Reasonable assumptions may need to be made to simplify the problem.

An example of a context rich problem is shown in Table 2-2B. This is the inclined-plane textbook problem rewritten in context-rich form. According to Heller and Hollabaugh,⁵⁹

Because context-rich problems are complex and involve making decisions about physics concepts and principles new to beginning students, they are difficult and frustrating even for the best students. In cooperative groups, however, students share the thinking load and can solve these problems. Because decisions must be made, the context-rich problems forced the groups to discuss physics issues while practicing effective problem-solving techniques. The group practice enhanced the students’ ability individually [as well] ...

The students have to pool what they know of the actual behavior of objects and the physics principles and concepts that describe this behavior to solve context-rich problems. Figure 2-4 is an example of a good group solution to the traffic ticket

problem shown in Table 2-2B. The following is a description from Heller and Hollabaugh of how the group came up with this solution,

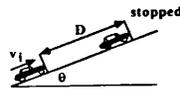
The students first sketched the situation and discussed what variable was needed to answer the question: “Will you fight the traffic ticket in court?” They decided they should calculate the initial velocity of the car just before the brakes were applied to see if this velocity was above the speed limit of 25 mph. After drawing the kinematics diagram, they then discussed what information they needed to find the initial velocity. They decided they could ignore the information about the child, since “*the car stopped before it hit the child.*” They then spent several minutes drawing free body diagrams of the car and discussing whether they needed to use static friction, kinetic friction, or both. During this discussion, they referred several times to the friction experiments they were doing in the laboratory. Once this issue was resolved and the force diagram agreed upon, they systematically planned a solution following the planning procedure modeled during lectures.

Notice that this group focused on “what physics concepts and principles should be applied to this problem” rather than “what formulas should we use.” While the students attitudes towards using the prescribed strategy for context-rich problems improved, they still found using the strategy “annoying” or “frustrating” to use on simple textbook problems because the strategy required them to write down more than they thought was necessary. This reaction is particularly interesting since Heller and Hollabaugh note that “these students were not usually successful at solving these problems using the formulaic strategy they preferred.” However these same students did agree that the prescribed strategy was useful for solving the more difficult textbook problems in addition to individual and group context-rich problems.

Heller and Hollabaugh’s study strongly suggests one reason why students who do well on traditional textbook problems may not do as well on qualitative problems.

Figure 2-4. Student group solution to the context-rich problem shown in Table 2-2b.⁵⁸

Visualize:

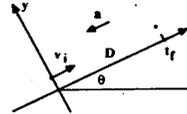
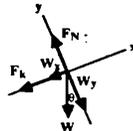


- $\theta = 20^\circ$
- $D = 50 \text{ ft}$
- speed limit = 25 mph
- $\mu_k = 0.60$
- $m_{\text{car}} = 1570 \text{ kg}$
- $m_{\text{driver}} = 130 \text{ lbs}$

Question: Is the speed faster or slower than 25 mph?

Physics Description:

Free-body Diagram Force Diagram



- W = weight of car and driver
- F_N = normal force
- F_k = kinetic force of friction
- v_i = initial velocity of car
- v_f = final velocity of car (0)
- t_i = initial time when brakes slammed on (0)
- t_f = final time when car stopped

Question: Is v_i less than 25 mph?

General Principles: $\Sigma F_x = ma_x$ $a_x = \frac{\Delta v_x}{\Delta t}$
 $W = mg$ $v_f = \frac{v_f + v_i}{2}$
 $F_k = \mu_k F_N$ $v_r = \frac{\Delta r}{\Delta t}$

Plan:

1. To find v_i
 $a_x = \frac{v_f - v_i}{t_f - t_i}$
2. To find t_f
 $\bar{v} = \frac{D}{t_f - t_i}$
3. To find \bar{v}
 $\bar{v} = \frac{v_f + v_i}{2}$
4. To find a_x
 $\Sigma F_x = -F_k - W_x = ma_x$
5. To find W_x
 $\sin\theta = \frac{W_x}{W} = \frac{W_x}{mg}$
6. To find F_k
 $F_k = \mu F_N$
7. To find F_N
 $\Sigma F_y = F_N - W_y = 0$
8. To find W_y
 $\cos\theta = \frac{W_y}{W} = \frac{W_y}{mg}$

Unknowns

a_x, v_i, t_f

\bar{v}

F_k, W_x

F_N

W_y

There are 8 equations and 8 unknowns:

Solve #8 for W_y , substitute into #7 to find F_N . Substitute F_N into #6 to find F_k . Solve #5 for W_x , substitute F_k and W_x into #4 and isolate a_x . Equate #2 and #3 and solve for t_f . Substitute t_f and a_x into #1 to find v_i .

Execute: (only last steps shown)

$$v_i = \sqrt{2Dg(\mu \cos\theta + \sin\theta)}$$

$$\text{Units: } \sqrt{(ft) \left(\frac{ft}{sec^2} \right)} = \sqrt{\frac{ft^2}{sec^2}} = \frac{ft}{sec} \quad \text{OK}$$

$$v_i = \sqrt{2 \cdot 50ft \cdot 32 \frac{ft}{sec^2} \cdot (0.6 \cdot 0.94 + 0.34)}$$

$$= 53.8 \text{ ft/sec}$$

Change to mph:

$$v_i = \left(\frac{53.8 \text{ ft}}{\text{sec}} \right) \left(\frac{3600 \text{ sec}}{1 \text{ hr}} \right) \left(\frac{1 \text{ mile}}{5280 \text{ ft}} \right)$$

$$= 36.7 \text{ miles/hr}$$

You were speeding -- you better pay the fine!

Because students don't use physical reasoning to solve problems and their grades depend on how well they do on the typical textbook problems, they may not see conceptual understanding and physical reasoning as important. This result is consistent with the findings of Maloney and Siegler as well as the previously described research characterizing novice problem solvers.⁶⁰

Maloney and Siegler gave novice undergraduate physics students a set of problems in which the students were asked to compare five objects and determine which objects had the largest and smallest values of either momentum or kinetic energy. Some of the questions were phrased in everyday language and some were phrased in explicit physics language asking about momentum and kinetic energy. The object of the study was to observe what strategies the students used to solve the problems and if the students had multiple strategies. Although Maloney and Siegler did observe students using multiple strategies, in both types of problems the strategies emphasized mathematical formulas, not physics concepts. They also found that while students tended to use momentum on most of the problems in everyday language, students tended use a different strategy if the question asked about kinetic energy explicitly. This result is one of many that indicate that students approach problems differently depending on cues in the problem.

3. Cues, triggers, and models used by students

In 1984, Anzai and Yokoyama presented results from a series of three studies of novice student representations or models of three physics problems and how readily they could be persuaded to change their models.⁶¹ They defined 3 types of student models:

experiential models, false scientific models, and correct scientific models. Experiential models are models that are derived from the students' experiences that do not involve scientific concepts or terms. False scientific models and correct scientific models both explicitly involve scientific concepts, terms, or relations but the false scientific model incorrectly characterizes the problem information.

The three physics problems they used are described below:

1. Yo-yo problem – what will happen if you pull the string of a wound yo-yo sitting on a horizontal surface, assuming the disks may roll but never slide?
2. Pulley problem – If two different mass blocks are hanging by a string on opposite sides of a pulley that is suspended from a spring balance, what does the balance read? Assume the pulley is massless.
3. Balance problem – For a two pan balance with a 1-kg mass on one side and a container (assumed to be massless) with 1 kg of water on the other, what happens to the balance when a 1 kg ball suspended by a string is completely immersed in the water?

In addition to these three problems, they also constructed augmented versions that gave physics cues on how to solve the problems. Anzai and Yokoyama used these problem in a series of studies using these problems with experts and novices including 216 university freshman enrolled in a college physics course.

The results of Anzai and Yokoyama's studies can be summarized as follows:

- Experts would generate one model that they would use to solve the problem. The novices would generate several models (different from the expert models) and compare them before deciding which one to use to solve a problem. The novice models were primarily either experiential or scientifically false.
- The novices generated experiential models for the basic pulley problem and the basic yo-yo problem, but they generated false scientific models for the basic balance problem. With additional cues the novices were able to generate correct scientific models for the augmented yo-yo and pulley problems, but not for the balance problem. This result suggests that false scientific models are more insensitive to physical cues. Further study with the balance problem showed that while most students (86%) were able to understand the buoyant force on the ball, many of the students (44%) did not recognize the reaction force on the container of water.

- Certain physical cues were more helpful than others in getting the students use the correct scientific model with the yo-yo and the pulley problem. In one case with the yo-yo problem, two different physically relevant cues were necessary. In another case with the pulley problem, tension cues improved student performance significantly but acceleration cues did not. These results suggest that novices have fragmented, incomplete knowledge and that the model they use to solve a problem strongly depends on the cues given in the problem.

In a related study of knowledge and information processing involved in solving mechanics problems, Hegarty argued that novice problem solvers have two distinct sets of knowledge from which they construct problem representations.⁶² The two types of knowledge are intuitive knowledge based on what people learn from observing the world around them and theoretical knowledge that is acquired from formal instruction. Hegarty claims that for novice problem solvers these two types of knowledge are not integrated and so novices must choose one or the other. In contrast, experts develop representations from a knowledge base where everyday and formal knowledge are integrated.

Hegarty then argues that novices' conceptual knowledge changes in three ways as they learn mechanics:

1. The level of specificity of concepts broadens and is tied to underlying physical principles rather than to surface features.
2. Concepts change from qualitative to quantitative.
3. Concepts are applied more consistently to appropriate situations.

The problem solving studies discussed above indicate that students' conceptual understanding and their conceptual models play a major role in how they look at and how they solve physics' problems. In order to evaluate what students are learning, it is important to consider what we know about students' understanding of physics concepts. It is also interesting to note that in some instances students separate what they know

from their own experiences, intuitive knowledge, and what they learn from formal instruction, theoretical knowledge. We will refer to the link between these two types of knowledge as a reality link, the link between what a student learns in classes and the student's real world.

III. CONCEPTUAL UNDERSTANDING

Around the same time researchers began studying how people solve physics problems, physics education researchers such as McDermott,⁶³ Clement,⁶⁴ Viennot,⁶⁵ Hestenes & Halloun^{66,67} and others⁶⁸ began studying college students' conceptual understanding of physics in the area of mechanics. Using a combination of demonstration interviews (see chapter 7) and specially designed problems (see chapter 6) they began studying students' understanding of selected concepts in mechanics like velocity, acceleration, and force in specific contexts. Here again, they found that the key point is that there are significant differences between experts and novices. Like problem solving, the main difficulty in helping novices become experts is teaching them in ways that take into account their existing prior knowledge. This section will summarize what is known about the initial state of students' conceptual understanding and how to help them learn a good functional understanding of physics concepts. The focus will be on the best understood and longest studied areas of students' conceptual understanding, kinematics and Newton's laws of motion.

A. Kinematics and Newton's Laws

In the early 1980s, McDermott and other physics education researchers found that each student does not come into a physics course as a blank slate. The students

bring with them their own system of common sense beliefs and intuitions about how the world works derived from extensive personal experience. Furthermore, they found that many students' common sense beliefs are very stable, incompatible with the physics taught in the introductory course, and appear to outlast instruction. These research results led Hestenes *et al.* to claim that "instruction that does not take the students initial state of conceptual understanding into account is almost totally ineffective, at least for the majority of the students."⁶⁹ Traditional instruction does little to change students' common-sense beliefs causing some of them to misinterpret the course material. However, many students have the same types of common-sense beliefs. This makes it possible to design curriculum that can take the more prevalent common-sense beliefs into account and help students learn concepts more effectively. There are three frequently found types of student difficulties with common-sense beliefs of motion and force: language, pre-Newtonian views of motion and force, and representations.

1. The language of physics

The language difficulty occurs mainly because many words used in the basic description of mechanics are also used in common everyday language. The way students use words like force, momentum, energy, acceleration, speed, and velocity in common speech can cause difficulties in the context of the physics class. Many of these words have different meanings and connotations in common speech and many are used interchangeably without regard to the meaning they have for physicists. As a result, students often use the language of physics either without understanding the meaning of the words in the physics context⁷⁰ and/or without differentiating between words for related concepts. For example, many students have trouble differentiating between

distance, velocity and acceleration and equate them all with a generalized idea of “motion.”⁷¹

2. Common sense beliefs on force and motion

In studying students’ common sense beliefs and building on the earlier studies, Halloun and Hestenes found from interviews and diagnostic concept tests (see chapter 4) that the majority of responses from 478 students on force and motion questions at the beginning of a University Physics class were consistent with either pre-Newtonian (83% on the diagnostic) or Newtonian (17%) models of motion.⁷² Moreover, nearly every student used a mixture of models of motion and appeared to apply the models inconsistently in different contexts. Interviews were conducted with 22 of the students to probe their common sense beliefs more deeply and verify the diagnostic results.

Halloun and Hestenes caution that the common-sense alternatives to Newtonian views are not just misconceptions. Many of these common-sense beliefs were held by some of the greatest intellectuals in the past including Galileo and even Newton.⁷³ Many are reasonable hypotheses grounded in everyday experiences. For example, the common-sense belief that something must cause the motion of an object is due to the observation that some force must be applied to most objects like cars, trains, and boxes to keep them in motion at constant speed because of frictional effects.

The fact that the students apply their common-sense beliefs inconsistently implies that their knowledge structure coming into the class is fragmented, incoherent, and context dependent. Studies by Minstrell⁷⁴ and diSessa⁷⁵ indicate that the students’ common-sense beliefs can indeed be characterized as loosely organized, ill-defined bits and pieces of knowledge that are dependent upon the specific circumstance in question.⁷⁶

(This context dependence is related to the issue of cues and triggers discussed in the problem-solving section of this chapter.) diSessa has found that many student common sense beliefs are rooted in pieces of knowledge he calls “psychological primitives” or “p-prims.” P-prims are simple, general isolated pieces of mental models that are cued by particular situations and are used to explain the events of the situation. They are usually strongly tied to real world experiences. The common sense example discussed above is an expression of the “force as mover” p-prim and the “continuous force” p-prim. The “force as mover” p-prim is the belief that objects go in the direction they are pushed. This p-prim tends to be used when an object is given a short, instantaneous push. The “continuous force” p-prim is the belief that a force is required to keep an object moving. This p-prim tends to be used in situations where the force is continuous rather than instantaneous.

Studies using pre and post testing and/or interviews have shown that many students still have these common-sense beliefs after traditional instruction.⁷⁷ In their interview study on students’ common sense beliefs, Halloun and Hestenes found that as a rule,

students held firm to mistaken beliefs even when confronted with phenomena that contradicted those beliefs. When a contradiction was recognized or pointed out, they tended at first not to question their own beliefs, but to argue that the observed instance was governed by some other law or principle and the principle they were using applied to a slightly different state. ... Careful interviews of students who have just witnessed a demonstration are enough to make one dubious about the effectiveness of typical classroom physics demonstrations in altering mistaken physics beliefs. We doubt that a demonstration can be effective unless it is performed in a context that elicits and helps to resolve conflicts between common sense and specific scientific concepts.

This last finding of the ineffectiveness of typical demonstrations to resolve conflicts between common-sense beliefs and specific physics concepts has also been reported in studies by Kraus *et al.*⁷⁸ as well as Redish *et. al.*⁷⁹

Recent studies by Francis and Larson⁸⁰ at Montana State University have shown that even after students appear to have acquired a Newtonian view of linear force and motion, rotational analogs to the students' common sense beliefs from linear motion appear when the students begin discussing rotational motion and dynamics. This suggests that students actually hold onto both the Newtonian concepts and the common sense beliefs and that their response will depend on what is triggered by the cues of the situation. It further suggests that in situations outside the context where they learn specific physics concepts, students have a tendency to revert to their common-sense beliefs.

3. Representations

In addition to the above mentioned difficulties with language and concepts, students also have difficulties with the abstract representations of physics such as graphs, equations, free-body diagrams, and vectors. One of the main goals of the hidden curriculum is to help students become fluent with the multiple representations of physics, a necessity for students trying to develop a robust, functional knowledge of physics. There is a lot of research on this issue by both math and physics education researchers. In this dissertation and this section, we will only consider student difficulties with understanding graphs since graphs are one of the most useful and powerful representations of ideas and data, both in class and in everyday life.

Several studies have found that students coming into introductory physics classes understand the basic construction of graphs but have difficulty applying their understanding to the tasks they encounter in physics.^{81, 82} The two most common types of student errors are thinking of a graph as a literal picture of an object's motion and confusing the meaning of the slope of the line with the height of the line. An example of the former is when a student asked to draw a velocity vs. time graph of a bicycle going along on a hilly road draws a velocity graph that resembles the hills and valleys traversed by the bicycle.⁸³ An example of the latter is when students asked to find the point of maximum rate of change indicate the point of largest value. In general, students tend to find interpreting slopes more difficult than individual data points. They also have difficulty separating the meanings of position, velocity, and acceleration graphs.

Beichner at North Carolina State University developed a multiple-choice diagnostic test for studying the prevalence of student difficulties with the understanding of kinematics graphs.⁸⁴ In a study of 900 students at both the high school and college level, there was no significant difference in the overall score between the high school and the college students. However, the calculus-based students did score significantly better than the algebra/trig students (with a mean of 9.8 vs. 7.4 out of a maximum of 21). In addition, he uncovered a consistent set of student difficulties with graphs of position, velocity, and acceleration. One of the main difficulties was that approximately 25% of the students believed that switching variables would not change the appearance of the graph. This is an indication that the students are interpreting the graphs as pictures. It is interesting to note that the students who could correctly translate from one variable to another had the best scores on this diagnostic test. Another result was that 73% of the

students correctly identified the slope of a line passing through the origin while only 25% were able to do so for a line that did not go through the origin. Approximately 25% of the students gave responses that are consistent with the slope/height mix-up described above. He also found that the students had trouble using the area under the curve to go from one graph to another. About one third of the students gave responses that used the slope of the line when they needed to find the area, and less than a third of the students were correctly able to determine the change in velocity from an acceleration graph.

Although Beichner did not use interviews to see why students answered the way they did, a similar study by Thornton of 10 multiple-choice velocity and acceleration graph questions did.⁸⁵ Thornton found student error rates after traditional instruction of about 40% on the velocity questions and 70-95% error rates on the acceleration questions. The instructors of these classes felt that these questions were simple (they expected error rates of 5-10%) and that students who were unable to answer these questions correctly had a very poor understanding of kinematics. Thornton noted that the problem was not that the students were unable to read graphs. Almost all (95%) of the students could answer similar questions about distance graphs correctly and interviews with the students showed that the students were picking graphs consistent with their verbal or written explanations of velocity and acceleration. This implies that students' difficulties with kinematics graphs are directly related to their conceptual difficulties with acceleration and velocity.

Research into students' conceptual understanding has extended to many areas besides mechanics and continues to be an active area of PER. Some of the other areas include:

- energy and momentum⁸⁶
- heat and temperature⁸⁷
- mechanical waves⁸⁸
- electricity and magnetism⁸⁹
- light and optics⁹⁰

In the next section I will discuss research on helping students to improve their conceptual understanding.

B. Mechanisms for Changing Students' Beliefs

In order to see how student beliefs can be changed, I need to discuss two mechanisms for learning introduced by Piaget, “assimilation” and “accommodation.”⁹¹ Piaget defines assimilation as the process by which people learn new ideas that match or extend on their existing conceptual knowledge. He defines accommodation as the process where people learn new ideas that don't fit into their existing conceptual knowledge either because the ideas are completely new or because the idea conflicts with what they already know. Students find it much easier to learn physics concepts that fit their view of how things work, i.e. to assimilate new ideas.⁹² Accommodation tends to be much harder because the students must change or rethink their existing views. One reason for this is that often students will perceive and interpret what they learn in a way that makes sense in terms of their existing views. For example, in the case of the demonstration studies described previously, many students tried to assimilate rather than accommodate what they observed by interpreting it either as an example that demonstrates what they believe or as a special case that is unrelated. This tendency to assimilate rather than accommodate is one reason that students' conceptual knowledge may contain contradictory elements.

However, even though helping students learn to change their conceptual understanding is difficult, it is not impossible. Research has shown that it is possible to stimulate conceptual change for most of the students in a class.⁹³ In his review of cognitive science research relevant to physics instruction, Redish notes that the mechanism for conceptual change appears to critically involve prediction and observation.⁹⁴ “The prediction must be made by the individual and the observation must be a clear and compelling contradiction to the existing [conceptual knowledge].”

He notes Posner *et al.*'s suggestion that in order to change students' existing conceptual understanding the proposed replacement must have the following characteristics:⁹⁵

- The replacement must be understandable.
- The replacement must be plausible.
- There must be a strong conflict with predictions based on the subject's existing conceptual understanding.
- The replacement concept must be seen as useful.

Redish adds, “The clearer the prediction and the stronger the conflict, the better the effect.” However, since students can hold conflicting ideas in their models at the same time, it is crucial that the students be made to resolve the conflict for conceptual change to be effective.

Two of the three research-based curricula being studied in this dissertation, *Tutorials* developed by the McDermott PER group at University of Washington⁹⁶ and *Workshop Physics* developed by Priscilla Laws at Dickinson College,⁹⁷ were specifically designed to improve students' conceptual understanding of physics based on the research discussed in this section. (Both curricula are described in detail in chapter 8.) Both

curricula use strategies that resemble Posner *et al.*'s four conditions of conceptual conflict and change.

In the *Tutorial* curricula, traditional recitations are replaced by groups of three and four students working through a specially designed worksheet by consensus. The tutorials use a strategy of elicit, confront, and resolve. The worksheet problems put the students into situations where they tend to use their common sense beliefs as part of their reasoning, then put them in a situation where they are forced to recognize an inconsistency between their reasoning and something they believe is true, and finally helps them to reconcile the inconsistency.

In the *Workshop Physics* curricula, the traditional lecture/lab/recitation format is abandoned in favor of an all-laboratory approach. Each laboratory unit is designed to take the collaborative student groups through a four-part process that can be summarized as follows:

1. Students make predictions about a physical situation.
2. The students perform discovery-oriented experiments and try to reconcile the differences between their experimental observations and their predictions.
3. The students develop definitions and equations from theoretical considerations.
4. Finally, they perform experiments to verify their theoretical model and apply their understanding of the phenomenon to the solution of problems.

While these two curricula seem very different, there are some common factors between them. Both of them build on students' existing common sense beliefs to help them develop a physicists' understanding of the concepts. They are also constructed on the principle that the students must be "engaged" in the learning process by thinking about situations, committing to a prediction of what will happen, discussing that

prediction with their fellow students, and resolving that prediction with what really happens.

IV. EXPECTATIONS AND EPISTEMOLOGY

Students bring more than just naive or novice views of problem solving and physics concepts to the introductory physics classroom. Each student, based on his or her own experiences, also brings to the class a set of attitudes, beliefs, and assumptions about what sorts of things they will learn, what skills will be required, and what they will be expected to do. In addition, their view of the nature of scientific information affects how they interpret what they hear. In this dissertation, I will use the phrase “expectations” to cover this rich set of understandings. In this dissertation, I consider what we might call students' cognitive expectations — expectations about their understanding of the process of learning physics and the structure of physics knowledge rather than about the content of physics itself. This is in contrast to more affective expectations such as students' motivation, preferences, and feelings about science and/or scientists, etc. While these are also important, they are not part of this study and have been probed extensively elsewhere.⁹⁸

As we saw in the studies of Hammer and Tobias, students' expectations about what they were learning and what they needed to do to succeed in the class seemed to affect their views on building conceptual understanding and problem solving as well as what they took away from the class. In particular, some of the students' expectations prevented the students from building a robust understanding of physics. Furthermore, it is reasonable to assume that one reason why many of Mazur's students learned to do the

traditional quantitative without learning the underlying concepts is related to these students' expectations of what they were supposed to learn.

The role of these types of epistemological⁹⁹ beliefs on adult learners in introductory undergraduate physics courses is not well understood. The few studies like Hammer's that exist indicate the effects may be profound and very much related to students' conceptual knowledge and problem solving skills. Furthermore, these studies indicate that like problem solving and conceptual understanding, introductory students' epistemological beliefs often differ from those of experts and may hinder their learning of physics. This section will summarize the nature and results of four of the more pertinent studies.

A. Previous Research on Cognitive Expectations in the Pre-College Classroom

There are a number of studies of student expectations in science in the pre-college classroom that show that student attitudes towards their classroom activities and their beliefs about the nature of science and knowledge affect their learning. Studies by Carey,¹⁰⁰ Linn,¹⁰¹ and others have demonstrated that many pre-college students have misconceptions both about science and about what they should be doing in a science class. Other studies at the pre-college level indicate some of the critical items that make up the relevant elements of a student's system of expectations and beliefs. For example, Songer and Linn studied students in middle schools and found that students could be categorized as having beliefs about science that were either dynamic (science is understandable, interpretive, and integrated) or static (science knowledge is memorization-intensive, fixed, and not relevant to their everyday lives).¹⁰² In a review of

student expectation studies in high school physics, Gunstone concludes: “The ideas and beliefs of learners about learning, teaching, and the nature of appropriate roles for learners and teachers are major influences is the likelihood of learners choosing to undertake the demanding and risky processes of personal and conceptual change.”¹⁰³

Expectation studies of high school mathematics classes by Schoenfeld¹⁰⁴ and the third National Assessment of Educational Progress¹⁰⁵ have found that students’ beliefs about mathematics and mathematical problem solving are shaped by their experiences in mathematics classrooms. Using national math assessments, surveys, interviews, and classroom observations, these two studies found that the majority of junior high school and high school students have the following beliefs about the nature of mathematics:

- Mathematics is mostly memorization
- Mathematics problems have one and only one correct solution and answer; the correct solution usually uses the rule the teacher has most recently demonstrated to the class.
- Students who have understood the mathematics they have studied will be able to solve any assigned math problem in five minutes or less. (Note: students with this belief will give up on a problem after a few minutes of unsuccessful attempts, even though they might have solved it had they persevered.)

Schoenfeld’s studies had two additional findings of typical student beliefs about mathematics that echo the expectations of Liza in Hammer’s study discussed earlier in this chapter:

- Ordinary students cannot expect to understand mathematics; they expect simply to memorize it and apply what they have learned mechanically without understanding.
- The mathematics learned in school has little or nothing to do with the real world.¹⁰⁶

In his classroom observations and interviews, Schoenfeld studied how student expectations affected their behavior in class and in problem solving. He concluded that, "Student's beliefs shape their behavior in ways that have extraordinarily powerful (and often negative) consequences."

These results led Edward Silver to use the phrase "hidden curriculum"¹⁰⁷ to describe this unintentional by-product of formal mathematics education.¹⁰⁸ In his words

Since the students' viewpoint represented by these statements is clearly inadequate, and potentially harmful to their future progress in mathematics, we need to focus our attention more clearly on those hidden products of the mathematics curriculum. . . . Our students may realize greater educational benefits from our attention to the hidden curriculum of beliefs and attitudes about mathematics than from any improvement we could make in the "transparent" curriculum of mathematics facts, procedures, and concepts.

B. Studies of Young Adults' Attitudes Towards Knowledge and Learning

Two important large scale studies that concern the general cognitive expectations of adult learners are those of Perry¹⁰⁹ and Belenky *et al.*¹¹⁰ Perry tracked the attitudes of about 100 Harvard and Radcliffe students on epistemology, morals, and general world outlook throughout their college career. The students filled out an attitudinal survey at the beginning of their college careers and were interviewed at least once or twice a year during their four years at college. Extending on Perry's study, Belenky *et al.* conducted in-depth interviews on similar issues, but with women from various walks of life. They tracked the views of 135 women in a variety of social and economic circumstances including 90 who were enrolled in one of six academic institutions varying from an inter-city community college to a prestigious four-year women's college. Twenty-five of the women in an academic setting were interviewed a second time one to five years later.

Both studies found evolution in the expectations of their subjects, especially in their attitudes about knowledge.¹¹¹ Both studies frequently found their young adult subjects starting in a "binary" or "received knowledge" stage in which they expected everything to be true or false, good or evil, etc., and in which they expected to learn "the truth" from authorities. Both studies observed their subjects moving through a "relativist" or "subjective" stage (nothing is true or good, every view has equal value) to a "consciously constructivist" stage. In this last, most sophisticated stage, the subjects accepted that nothing can be perfectly known, and accepted their own personal role in deciding what views were most likely to be productive and useful for them.

The two studies also had two similar findings regarding the progression of their subjects through these stages. One, that the progression through the stages was not always linear and some of the subjects stayed in the binary or relativist stages. Two, the subjects did not usually progress from one stage to next without some type of cognitive conflict where their previous epistemology was inadequate for their current situation. This is very similar to Posner *et al.*'s conditions of conceptual change discussed earlier in this chapter.

Although the Perry and Belenky *et al.* studies both focused on areas other than science,¹¹² most professional scientists who teach at both the undergraduate and graduate levels will recognize a binary stage, in which students just want to be told the "right" answers, and a constructivist stage in which the student takes charge of building his or her own understanding. Consciously constructivist students carry out their own evaluation of an approach, equation, or result, and understand both the conditions of validity and the relation to fundamental physical principles. Students who want to

become creative scientists will have to move from the binary to the constructivist stage. This is the transition that we will explore in chapter 10.

Another finding of the studies by Perry and Belenky *et al.* was that these expectation stages were not mutually exclusive and were often domain specific. An excellent introduction to the cognitive issues involved is given by Reif and Larkin¹¹³ who compare the spontaneous cognitive activities that occur naturally in everyday life with those required for learning science. They pinpoint differences and show how application of everyday cognitive expectations in a science class causes difficulties. Another excellent introduction to the cognitive literature on the difference between everyday and in-school cognitive expectations is the paper by Brown, Collins, and Duguid, who stress the artificiality of much typical school activity and discuss the value of cognitive apprenticeships.¹¹⁴

C. Students Expectations in Undergraduate Physics

The most relevant and valuable study for the expectation questions in this dissertation is Hammer's Ph.D. thesis at Berkeley.¹¹⁵ In his dissertation (which followed the preliminary study described earlier in this chapter), Hammer interviewed six students several times each from the first semester of the calculus-based sequence for engineering students at the University of California at Berkeley. Four of the students volunteered and two more agreed to participate after specifically being selected in order to add "good" students to the group. All six students had taken physics in high school and scored at least 700 on the math SAT. Each student was interviewed for approximately ten hours during the same semester. In these interviews, Hammer used three types of

activities to probe student expectations: open and semi-directed discussions, problem solving, and direct questioning. For the problem solving activities, he asked the students to solve the problems out loud. The problems included three specifically chosen to address student common-sense beliefs. The interviews were taped and transcribed, and students were classified according to their statements and how they approached the problems.

Hammer proposed three dimensions along which to classify student expectations of physics knowledge: beliefs about learning, beliefs about content, and beliefs about structure. The three dimensions are described in Table 2-3 below.

In their beliefs about how physics is learned, the type A students feel they need to make sense of the course material for themselves, while the type B students feel they have to trust what they learned from the authorities, i.e. the teacher and the text, and simply believe what they were given. This dimension is based largely on the binary or received knowledge state of Perry and Belenky *et al.* However, Hammer notes that while a student in the relativist or subjective state might feel that all views are equally valid, they may recognize that using the views of the instructor and the textbook (the classroom authorities) on tests and exams is expedient for getting a good grade in the course. The quote from Mazur's student about whether she should respond to the ungraded FCI according to what she was taught or by the way she really thinks is an example of this.

For their beliefs about the content of physics knowledge, the type A students feel that physics is coherent and fits together in a way they expect to understand and use. In contrast, type B students feel that knowing physics means remembering facts, and/or

they cannot solve a problem without knowing the “right” equation, i.e. they treat physics knowledge as a set of independent and unrelated pieces of information. Some other indicators of this dimension are that type A students believe that understanding

Table 2-3. Hammer’s expectation dimensions of students learning

Dimensions	Type A	Type B
1: Beliefs about learning	<u>independent</u> self-motivated, questions material until it makes sense to them	<u>by authority</u> takes what is given by instructor or text without evaluation
2: Beliefs about content	<u>coherent</u> believes physics can be considered as a connected, consistent framework	<u>pieces</u> believes physics can be treated as separate facts or “pieces”
3: Beliefs about structure	<u>concepts</u> focuses on conceptual understanding and connects formulas to intuition and underlying concepts	<u>formulas</u> focuses on memorizing formulas and using them in problem solving.

formula derivations is important and check for consistency in their work while type B students do not think derivations are important and are not bothered by apparent inconsistencies.

For their beliefs about the structure of physics knowledge, the type A students feel that the underlying concepts are what is essential and that the formulas are expressions of those concepts, while the type B students feel that the formulas are essential (at least for them), and the underlying concepts are not something that they need to bother with. Note that these three dimensions need not be independent.

Hammer classified students by the number of times something they said or did in the interviews indicated they held a particular type of expectation. He found that the students’ expectations were generally consistent across all the activities in the interview.

Of the two students from the earlier study, Liza was classified as Type B while Ellen was type A. In this study, the two “good” students were both type A and other four fell into category B for all the variables. The type A students earned an A and an A+ in the course, while the four type B students grades ranged from A- to C+.

In addition, all four of the type B students displayed misconceptions on at least one of three selected problems in interviews. In general, they displayed more misconceptions than the type A students and were usually unable to resolve them when challenged by Hammer. The two type A students displayed no misconceptions on the selected problems and in general were able to resolve their own misconceptions when challenged. It is not surprising that the two good students received a higher grade and showed fewer misconceptions, since they were selected based on their high grades on the first midterm. It is surprising that they are characterized by expectations different from the subjects that did less well in the class and who were unable to resolve their misconceptions.

It is interesting to note the one of the type A students, code named Ken, is an interesting counter example to many instructors’ suppositions about successful physics students. Ken had a weaker math and science background than two of the type B students. Also, he was more concerned with getting a good grade in the class than in understanding physics but he felt conceptual understanding was essential to getting a good grade. Last, he did not value derivations except as support for his conceptual understanding. His main advantage over the two students with stronger backgrounds seems to have been his expectations, particularly his need to build a good conceptual understanding of physics.

Hammer also made the following three general observations about the involvement of expectations in the subjects learning.

1. [Type B subjects] were quite casual about making and breaking associations between different aspects of their knowledge; [type A subjects] were much more careful about building and modifying their understanding.
2. [Type B subjects] were quick to decide that they understood new information, while [the type A subjects] were more reflective and questioning.
3. [Type B subjects] were reluctant to spend time working on problems they did not know how to solve, while [the type A subjects] seemed to consider these the most interesting [problems]. In part, this appeared to reflect different goals: [the type A subjects] appeared to have a goal of understanding the material, while [the type B subjects] did not always consider understanding important.

Note that the first observation would lead to significant differences in the two types of students' knowledge structures. The second observation is remarkably similar to the result of the studies by Chi *et al.*¹¹⁶ and Fergusson-Hessler and de Jong¹¹⁷ (discussed earlier in the problem solving section) that found that good students process readings and example problems more deeply than poorer students who just tend to read and accept. The third observation implies that the students' personal course goals may be hindering them from reaching our goal of robust functional understanding.

In summary, Hammer found that his six students had expectations about physics knowledge and learning that could be classified based on indications over many hours of interviews. He found that these indications were consistent across a wide range of activities. His results indicate that expectations seem to be involved in whether the subjects were able to solve his interview problems and in whether they developed a coherent, conceptual understanding of the course material. These findings support expectations as a valid perspective for evaluating student learning and reasoning. Due to

the limited scope of his study, it is impossible to tell the extent to which the three expectation variables would be correlated or what the distribution of the categories would be in a typical class.

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- ¹⁰⁵ T.P. Carpenter, M.M. Lindquist, W Matthews, and E.A. Silver, "Results of the third NAEP Mathematics Assessment: Secondary school," *Math. Teach.* **76**, 652-659 (1983); National Assessment of Educational Progress, *The Third National Mathematics Assessment: Results, Trends, and Issues* (Educational Commission of the States, Denver CO, 1983).
- ¹⁰⁶ A classic example of this is the example from the third National Assessment of Educational Progress where of the 70 % of the students who performed the calculation correctly for a problem inquiring about the number of buses for a task roughly two thirds of them wrote "31 remainder 12" or "31". Less than one third wrote down the correct answer of "32". See Ref. 105.
- ¹⁰⁷ H. Lin, "Learning physics versus passing courses," *Phys. Teach.* **20**, 151-157 (1982).
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- ¹¹⁰ M.F. Belenky, B.M. Clinchy, N.R. Goldberger, and J.M. Tarule, *Women's Ways of Knowing* (Basic Books, 1986).
- ¹¹¹ This brief summary is an oversimplification of a complex and sophisticated set of stages proposed in each study.
- ¹¹² Perry specifically excludes science as "the place where they do have answers." See Ref. 109.

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- ¹¹⁷ See Ref. 25.