Chapter 6: Using Complex Problems to Evaluate Coherence in Qualitative and Quantitaitve Knowledge

Chapter 6: Using Complex Problems to Evaluate Coherence between Qualitative and Quantitative Knowledge

Introduction

In addition to schemas centered around physics topics, the last chapter provided evidence that many of our students have isolated schemas for qualitative and quantitative knowledge. In this chapter we present results showing how students perform on qualitative questions and quantitative questions about various physics topics. The results we present in this chapter will show that the qualitative knowledge and quantitative knowledge our students possess about a particular topic often do not overlap, and may even be contradictory. For example, we have already seen how sometimes students write down the equation for Newton's 2nd law (NII) yet still believe that a net force is needed to keep an object moving at constant velocity. We will show that students often do not attach qualitative meaning to their quantitative knowledge.

Research on physics problem-solving has clearly demonstrated that experts tend to use their qualitative knowledge, more than novices, in solving quantitative problems.[1] In addition, Larkin has found that experts will use their qualitative understanding in formulating physical representations.[2] In this chapter we specifically focus on the coherence between qualitative knowledge and quantitative knowledge for novice problem-solvers. Our results indicate that students' qualitative and quantitative schemas for answering questions and problems are only weakly linked. We show that students often use either a qualitative schema to solve a problem or a quantitative schema, but rarely use integrated schemas.

This implies that giving our students either qualitative questions or quantitative questions on exams to evaluate their performance will only evaluate the quality of a particular schema; it will tell us little about the quality of the coherence in their physics knowledge. For instance, in an introductory physics course for pre-medical and engineering students, the students are usually evaluated by their performance on quantitative problems. Students who are good at formula manipulation are often rewarded by receiving good grades in the course.[3] To get a deeper understanding of our students' knowledge, physics education researchers try to incorporate different types of evaluation techniques to probe deeper into students' understanding.[4]

One common assumption instructors make is that the ability to solve traditional homework and exam problems implies qualitative understanding. There is by now an extensive literature that documents that quantitative skills do not always imply qualitative understanding and our results confirm this. This supports the claims of physics education researchers who have shown that students often pass the introductory physics course without a basic conceptual understanding of the material.[5] Students leave our courses with many of the same misconceptions with which they entered; but now they can solve traditional problems.

We also provide results that show that improving students' qualitative understanding does not imply that students will automatically use their qualitative understanding in solving quantitative problems. Therefore, innovative curricula that have been shown to improve our students' qualitative understanding are not sufficient to produce effective problem-solving in some contexts.

By applying the theoretical framework of schema theory to qualitative knowledge and quantitative knowledge we can begin to make sense of these issues. Schema theory helps us understand why success on quantitative problems tell us little about the quality of qualitative understanding, and why success on qualitative questions tell us little about the quality of problem-solving skills. Unlike expert problem-solvers who tend to integrate qualitative and quantitative knowledge, our students find it difficult and consider it unnecessary to go back and forth between these two types of knowledge.

At this point it will be useful to define what we mean by quantitative questions and qualitative questions. For experienced physicists this distinction is not necessarily an obvious one. An experienced physicist will usually see a problem as containing both qualitative and quantitative aspects. For instance, when presented with an equation such as $x - x_o = \frac{1}{2} at^2$ experienced physicists will integrate a story of the motion with the equation. They will picture an object starting at some position, from rest, moving along a straight line with increasing velocity. Novice physics students will often just see the equation as a way to solve for a particular variable; they do not consider the qualitative picture of what is happening important. For this dissertation we will define *quantitative questions* as questions that either require the students to solve for a given variable in terms of symbols or to actually calculate a specific value for a particular variable. Often these problems can be solved without requiring a qualitative understanding of the underlying physics; students often solve these problems by selecting a formula from a list (by identifying the presence of particular variables) and then manipulating equations. These types of problems are typically found at the end of the chapter in a physics textbook.[6] *Qualitative questions* are questions where the student reasons conceptually instead of algebraically about a given situation. Again, for some questions the distinction will be difficult. Although these definitions are simple, they will allow us to classify many questions as being either qualitative or quantitative.

In the paragraph above we have classified qualitative and quantitative questions simply by the questions themselves, but this is a simplification. A more accurate definition of a qualitative and a quantitative question would depend on the characteristics of the problem and the individual solving the problem. Simply stated, a question which activates qualitative knowledge is a qualitative question and a question that activates quantitative knowledge is a quantitative question. The particular knowledge an individual has will play a large role in which schema gets activated, or whether a schema with both qualitative and quantitative knowledge is activated.

To make the distinction clearer let's look at an example from Serway[7] shown in Figure 6 - 1. Based on our simple definition, using only the characteristics of the question, we would tentatively classify this problem as quantitative because the students are asked to calculate the value of the potential differences for a set of circuit elements. Most experts would see this question as more of a qualitative question. Rather than applying Kirchoff's voltage law on two different loops, they would reason conceptually about the current and the properties of the different circuit elements.

After a conceptual analysis they would then apply some simple formulas, instead of solving two simultaneous differential equations, to find an answer. Some students treat this question as quantitative, applying little conceptual reasoning, while others treat this question as qualitative. Two student responses are shown in

Figure 6 - 2 (a, b) depicting the different ways different students can approach this problem. One students clearly answers the questions using qualitative reasoning, while the other student clearly begins with a quantitative analysis.

The switch in the figure at right is thrown closed at t = 0. Before the switch is closed, the capacitor is uncharged and all currents are zero. Determine the currents in L, C, and R (a) the instant after the switch is closed and (b) long after it is closed.

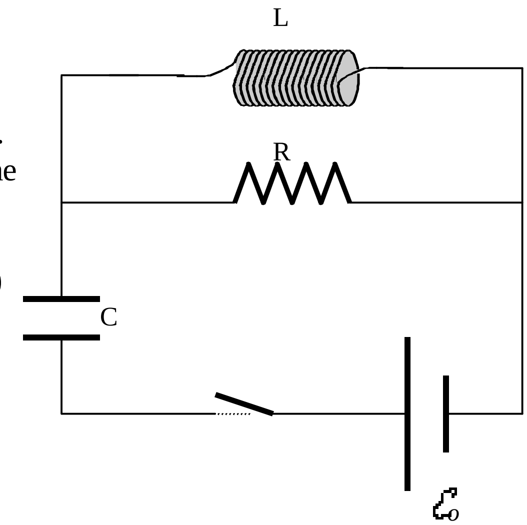
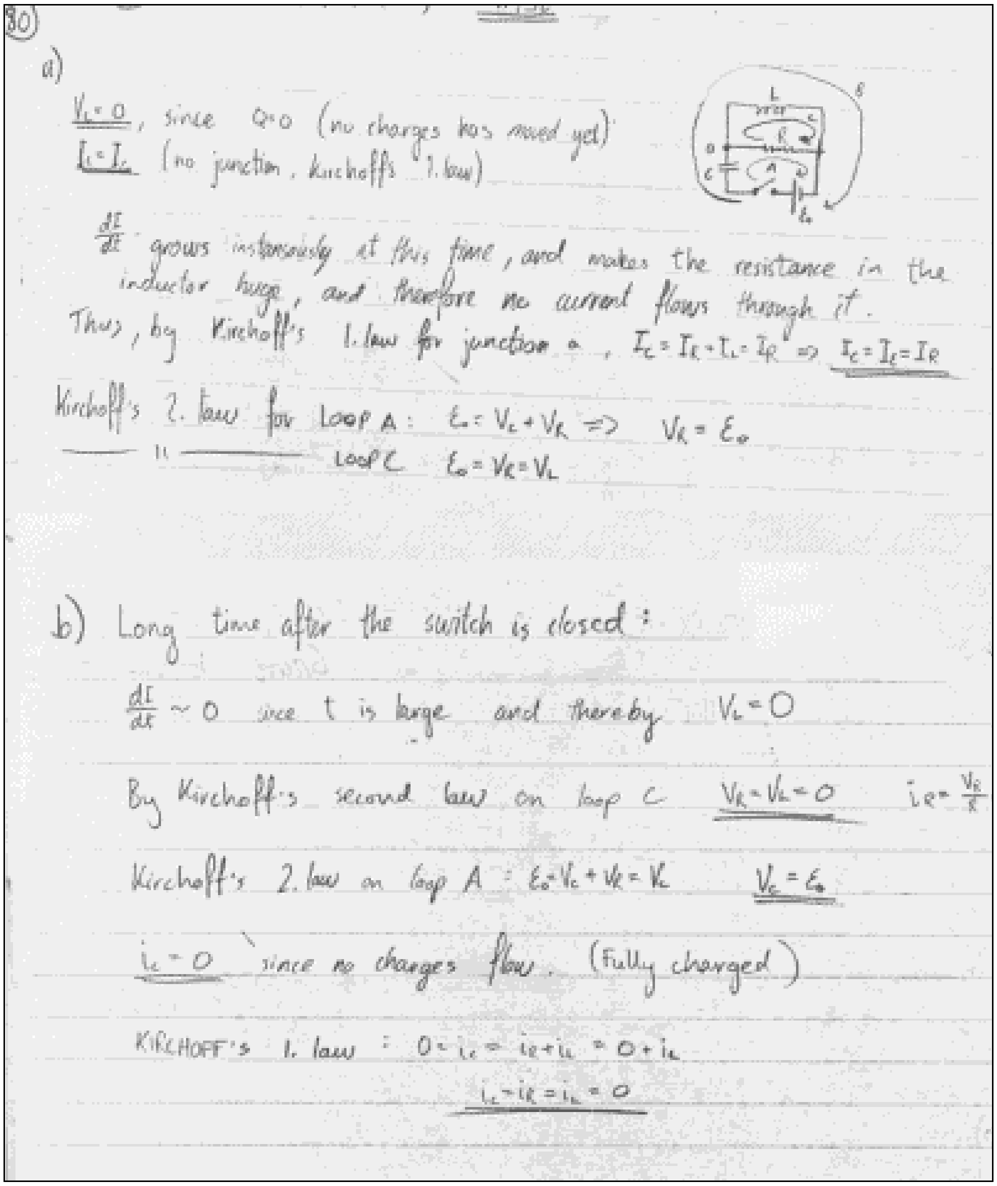


Figure 6 - 1

Inductive circuits problem from Serway. This problem can be interpreted as either a qualitative problem or a quantitative problem depending on the individual solving the problem.

We provide evidence for the presence of largely distinct qualitative and quantitative schemas with data from the three semesters of the engineering physics sequence at the University of Maryland (Physics 161, 262, and 263.)[8] This chapter examines responses on qualitative and quantitative parts to open-ended questions. We do not differentiate the students in the tutorial sections and the recitation sections for most of the studies in this chapter.

We demonstrate the fragmentation between the qualitative knowledge and quantitative knowledge by identifying two types of student responses. In the first case we observed that students can often solve quantitative questions correctly without a conceptual understanding of the physics involved in the question. In the second case we observe that students may have the correct conceptual understanding but not apply this conceptual understanding in solving a quantitative problem.


80)

a)

$V_c = 0$, since $Q = 0$ (no charges has moved yet)

$I_c = I_L$ (no junction, kirchoffs 1. law)

$\frac{dI}{dt}$ grows instantaneously at this time, and makes the resistance in the inductor huge, and therefore no current flows through it.

Thus, by Kirchoff's 1. law for junction a, $I_C = I_R + I_L = I_R \Rightarrow I_C = I_L = I_R$

Kirchoff's 2. law for LOOP A: $\mathcal{E}_0 = V_c + V_R \Rightarrow V_R = \mathcal{E}_0$

" LOOP C $\mathcal{E}_0 = V_R = V_L$

b) Long time after the switch is closed:

$\frac{dI}{dt} \sim 0$ since t is large and thereby $V_L = 0$

By Kirchoff's second law on loop C $V_R = V_L = 0$ $i_R = \frac{V_R}{R}$

Kirchoff's 2. law on loop A: $\mathcal{E}_0 = V_c + V_R = V_c$ $V_c = \mathcal{E}_0$

$i_c = 0$ since no charges flow. (Fully charged)

KIRCHOFF'S 1. law: $0 = i_c = i_R + i_L = 0 + i_L$

$i_c = i_R = i_L = 0$

Figure 6 - 2

(a)

Sample student response showing how students can treat one question either qualitatively or quantitatively. In this solution the student uses his qualitative understanding.

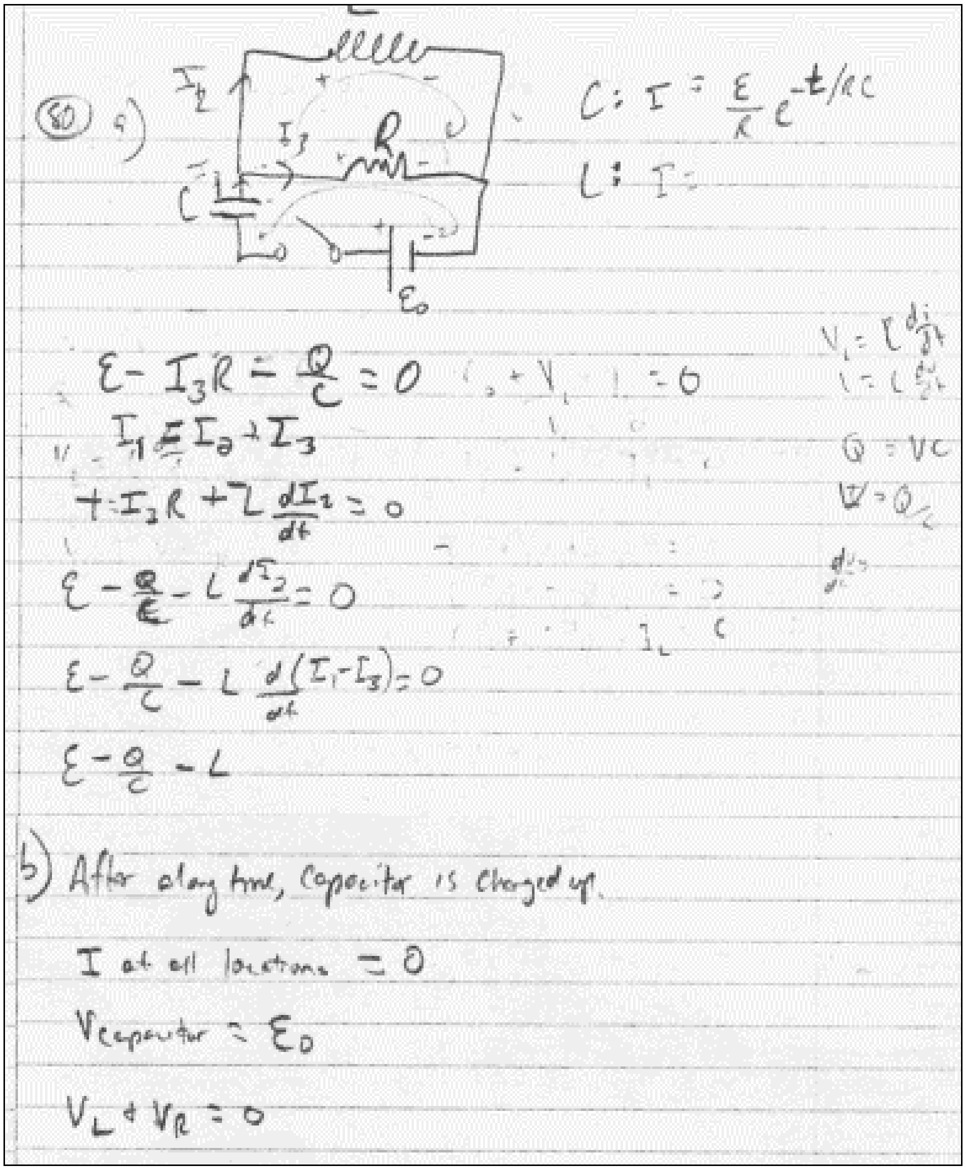


Figure 6 - 2

(b)

Sample student responses showing how students can treat one question either qualitatively or quantitatively. In this solution the student uses a quantitative analysis.

Sample Analysis

Before attempting to describe our large-scale studies we will first look in detail at a sample student response to an exam problem.[9] This allows the reader to become acquainted with some of the issues of interest. This analysis also clarifies what we mean by qualitative and quantitative questions and responses.

The question shown in Figure 6 - 3 was written by the author and was asked in the Physics 263 class at the University of Maryland. The question was asked on an exam and tests the students' knowledge of inductive circuits with a DC voltage source.

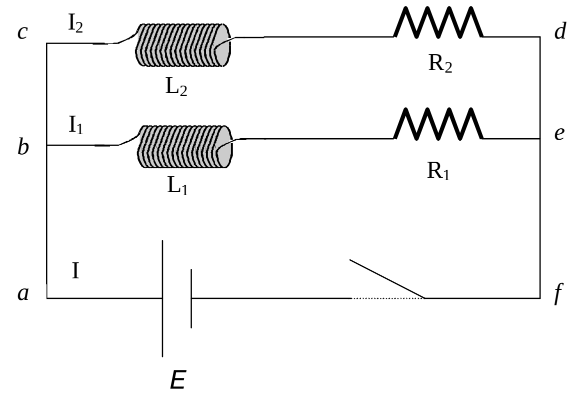
The problem begins with a quantitative question asking the students to write down the circuit equations using Kirchoff's Laws. The next three parts are qualitative questions where students are first asked to reason about the potential differences across each circuit element and then asked to sketch the graph of the current I_2, given the current I_1. The last part of the problem is a quantitative question asking about the energy stored in the inductors a long time after the switch has been closed.

For an expert problem solver none of the parts of this problem are exclusively qualitative or quantitative. The expert problem solver will use schemas containing both types of knowledge to solve each of the parts (if the responses are not obvious). We also expect the expert problem solver to apply qualitative and quantitative reasoning more consistently then our students, since these two types of content knowledge are more integrated in the expert.

One student's response to the exam questions is given in Figure 6 - 4. This student's responses on the quantitative and the qualitative questions are often contradictory. This student is not representative of the whole class or even most of the class, but the response provides us with an example of the type of incoherence we often observe when students are solving complex physics problems requiring both quantitative and qualitative skills.

The student's response in Figure 6 - 4 has been typed in order to make it easier to read. This student shows a clear distinction between his qualitative and quantitative knowledge. For the quantitative questions, parts a and e, the student correctly applies his quantitative knowledge. He uses Kirchoff's voltage law (KVL) over the two loops correctly, identifying the voltage drops across the inductors and the resistors. He also has the correct signs for each of the voltage drops. For the final part the student correctly uses the fact that the current will be a maximum a long time after the switch is closed. One may then conclude that this student has a good understanding of inductive circuits. Without solving the circuit equations, the student realized that the current will become constant after a long time, by either applying a piece of qualitative knowledge or remembering a fact that the student associates with this type of problem.

Consider the circuit shown at right consisting of a battery, two different inductors, and two identical resistors, where $\mathcal{E}$= 20 V, $R_1 = R_2 = 10\ \Omega$, $L_1 = 10\ \mu H$, and $L_2 = 5\ \mu H$. Before the switch is closed there is no current in the circuit. Assume that the battery, the wires, and the inductors have no resistance.

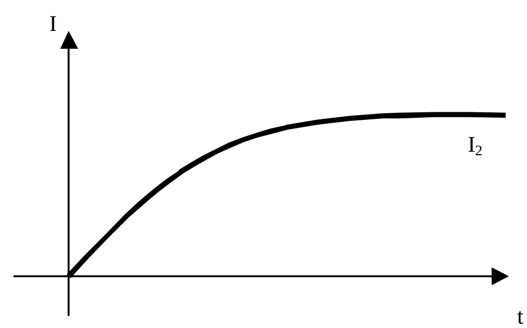


a. Use Kirchoff's Law to write the circuit equation for loop *acdf* and loop *abef*.

b. Rank the magnitudes of the potential differences over the battery, L_1, L_2, R_1, and R_2 **a very short time after** the switch is closed. If any of the potential differences are zero state that explicitly. Explain your reasoning.

c. Rank the magnitudes of the potential differences over the battery, L_1, L_2, R_1, and R_2 **a long time after** the switch is closed. If any of the potential differences are zero state that explicitly. Explain how you know.

d. The figure at right shows the current I_2 as a function of time. Redraw this figure in your exam booklet. On the same figure sketch the graph of I_1. Be sure to make your sketch consistent with the sketch of the current I_2 and be sure to label each of the currents. Explain your reasoning on qualitative grounds using your answer to part a. (You do not need to solve the circuit equations.)



e. Calculate the energy stored in each inductor **a long time** after the switch has been closed. Show all work.

Figure 6 - 3

Inductive circuits problem asked as an exam question in the physics 263 class. This question is an example of a tutorial question with quantitative parts.

When we analyze the qualitative questions we see that the student's qualitative understanding has serious deficiencies, despite his correct answers to the quantitative questions. In part b, a short time after the switch is closed, the student states that the change in current is zero, therefore the voltage drop over the inductors is zero. Here, the student may be confusing current with change in current. The confusion of a quantity and its rate of change is a common error, and appears in the student confusion of acceleration and velocity.[10] The next part of the problem asks about the voltage drops a long time after the switch has been closed. The student states that the voltage drops over the inductors are the largest because of the current in the inductors, even though he states that $V_L = -L\, dI/dt$ and in part d he states that the current will reach a maximum value of $\mathcal{E}/R$.

a. For loop acdf, $\mathcal{E} - L_2\, dI_2/dt - I_2 R_2 = 0$

For loop abef, $\mathcal{E} - L_1\, dI_1/dt - I_1 R_1 = 0$

(since R_1=R_2 we can replace both with R_2 but since the diagram has both R_1 and R_2 I'll leave it in.)

b. $|V_{R1}|=|V_{R2}|>|V_{L1}|=|V_{L2}|=0$
A short time after the switch is closed the change in current across the inductor is zero and V_L=0. V_{R1}=V_{R2} because $R_1 = R_2$.

c. $|V_{L1}|>|V_{L2}|>V_{R1}=V_{R2}=0$
$V_{L2} > V_{L1}$ because I_1=I_2 so the only difference is their L and L_1>L_2. The inductor produces a magnetic field thanks to the current and the induced current. The V_{R1} and V_{R2} are zero because all the current is going through the inductors.

d. $\mathcal{E} - L\, dI/dt - IR = 0$, $\mathcal{E}/R - L/R\, dI/dt - I = 0$

e. $U_f = \frac{1}{2} L I_f^2$
A long time after the switch has been closed *I = Imax*
Imax = Qω=$\mathcal{E}$*/R*
Therefore
$U_{f1} = \frac{1}{2}(10X10^{-6})(20/10)^2 I_f^2$, $U_{f2} = \frac{1}{2}(10X10^{-6})(20/10)^2 I_f^2$
= 2 X 10^{-5} *Joules*

Figure 6 - 4

Student solution to the question in Figure 6 - 3.

When using Kirchoff's voltage law (KVL) in part a, the student presumably uses the fact that the currents are the same in series. He uses I_1 to describe the current across the leg of the circuit *be* and I_2 to describe the current across leg *cd*. This correct application of KVL is contradicted in part c when the student states that the voltage drops over the resistors are zero because all the current is going through the inductors. This provides us with another example of how a question may cue a qualitative schema activating a different response than a question that may cue a quantitative schema. Besides having contradicting qualitative and quantitative knowledge, a student might also possess contradicting qualitative knowledge that can be activated by different cues.

It is important to note that we are not simply looking at whether students score better on quantitative questions or qualitative questions testing the same ideas. We would like to understand the structure of their qualitative and quantitative knowledge and identify consistency and inconsistency in the students' reasoning. The above example shows that the student directly contradicts his qualitative statements and quantitative statements even when the statements deal with the same physical phenomena. The following examples are done on a larger scale to show the pervasiveness of these difficulties.

Momentum Question

The question shown in Figure 6 - 5 was asked in the non-tutorial Physics 161 class at the University of Maryland.[11] This question was posed on the final exam for the class. It consists of two qualitative questions and two quantitative questions.

Two blocks collide on a frictionless surface. After the collision the two blocks stick together. Block A has a mass M and is initially moving at speed v in the $+x$ direction. Block B has a mass $2M$ and is initially at rest.

a) Draw a free body diagram for each block at an instant ***during*** the collision. Rank the magnitudes of the horizontal forces in your free body diagram.
b) What is the final velocity of the blocks after the collision? Show all work.
c) Calculate the change in momentum of block A. Calculate the change in momentum of block B. Show all work.
d) Can a system whose momentum is conserved be made up of smaller systems whose individual momenta are not conserved? Explain your reasoning.

Figure 6 - 5

Question on momentum conservation asked on the Physics 161 final exam.

The students are first asked to draw a free-body diagram for the blocks during the collision. They are then asked about the final velocity of the two blocks after the collision. Since the collision is inelastic, students can determine the velocity of the blocks using the principle of conservation of momentum. In the next part, the students are asked about the changes in momentum of both blocks, A and B. The final part is a qualitative question testing the students on the concept of conservation of momentum.[12] Parts b and c are quantitative questions requiring the application of conservation of momentum. The last three questions require the application of the principle of conservation of momentum.

In part a, students are asked to draw a free-body diagram for both blocks during the collision. Only about 38% of the students had correct free-body diagrams for the blocks during the collision and 15% of the students did not include a free-body diagram. The two most common errors were to include a force on block A in the direction of A's initial motion (9%) or to omit the horizontal forces during the collision (7%), accounting for 16% of the errors. Each of the remaining errors accounted for fewer than 5% of the responses.

In the ranking task, many students (65%) left the question unanswered, possibly because students in a traditional physics class often have little practice with ranking tasks. Only 18% of the entire class correctly ranked the NIII force pairs. The most common incorrect answer, given by 9% of the class was that the force exerted on block B by block A was larger. This is consistent with previous studies on student understanding of the third law.[13]

Students performed better on the quantitative questions in parts b and c. In part b, 81% of the students calculated the final velocity of the blocks correctly. The most common error, given by 11% of the students, was using conservation of energy, instead of conservation of momentum. These results would indicate to most instructors that students have a good understanding of the conservation of momentum. It shows that most of the students were capable of deciding when conservation of energy can and cannot be applied. In part c, 37% of the students calculated the change in momentum of each block correctly. Of the students who answered incorrectly, only 2% of the students stated that the change in momentum of each block was zero. Therefore, about 90% of the students in the class obtained non-zero responses for the change in momentum of each block.

In part d, students are given a general qualitative question about the conservation of momentum. Recall that about 81% of the students used the principle of conservation of momentum in part b and about 90% of the students calculated non-zero answers for the change in momentum of each individual block. We would therefore expect that most of the students would answer "Yes" to the question in part d, possibly citing their answer to part c as evidence. We find that only 53% of the students answered correctly, while 41% of the students answered "No". This result shows a clear distinction between students' qualitative and quantitative knowledge. It shows that having contradictory statements for these two questions is acceptable for many of the students. Either the students are not aware of the contradiction or they recognize the contradiction but do not attempt to resolve it. Table 6 - 1 summarizes student performance on the momentum problem for parts b, c, and d. Responses are divided into correct and incorrect with special cases of incorrect responses included in parenthesis.

N=115

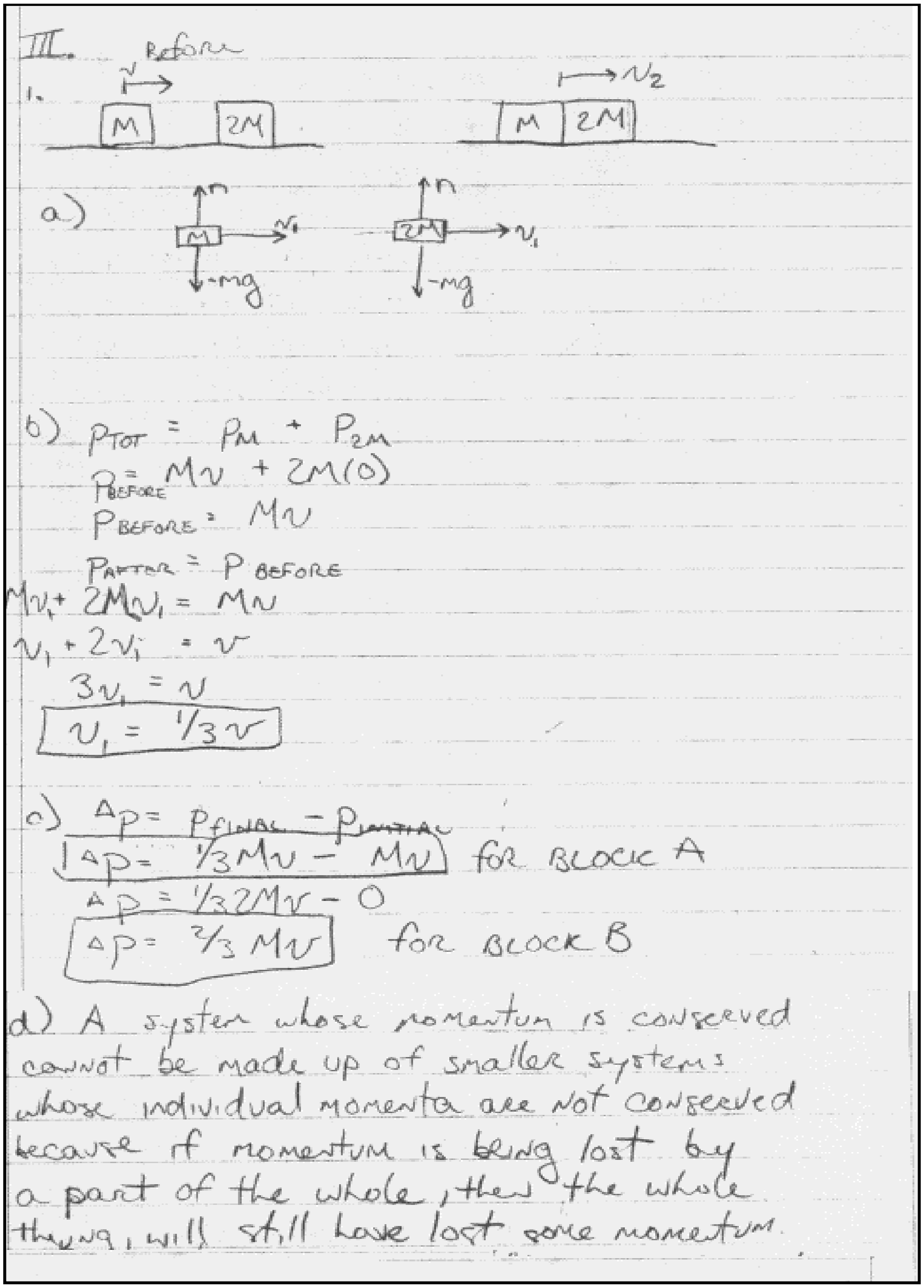
	b) Calculate the final velocities.	c) Calculate Δp for blocks A and B	d) If $\Delta p_{sys} = 0$ can Δp_A and Δp_B be non zero.
Correct	81%	37%	53%
Incorrect	19%	63% (49% had non zero)	47% (41% had No)

Table 6 - 1

Summary of student performance on the momentum question (parts b, c, and d.)

Figure 6 - 6 shows a sample student response to the problem. Although this student answered the quantitative questions correctly using the principle of conservation of momentum, his response on the qualitative questions is incorrect. The free-body diagrams for both blocks include vectors labeled *v*, which most likely represent the final velocity, and not the forces on the blocks. On part d this student directly contradicts his responses to parts b and c. His response to part d also shows that the confusion does not come from a misconception about the term "system." He believes that if one part of the system loses momentum the entire system must also lose momentum. He is therefore not considering that if one part of the system loses momentum another part of the system can gain that momentum. There were a number of students who made this error.

To get a clearer picture of how students are responding to the quantitative question in part c and the qualitative question in part d we concatenated their responses on these two questions. The results of this comparison are shown in Table 6 - 2. (The questions for part c and d are paraphrased in the table.) The darkly shaded cells of the table represent the percentage of students who were consistent in their responses to the two questions. The lightly shaded cells represent the percentage of students who answered the two questions inconsistently. As an example, we see from the table that 37% of the students obtained the correct answer for the quantitative question (part c). Of those students 19% of the class answered the qualitative question consistently, stating that a system whose momentum is conserved can be made up of smaller systems whose momenta are not conserved. We also see 16% of the class answered inconsistently, stating that a system who's momentum is conserved must be made of smaller systems whose momenta are conserved, even though they had shown in part c that the change in momentum of block A and block B were non-zero for this situation.



III. Before

1. $\vec{v}$ M 2M — v_2 M 2M

a) n, v_1, M, -mg; n, 2M, v_1, -mg

b) $p_{TOT} = p_M + p_{2M}$

$p_{BEFORE} = Mv + 2M(0)$

$p_{BEFORE} = Mv$

$p_{AFTER} = p_{BEFORE}$

$Mv_1 + 2Mv_1 = Mv$

$v_1 + 2v_1 = v$

$3v_1 = v$

$v_1 = 1/3\, v$

c) $\Delta p = p_{FINAL} - p_{INITIAL}$

$\Delta p = 1/3\, Mv - Mv$ for BLOCK A

$\Delta p = 1/3\, 2Mv - 0$

$\Delta p = 2/3\, Mv$ for BLOCK B

d) A system whose momentum is conserved cannot be made up of smaller systems whose individual momenta are not conserved because if momentum is being lost by a part of the whole, then the whole thing, will still have lost some momentum.

Figure 6 - 6

Sample solution to the momentum exam question shown in Figure 6 - 5.

Quantitative Question Response (c) Calculate ΔPA, ΔPB.	**Qualitative Question Response (d)** If $\Delta P_{sys} = 0$ can ΔP_A and ΔP_B be non zero?			**N=115**
	Correct: Yes	Incorrect: No	Incorrect: Other	TOTAL
Correct	19%	16%	2%	37%
Incorrect: Non Zero	27%	21%	1%	49%
Incorrect: Zero	1%	1%	0%	2%
Other	8%	3%	2%	13%
TOTAL	55%	41%	4%	**100%**

Table 6 - 2

Table showing how students answered different parts of the question in Figure 6 - 5. The results show that many students answer inconsistently on the quantitative and qualitative parts of the question.

Circuits Question

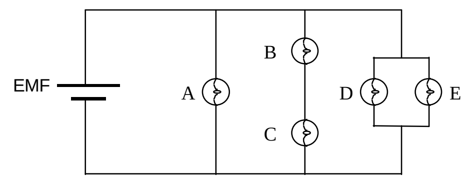
The mixed qualitative and quantitative question, shown in Figure 6 - 7, was asked in the physics 262 class at the University of Maryland. The results show that students can perform reasonably well on a quantitative question yet they can have serious conceptual difficulties.[14] This question is similar to the question presented by Eric Mazur[15] at Harvard University and a pair of questions given by Steve Kanim[16] at the University of Washington.[17]

The question shown in Figure 6 - 7 was given on the final exam in two Physics 262 classes. Each class consisted of three hours of lecture, two hours of lab, and either one hour of tutorial or one hour of traditional recitation each week. Students in the tutorial section had no specific tutorial instruction on circuits.

Part A of the problem is a qualitative question that comes from the University of Washington PEG.[18] Part B is a standard quantitative question. The correct solution to part A is that bulbs A, D, and E all have equal brightness because they are all connected in parallel and therefore each bulb will have the same voltage drop as the battery. Bulbs B and C would be dimmer since the resistance in that leg of the circuit is greater and the voltage drop over both bulbs is the same as the battery. One possible solution to part B would involve a straightforward application of Kirchoff's voltage rules. Another correct method used by students was to calculate the

In this problem, assume that the battery is ideal and that all wires have zero resistance.

A. Consider the circuit shown at right containing 5 identical bulbs labeled A-E. Rank the brightness of the bulbs from most bright to least bright. If any bulbs are equally bright, state that explicitly. Explain your reasoning.



B. For the circuit shown at right, find the current through R_1, the 10 ohm resistor. Show how you arrived at your answer.

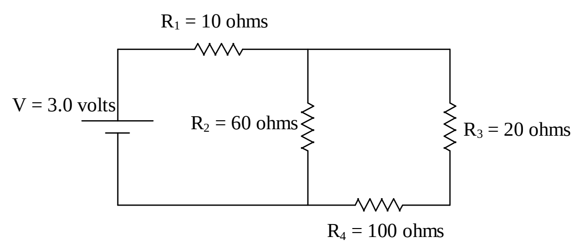


Figure 6 - 7

A final exam question with a qualitative question (A) and a quantitative question (B). Students performed better on the quantitative question despite it being more difficult than the qualitative question.

equivalent resistance in the circuit in order to find the current through R_1. Most experienced physicists would rate the quantitative question (part B) to be more difficult than the qualitative question (part A.)

The results from the two classes are combined in Table 6 - 3. The results indicate that students had significantly more difficulty with the qualitative question. Only 17% of the students answered this question correctly. These results are consistent with the results from the University of Washington PEG where 15% of their population answered this question correctly.[19] Their results also show that students have some fundamental misconceptions with the topics of current and voltage.

The work done by the University of Washington Physics Education Group on the topic of electric circuits gives us an idea of why some of the errors on the conceptual questions were made. Some incorrect models used by students include a model where current gets used up in a bulb and a model where the battery is a constant current source.

Only 44% of the students at Maryland answered the quantitative question correctly. Although performance was better on the quantitative question (part B), the

N=131

Part A: Qualitative	Correct: A=D=E>B=C	Incorrect: D=E>A>B=C	Incorrect: A>B=C>D=E	Incorrect: Other
	17%	21%	19%	43%

Part B: Quantitative	Correct	Incorrect: V_{batt}/R_1	Incorrect: R_{eq} wrong	Incorrect: Other
	44%	21%	12%	23%

Table 6 - 3

The types of responses given on the two circuits questions and the percentages of students giving each type of response.

percentage correct is far from where we would like it to be. The most common error involved the students' incorrect application of Ohm's Law. Students who made this error divided the battery voltage by the resistance, R_1. One possible explanation for this is that students are applying Ohm's Law without attaching conceptual meaning to it. Another explanation is that the students were applying the model where the battery acts as a constant current source, independent of the circuit.

The most relevant result to the issue of coherence between qualitative and quantitative schema is that most of the students who answered this question applied different models when solving parts A and B. Applications of Kirchoff's Rules and calculations of equivalent resistances tell us little about student reasoning. We see from this example that students can have incorrect models for current and voltage yet still correctly apply algorithmic methods, such as Kirchoff's Voltage Rules to solve standard problems.

N=131

Quantitative Question Response (c) Find the current through R_1.	**Qualitative Question Response (d)** Rank the brightness of the five bulbs. Correct	Incorrect	TOTAL
Correct	11%	34%	45%
Incorrect	6%	50%	56%
TOTAL	17%	84%	**100%**

Table 6 - 4

Performance on the qualitative and the quantitative parts of the circuits question.

Table 6 - 4, on the previous page, shows how students performed on the qualitative part and the quantitative part of the circuits problem. We also see that students who have a good qualitative understanding of the material but do not necessarily apply their qualitative understanding to quantitative problems. We see that 35% of the students who answered the qualitative question correctly answered the quantitative question incorrectly. It is also interesting to note that only 11% of the students answered correctly on both the qualitative and the quantitative questions.

Electric Potential Problem

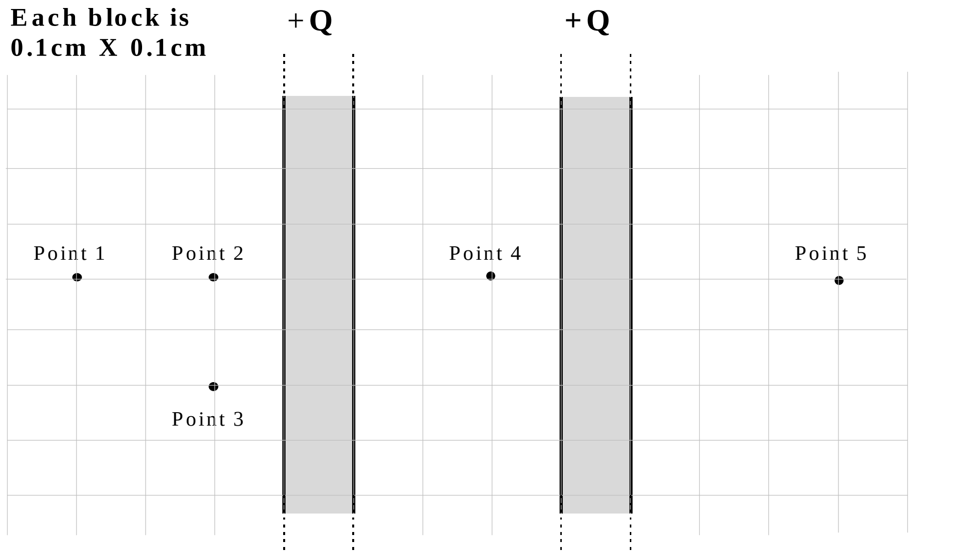
The three examples given above concentrated on students who could solve quantitative problems without a deep qualitative understanding of the material. These results support claims that have previously been reported in the literature. [20] A more surprising result that is usually not discussed, is that students can have the correct qualitative understanding, but do not transfer this qualitative knowledge to quantitative questions.

These results are important because some instructors view the introductory physics course as simply a place for students to develop analytic skills to solve problems. This study shows that good qualitative understanding, although necessary for effectively problem-solving, is not sufficient; a students' qualitative and quantitative schema must be connected. We believe that good analytic skills consist of not merely being able to perform algebraic manipulations; good analytical skills involve applying conceptual understanding and reasoning to solve quantitative problems.

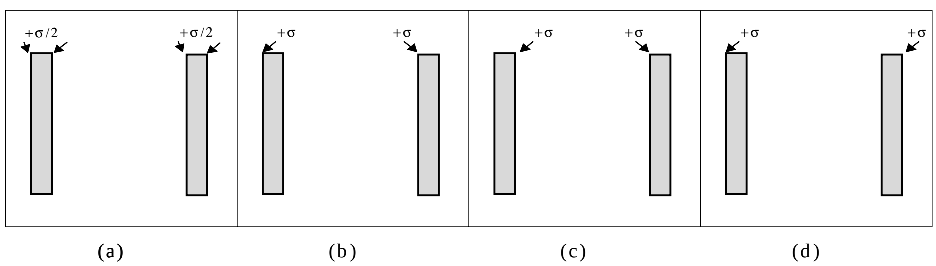
The following example was written by the author and was asked in the physics 262 class at UMd with tutorials as a tutorial homework bridging problem.[21]. The question is shown in Figure 6 - 8. We concentrate on the student responses to parts b and d although the responses on the other parts provide us with additional information about student understanding of this topic. We analyzed 56 student solutions.

The qualitative parts of this question have the students sketching electric field vectors at different positions and identifying the charge distribution on the metal slabs. Students performed well on the qualitative questions. Field vectors were, for the most part, drawn in the correct direction and had the correct magnitude. The most common error was drawing field vectors that changed in magnitude as you looked at points farther and farther away from the plates. About 80% of the students correctly indicated that the field would be zero at point 4, between the slabs, even though the point was not halfway between the slabs. This result will be particularly interesting when we look at how the students answered the final quantitative part of the problem. Students also performed well on the question about the charge distribution. We saw 88% of the students answering part a correctly.

Consider two very large conducting plates a distance D apart. Both plates have a charge of +Q on them. See figure at right.



a. Which diagram below shows the correct charge distribution on the plates? Explain your reasoning.



b. Draw vectors on the diagram above representing the electric field at points 1, 2, 3, 4, and 5. If the field is zero at any point indicate that explicitly.

c. A small charge q, initially at rest at point 1, is moved by a hand so that it comes to rest at point 3. Write an equation for the work done on the charge.

d. If the charge density, σ, is 2×10^{-9} C/m^2, determine the potential difference between points 1 and 5. Show all work.

Figure 6 - 8

Electric potential bridging problem asked as part of the tutorial homework assignment.

Students did not perform as well on the quantitative questions about the work done on a point charge to move it from point 1 to point 3 and the question about the potential difference from point 1 to point 5. Many students gave responses that contradicted their qualitative answers. In part c, 20% of the students wrote down the work done on a point charge to move it in the field of another point charge, instead of in the field due to parallel plates. Only about 32% of the students answered part c correctly, by treating the field as constant and taking into account the dot product for the force and displacement. The results from part d are even more striking. Even though about 80% of the students indicated that the field was zero between the two plates and was pointing in opposite directions on either side of the two plates, many of these students considered **E** to be constant when calculating the potential in part d. Our results show that 27% of the students treated the field as constant in the equation, $\int_1^5 E \cdot dl = -V$. Most of these students (21% of the entire class) stated, in part b, that the field between the two plates was zero, which indicates that they are not using their qualitative knowledge of the electric field in the equation. Table 6 - 5 summarizes the student performance on this problem. The table also lists some of the specific errors made by the students.

N=56

	a) Describe the charge distribution.	b) Draw field vectors at the points indicated.		c) Calculate the work from 1 to 3.	d) Calculate the potential from 1 to 5.
		Point 4	General		
Correct	88%	82%	63%	32%	41%
Incorrect	12%	18%	37%	68%	59%

(21% had vectors with different magnitudes, 7% drew correct field lines)

(20% used work done by a point charge, 11% had no dot product)

(27% treated the **E** field as constant in potential equation.)

Table 6 - 5

Summary of the results on the electric potential problem.

Instructors rarely attach conceptual meaning to equations in the physics course, and when they do, it is usually stated and not written on the board. This can lead to students creating separated schema for the qualitative and quantitative aspects of a concept or principle. In this situation we see that even though we can improve student qualitative knowledge, through the use of modified curriculum, there is no guarantee that these concepts will transfer to quantitative problems. In order for students to become expert problem solvers, not only do students have to develop better qualitative knowledge, they also have to be able to transfer this qualitative knowledge to new situations such as traditional problems.

Interference Problem

The next example shows that students in the top of the class (measured by total overall score) made the error of not connecting the mathematics with the concepts more often than the other students. This is surprising because instructors usually expect the students in the top of the class to have more coherence between the equations and the concepts and principles. In addition this particular error can be classified as a sophisticated type of error because of the level of reasoning involved in the solution. These results show that coherent qualitative and quantitative knowledge can benefit the entire student population, not just the students in the middle and bottom of the class.

The interference problem, shown in Figure 6 - 9, was posed on an exam in the physics 263 class at UMd, with tutorials. Students received instruction on double slit interference both in lecture and in the tutorial class. The physics 263 class at the University of Maryland uses four tutorials written by the PEG at the University of Washington. These tutorials are dedicated to the wave properties of light and physical optics. They have since been revised by the PEG and are included in *the Tutorials in Introductory Physics* book.[22] We will concentrate on part b ii of the problem.

The correct answer to part b ii can be obtained qualitatively by noticing that as the wavelength increases, the path difference, *ΔD*, required for the first minimum also increases. A larger value of *ΔD* means that the angle between the vertical and the path from the slits to the first minimum is greater. Therefore the point P_1 would be between the first minimum and the central maximum. A solution to the problem is shown in Figure 6 - 10. The particular error we will be concerned with shows the lack of coherence between the equation $\left(n+\frac{1}{2}\right)\lambda = d\sin\theta$ and the conceptual aspects of the double slit interference problem. Since we are given the distance to the first minimum, substituting n = 0 into the equation gives the relevant equation; we have $\lambda/2 = d\sin\theta$ for this situation. Approximately 17% of the students made the mistake of substituting 2λ for λ since they were told that the wavelength of light was increased to 1000 nm. This left the students with the following equation, $\lambda = d\sin\theta$ which they stated was the condition for a maximum. These students therefore concluded incorrectly that point P_1 became the position for maximum constructive interference. The students who made this error were no longer considering the point P_1, since they changed the path difference, *ΔD*. We refer to this error as “equation-concept.” Again, this is a sophisticated mistake, but a fairly common and profound error. Students who answered this way were obviously using the concepts; they were using the fact that $\Delta D = \lambda/2$ for destructive interference and $\Delta D = \lambda$ for constructive interference. Students were therefore applying some concepts but the concepts were not correctly associated with the equation. Two sample student responses are shown in Figure 6 - 11 depicting this error.

Consider a plane wave of monochromatic green light, $\lambda = 500$ nm, that is incident normally upon two identical narrow slits (the widths of the individual slits are much less than λ). The slits are separated by a distance $d = 30$ µm. An interference pattern is observed on a screen located a distance L away from the slits. On the screen, the location nearest the central maximum where the intensity is zero (i.e., the first dark fringe) is found to be 1.5 cm from this central point. Let this particular position on the screen be referred to as P_1.

a. Calculate the distance, L, to the screen. Show all work.

b. In each of the parts below, one change has been made to the problem above (in each case, all parameters not explicitly mentioned have the value or characteristics stated above). For each case, explain briefly whether the light intensity at location P_1 would remain zero or not. If not, will P_1 become the location of a maximum constructive interference (bright) fringe? In each case, explain your reasoning.

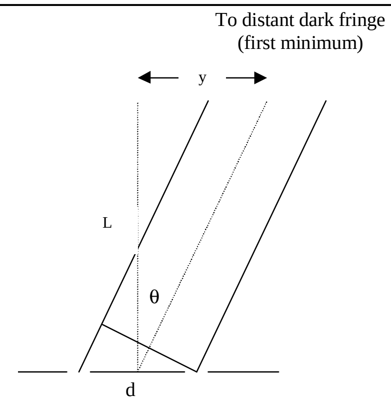
    i. One of the two slits is made slightly narrower, so that the amount of light passing through it is less than that through the other.
    ii. The wavelength is doubled so that $\lambda = 1000$ nm.
    iii. The two slits are replaced by a single slit whose width is exactly 60 mm.

Figure 6 - 9

Exam question on physical optics. We will focus on the student responses to part B ii.

The graph in Figure 6 - 12 shows the percentage of students answering correctly, and answering incorrectly vs. total score in the class. The total score in the class is based on homeworks, exams, labs, and a quiz. The graph also shows the percentage of students making the "equation-concept" error we discussed earlier. The graph is constructed in the following way. We first take the student with the highest score in the class and see the type of response he or she gave. The second point represents the top two students and the percentage of correct, incorrect, and "equation-concept" responses given. The third data point represents the top three student's responses and so forth. The final point represents the percentage of correct, incorrect, and "equation-concept" responses for the entire class. We can therefore look for trends in the responses for the students based on their overall score in the class.

a. The distance L to the screen can be obtained from $\left(\frac{1}{2}\right)\lambda = d\sin\theta$ since we know the distance to the first dark fringe. Since θ is small we have $L = \dfrac{d y_1}{\frac{1}{2}\lambda} = 1.8\ m$



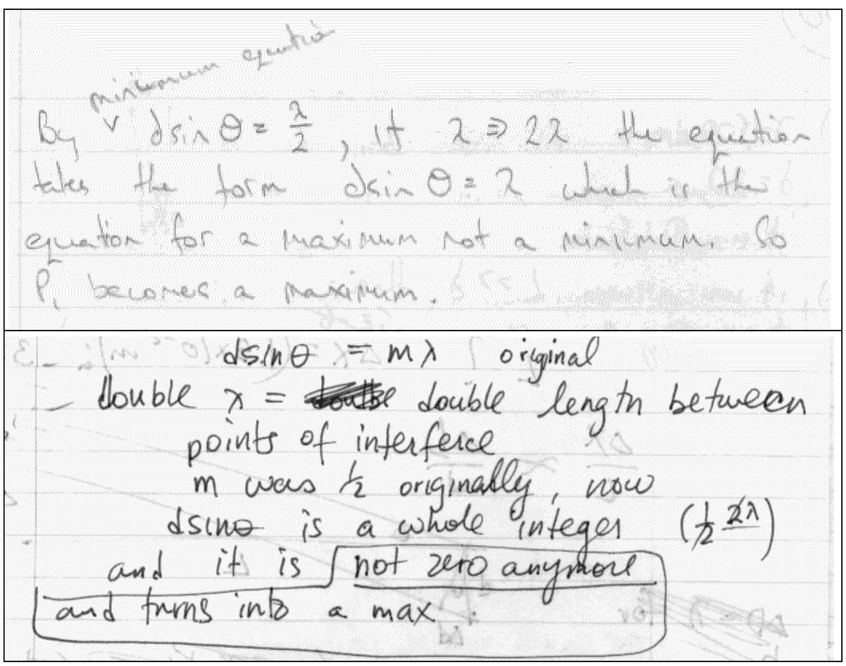
b. i. If one slit is made narrower we will no longer have complete destructive interference although the point will still be a minimum.

ii. If l is doubled the ΔD would have to increase for the first minimum therefore P_1 would be in-between a minimum and a maximum.

iii. The 1st minimum will still be in the same place since the ΔD between the path beginning at the end of the slit and the path beginning at the middle of the slit must be λ/2 and the d for both cases is 30 mm.

Figure 6 - 10

Solution to the problem in Figure 6 - 9.


By minimum equation d sin θ = λ/2, if λ ⇒ 2λ the equation takes the form d sin θ = λ which is the equation for a maximum not a minimum. So P1 becomes a maximum.

d sin θ = mλ original
double λ = ~~double~~ double length between points of interfece
m was ½ originally, now
d sin θ is a whole integer (½ 2λ)
and it is not zero anymore
and turns into a max.

Figure 6 - 11

Two sample student responses showing the "equation-concept" error.

The data series representing the correct responses and incorrect responses behaves as we would expect. The percentage of correct responses increases as we move to the top of the class and the percentage of incorrect responses decreases as we move to the top of the class. The percentage of "equation-concept" type responses is surprising because this particular error actually increases as we move to the top of the class. This result shows that even the more successful students fail to link the equations to concepts. The data also indicates that the top students in the class may rely on equations more than the other students in the class, possibly because of the way they view physics knowledge and the way they view the applications of physics knowledge.[23]

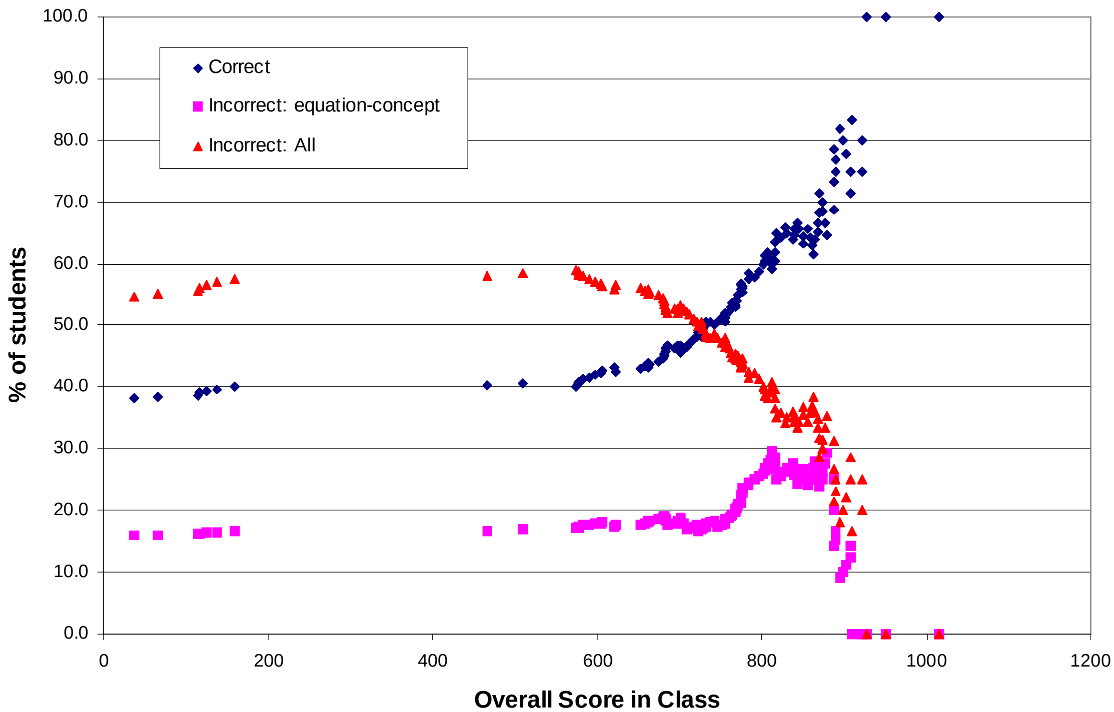


Figure 6 - 12

Graph showing the types of responses students gave versus their total score in the class. The percentage of students making the "equation-concept" error increases as the student score increases.

It would be too simple to say that these students are applying these equations blindly. Their performance on other measures and even their responses on this question show that these students do think qualitatively about equations. One explanation for these results is that the equations cue incorrect conceptual knowledge, which may not be linked to the correct conceptual knowledge. The two sample student responses shown in Figure 6 - 11 indicate that some students are using the concepts but not using the concepts in conjunction with the equation. Physics education researchers have demonstrated that students can have many models at one time and that these models may even be contradictory.[24]

Summary

This chapter has shown that students have difficulty developing coherence between qualitative knowledge and quantitative knowledge in many areas of physics. Although many instructors assume that proficiency in solving textbook problems means that students understand the underlying principles and concepts, physics education research has demonstrated that this is not the case. Similarly many instructors feel that success on conceptual questions implies that students will be able to solve quantitative problems. Again, we see that this is not the case. In order for students to develop coherence in their content knowledge it is necessary for them to integrate their qualitative schema and their quantitative schema.

The examples presented in this chapter show that introductory physics courses often fail to accomplish the goal of developing coherence between qualitative and quantitative schemas. This incoherence persists from the beginning of the introductory engineering sequence until the end.[25] In addition, the data shows that students at the top of the class might also benefit from instruction aimed at developing coherence.

One might argue that genuine quantitative knowledge is constructed from qualitative knowledge and the quantitative problems are drawing out the qualitative difficulties the students are having. Therefore the students who are responding correctly on the qualitative questions, but not on the quantitative questions, actually do not have a good qualitative understanding. Although we agree that the students do not have a deep conceptual understanding of the material, where they are tying together qualitative and quantitative ideas, we believe that their correct qualitative responses can be used for the quantitative questions if the connection between these two schemas is made. So although these students do not possess a deep conceptual understanding of the material, their correct responses on the qualitative questions can serve as a resource for the quantitative questions.

[1] See F. Reif, "Teaching problem solving – A scientific approach," The Physics Teacher, 310-316 (1981); D. P. Simon and H. A. Simon, "Individual differences in solving physics problems," In *Children's Thinking: What develops?* R. S. Siegler (Ed.) (Lawrence Erlbaum, NJ, 1978), pp. 325-348; M. T. H. Chi, P.S. Feltovich and R. Glaser, "Categorization and representation of physics problems by experts and novices," Cognitive Science, **5**, 121-152 (1981); J. H. Larkin and F. Reif, "Understanding and teaching problem solving in physics," European Journal of Science Education, **1,** (2), 191-203 (1979).

[2] See chapter 2 and J. H. Larkin, "The role of problem representation in physics." In *Mental models*, D. Gentner and A. L. Stevens (Eds.) (Lawrence Erlbaum, NJ, 1983), pp. 75-98.

[3] D. Hammer, "Two approaches to learning physics," Phys. Teach. 27 (9) 664-670 (1989).

[4] These techniques are described in chapter 3.

[5] E. Mazur, *Peer Instruction*, (Prentice Hall, NJ, 1997).

[6] More complex questions combining algebraic manipulation with qualitative reasoning are present in most introductory textbooks. In our experience these are rarely assigned and when they are, are successfully solved by few of the students.

[7] R. A. Serway, *Physics for Scientists and Engineers*, (Saunders College, Philadelphia, 1996).

[8] The engineering physics course at the University of Maryland is described in chapter 3 of this dissertation.

[9] It is often beneficial for physics education researchers to first try to characterize particular students' responses before attempting studies, which characterize the class. This allows us to get a deeper understanding of what our students are doing and why they are doing it.

[10] J. Torney-Purta, "Schema Theory and Cognitive Psychology: Implications for Social Studies," Theory and Research in Social Education, **19** (2), 189-210 (1991), I. A. Halloun and D. Hestenes, "Common sense concepts about motion," Am. J. Phys. 53 (11) 1056-1064 (1985).

[11] See chapter 3 for more information on the Physics 161 class at UMd.

[12] A similar question is asked in the Conservation of Momentum Tutorial from the University of Washington referenced elsewhere. The question was chosen, in part, because it proved to be a sticking point for the students in the tutorial class. Note that this particular class did not use the tutorial curriculum.

[13] See D. P. Maloney, "Rule-governed approaches to physics: Newton's Third Law," Phys. Educ. **19**, 37-42 (1984); and E. F. Redish, J. M. Saul, and R. N. Steinberg, "On the effectiveness of active-engagement microcomputer-based laboratories," Am. J. Phys. **66** (3), 212-224 (1998).

[14] See P.S. Shaffer, "Research as a guide for improving instruction in introductory physics," Ph.D. dissertation, Department of Physics, University of Washington, (1993).

[15] See Ref. 5.

[16] S. Kanim, "An investigation of Student difficulties in qualitative and quantitative problem solving: Examples from electric circuits and electrostatics," Ph.D. dissertation, Department of Physics, University of Washington, (1999).

[17] Steve Kanim has done extensive work on student responses to paired, qualitative and quantitative questions, in the contexts of electric circuits and electrostatics.

[18] See L.C. McDermott and P.S. Shaffer, "Research as a guide for curriculum development: An example from introductory electricity, Part I: Investigation of student understanding." Am. J. Phys. **60** (11), 994-1002 (1992); Erratum to Part I, Am. J. Phys. **61** (1), 81 (1993) and P.S. Shaffer and L.C. McDermott, "Research as a guide for curriculum development: An example from introductory electricity, Part II: Design of instructional strategies." Am. J. Phys. 60 (11), 1003-1013 (1992).

[19] For a more detailed account of this research see reference 18.

[20] See I. A. Halloun and D. Hestenes, "Common sense concepts about motion," Am. J. Phys. **53** (11) 1056-1064 (1985); L. C. McDermott, "Millikan Lecture 1990: What we teach and what is learned – closing the gap," Am. J. Phys. **59** (4) 301-315 (1991).

[21] Bridging problems are discussed in detail in chapter 3 of this dissertation.

[22] L.C. McDermott, P.S. Shaffer, and the PEG, *Tutorials in introductory Physics*, (Prentice Hall, NY, 1997).

[23] These issues are discussed in D. Hammer, "Epistemological beliefs in introductory physics," Cognition and Instruction, **12** (2) 151-183 (1994).

[24] A number of researchers have discussed how different knowledge can be cued. See M. C. Wittmann, "Making Sense of how students come to an understanding of physics: An example from mechanical waves," Ph.D. dissertation, Department of Physics, University of Maryland, College Park, (1998); and B. Sherin, "The Symbolic Basis of Physical Intuition: A study of two symbol systems in physics instruction," Ph.D. dissertation, School of Education, University of California, Berkley, (1996). Wittmann describes how different types of questions cause students to bring up different models. He also shows that multiple models can brought up at the same time for some students. Bruce Sherin discusses how equations may cue certain p-prims and how certain p-prims may cue certain equations.

[25] Our results have also shown that these difficulties exist in physics graduate students also.