Chapter 7: How does Conceptual Instruction Affect Coherence Between Qualitative and Quantitaitve Knowledge?

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Introduction

This chapter documents the evolution of coherence between qualitative and quantitative knowledge as students go through a modified curriculum. By tracking one group of students through the physics 161 course we can look at the dynamics of how coherence develops between students' qualitative and quantitative schemas. Redish et al. have examined how students view coherence before students begin the physics course and after students have completed the course. They report that for traditional classes, students' views of coherence move away from an expert's views on coherence.[1] In particular students tended to shift toward believing that physics can be treated as unrelated facts or pieces.

In this study we track a class of engineering students as they go through *Tutorials in Introductory Physics*.[2,3] The previous three chapters have demonstrated that student content knowledge is often fragmented. We have seen that students often exhibit local coherence and use schemas consisting of either qualitative or quantitative knowledge, but they rarely use schemas containing integrated knowledge. This chapter will discuss some of our results about how a course which includes conceptual instruction can lead to some students forming links between the conceptual knowledge they develop and quantitative problem-solving. Even though we see some improvements in student coherence, the results are by no means dramatic and there is definite room for improvement. Our data will show some of the limitations in relying on the tutorial curriculum for building coherence between qualitative and quantitative knowledge.

We had the opportunity to give a similar exam problem to a class of Physics 161 students twice during the semester. The problem focused on Newton's 2nd law (NII) and tension. Students were given a problem involving two blocks attached by a massless string on a frictionless surface with force applied to the first block. This problem was included on the first exam in the Physics 161 course, after the students had traditional instruction on the material, but before they had tutorial instruction. The problem has been repeated on the third exam after the students went through the tutorial curriculum. The question tested the students' qualitative and quantitative understanding of NII and tension. In addition it tested whether students could apply their qualitative understanding of NII (i.e. the FBD) to a quantitative application of NII.

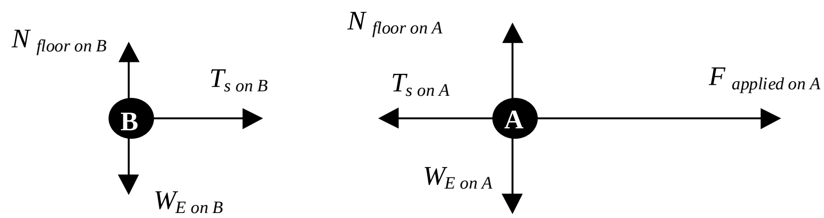
Context

In the spring '98 semester there were three 161 classes, all using the tutorial curriculum. The same professor taught two of the classes (Classes A, B). The NII-tension problem was asked on the first exam and the third exam in two of the three classes (Class A, B) of the Physics 161 class. The third class (Class C) was asked the problem on the third exam only. The wording of the question changed slightly from version 1 (Exam 1) to version 2 (Exam 3) and the labels on the blocks changed from version 1 to version 2. Version 2, along with a model solution is shown in Figure 7 - 1.[4] Version 1 was written by the instructor for the course, while version 2 was written by the University of Maryland PERG.

Because we are looking at the dynamics of student coherence it is important to identify when each type of instruction occurred in the class. A timeline for the course is shown in Table 7 - 1. The type of instruction is listed in the far-left column and the two columns on the right indicate when the specific type of instruction began and ended. (For items that only lasted one day, such as homework due dates and exams, there is no ending date.) In addition, although tutorials occurred over two days, each student only went to one (1-hour) section each week. There were three (1-hour) lectures a week for the students in Classes A and B, and two (1.5-hour) lectures a week for students in Class C.

Two blocks of masses m_A and m_B, connected by a massless string, are sliding to the right on a horizontal surface with no friction. Block *A* is pulled by a hand with a force $\mathbf{F}_{\text{hand}}$.

A. Draw separate free-body diagrams for each of the blocks.



B. Find the tension in the string that connects the two blocks.

$$\sum F_A = F_{applied} - T_{s\ on\ A} = m_A a \text{ and } \sum F_B = T_{s\ on B} = m_B a$$

$$\text{and } T_{s\,on\,A} = T_{s\,on\,B} = T \text{ therefore}$$

$$F_{applied} = (m_A + m_B)a \text{ so } T = {m_B\, a}\big/{(m_A + m_B)}$$

Figure 7 - 1

The NII-Tension problem, given on the first and third exams, with a model solution. The wording and formatting of the problem changed slightly from the first exam to the third exam.

All textbook homework for the class came from the Serway text.[5] For the homework entries, the chapter from which the problems came is listed as well as the number of problems from that chapter. For instance, *HW4 due, Chap 4(2), 5(6)* means that homework assignment 4 included two questions from chapter four and six questions from chapter five. Newton's laws are covered in Chapter 5 of Serway.

The topics for the tutorial entries are listed in parentheses. Because the different classes went through slightly different tutorial curricula some entries have multiple topics. For instance *TUT (NIII,Tension)* means that the students in some classes had gone through a NIII tutorial and the other students had gone through the tension tutorial. It should also be noted that the names of the tutorials provide the reader with a very limited idea about the content of the tutorial. The tension tutorial, for example, is designed to also help students with the related concepts of NII and NIII. Other tutorials try to link related concepts in a similar way.[6]

The first exam was administered following all lecture instruction and after the first homework assignment *(HW 4)* on Newton's Laws was due. In particular, one question on the homework was very similar to this exam problem. The question involved two blocks being pushed by a hand on a frictionless surface. It involved the same physical concepts except that the normal force between the blocks played the role of the tension force on the homework question. The second homework *(HW 5)* on

	Class A,B	**Class C**
Type of instruction	Date Begin - End	Date: Begin - End
Lecture (Newton's Laws)	9/18-9/30	9/22-9/29
Tutorial (Forces, FBD's)	9/29-9/30	9/29-9/30
HW4 due, Chap 4(2), 5(6)	9/30	9/29
Exam I	10/2	10/1
HW5 due, Chap 5(6), 6(2)	10/6	10/5
Tutorial (NII, NIII)	10/6-10/7	10/6-10/7
Tutorial (NIII, Tension)	10/13-10/14	10/13-10/14
Tutorial (Air Resistance)	10/20-10/21	10/20-10/21
Tutorial (Problem Solving)	10/27-10/28	10/27-10/28
Exam III	12/4	12/3

Table 7 - 1

Table showing a timeline for the section of the course covering Newton's Laws and Tension.

Newton’s Laws was due after Exam 1. *HW 4* had one problem dealing explicitly with tension while *HW 5* had three problems explicitly dealing with tension. Tutorial instruction on Newton’s Laws prior to the examination consisted of a single tutorial dealing with forces.

By the third exam the students had gone through a number of tutorials dealing with Newton’s Laws. In particular, class A had a tutorial dealing with the subject of tension and the application of NII and NIII. The University of Washington Physics Education Group (PEG) created the bulk of the tutorials on Newton’s Laws that were used in the three classes. The University of Maryland PERG created the remaining tutorials. Tutorials from the UW PEG, done by Class A, included tutorials on forces, NII and NIII, and Tension. Tutorials from the UMd PERG, done by Class A, included tutorials on air resistance and mechanics problem-solving.

The question we posed is similar to a question posed by the UW PEG. They describe some of their research on student understanding of Newton's Laws and Tension in their 1994 paper, "Research as a guide for teaching introductory mechanics: An illustration in the context of Atwood's machine."[7] They state that student difficulties with tension come from general difficulties with Newton's laws and acceleration and more specific difficulties about tension. The question shown in Figure 7 - 2 was asked by the PEG to elicit the difficulties students were having. The students who participated in this study were students at the University of Washington, who were enrolled in the calculus based physics course. The question involves two blocks connected by a string being pulled across a table by another string. The students were told that the mass of block A was less than the mass of block B and that the strings were massless. General errors documented by the PEG included “failing to (1) isolate an appropriate system, (2) identify correctly all the forces present, (3) discriminate properly between third law force pairs, and (4) recognize that it is the net force on a system that determines the acceleration."[8,9] Specific errors with the concept of tension included the idea that a string transmits a force. Students who stated that

Students were asked to compare

- the acceleration of block A with the acceleration of block B, and
- the force exerted by string 1 on Block A with that exerted by string 2 on block B.

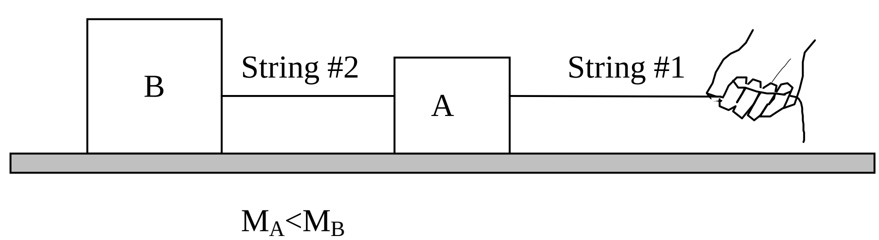


Figure 7 - 2

Question asked by the University of Washington PEG. The question is similar to the question posed in Figure 7 - 1.

the two tensions were equal (20%) seemed to have this particular belief.[10] Our data from the University of Maryland supports these results.

Pre-Tutorial responses

Student responses on version 1 of the problem, given at Maryland, showed that the students had many of the same difficulties that were reported by the Washington PEG. To get an idea of some of the difficulties the students were having we will concentrate on the responses given by Class A (N = 92). We concentrated on Class A because those students took the exam before students in class B and there were no absolute measures taken to ensure that students in class B had no prior knowledge about the exam. The data shows that these students did not connect their conceptual understanding with the quantitative calculation of the tension in the string. Performance on the qualitative part (free-body diagram) was good, but very few students answered the quantitative question correctly.

On the free-body diagrams, students had the most difficulty with the tension forces on each block. The tension force on the first block was drawn correctly by 70% of the students, while 12% of the students were missing the tension force on the first block. An additional 10% drew a line connecting the two blocks but did not include any direction for the force. (It is also possible that this line did not represent a force. It could have been included as simply part of the picture.) On the second block, 80% of the students drew the tension force correctly and 9% of the students drew a line with no direction. In addition to these errors, 4% of the students drew an extra force on their free-body diagrams. The extra forces were usually added to the 2nd blocks free-body diagram but this extra force was not labeled consistently from one student to another. The other forces that were acting on the blocks, including the weight, normal, and the applied force, were drawn correctly by over 90% of the class. Figure 7 - 3 summarizes how the students performed on the free-body diagrams. (Figure 7 - 4 shows how students performed post tutorial on the free-body diagrams. Solid vectors represent correct forces and dotted vectors represent incorrect forces.

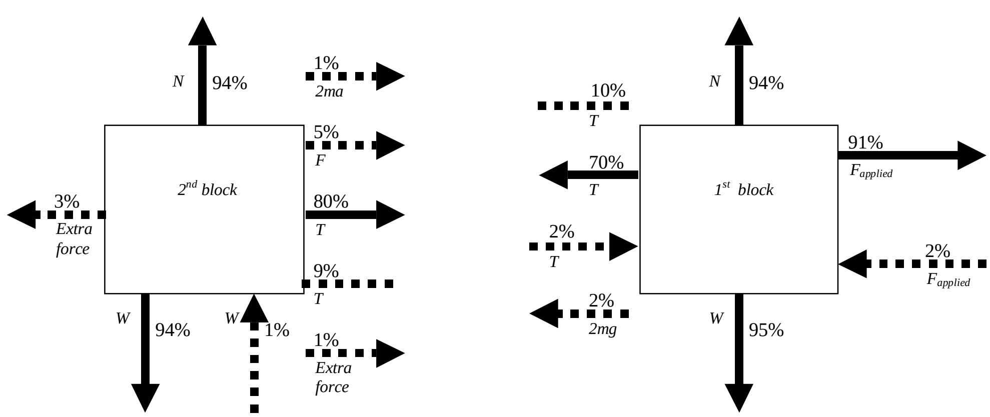


Figure 7 - 3

Student performance on the free-body diagrams for each block on Exam 1 (pre-T). Solid vectors represent correct forces and dotted vectors represent incorrect forces.

Only 7% of the students answered correctly on the quantitative question, where they are asked to calculate the tension in the string. These results show that a correct free-body diagram had little effect on how students answered the quantitative question. Possibly because of the wording in the problem, some students obtained an answer for the tension that did not include all the given quantities. We therefore created a category called correct (general) which included students who solved for the tension correctly and students who summed the forces correctly but did not solve for the tension in terms of just the given quantities (These responses were classified as "correct-unfinished.") The correct (general) category accounted for 29% of the students.

There were two profound errors that indicated that students were not attaching a correct conceptual meaning to the algebraic form of NII. The first error, "eq," involved the students setting the applied force equal to the tension force. The second error, "s," involved the students summing the forces on a particular system and setting it equal to the mass of a different system times the acceleration of that system. A student might sum the forces on the first block yet set that equal to the mass of both blocks times the acceleration.

Figure 7 - 5 shows examples of four types of responses that students gave on this problem. Many of the students answering incorrectly could not be categorized due to the nature of their responses. The first part of Table 7 - 2 shows how the students answered the quantitative question on exam 1.

Post-Tutorial instruction

Student performance on version 2 of the question, which was asked on exam 3, was better than their performance on version 1. Although performance improved on both the qualitative part, where they are asked to draw free-body diagrams for each block, and on the quantitative part, where they were asked to calculate the tension, the improvement was not dramatic. Again students performed well on the free-body diagrams. On version 2 all forces were drawn correctly by more than 90% of the students, although 4% of the students still drew an extra horizontal force. Figure 7 - 4

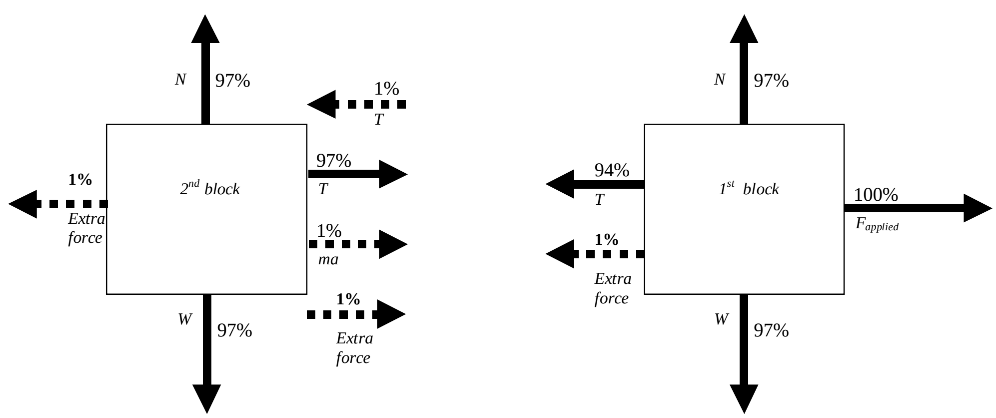


Figure 7 - 4

Student performance on the free-body diagrams for each block on exam 3 (post-T). Solid vectors represent correct forces and dotted vectors represent incorrect forces.

shows the results on the free-body diagrams. There was also improvement on the quantitative question. The second row of Table 7 - 2 shows the results for the calculation of the tension.

It is difficult to say why the students are performing better on the third exam. Besides having additional tutorials on NII the students had an additional homework assignment involving NII problems. In addition, the students were seeing the exam problem for the second time and the instructor in the course went over the first exam in Classes A and B. Each of these components most likely had some effect on how the students performed on the third exam.

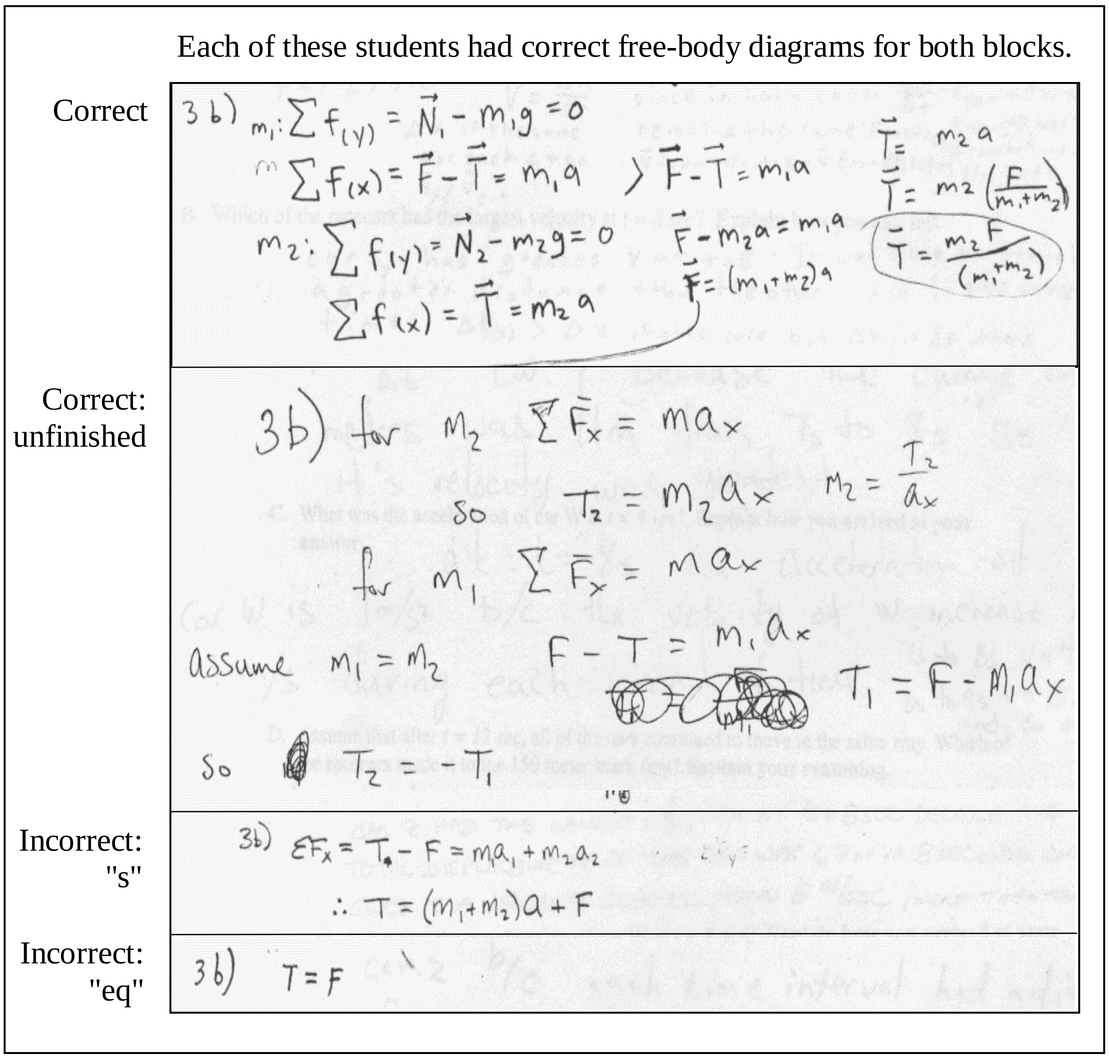


Figure 7 - 5

Examples of four different types of responses for the quantitative part of the question. Despite the different responses, each of these students had the correct free-body diagram for part (a). Note that the correct general category includes both the correct responses shown above.

We believe that seeing the problem a second time did not benefit the bulk of the class. Class C was presented this question only once, on the third exam. Their performance on this question was slightly worse than the performance from class A and Class B, but Class C performed worse on many types of measures throughout the semester. In particular the FCI provides a somewhat standardized measure of overall performance in the class. Students were given the FCI both before and after instruction. Students in all three classes performed about the same on the pre-test, indicating that the three populations were similar. The quantity often used to evaluate instruction is the gain on the FCI from pre-instruction to post-instruction. Classes A and B had a gain of 0.32 and class C had a gain of 0.25. So although class C performed worse on the exam question, since they performed worse on other measures also, we believe that seeing the question twice is not the main cause of the improvement we observe in class A's performance.

Calculate the tension in the string.	Correct: General	Incorrect: eq/s	Incorrect: Other	
Exam 1 (pre)	29% ± 5%	24% ± 4%	48% ± 5%	**N=92**
Exam 3 (post)	49% ± 6%	25% ± 5%	26% ± 5%	**N=76**

Table 7 - 2

Class A's performance on the quantitative part of the problem, where students are asked to calculate the tension in the string.

Case Examples

To better understand how student's progress from pre tutorial (pre-T) to post tutorial (post-T) it will be helpful to look at some sample student responses. Figure 7 - 6 shows the responses of two different students on exam 1 and exam 3. These two students show two different types of progression from the first exam to the third exam.

Student A's response on the free-body diagram is incorrect on exam 1; he neglects to include the tension force pulling back on the first block (block 1). His response on the quantitative part, on exam 1, is also incorrect; he states that the tension force is equal to the applied force. These responses also show that he is not relating the free-body diagram to the quantitative form of NII.

Student A performs much better on exam 3. He correctly sketches the free-body diagrams and seems to connect the quantitative representation of NII with the diagram.

On exam 1student B also answers the qualitative part of the question incorrectly. He makes the same error on the quantitative part that student A made. Again the qualitative representations is inconsistent with the quantitative representation.

On exam 3, student B does include correct free-body diagrams for each block. (Note that the blocks for student B are reversed compared to student A). He also seems to use NII to answer the quantitative part, by setting the force equal to the mass times the acceleration, unlike his response on exam 1 where his answer seemed to have little to do with NII. Unfortunately this student seems to have difficulty isolating the relevant system, and he states that the tension force is equal to the applied force. This student makes the same error on the quantitative part of the question on exam 3 that he made on exam 1, despite his now correct free-body diagram.

The responses on exam 1 for the qualitative and the quantitative questions are similar for the two students. It is clear that the free-body diagrams are not being used correctly, if at all, when the students are trying to solve for the tensions.

Pre-post Progression

At this point we examine the pre-post progression of the entire class. Students' responses from exam 1 (pre-Tutorial) to exam 3 (post-T) were classified into categories representing the progression from pre-T to post-T. We did this for the students' responses on the qualitative and the quantitative parts of the question. The categories are:

- "c" meaning that the student responded correctly on both pre-T and post-T,
- "+" meaning better on post-T than pre-T,
- "0" meaning same (but incorrect) on pre-T and post-T, and
- "-" meaning worse on post-T than pre-T.

It should be noted that these categories are general. For instance, assigning a "0" to a student's quantitative response means the student answered incorrectly pre-T and post-T; it doesn't necessarily mean that the student made the same type of error both times. Classifying each type of response according to the particular answer would require too many categories and therefore make the interpretation of results very difficult.

We introduce a *pre-post progression table* to show how students perform on the qualitative part and the quantitative part of the questions both before and after the tutorial curriculum. The pre-post progression table therefore tells us about the change in the student responses. Student performance is characterized by four categories: correct, same, better, and worse. A matrix is then formed showing the dynamics of how students perform pre-tutorial (pre-T) and post-tutorial (post-T)[11] on the qualitative part and the quantitative part. This allows us to get a picture of the types of links students are developing after going through the tutorial curriculum.

The table shows how the students in the class progressed both qualitatively and quantitatively from the beginning of the course (Exam 1) to the end of the course (Exam 3). The qualitative responses are listed in the rows and the quantitative responses are listed in the columns. At this point it will be helpful to show where our case studies, shown in Figure 7 - 6, belong in the pre-post progression table.

	Student A	Student B
Exam 1 Response Pre-Tutorial		
Exam 3 Response Post-Tutorial		

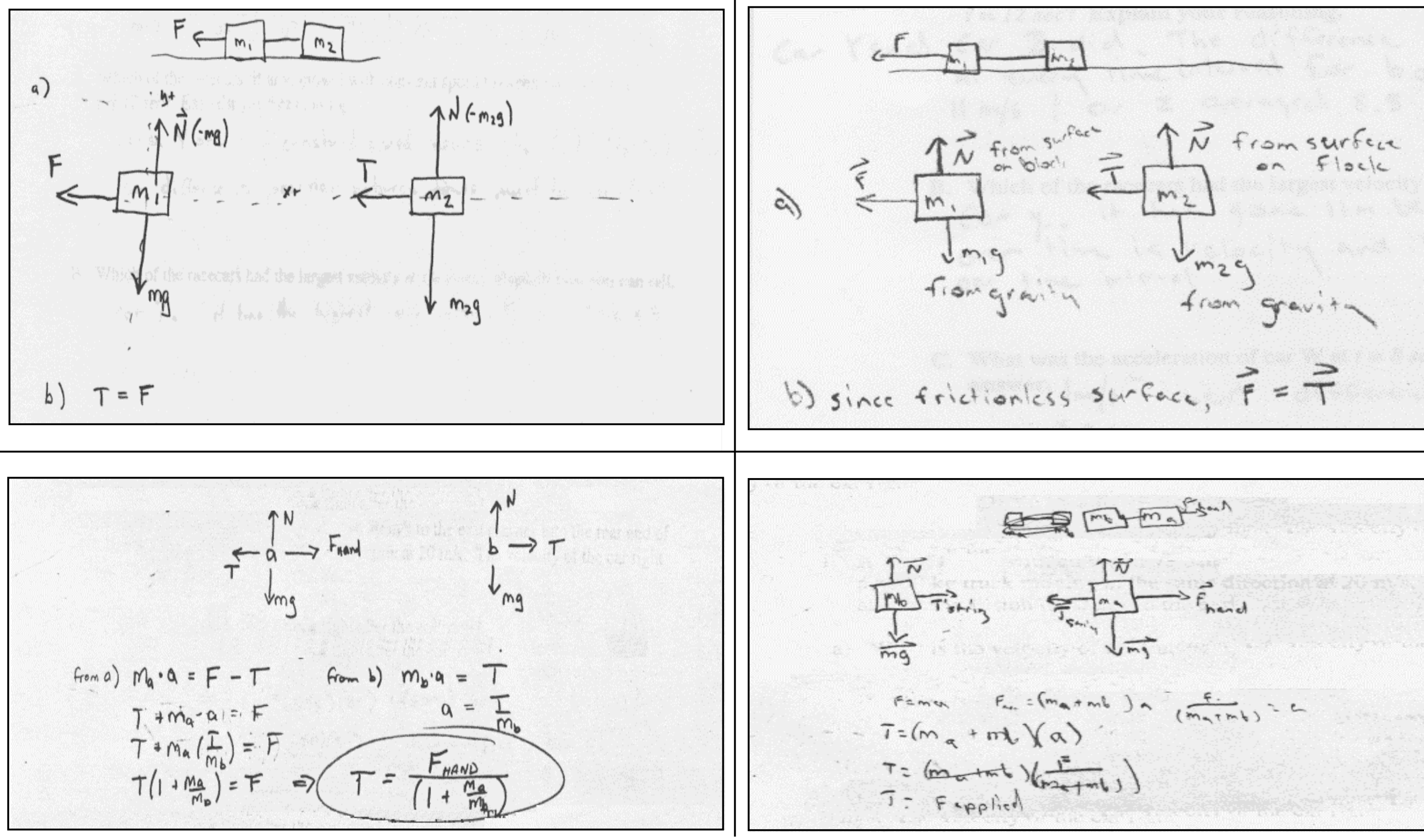


Figure 7 - 6

Examples of how two students responded on exam 1 and exam 3.

Student A's free-body diagrams improved from the pre-T version to the post-T version. In the pre-T version student A's free-body diagram was missing the tension force on the first block, while on the post-T version all the forces are included on the free-body diagrams. We would therefore record a "+" for the qualitative response. The quantitative response also improved. In particular we see student A correctly using the free-body diagram to obtain the algebraic form of NII on exam 3. This is coded as a "+" for the quantitative question. This student would therefore be in the "+" • "+" section of the table. The table shows that 9% of the students were in this category. Student B would instead be coded as a "+" • "0" indicating that his free-body diagram was better on post-T, but his quantitative response did not improve. On both the pre-T and post-T version the student stated that the tension is equal to the applied force. The "+ " • "0" section of the table includes 16% of the students in the class.

The table gives us an indication of how the student's qualitative schema and quantitative schema are linked. In particular the students who answered the qualitative question correctly both pre-T and post-T provide some useful insights. Despite the fact that so many students had correct free-body diagrams in both versions of the question, many students were unable to use the free-body diagrams correctly in order to solve for the tension in the string. These students had trouble making the connection between the free-body diagram representation and the quantitative representation of NII.

Pre-post progression	**Quantitative Question**				**N=76**
Qualitative Question	c	+	0	-	TOTAL
c	1%	24%	27%	5%	57%
+	1%	9%	16%	5%	31%
0	0%	0%	7%	0%	7%
-	0%	4%	1%	0%	5%
TOTAL	2%	37%	51%	10%	**100%**

Table 7 - 3

Pre-post progression table showing how students answered both the qualitative part and the quantitative part of the problem on both exam 1 and exam 3.

One would hope that conceptual exercises such as the tutorials help the students make the bridge between these two representations. Table 7 - 3 shows that some students are improving on the quantitative question. In particular, 24% of the students had the qualitative question correct both pre-T and post-T, and improved their quantitative responses on the post-T. There are a number of factors that could have led to these improvements and although this result is encouraging, we cannot say what factors were responsible for the improvements. It is possible, although unlikely, that students are simply remembering the correct answer on the first exam. Students in the "c " • "0" give us more information. These students answered the quantitative question wrong on both the pre-T and the post-T versions despite having the correct free-body diagrams on both versions. This result points out a limitation in the tutorial curriculum. The links between the qualitative representation and the quantitative representation are not being made by these students.

There were also a large number of students who improved on the qualitative part of the problem but did not improve on the quantitative part. These students are represented by the "+" • "0" and the "+" • "-" elements in Table 7 - 3. This type of progression accounted for 21% of the students in the class. This result is consistent with the results on the electric potential problem, discussed in chapter 5. In that example, many students had a good qualitative understanding of the electric field but had trouble linking it to the equation for the potential. These results show that even if qualitative understanding improves, it does not guarantee an improvement in quantitative problem-solving.

From their written responses it is hard to determine why the students performed so poorly on the quantitative question. Some of the students may be using NII haphazardly and not attaching meaning to the algebraic representation. In addition, some of the students may be attaching an incorrect qualitative understanding to either the free-body diagram or the algebraic representation. One-on-one interviews could be used to probe deeper into some of these issues.

Summary

We were able to determine the dynamics of coherence between qualitative knowledge and quantitative knowledge by looking at student responses to a question containing both qualitative and quantitative parts, both before and after a modified curriculum. We examined whether the tutorial curriculum, developed by the University of Washington PEG, helped students with the concepts of NII and tension and the application of these concepts to quantitative problem-solving.

Although the tutorial curriculum is effective in helping students obtain a qualitative understanding, there is only a small improvement in quantitative problem-solving on this problem; and this improvement may not be due entirely to the tutorial.[12]

Performance on the qualitative question was good both before and after tutorial instruction. We saw that 57% of the students answered correctly on the free-body diagram both pre-T and post-T and 88% of the students answered correctly post-T. On the quantitative question only 2% of the students answered correctly both pre-T and post-T and 39% of the students answered correctly on the post-T.

Our study shows that the tutorial curriculum is not sufficient for many of students to make the connections between their qualitative knowledge and their quantitative knowledge in this context. Many students still made errors on the quantitative part of the question, where they are asked to calculate the tension, despite having correct free-body diagrams. There are 27% of the students in the category "c" • "0" indicating that even though they had little difficulty with the free-body diagrams both pre-T and post-T they could not apply the free-body diagram correctly to solve for the tension in the string.

But some students are making the connections between their qualitative knowledge and their quantitative knowledge by the third exam. Students who have correct free-body diagrams both pre-T and post-T and show improvement on the quantitative part of the question are the strongest evidence for these links. Although most students taking exam 1 had a correct free-body diagram, they had difficulty using the free-body diagram to solve for the tension.

[1] E. F. Redish, J. M. Saul, and R. N. Steinberg, "Student expectations in introductory physics," Am. J. Phys. **66** (3), 212-224 (1998).
[2] L. C. McDermott, P. S. Shaffer, and the PEG, *Tutorials in introductory Physics*, (Prentice Hall, NY, 1997).
[3] For more information on the Tutorial curriculum see chapter 3 of this dissertation and P. S. Shaffer, "Research as a guide for improving instruction in introductory physics," Ph.D. dissertation, Department of Physics, University of Washington, (1993).
[4] For a research paper on student understanding of the concept of tension which includes an identical question see L. C. McDermott, P. S. Shaffer, and M. D. Somers, "Research as a guide for teaching introductory mechanics: An illustration in the context of the Atwood's machine," Am. J. Phys. 62 (1) 46-55 (1994).
[5] R. A. Serway, *Physics for Scientists and Engineers*, 4th edition, (Saunders College, Philadelphia, 1996).
[6] For the actual topics see L. C. McDermott, P. S. Shaffer, and the PEG, *Tutorials in introductory Physics*, (Prentice Hall, NY, 1997).
[7] See Ref. 4.
[8] See Ref. 4.
[9] Many researchers have documented these difficulties and others in introductory mechanics. The reader is referred to Ref. 4 for additional sources.
[10] For more detail see Ref. 4.
[11] Note that pre-T means that the question was asked before most of the tutorials were given. The students did have a single tutorial on forces before exam 1. See the timeline on page 160 and the context for more detail.
[12] Others have begun to study these issues also. Steve Kanim has designed supplemental homework assignments that have proven effective in helping students link the concepts from tutorial to quantitative problem-solving. See S. Kanim, "An investigation of student difficulties in qualitative and quantitative problem solving: Examples from electric circuits and electrostatics," Ph.D. dissertation, Department of Physics, University of Washington, (1999).